

Mechanics I

CHAPTER 6

Newton's Laws of Motion (Without Friction)

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6

Newton's Laws of Motion (Without Friction)

INTRODUCTION

In kinematics, we dealt with the motion of particles based on the definitions of position, velocity, and acceleration without analyzing its cause. We would like to be able to answer general questions related to the cause of motion such as “what mechanism causes change in motion?” and “why do some objects accelerate at higher rates than others?” In this section, we will discuss the causes of change in the motion of particles using the concepts of force and mass. We will discuss the three fundamental laws of motion, which are based on experimental observations and were formulated by Sir Isaac Newton.

NEWTONIAN MECHANICS

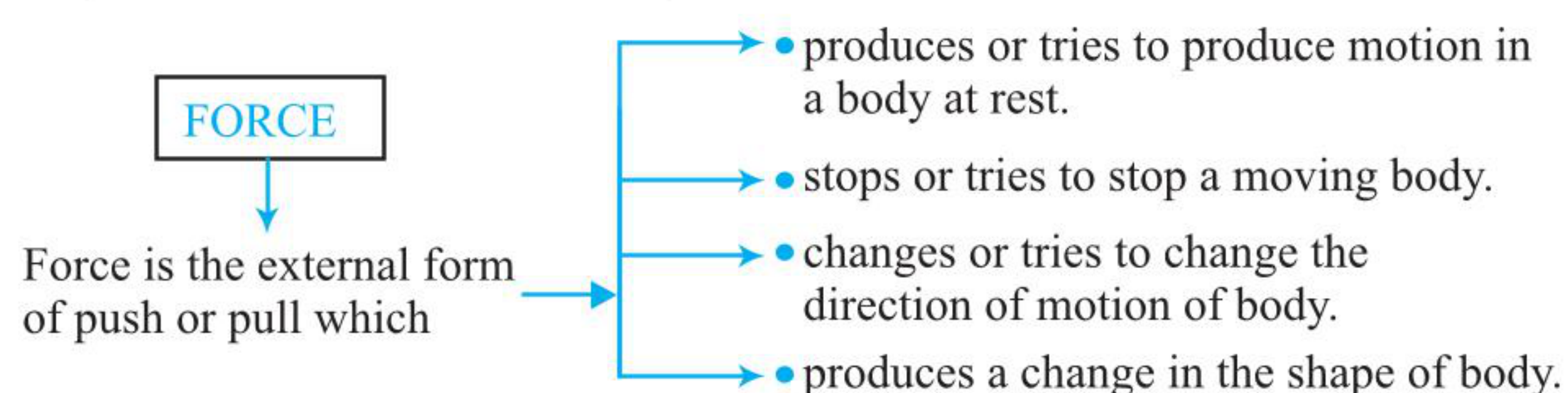
How are force and acceleration related?—This relationship was first understood by Isaac Newton. The study of that relation, as Newton presented it, is called *Newtonian mechanics*. Newtonian mechanics covers the discussion of the movement of objects under the influence of forces by making use of Newton's three laws.

If the speeds of the interacting objects are very large, comparable to the speed of light, Newtonian mechanics is not applicable. In that case we must replace Newtonian mechanics with Einstein's special theory of relativity, which holds at any speed, including those near the speed of light. If the interacting objects are on the scale of atomic structure, we must replace Newtonian mechanics with quantum mechanics.

CONCEPT OF FORCE

How a body moves? It is determined by the interaction of the body with its environment. This interaction is called force. The concept of force gives us a quantitative description of the interaction between two bodies or between a body and its environment. As a result of everyday experiences, everybody has a basic understanding of the concept of force. When you push or pull an object, you exert a force on it. You exert a force when you throw or kick a ball. In these examples, the word “force” is associated with the result of muscular activity and with some change in the state of motion of an object. However, force does not always cause an object to move.

For example, as you sit reading a book, the gravitational force acts on your body and yet you remain stationary. You can push on a heavy block of stone and yet fail to move it.



CLASSIFICATION OF FORCES

Based on the nature of the interaction between two bodies, forces may be broadly classified as under:

Contact forces: Tension, normal reaction, friction, etc. These forces act between bodies in contact.

Field forces (Non-contact forces): Weight, electrostatic forces, etc. Forces that act between bodies separated by a distance without any actual contact.

Since we will encounter these forces in our analysis, we will briefly discuss each force and how it acts between two bodies, its nature, etc., and how we are going to take it into account. We will discuss some special forces.

NEWTON'S LAWS OF MOTION

The entire classical mechanics is based upon Newton's laws of motion. These, in fact, are simply known as laws of motion. These laws provide the basis for understanding the effect that forces have on an object.

NEWTON'S FIRST LAW OF MOTION

According to this law, a body continues to be in its state of rest or uniform motion along a straight line unless it is acted upon by some external force to change the state.

An object lying anywhere keeps on lying there unless someone moves it. For example, a chair, a table, a bed, etc., cannot change their position on their own, i.e., a body at rest cannot start moving on its own. The reverse is also true, though it is slightly difficult to perceive. If there were no forces of friction and air resistance, etc., a body moving uniformly along a straight line will never stop on its own.

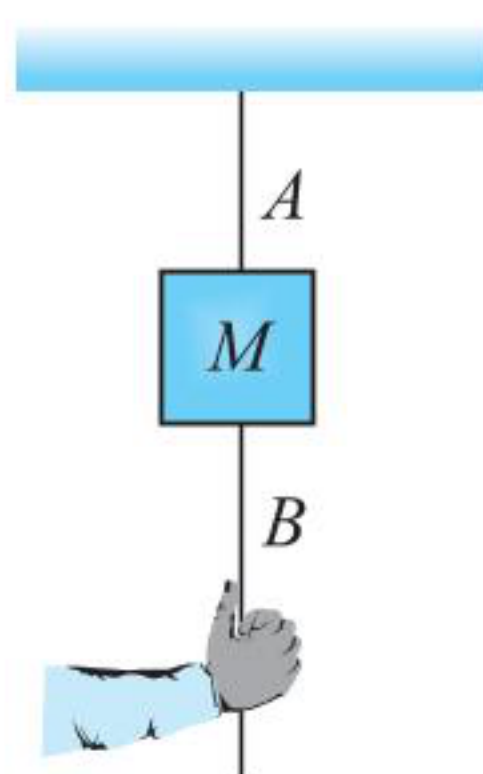
This means a body, on its own, cannot change its state of rest or state of uniform motion along a straight line. This inability of a body to change by itself its state of rest or state of uniform motion along a straight line is called inertia of the body. Hence, Newton's first law defines inertia and is rightly called the *law of inertia*.

Quantitatively, the inertia of a body is measured by the mass of the body. The heavier the body, the greater the force required to change its state and hence, the greater is its inertia. The reverse is also true. Inertia are of three types: inertia of rest, inertia of motion, and inertia of direction.

Inertia of rest It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

Examples

1. A person who is standing freely in bus is thrown backward when the bus starts suddenly.
2. In the arrangement shown in figure,
 - (a) If string B is pulled with a sudden jerk, then it will experience tension while due to the inertia of rest of mass M , this force will not be transmitted to string A and so string B will break.
 - (b) If string B is pulled steadily, the force applied to it will be transmitted from string B to A through mass M and as tension in A will be greater than that in B by Mg (weight of mass M), string A will break.
3. If we place a coin on a smooth piece of card board covering a glass and strike the card board piece suddenly with a finger, the cardboard slips away and the coin falls into the glass due to the inertia of rest.



Inertia of motion It is the inability of a body to change itself its state of uniform motion, i.e., a body in uniform motion can neither accelerate nor retard by its own.

Examples

1. A person jumping out of a moving train may fall forward.
2. An athlete runs a certain distance before taking a long jump. This is because the velocity acquired by running is added to the velocity of the athlete at the time of jump. Hence, he can jump over a longer distance.

Inertia of direction It is the inability of a body to change by itself the direction of motion.

Examples

1. The rotating wheel of any vehicle throw out mud, if any, tangentially due to directional inertia.
2. When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

Important Points:

- According to Newton's first law, a state of rest (zero velocity) and a state of constant velocity are completely equivalent, in the sense that neither one requires the application of a net force to sustain it. An object continues in a state of rest or in a state of motion at a constant velocity (constant speed in a constant direction), unless compelled to change that state by a net force.
- Newton's first law says there are two possible states for an object with no net force on it. An object at rest is said to be in static equilibrium. An object moving with constant velocity is said to be in dynamic equilibrium.

INERTIAL FRAME OF REFERENCE

We can observe a moving object from any number of reference frames. Newton's first law is not true in all reference frames, but we can always find reference frames in which it (as well as the rest of Newtonian mechanics) is true. Such special frames are referred to as **inertial reference frames**, or simply **inertial frames**. The acceleration of an inertial reference frame is zero, so it moves with a constant velocity. All of Newton's laws of

motion are valid in inertial reference frames, and when we apply these laws, we will be assuming such a reference frame. In particular, the earth itself is a good approximation of an inertial reference frame. Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame. Clearly, Newton's first law does not hold for observers who use the accelerating frame of reference. As a result, such a reference frame is said to be *non-inertial*. All accelerating reference frames are *non-inertial*.

Linear Momentum, p Linear momentum is a vector quantity. It is the quantity of motion in a body. It is given by the product of mass and velocity. Thus,

$$p = mv$$

In vector form, $\vec{p} = m\vec{v}$. The direction of $\vec{p}$ is same as that of $\vec{v}$.

NEWTON'S SECOND LAW OF MOTION

According to this law, the rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the force applied.

When an unbalanced force is applied on a body, the momentum of the body changes. The rate of change of momentum with respect to time is defined as the net external force acting on the body, i.e., $\vec{F}_{\text{ext}} = d\vec{p}/dt$, where linear momentum $\vec{p} = m\vec{v}$, m is the mass of the body, and $\vec{v}$ is the instantaneous velocity.

$$\vec{F}_{\text{ext}} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

$$\text{If } m \text{ is constant, i.e., } \frac{dm}{dt} = 0 \Rightarrow \vec{F}_{\text{ext}} = m\vec{a}$$

If the mass of the body is constant, the acceleration of the body is inversely proportional to its mass and directly proportional to the resultant force acting on it, i.e., $\sum \vec{F} = m\vec{a}$. This vector equation is equivalent to three algebraic equations:

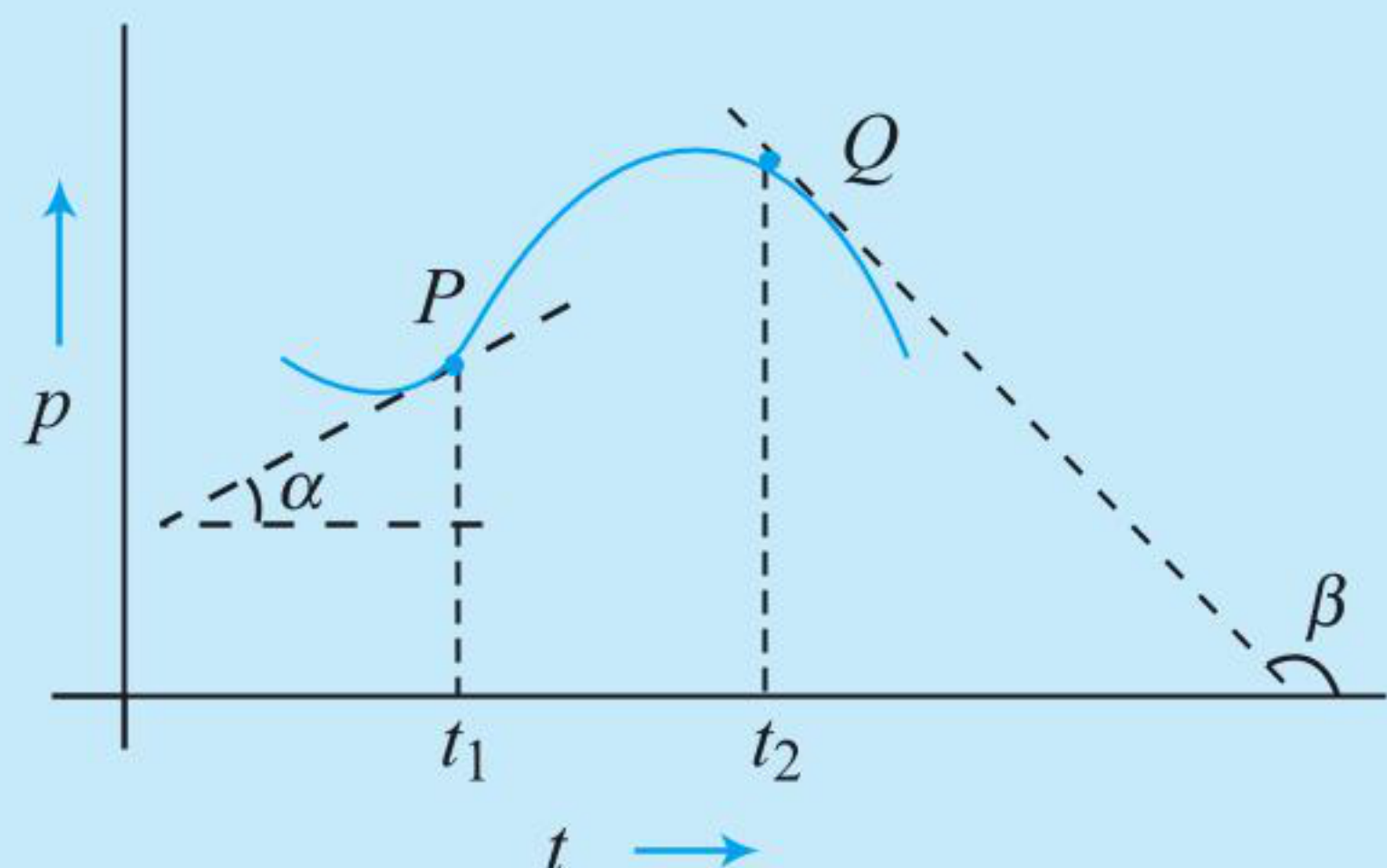
$$\sum F_x = ma_x, \sum F_y = ma_y, \text{ and } \sum F_z = ma_z$$

Important Points:

- $\vec{F} = 0$, second law gives $\vec{a} = 0$; therefore, it is consistent with the first law.
- The second law of motion is a vector law. It means whatever be the direction of the instantaneous velocity of a particle, if any net external force is acting on it, this force will change only that component of the velocity which is in the direction of force.
- Strictly speaking, this law applies to particles, i.e., point masses only. However, with the introduction of the concept of the center of mass, this law can now be applied in case of extended bodies or a system of point masses also. You will study this in the chapter on systems of particles and rotational motion.

- The second law is a local law. This means that it applies to a particle at a particular instant without taking into consideration any history of the particle or its motion.

- As $\vec{F} = \frac{d\vec{p}}{dt}$, where $\vec{p}$ denotes momentum, the slope of momentum versus time graph gives force. In figure, $\tan \alpha$ gives the force at $t = t_1$ and $\tan \beta$ gives the force at $t = t_2$. The force $\vec{F}$ in the law stands for the net external force. Any internal forces in the system are not included in $\vec{F}$.



PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The second law suggests: $\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$;

If no external force acts on the system $\vec{F}_{\text{net}} = 0$, then $\vec{p} = \text{constant}$. It means if no external force acts on a system, the linear momentum of the system will remain constant. This statement is called **principle of conservation of momentum**.

Practical Applications of the Principle of Conservation of Linear Momentum

- When a bullet is fired from a gun, the gun recoils or gives a kick in backward direction. Let M be the mass of gun and m , the mass of the bullet. Initially, both the gun and the bullet are at rest. On firing the gun, suppose that the bullet moves with a velocity $\vec{v}$ and the gun moves with velocity $\vec{V}$.

According to the principle of conservation of momentum, total momentum of gun and bullet before firing = total momentum of gun and bullet after firing.

$$\text{i.e. } 0 = M \vec{V} + m \vec{v}$$

$$\text{or, } \vec{V} = -\frac{m}{M} \vec{v}$$

The negative sign shows that $\vec{V}$ and $\vec{v}$ are in opposite direction, i.e. as the bullet moves forward, the gun will move in backward direction. The backward motion of the gun is called recoil of the gun.

- When a man jumps from a boat to the shore, the boat slightly moves away from the shore. Initially, the total momentum of the boat and the man is zero. When the man jumps from the boat to the shore, total momentum of man and the boat will be zero only if the boat moves in opposite direction.
- A rocket works on the principle of conservation of momentum. As the fuel in the rocket undergoes combustion,

the burnt gases leave the body of the rocket with a large velocity in downward direction and thus provide upthrust to the rocket. If we assume that the fuel is burnt at a constant rate, then the rate of change of momentum of the rocket will be constant. As more and more fuel gets burnt, the mass of the rocket goes on decreasing and it leads to increase in the velocity of the rocket more and more rapidly.

It may be pointed out that rocket propulsion is an application of the principle of conservation of momentum to a situation, in which the mass of the system goes on changing.

- If an astronaut in open space, away from space ship, wants to return to his space ship, he can do so by throwing something in a direction opposite to that in which the space ship is moving. When the astronaut throws some object away from the space ship, he himself will recoil i.e. will move in the opposite direction. Due to this, the astronaut will move towards the space ship.

IMPULSE

According to Newton's second law, the momentum of a particle changes if a net force acts on the particle. Knowing the change in momentum caused by a force is useful in solving some problems. To build a better understanding of this important concept, let us assume that a net force $\Sigma \vec{F}$ acts on a particle and that this force may vary with time. According to Newton's second law,

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} \text{ or } d\vec{p} = \Sigma \vec{F} dt \quad \dots(i)$$

We can integrate this expression to find the change in the momentum of a particle when the force acts over some time interval. If the momentum of the particle changes from $\vec{p}_i$, at time t_i to $\vec{p}_f$, at time t_f , integrating Eq. (i) gives

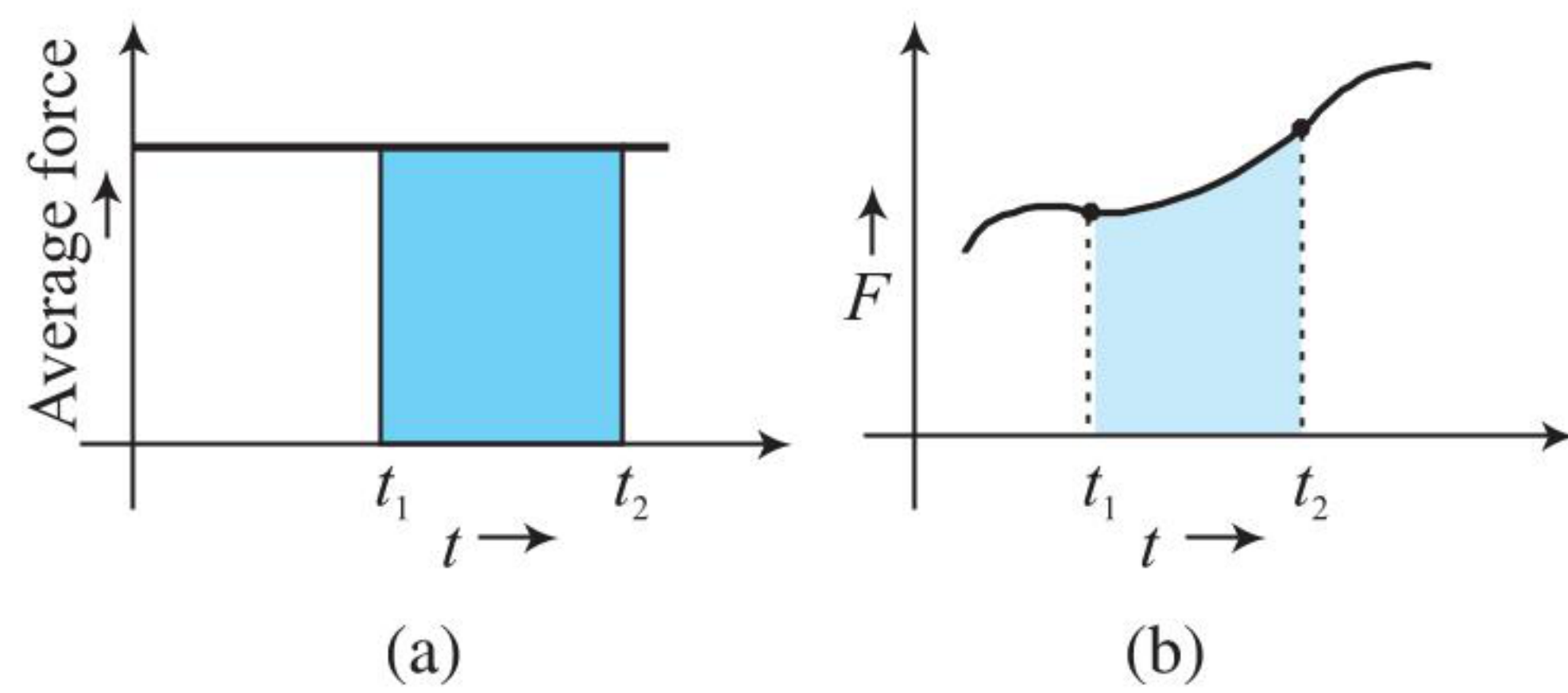
$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \Sigma \vec{F} dt \quad \dots(ii)$$

To evaluate the integral, we need to know how the net force varies with time. The quantity on the right side of this equation is a vector called the impulse of the net force $\Sigma \vec{F}$ acting on a particle over the time interval $\Delta t = t_f - t_i$.

$$\vec{J} = \int_{t_i}^{t_f} \Sigma \vec{F} dt \quad \dots(iii)$$

From its definition, we see that impulse $\vec{J}$ is a vector quantity having a magnitude equal to the area under the force–time curve as described in figure. It is assumed the force varies with time in the general manner shown in figure and is non-zero in the time interval $\Delta t = t_f - t_i$. The direction of the impulse vector is same as the direction of the change in momentum. Impulse has the dimensions of momentum, that is, ML/T . Impulse is not a property of a particle; rather, it is a measure of the degree to which an external force changes the particle's momentum.

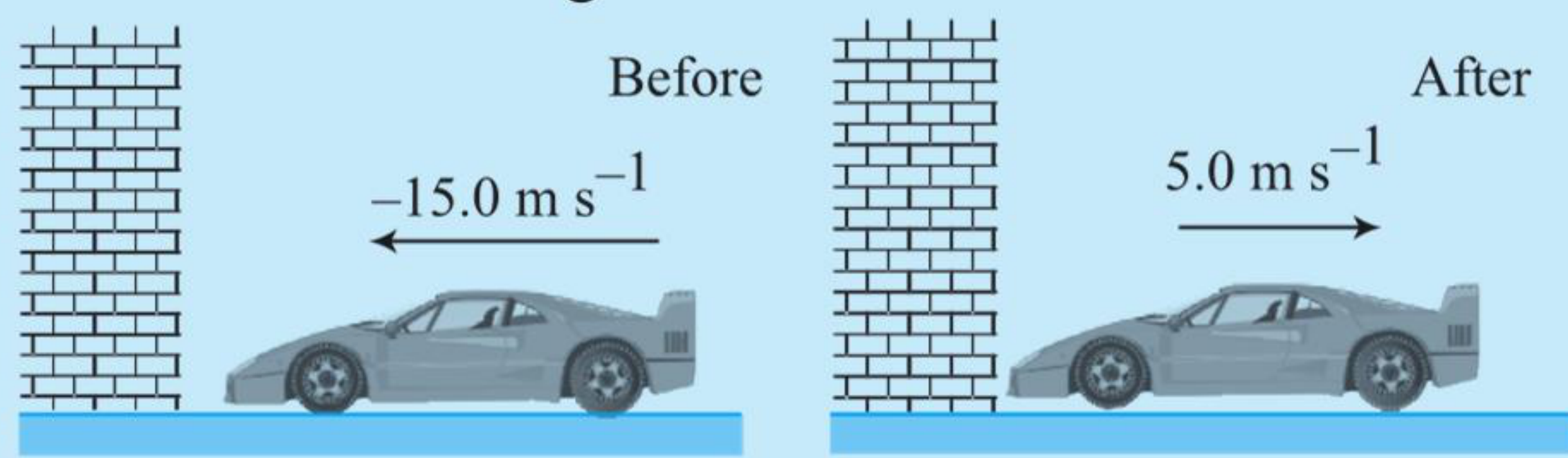
Equation (ii) is an important statement known as the impulse–momentum theorem: if we plot a graph between average force and time [Figure], the area under the curve will give the impulse imparted during the time interval under consideration.



$$J = \int_{t_1}^{t_2} F dt = \int_{t_1}^{t_2} dp = \text{Total change in linear momentum} = \text{Impulse}$$

ILLUSTRATION 6.1

In a particular crash test, a car of mass 1500 kg collides with a wall as shown in figure. The initial and final velocities of the car are $\vec{v}_i = -15.0\hat{i} \text{ m s}^{-1}$ and $\vec{v}_f = 5.00\hat{i} \text{ m s}^{-1}$, respectively. If the collision lasts 0.150 s, find the impulse caused by the collision and the average force exerted on the car.



Sol. The collision time is short, so we can imagine the car being brought to rest very rapidly and then moving back in the opposite direction with a reduced speed.

Let us assume that the force exerted by the wall on the car is large compared with other forces on the car (such as friction and air resistance). Furthermore, the gravitational force and the normal force exerted by the road on the car are perpendicular to the motion and, therefore, do not affect the horizontal momentum. Therefore, we categorize the problem as one in which we can apply the impulse approximation in the horizontal direction.

Evaluating the initial and final momenta of the car, we get

$$\vec{p}_i = m\vec{v}_i = (1500 \text{ kg})(-15.0\hat{i} \text{ m s}^{-1}) = -2.25 \times 10^4 \hat{i} \text{ kg m s}^{-1}$$

$$\vec{p}_f = m\vec{v}_f = (1500 \text{ kg})(5.0\hat{i} \text{ m s}^{-1}) = 0.75 \times 10^4 \hat{i} \text{ kg m s}^{-1}$$

Impulse on the car:

$$\begin{aligned} \vec{J} &= \Delta \vec{p} = \vec{p}_f - \vec{p}_i \\ &= 0.75 \times 10^4 \hat{i} \text{ kg m s}^{-1} - (-2.25 \times 10^4 \hat{i} \text{ kg m s}^{-1}) \end{aligned}$$

$$\Rightarrow \vec{J} = 3.00 \times 10^4 \hat{i} \text{ kg m s}^{-1}$$

The average force exerted by the wall on the car

$$\vec{F}_{\text{avg}} = \frac{\Delta \vec{p}}{\Delta t} = \frac{3.00 \times 10^4 \hat{i} \text{ kg m s}^{-1}}{0.150 \text{ s}} = 2.00 \times 10^5 \hat{i} \text{ N}$$

ILLUSTRATION 6.2

A body of mass $m = 1 \text{ kg}$ falls from a height $h = 20 \text{ m}$ from the ground level.

(a) What is the magnitude of total change in momentum of the body before it strikes the ground?

(b) What is the corresponding average force experienced by it? ($g = 10 \text{ m s}^{-2}$)

Sol.

(a) The body falls from rest ($u = 0$) through a distance h before striking the ground, the speed v of the body is given by kinematical equation: $v^2 = u^2 + 2as$;

Putting $a = g$ and $s = h$, we obtain $v = \sqrt{2gh}$

Thus, the magnitude of the total change in momentum of the body just before it strikes the ground $= \Delta p = |mv - 0| = mv$

where $v = \sqrt{2gh} \Rightarrow \Delta p = m\sqrt{2gh}$

$$\text{or } \Delta p = (1) \sqrt{(2 \times 10 \times 20)} \text{ kg m s}^{-1} = 20 \text{ kg m s}^{-1}. \quad \dots(i)$$

(b) We define average force experienced by the body,

$$\vec{F}_{\text{av}} = \frac{\Delta \vec{p}}{\Delta t}; \Delta t = \text{time of motion of the body} = t(\text{say}).$$

We know $\Delta p = 20 \text{ kg m s}^{-1}$ from (i). Now, let us find t using the facts given in problem. From kinematics, we know

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \quad (\text{here } u = 0, s = h, \text{ and } a = g) \\ \Rightarrow h &= \frac{1}{2}gt^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}} \end{aligned}$$

$$\therefore F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{\Delta p}{t}$$

Putting the general values of Δp and t , we get

$$F_{\text{av}} = \frac{m\sqrt{2gh}}{\sqrt{2h/g}} = mg \Rightarrow \vec{F}_{\text{av}} = m\vec{g}$$

where mg is the weight of the body and $\vec{g}$ is directed vertically downward. Therefore, the body experiences a constant vertically downward force of magnitude mg .

ILLUSTRATION 6.3

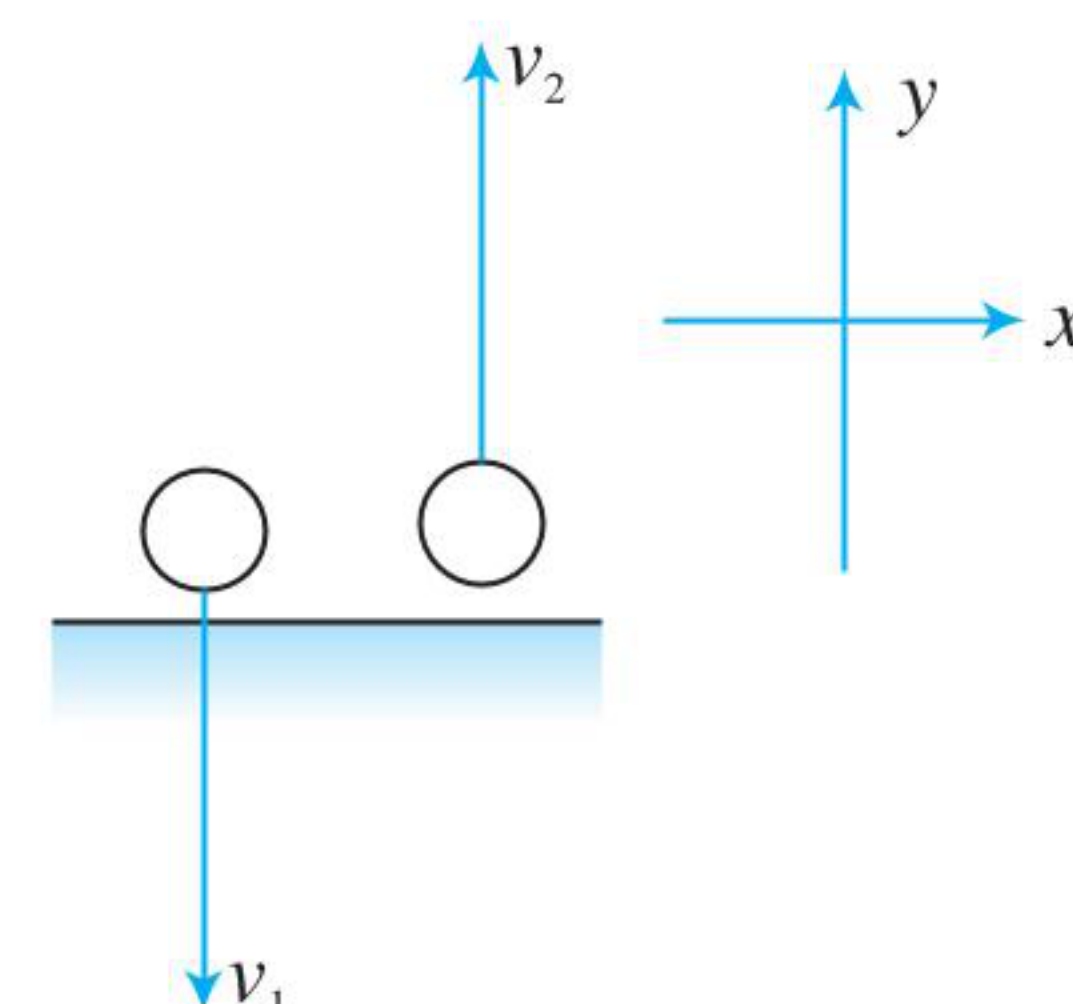
An iron ball of mass $m = 50 \text{ g}$ falls from a height of $h_1 = 5 \text{ m}$ and rises upto $h_2 = 3.2 \text{ m}$ after colliding with the horizontal surface. If the time of contact of the ball is $\Delta t = 0.02 \text{ s}$, find the average contact force exerted on the ball by the horizontal surface.

Sol. The change in linear momentum during collision is

$$\begin{aligned} \Delta \vec{p} &= m\vec{v}_2 - m\vec{v}_1 = m(\vec{v}_2 - \vec{v}_1) \\ &= m[v_2\hat{j} - (-v_1\hat{j})] = m[(v_1 + v_2)]\hat{j} \end{aligned}$$

The average force during the collision is

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m(v_1 + v_2)\hat{j}}{\Delta t}$$



We can calculate v_1 and v_2 by using kinematics.

Using $v^2 = u^2 + 2as$

Putting $a = g$ and $s = h$, we get

$$v_1 = \sqrt{2gh_1}, \quad v_2 = \sqrt{2gh_2}$$

or
$$\vec{F} = \frac{m(\sqrt{2gh_1} + \sqrt{2gh_2})\hat{j}}{\Delta t}$$

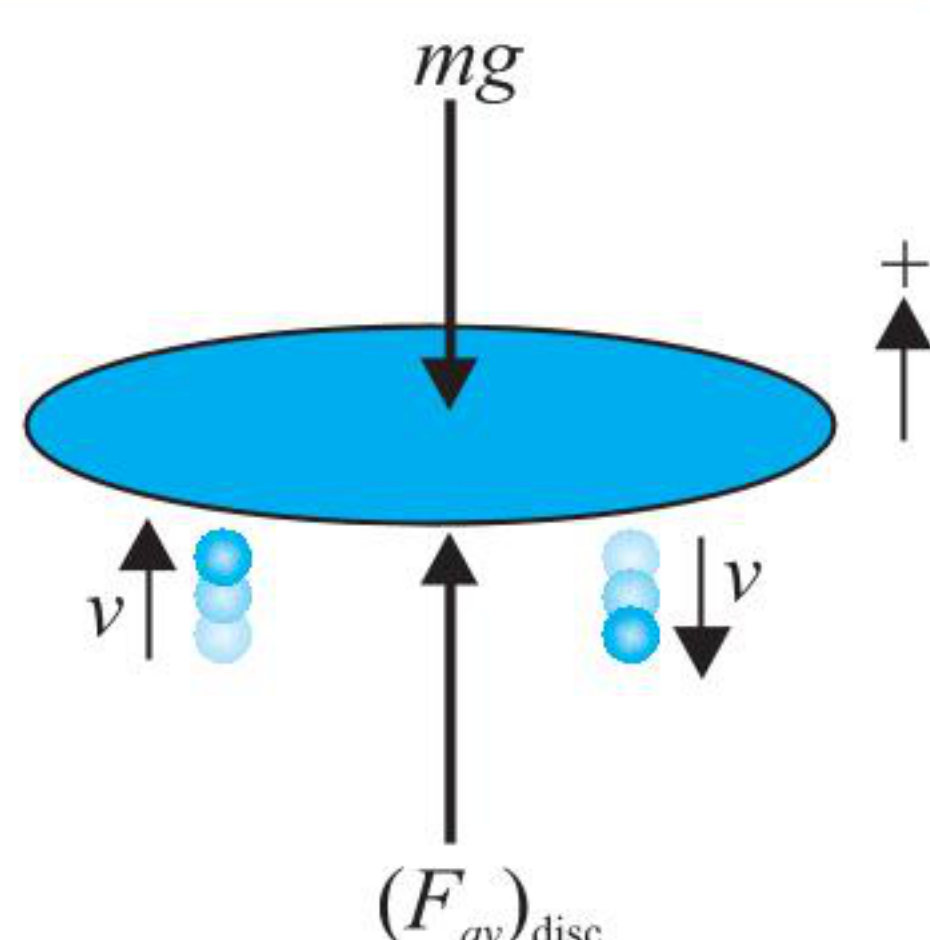
$$= \left(\frac{50}{1000}\right) \frac{(\sqrt{2 \times 10 \times 5} + \sqrt{2 \times 10 \times 3.2})}{0.02} \hat{j} = 45 \hat{j} \text{ N}$$

During collision, the horizontal surface pushes the ball up with an average force of 45 N.

ILLUSTRATION 6.4

A disc of mass 10 g is kept floating horizontally by throwing 10 marbles s^{-1} against it from below. If the mass of each marble is 5 g, calculate the velocity with which the marbles are striking the disc. Assume that the marbles strike the disc normally and rebound downward with the same speed.

Sol. As 10 marbles strike against the disc per second, the above change in momentum of 10 marbles takes place in 1 s. Therefore, rate of change of momentum of the marbles or the force exerted by the marbles on the disc will balance the weight of the disc.



Let v be the speed with which the marbles strike the disc and taking upward direction as positive.

Then, initial momentum of 10 marbles (striking s^{-1}), $p_1 = 10(mv)$

Final momentum of 10 marbles (striking s^{-1}),

$$p_2 = 10(-mv) = -10mv$$

The change in the momentum of the 10 marbles per second,

$$\Delta p = p_2 - p_1 = -10mv - (10mv) = -20mv$$

Therefore, rate of change of momentum of the marbles or the average force exerted by the disc on the marbles,

$$\left(\vec{F}_{\text{av}}\right)_{\text{mar}} = \frac{\Delta \vec{p}}{\Delta t} = \frac{-20mv}{1} = -20mv \text{ (Downward direction)}$$

Hence force applied by marbles on the disc will be in upward

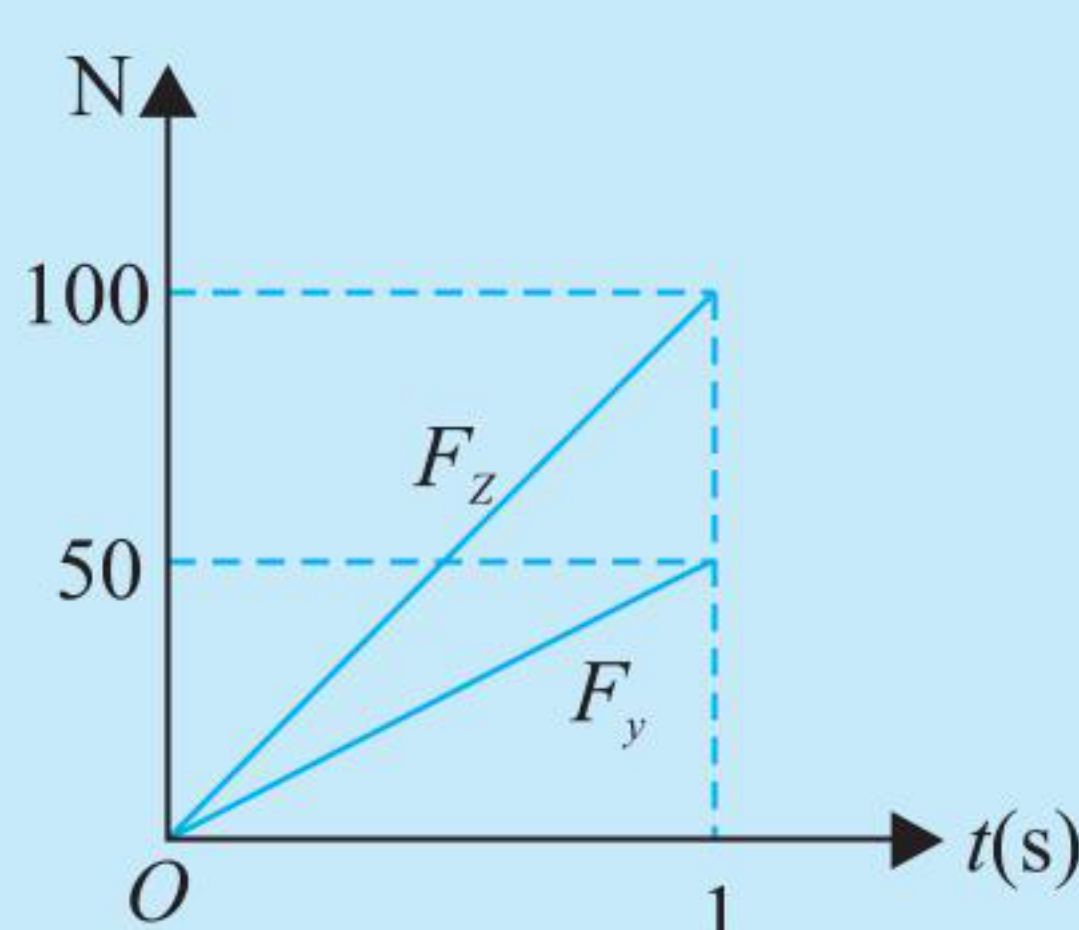
direction. Hence, $\left(\vec{F}_{\text{av}}\right)_{\text{disc}} = 20mv$

Since the disc remains floating, $\left(\vec{F}_{\text{av}}\right)_{\text{disc}} = \text{weight of the disc} = Mg$

$$\text{or } 20 \times 5 \times 10^{-3}v = 10 \times 10^{-3} \times 10 \Rightarrow v = 1 \text{ m/s}$$

ILLUSTRATION 6.5

A particle is acted upon by a force for 1 second whose x -component remains constant at $F_x = 5\sqrt{19}$ N but y and z components vary with time as shown in the graph. Calculate the magnitude of change in momentum of the particle in 1 sec. What angle does the impulse make with x -axis?



Sol. We know impulse $(\vec{J}) = \text{Change in linear momentum } (\Delta \vec{P})$

$$\text{or } \vec{J} = \Delta \vec{P} = \int \vec{F} dt = \hat{i} \int F_x dt + \hat{j} \int F_y dt + \hat{k} \int F_z dt \quad \dots(i)$$

Here, the area under F_y - t graph gives $\int F_y dt$ and the area under F_z - t graph gives $\int F_z dt$.

$$\vec{J} = \Delta \vec{P} = \hat{i} \left(\int_0^1 5\sqrt{19} dt \right) + \hat{j} \left(\frac{1}{2} \times 50 \times 1 \right) + \hat{k} \left(\frac{1}{2} \times 100 \times 1 \right)$$

$$\vec{J} = (5\sqrt{19}\hat{i} + 25\hat{j} + 50\hat{k}) \text{ Ns}$$

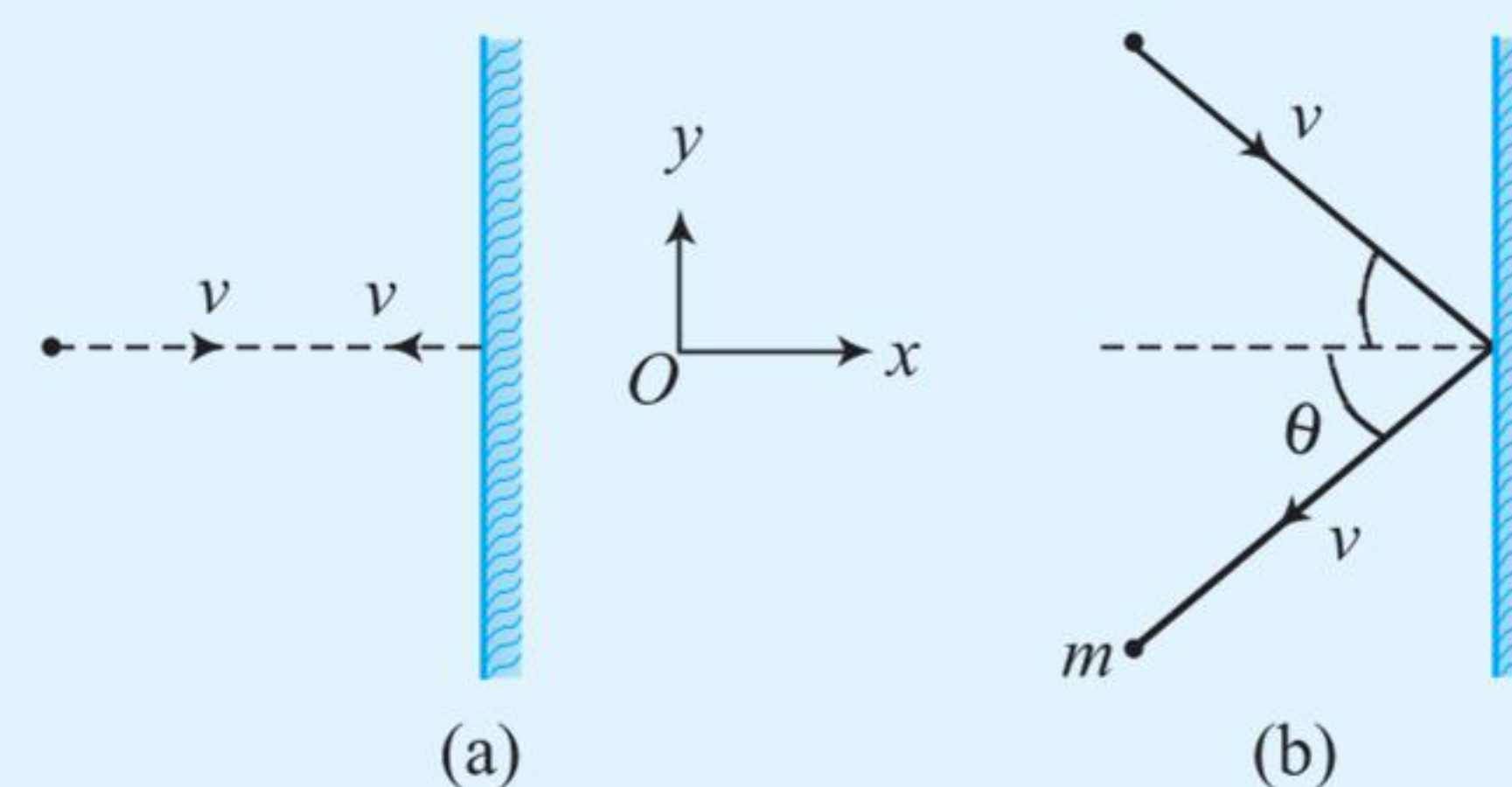
$$\therefore |\vec{J}| = \sqrt{(5\sqrt{19})^2 + 25^2 + 50^2} = 60 \text{ Ns}$$

If angle made by $\vec{J}$ with x axis is θ , then

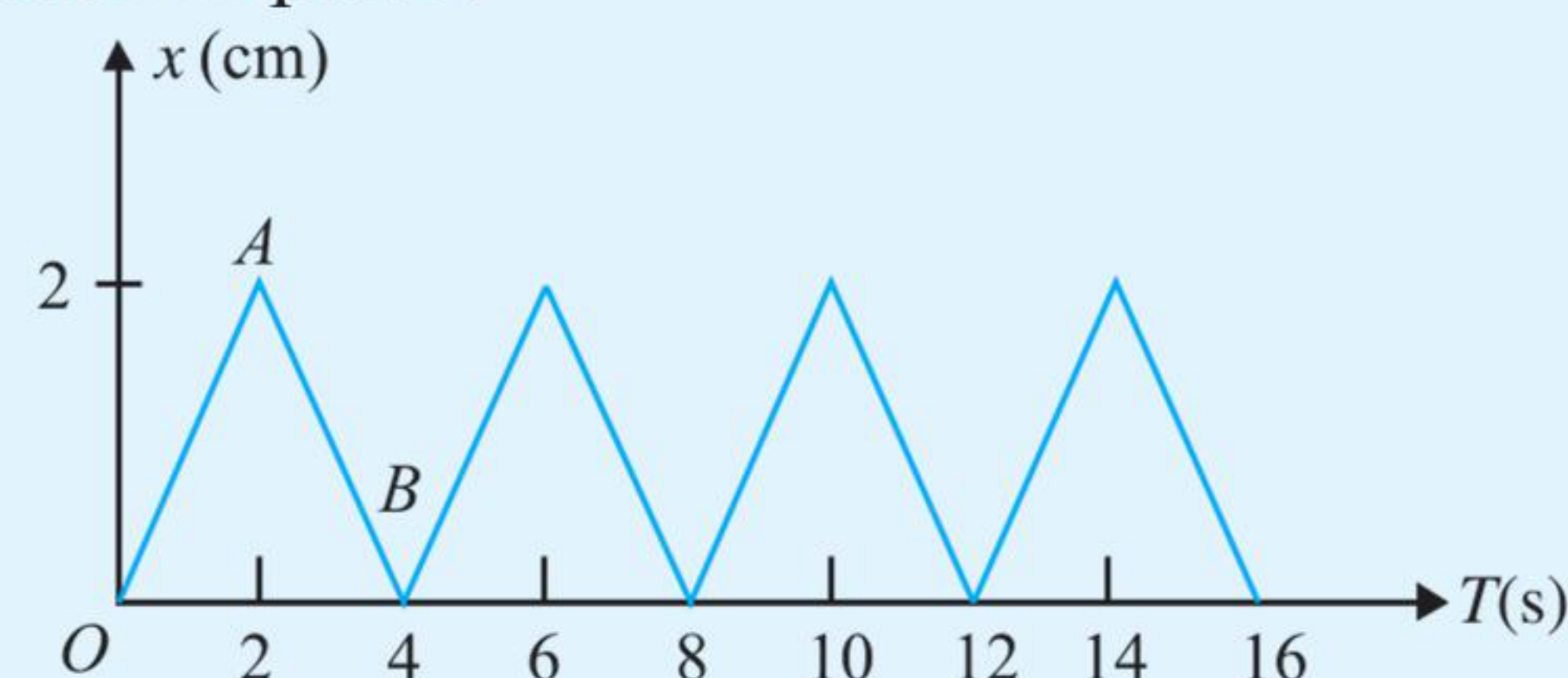
$$\cos \theta = \frac{5\sqrt{19}}{60} = \frac{\sqrt{19}}{12} \Rightarrow \theta = \cos^{-1} \left(\frac{\sqrt{19}}{12} \right)$$

CONCEPT APPLICATION EXERCISE 6.1

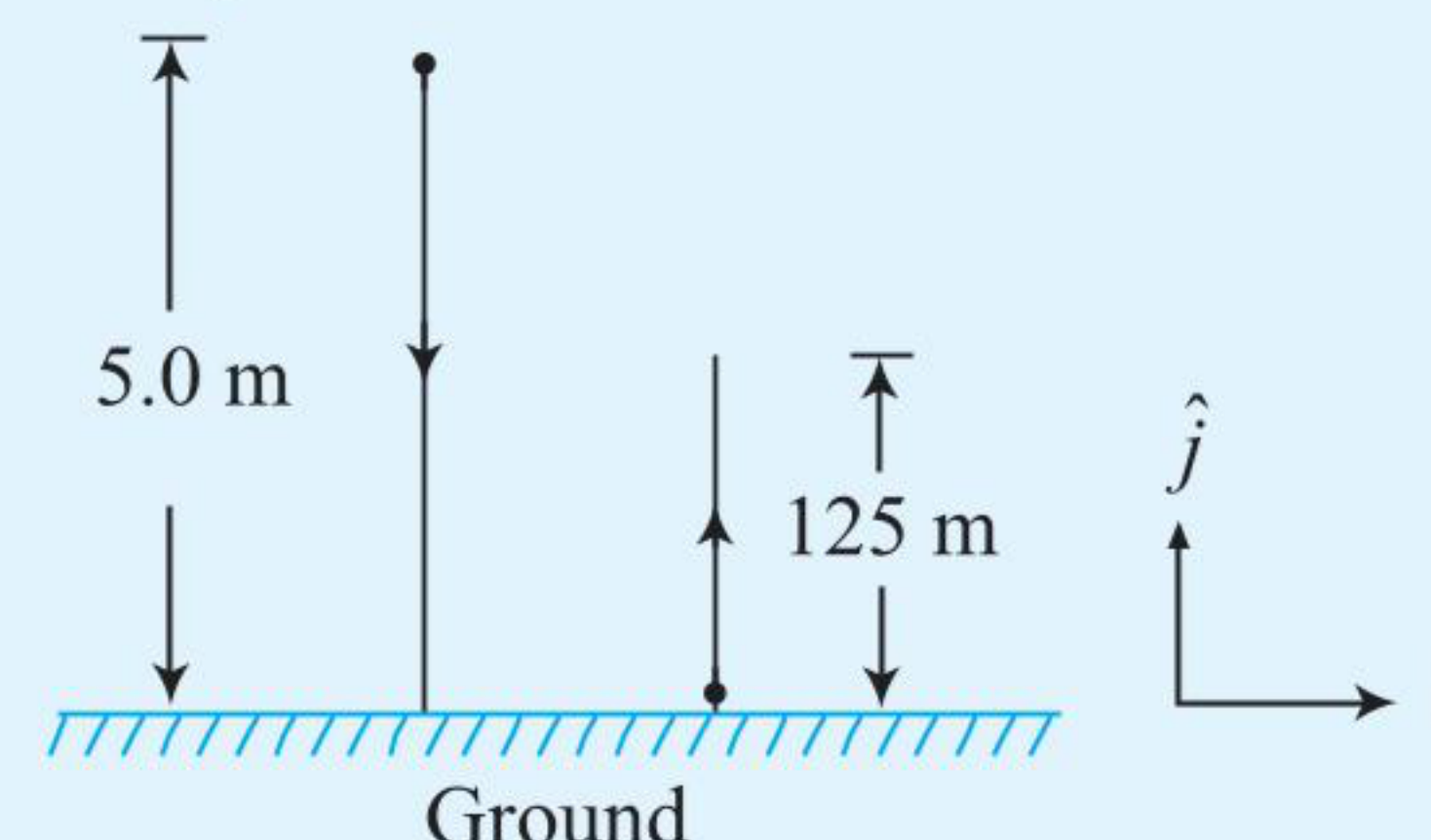
- As shown in figure, two identical balls strike a rigid wall with equal speeds but at different angles of incidence. They are reflected back without any loss in speed.



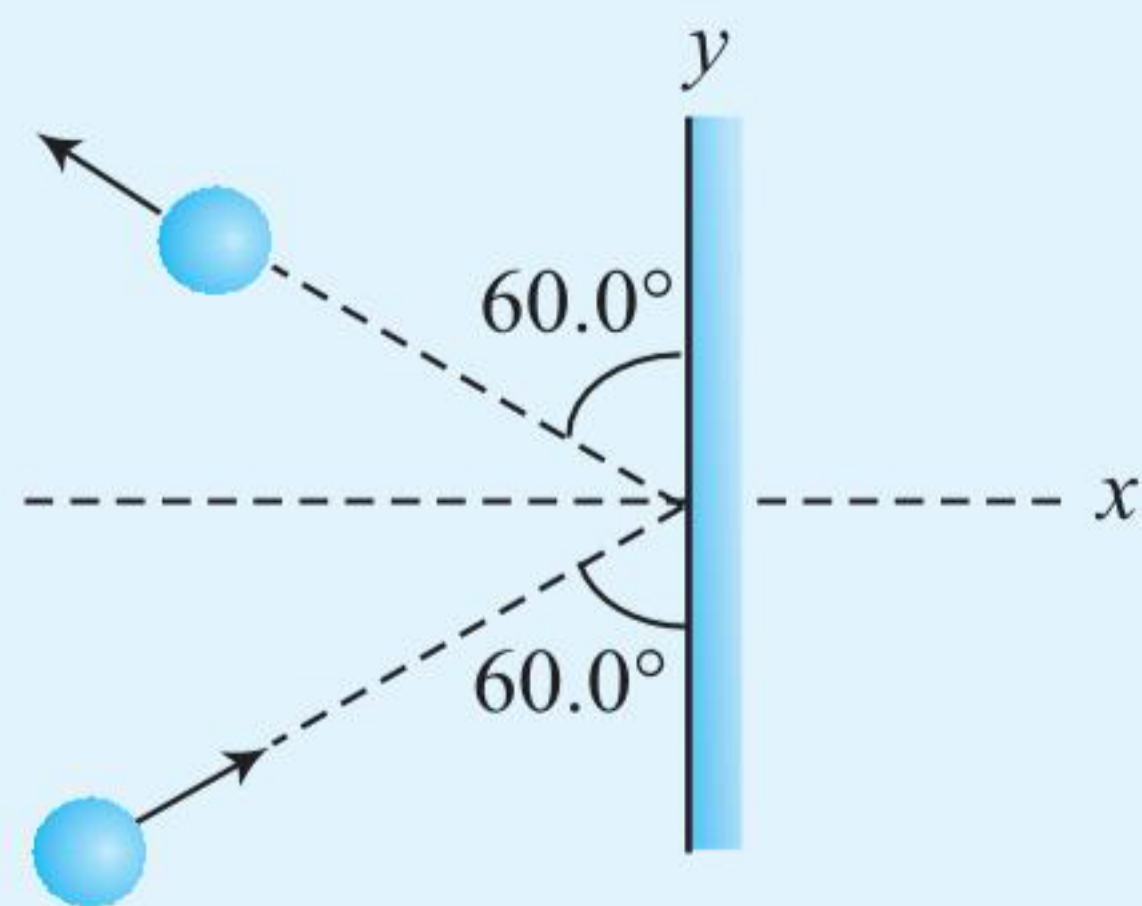
- Determine the direction of force exerted by each ball on the wall.
 - Determine the ratio of impulse imparted by the two balls on the wall in both cases.
- Figure shows the position-time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the particle? What is the magnitude of each impulse?



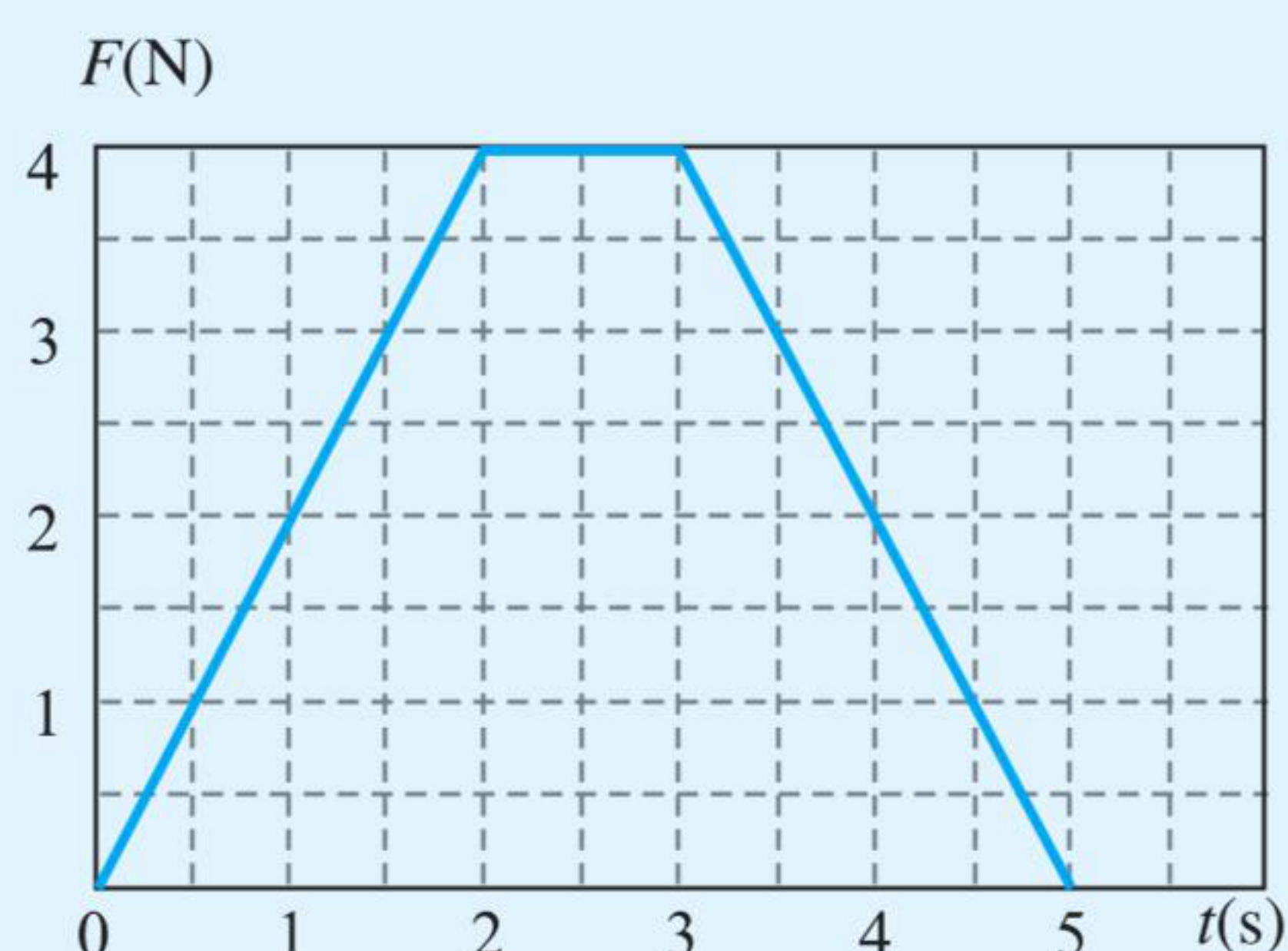
- A rubber ball of mass 50 g falls from a height of 5 m and rebounds to a height of 1.25 m. Find the impulse and the average force between the ball and the ground if the time for which they are in contact was 0.1 s.



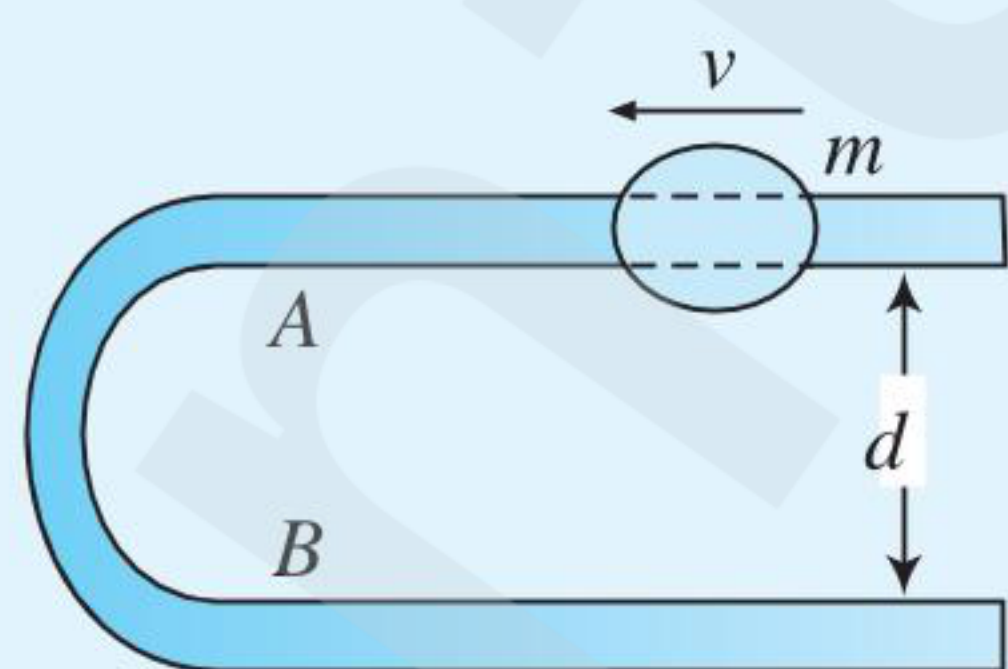
4. A 3-kg steel ball strikes a wall with a speed of 10.0 ms^{-1} at an angle of 60.0° with the surface of the wall. The ball bounces off with the same speed and same angle. If the ball was in contact with the wall for 0.2 s , find the average force exerted by the wall on the ball.



5. The magnitude of the net force exerted in the x direction on a 2.50-kg particle varies with time as shown in figure. Find (a) the impulse of the force, (b) the final velocity the particle attains if it is originally at rest, (c) its final velocity if its original velocity is -2.0 ms^{-1} , and (d) the average force exerted on the particle for the time interval between 0 and 5 s .



6. During a heavy rain, hailstones of average size 1.0 cm in diameter fall with an average speed of 20 ms^{-1} . Suppose 2000 hailstones strike every square meter of a $10 \text{ m} \times 10 \text{ m}$ roof perpendicularly in one second and assume that the hailstones do not rebound. Calculate the average force exerted by the falling hailstones on the roof. Density of the hailstones is 900 kg m^{-3} .
7. A U-shaped smooth wire has a semi-circular bending between A and B as shown in figure. A bead of mass m moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B . Find the average force exerted by the bead on the part AB of the wire.
8. Wind with a velocity 100 km h^{-1} blows normally against one of the walls of a house with an area of 108 m^2 . Calculate the force exerted on the wall if the air moves parallel to the wall after striking it and has a density of 1.2 kg m^{-3} .



ANSWERS

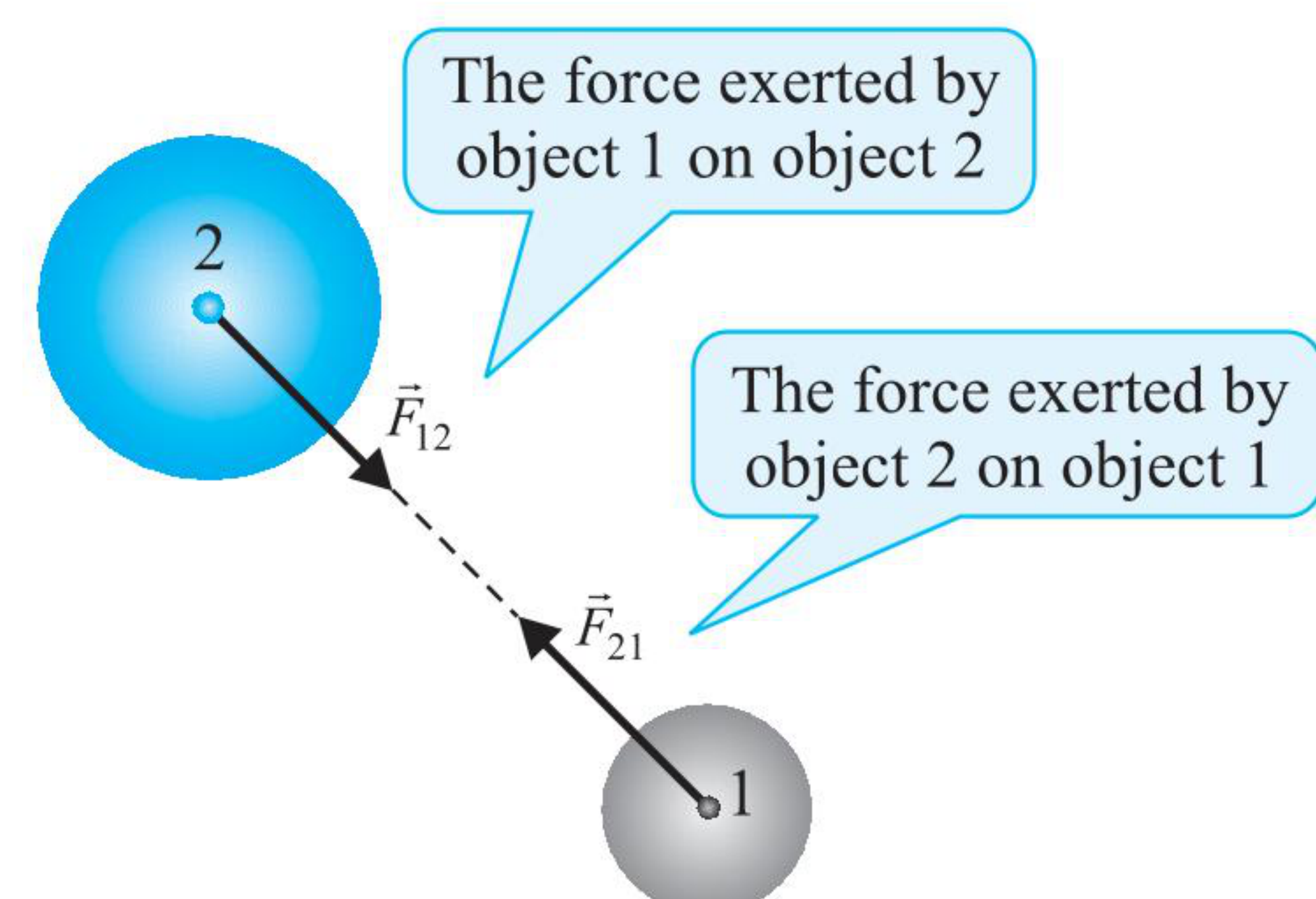
1. (a) Force will be along x -direction on the wall.
 (b) $\frac{2mv}{2mv \cos \theta}, \frac{1}{\cos \theta}$
2. $8 \times 10^{-4} \text{ N s}$, 2 s 3. $0.75 \hat{j} \text{ N s}$, 7.5 N 4. $150\sqrt{3} \text{ N}$
5. (a) 12 N s (b) 4.8 ms^{-1} (c) 2.8 ms^{-1} (d) 2.40 N
6. 1900 N 7. $\frac{4mv^2}{\pi d}$
8. $1.2 \approx 10^5 \text{ N}$

NEWTON'S THIRD LAW OF MOTION

Force is always a two-body interaction. The first law describes qualitatively and the second law describes quantitatively. What happens to a body if a force acts on it? Both these laws do not reveal anything about what happens to the other body participating in the interaction responsible for the force. For understanding the force during two-body interaction, let us do one activity. If you press against a corner of a book with your fingertip, the book pushes back and makes a small dent in your skin. If you push harder, the book does the same and the dent in your skin is a little larger. This simple activity illustrates that forces are interactions between two objects: when your finger pushes on the book, the book pushes back on your finger. This important principle is known as Newton's third law, which states that

If two objects interact, the force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1:

$\vec{F}_{12} = -\vec{F}_{21}$, where the minus sign means that these two forces are in opposite directions. The force that object 1 exerts on object 2 is popularly called the *action force*, and the force of object 2 on object 1 is called the *reaction force*. Note that these two forces do not act on the same body but are the forces with which two bodies act on each other.



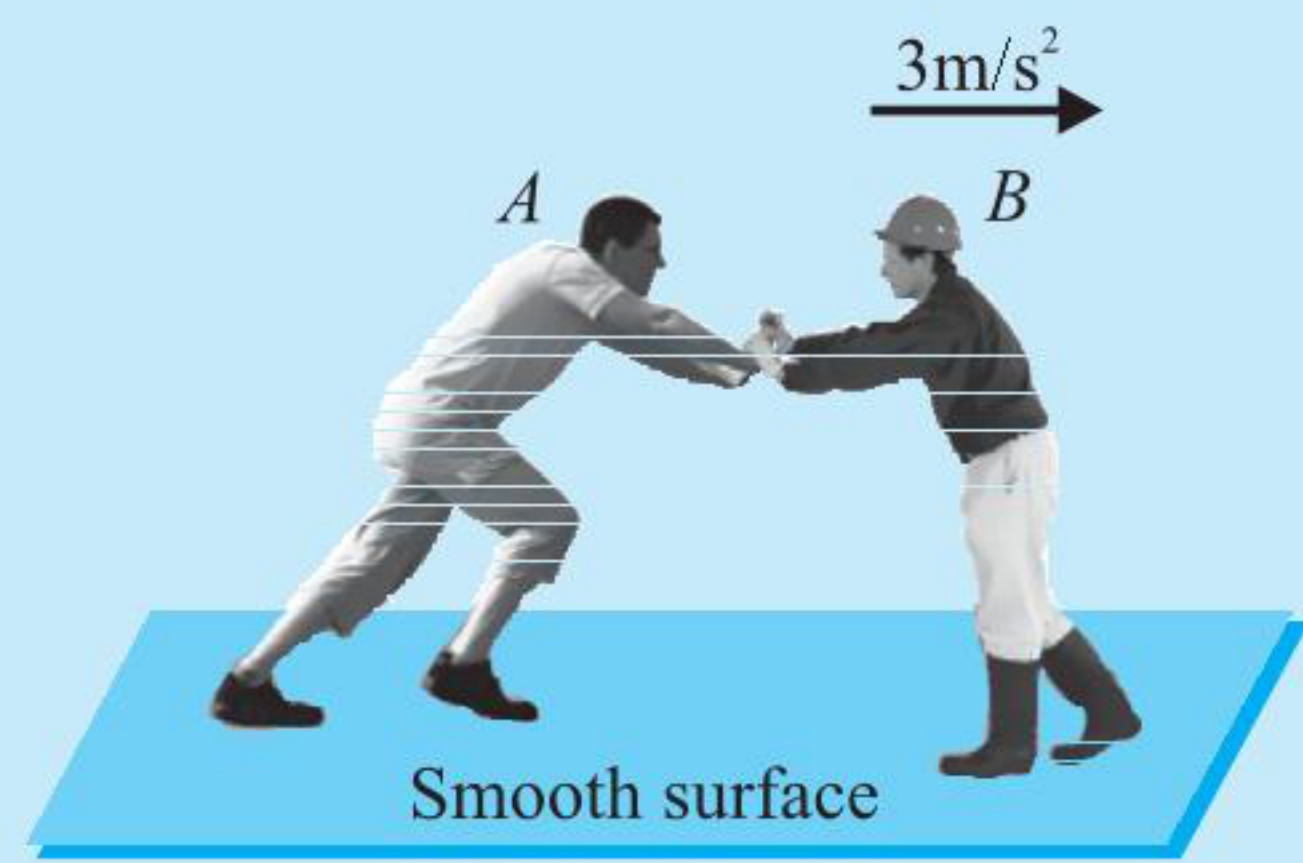
To every action, there is always an opposed equal reaction. It does not matter which force we call action and which we call reaction. The important thing is that they are co-pairs of a single interaction, and neither force exists without the other.

- When you walk, you interact with the floor. You push against the floor, and the floor pushes against you. The pair of forces occurs at the same time (they are simultaneous).
- Likewise, the tires of a car push against the road while the road pushes back on the tires—the tires and road simultaneously push against each other.
- In swimming, you interact with water, pushing water backwards, while water simultaneously pushes you forward—you and water push against each other.

The reaction forces are what account for our motion in these examples. These forces depend on friction; a person or car on ice, for example, may be unable to exert the action force to produce the needed reaction force. Neither force exists without the other.

ILLUSTRATION 6.6

Man 'A' of mass 60 kg pushes the other man 'B' of mass 75 kg due to which man 'B' starts moving with acceleration 3 m/s^2 . Calculate the acceleration of man 'A' at that instant.

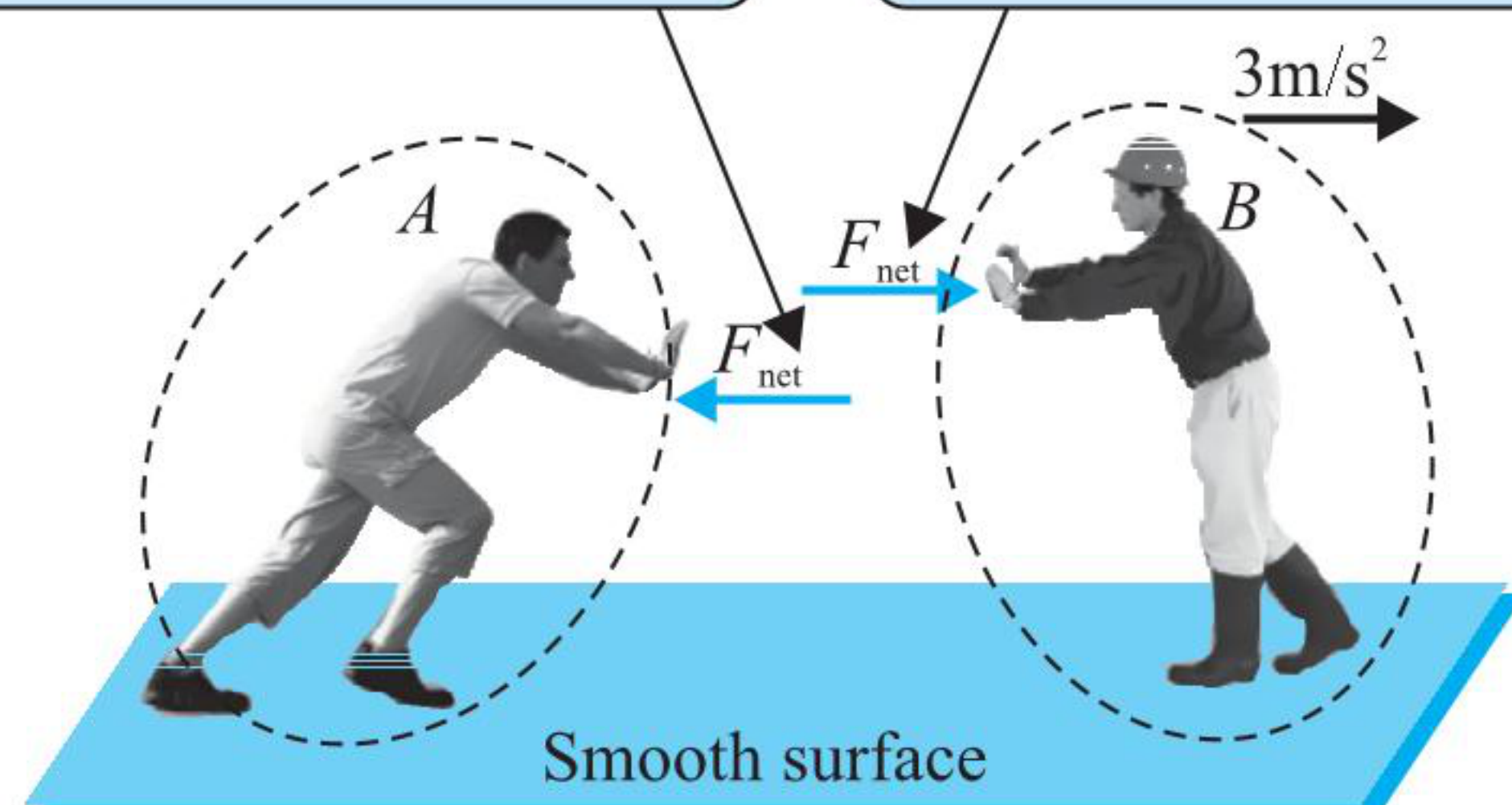


Sol. If man 'B' accelerates with acceleration 3 m/s^2 , the net force on man 'B' should be in the direction of acceleration and it should be

$$F_{\text{net}} = m_B a_B = 75 \times 3 = 225 \text{ N}$$

This reaction force will be equal in magnitude but acts in opposite direction on mass 'A'.

This is the action force applied by man 'A' on man 'B'. The reaction of this force will act on man 'A'.



According to Newton's third law of motion, if two objects interact, the force $\vec{F}_{AB}$ exerted by object A on object B is equal in magnitude and opposite in direction to the force $\vec{F}_{BA}$ exerted by object B on object A,

From F.B.D of man 'A'

$$a_A = \frac{225}{60} = \frac{15}{4} \text{ m/s}^2$$

FREE-BODY DIAGRAMS

In free-body diagrams (FBDs), the object of interest is isolated from its surroundings, and the interactions between the object and the surroundings are represented in terms of forces.

After knowing the nature of different forces, let us draw a "free-body diagram." The phrase itself reveals that we must mentally free (isolate) the bodies (or particles) in the system. Then consider all the forces acting on all the particles of the system.

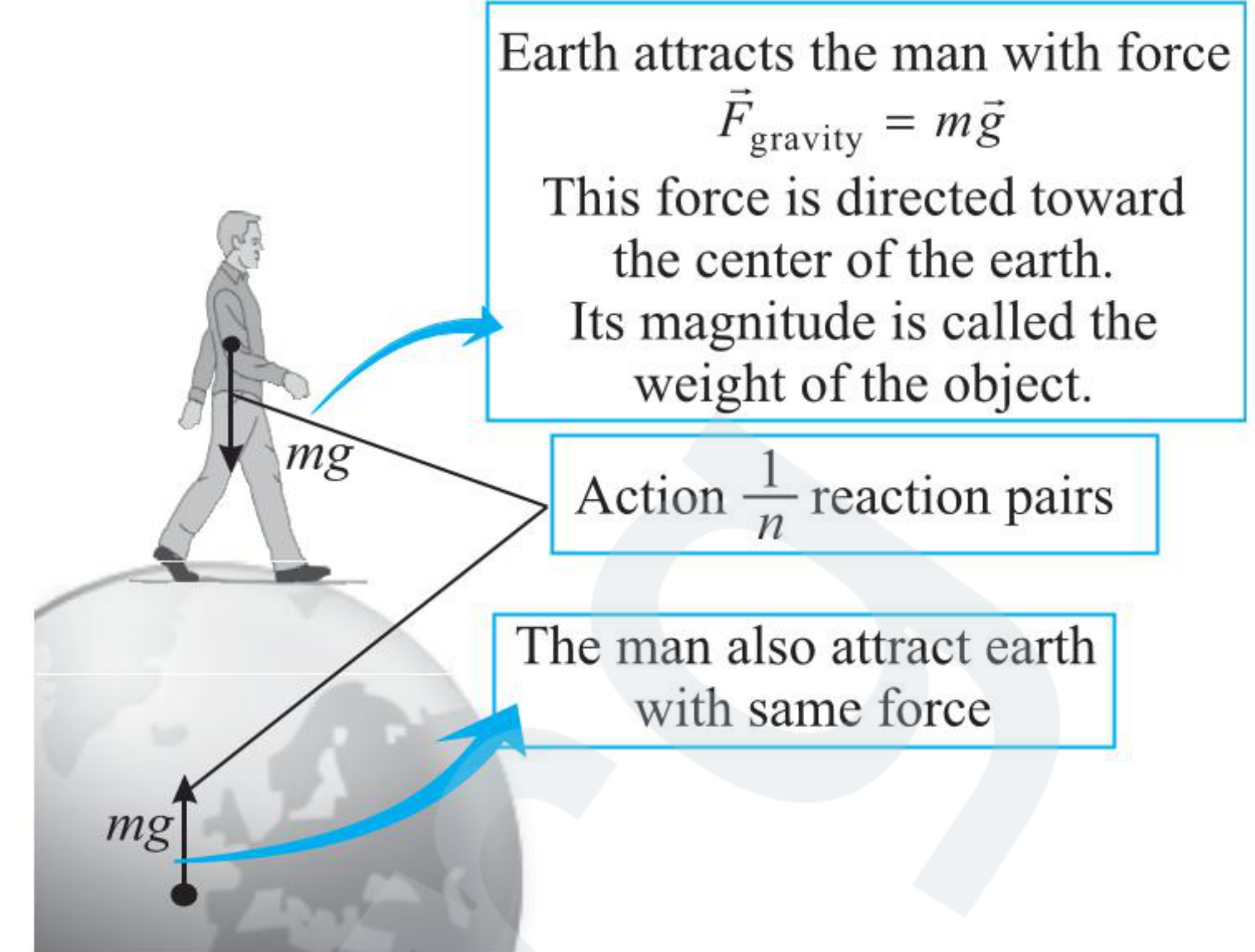
The common forces encountered in mechanics and representing these forces through free-body diagrams are

1. Weight
2. Normal force
3. Tension
4. Frictional force
5. Elastic spring force

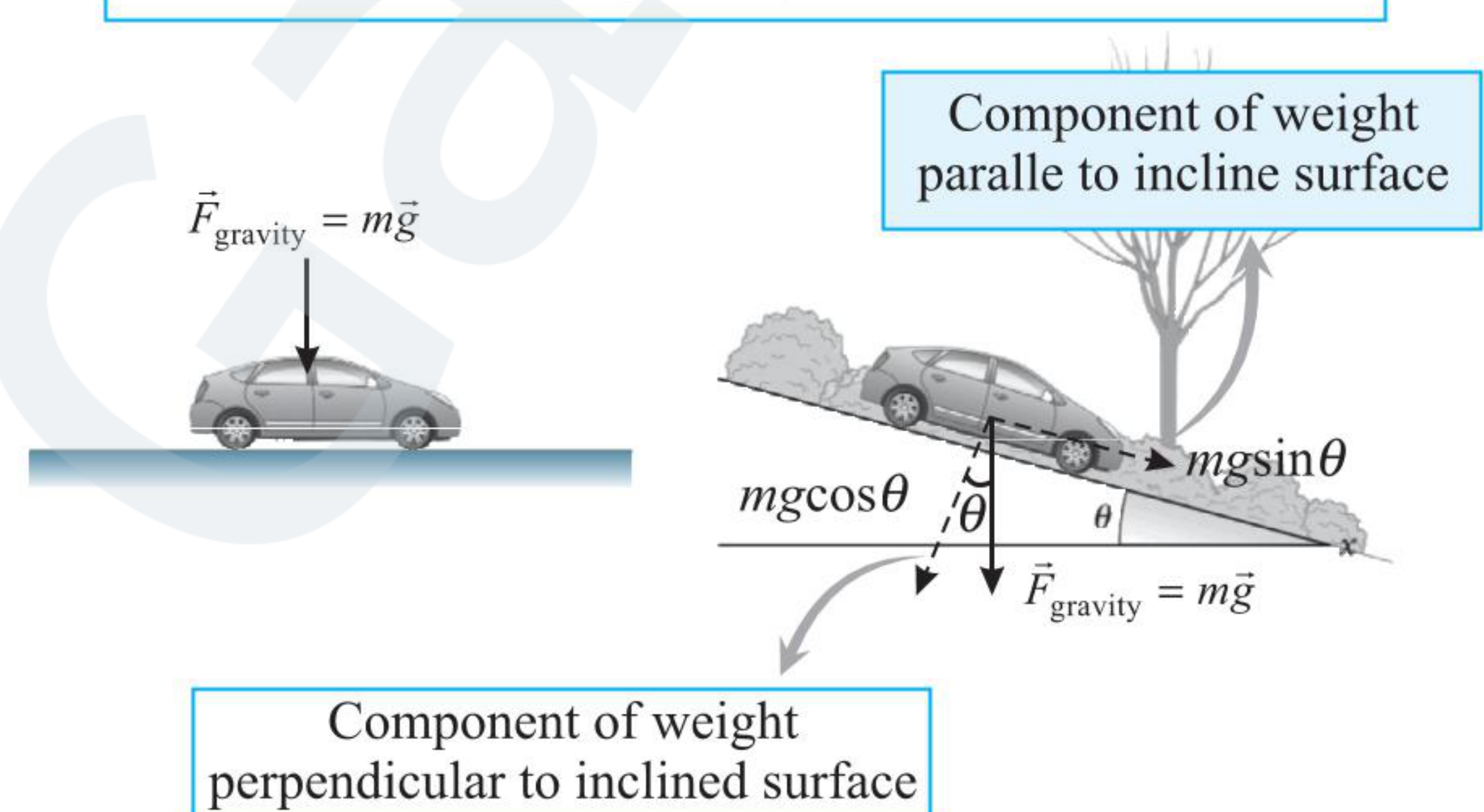
WEIGHT

Weight is the gravitational force with which the Earth pulls an object. The weight of an object can be written as the acceleration due to gravity, which is always directed towards the center of the Earth. For a small stretch of the Earth's surface, which can be considered to be flat, the acceleration due to gravity g can be taken to be uniform, pointing vertically downwards, and in this situation, the weight of the object is mg , directed vertically downwards.

Gravitational pull from the earth



We show the gravitational force in "Free body diagram" by arrow pointing downwards.

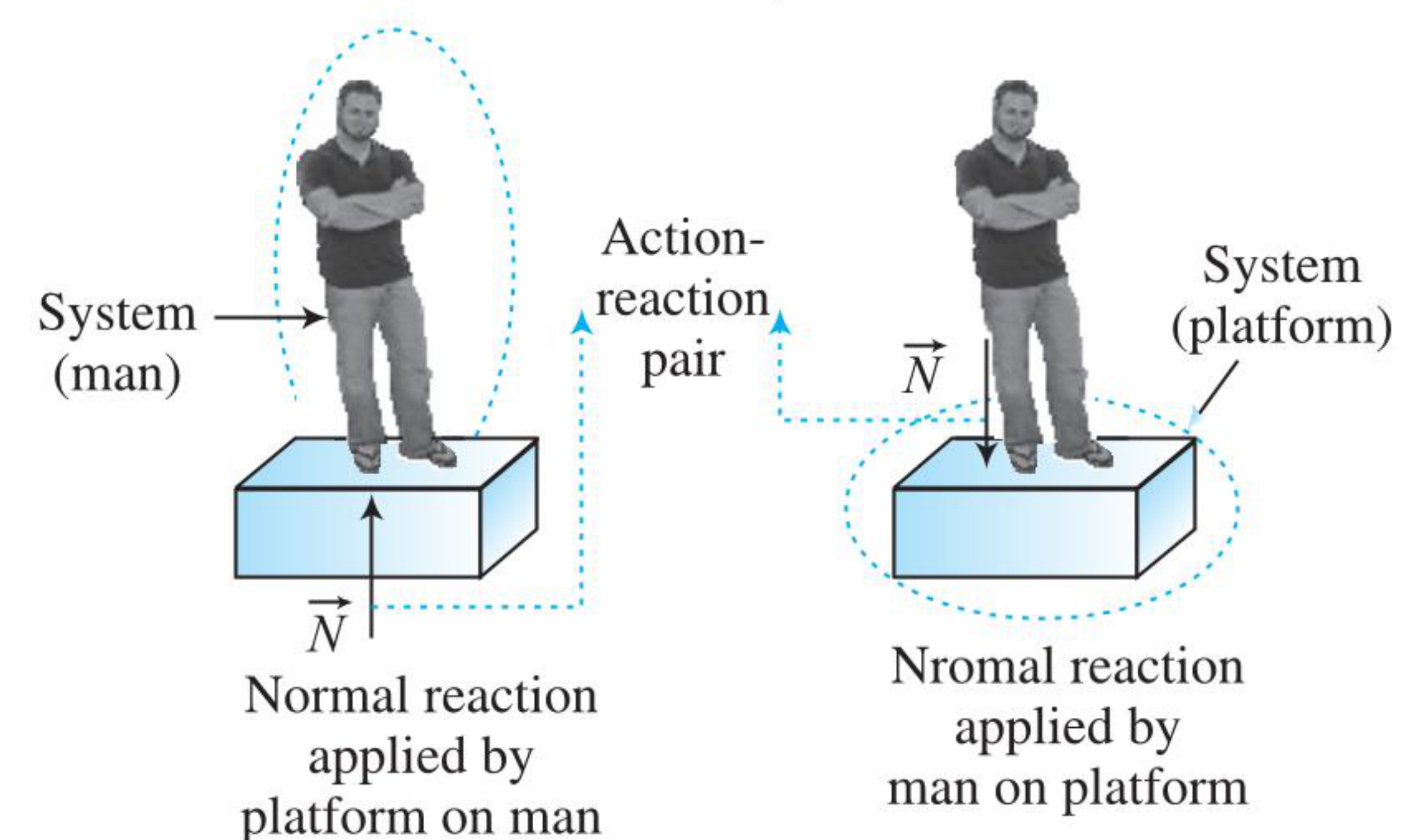


NORMAL FORCE

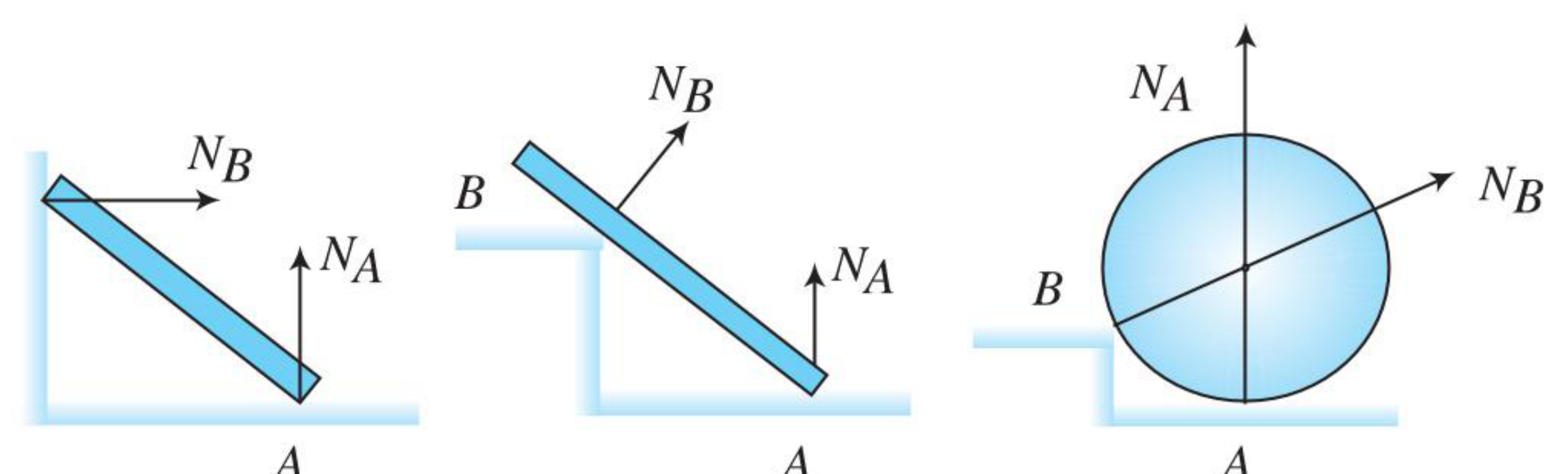
Whenever two surfaces are in contact, they press (or push) each other by a force called contact force. The component of the contact force perpendicular to the surface is called *normal reaction* and along the surface of contact is called *frictional force*.

The normal reaction to the contact surface can be computed by using $\sum \vec{F} = m\vec{a}$. Figure shows action-reaction pairs of normal forces for two physical situations.

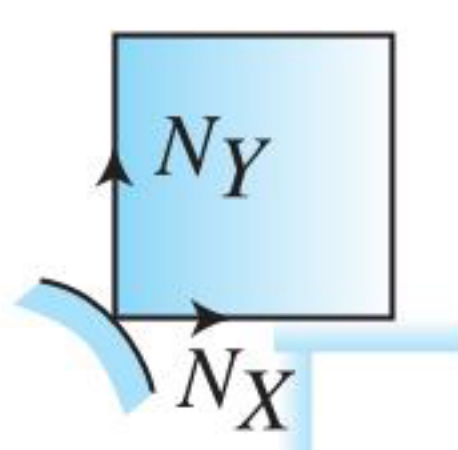
Normal forces on a block are drawn normal to the contact surface directed towards the block, and $N \geq 0$.



REPRESENTING NORMAL REACTION IN DIFFERENT SITUATIONS



If the direction of contact force cannot be determined, it should be shown as two components (as shown in figure).

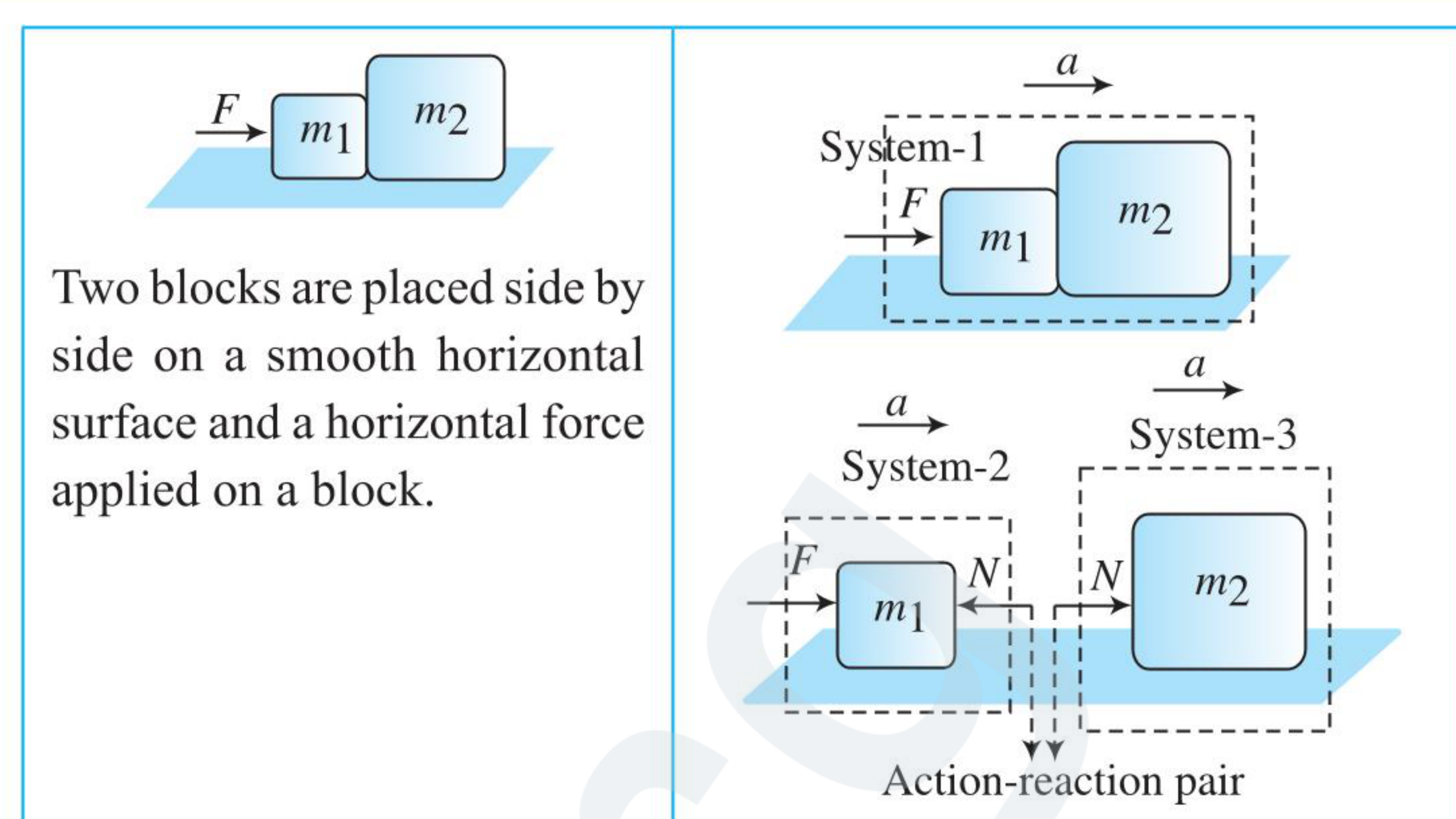


Important Points:

- Normal force will be perpendicular to the surface of contact.
- Normal reaction is a pushing force.
- If perpendicular to the surface of contact cannot be drawn, the normal force will act perpendicular to the surface of the body.
- If neither can be done, normal force has to be drawn as two components—one in the x -direction and one in the y -direction. Remember there is no relation between the forces acting along the x - and the y -directions. They are independent of each other.
- The number of normal forces acting on a body depends on the number of points or surfaces of contact.

REPRESENTING NORMAL REACTIONS AND WEIGHT IN FREE-BODY DIAGRAM

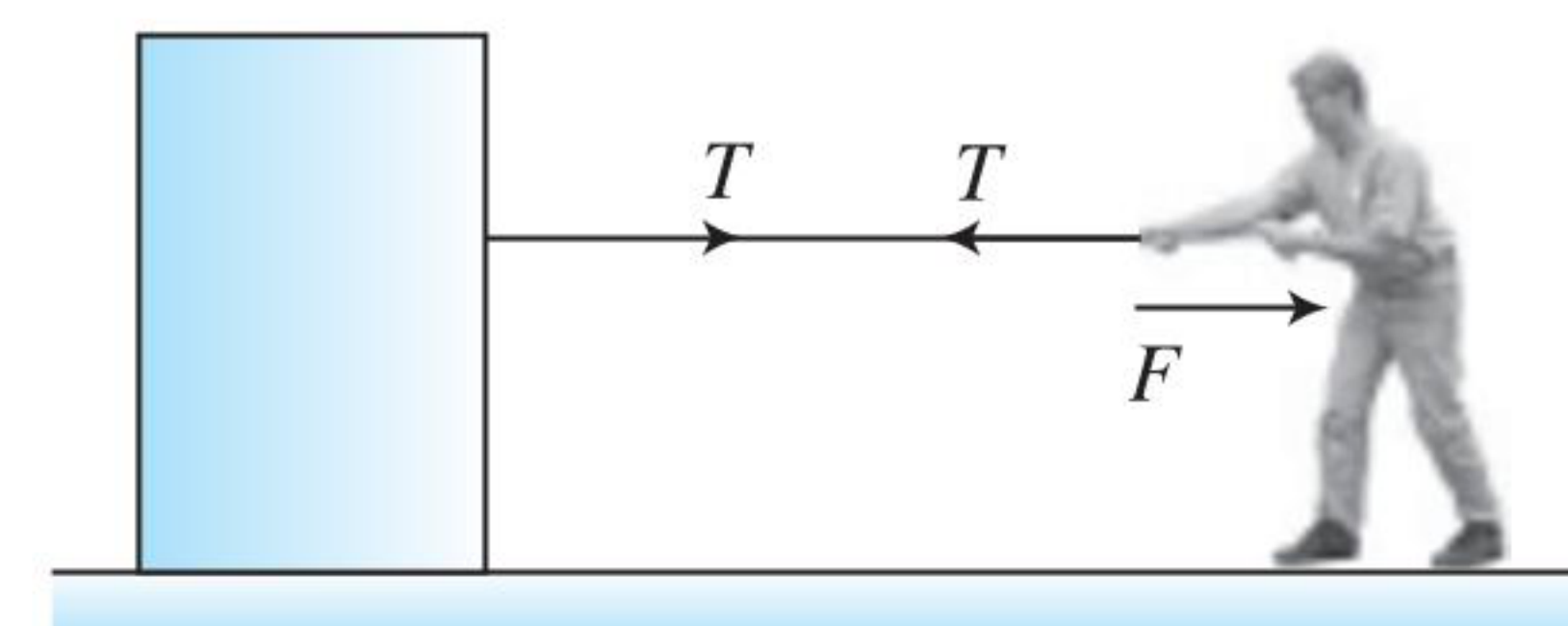
Situation	FBD
A block of mass m is placed on smooth surface.	Free body diagram of block Action and reaction pairs Free body diagram of platform
Let us consider two blocks of masses M and m placed as shown in figure and a force of magnitude F acting on M.	Free body diagram of $M + m$ Free body diagram of m Free body diagram of M Action-reaction pair
A block is released from a fixed smooth inclined plane.	Action-reaction pairs Free body diagram of block



TENSION

It is quite practical that we can pull objects by a string, but we cannot push objects by the string. This gives us an idea that a string can pull but cannot push. Tension force is an inter molecular force between the atoms of a string, which acts or reacts when the string is stretched. There are some important points to remember about the tension in a string, which are helpful in drawing free-body diagram of the bodies in a system.

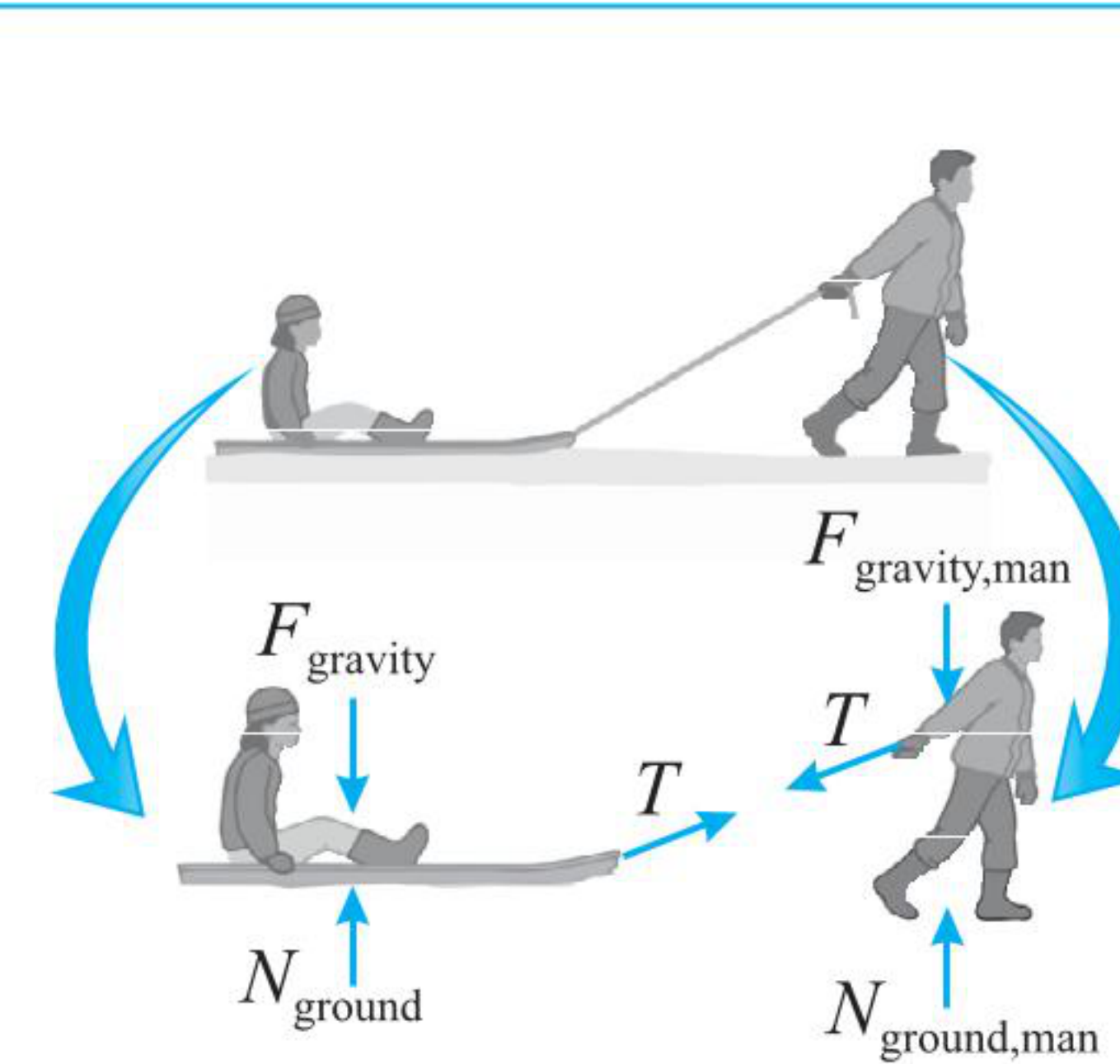
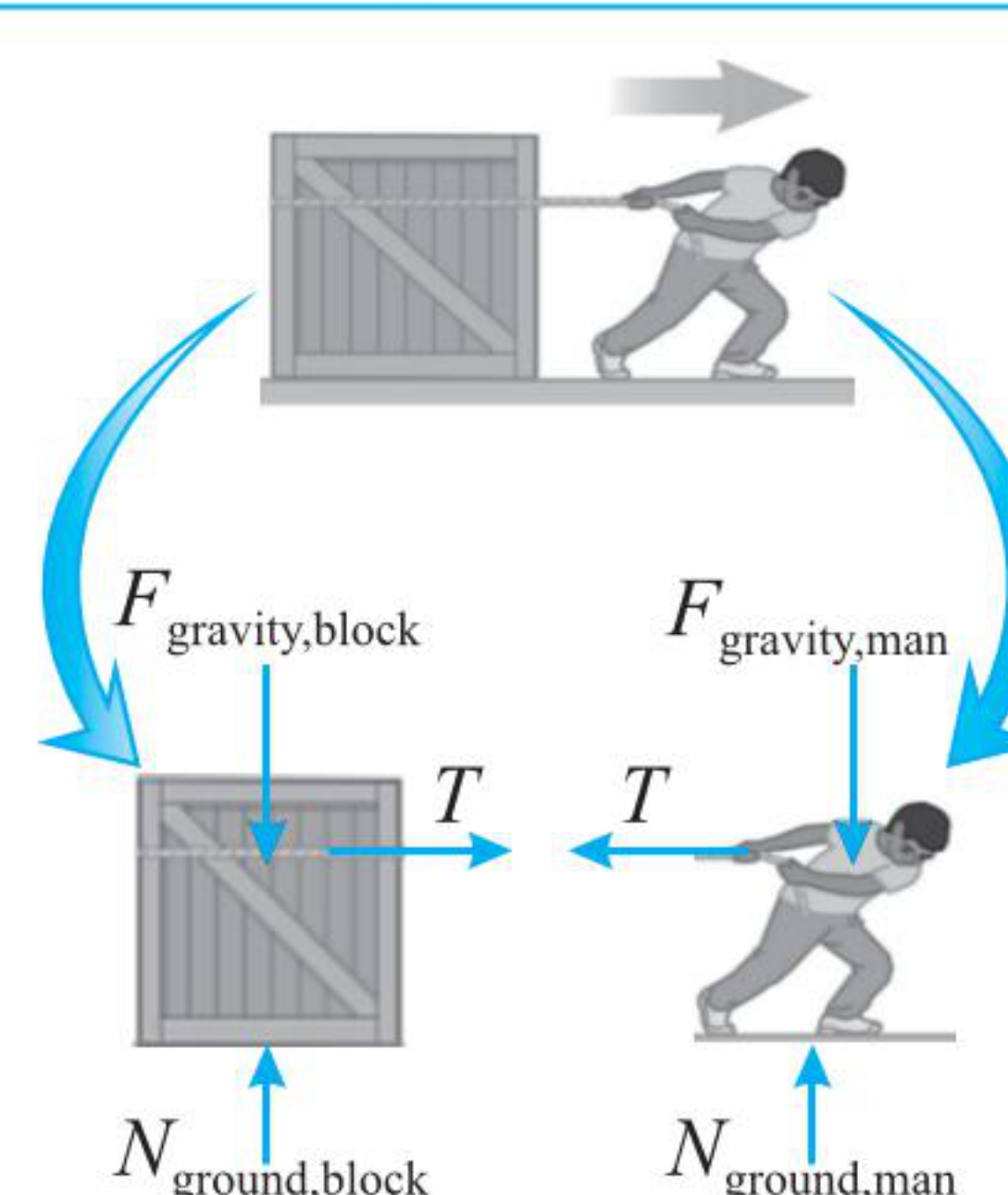
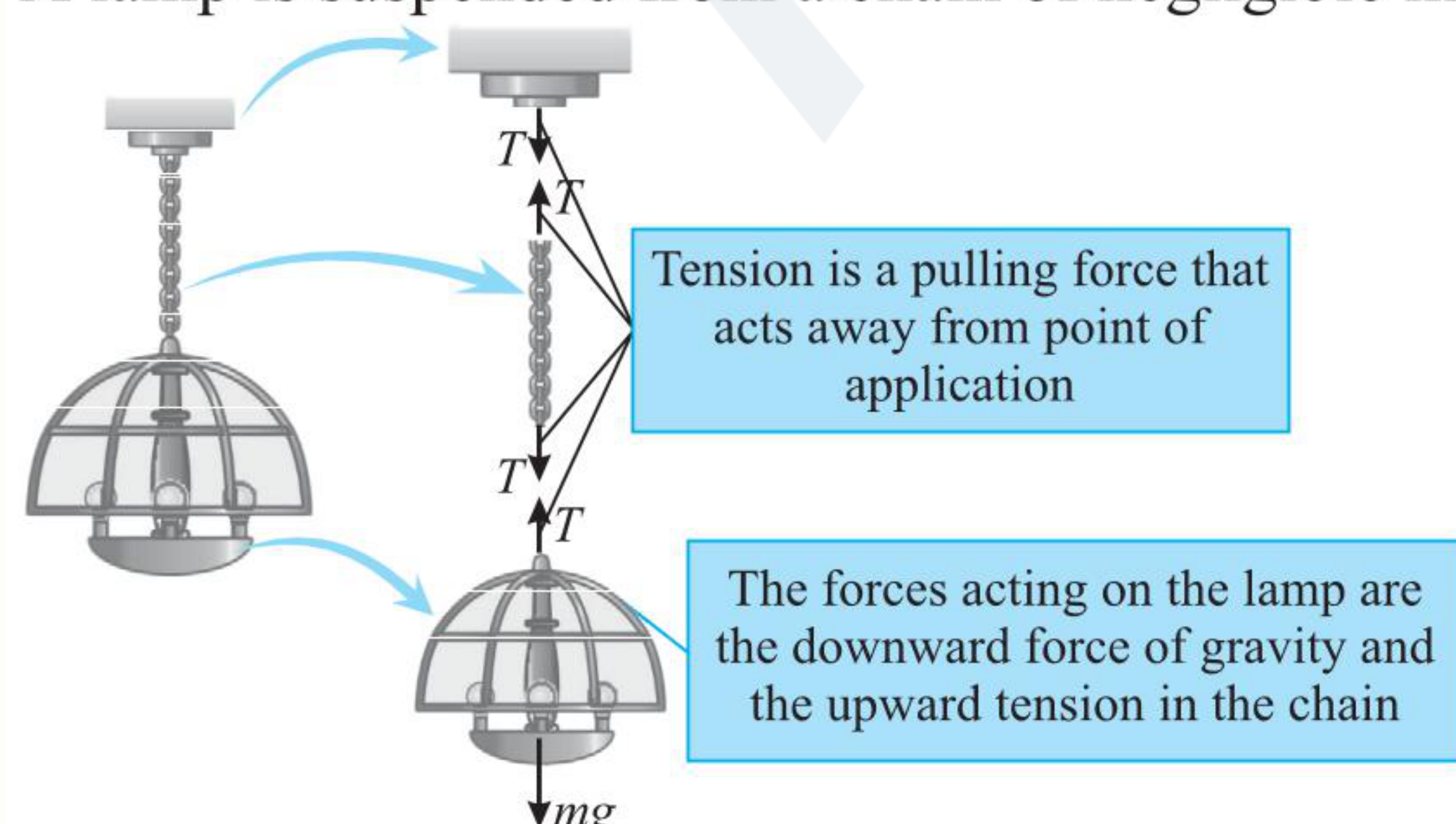
- The force of tension acts on a body in the direction away from the point of contact or tied ends of the string. For example consider figure. A man pulls a box with a string. The tension in string acts on the box towards right or in the direction away from the tied point and on the man it is again away from it. The way of showing the direction of tension is shown in figure.



- If string is massless and frictionless, the tension throughout the string remains constant as shown in figure. But if the string is massless and not frictionless, at every contact in the length of the string, tension changes and if it is not light, tension at each point will be different depending on the acceleration of the string.
- If a force is directly applied on a string, as say a child is pulling a tied string from the other end with some force, the tension in the string will be equal to the applied force. Irrespective of the motion of the pulling agent in figure, the man is applying a force F on string. Thus, the tension in string will be equal to this force, irrespective of whether the box will move or not, man will move or not.

Tension force of strings

A lamp is suspended from a chain of negligible mass.



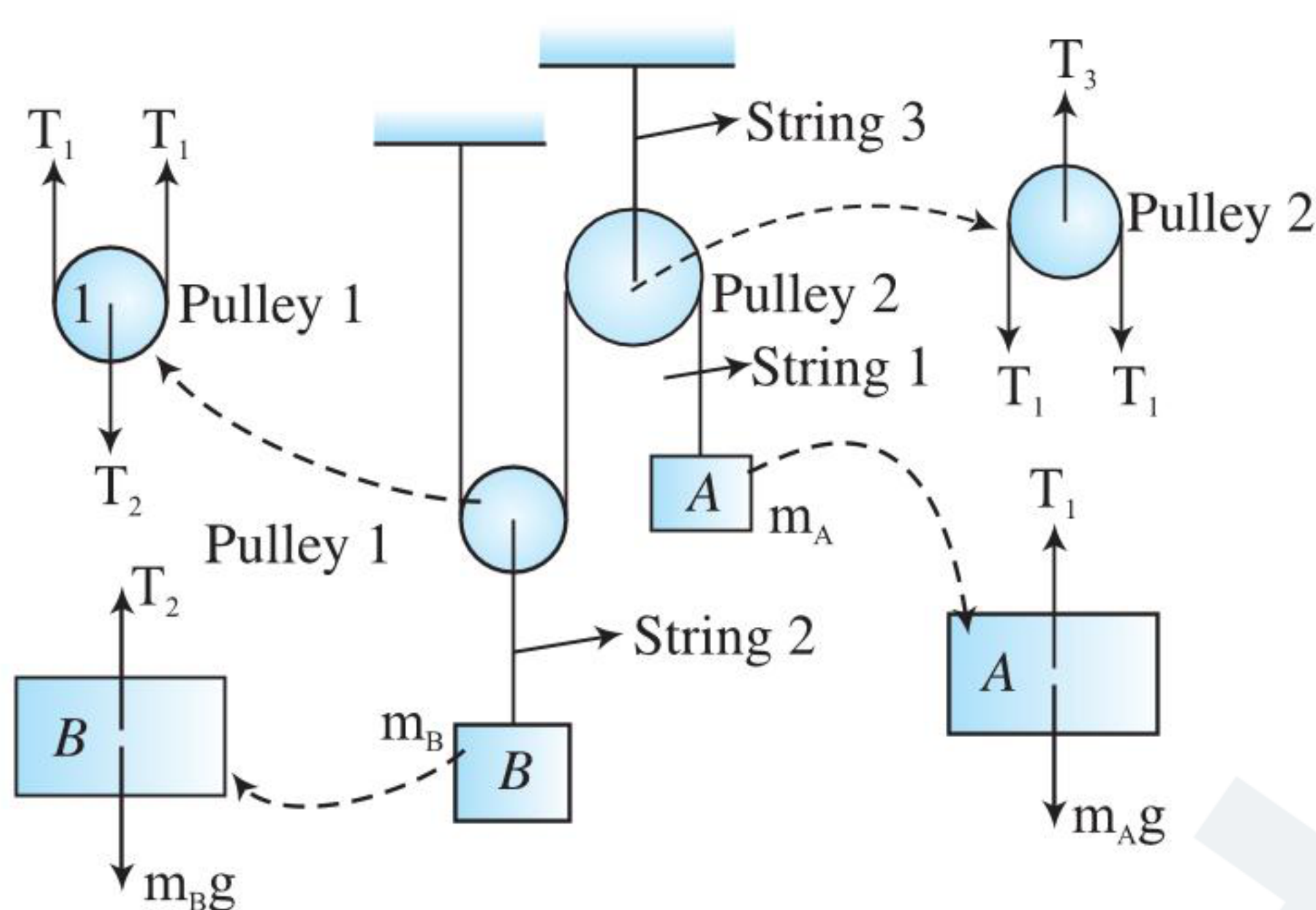
Important Points:

- String is assumed to be inextensible (perfectly elastic) unless stated. This is why the magnitude of acceleration of any number of masses connected through string is always same.
- String is assumed to be massless unless stated. This is why the tension in it everywhere remains the same and equal to applied force. However, if a string has a mass, the tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.

REPRESENTATION OF TENSION FORCE IN DIFFERENT SITUATIONS

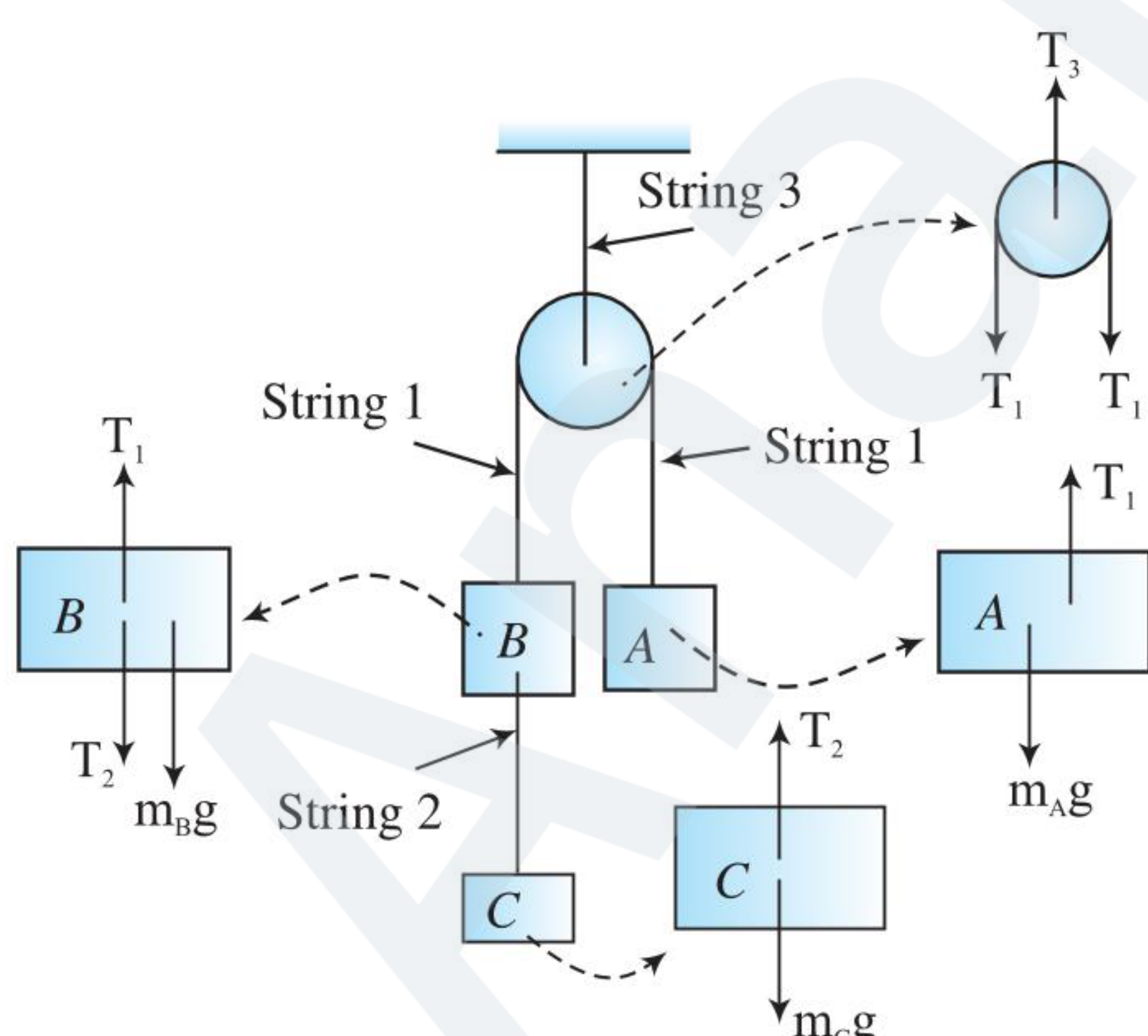
Two blocks of masses m_A and m_B are arranged in the diagram as shown. the free-body diagrams of

- Block m_A
- Block m_B
- Pulley 1
- Pulley 2



Three blocks of masses m_A , m_B , and m_C are arranged in the diagram as shown. The free-body diagrams of

- Block m_A
- Block m_B
- Block m_C
- Pulley



Important Points:

- Tension force always pulls a body.
- Tension can never push a body.
- Tension across massless pulley or frictionless pulley remains constant.
- Ropes become slack when tension force becomes zero.

FRICTION

When a surface of a solid body slides (or has a tendency to slide if it does not actually slide) on a surface of another solid body, its (surface's) motion (or the tendency of motion) is opposed. The force which opposes the relative motion (or tendency of the relative motion) between the contact surfaces is called *frictional force*.

The origin of frictional force is a complicated matter. Friction is a consequence of molecular interaction originating in the realm of molecules and atoms because of electrical reasons. On an atomic level, both surfaces of contact are irregular. There are many points of contact where atoms seem to cling together, and when the surface is pulled along, the atoms snap apart and vibration ensues.

If we push a block on the table, the table starts pushing the block back along the surface of contact (tangentially). Hence, we can call this contact force *tangential contact*.

Situation	FBD
A man is pushing a block placed on a rough horizontal surface.	
A block is sliding down on a rough inclined plane.	Action-reaction pairs of friction force Free body diagram of block

EQUILIBRIUM OF A PARTICLE

The equilibrium of a particle in mechanics refers to the situation when the net external force acting on the particle is zero.

The above condition is correct and complete as far as the equilibrium of a particle (i.e., a point mass) is concerned. In case of rigid bodies (i.e., extended bodies), there are two conditions to be satisfied for such bodies to have equilibrium. First, the net external force acting on the body should be zero. Second, the net external torque acting on the body should be zero.

CONCURRENT FORCES

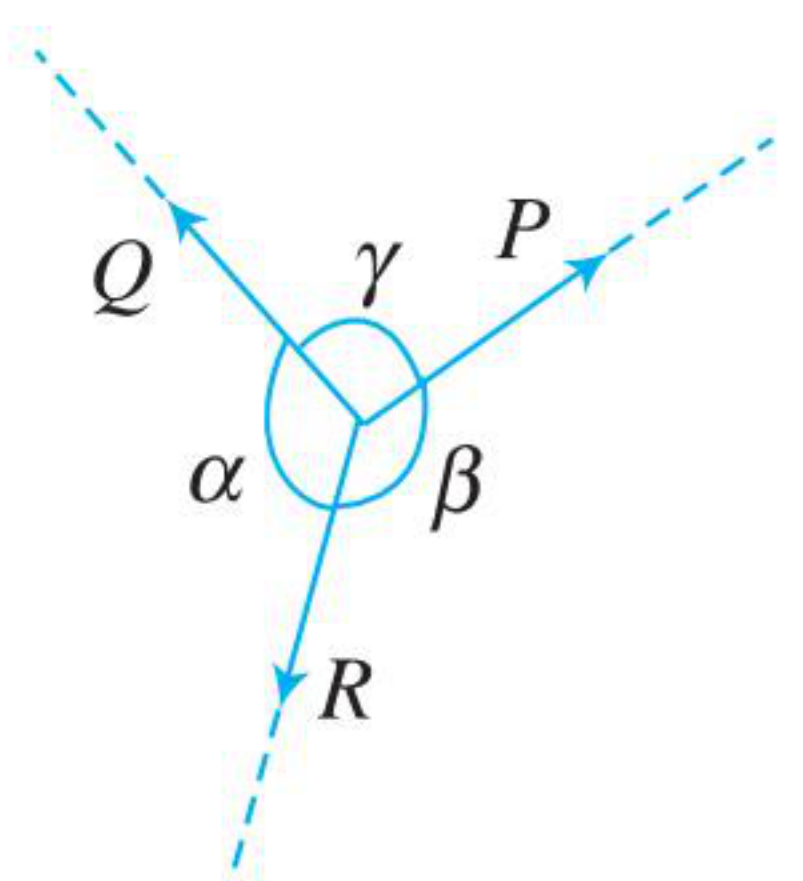
If two or more forces act on the same particle, we call them concurrent forces.

LAMI'S THEOREM

If three concurrent forces P , Q , and R acting on a particle keep the particle in equilibrium, then Lami's theorem states

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

Here α is the angle opposite to $\vec{P}$, β is the angle opposite to $\vec{Q}$, and γ is the angle opposite to $\vec{R}$.



Note: If concurrent forces are coplanar but more than three, then it is generally convenient to resolve all of them along two mutually perpendicular directions and then the resultant of each set of these resolved components will be zero.

$$\sum F_x = 0; \sum F_y = 0$$

If the forces are not coplanar, then they can be resolved along any three mutually perpendicular directions and then the following conditions apply:

$$\sum F_x = 0; \sum F_y = 0; \sum F_z = 0$$

PROBLEM-SOLVING BY APPLYING NEWTON'S LAWS

The following procedure is recommended when dealing with problems involving Newton's law.

Step 1: Draw a simple, neat diagram of the system to help establish the mental representation. Establish convenient coordinate axes for each object in the system.

Step 2: If an acceleration component for an object is zero, it is modeled as a particle in equilibrium in this direction and $\sum \vec{F} = 0$. If not, the object is modeled as a particle under a net force in this direction and $\sum \vec{F} = m\vec{a}$.

Step 3: Isolate the object whose motion is being analyzed. Draw arrows on your sketch to show the direction of each force acting on the object. Arrows are drawn to represent direction of forces acting on the body. This diagram is called direction of forces acting on the body. This diagram is called a free-body diagram. Draw a free-body diagram for this object. For systems containing more than one object, draw separate free-body diagrams for each object. Do not include in the free-body diagram forces exerted by the object on its surroundings. Find the components of the forces along the coordinate axes. Apply Newton's second law, $\sum \vec{F} = m\vec{a}$, in component form. Check your dimensions to make sure all terms have units of force.

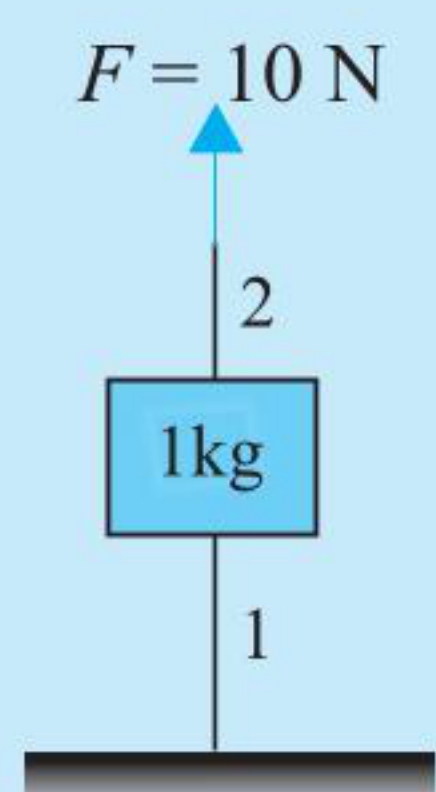
Step 4: Solve the component equations for the unknowns. Remember that to obtain a complete solution, you must have as many independent equations as you have unknowns.

Step 5: Make sure your results are consistent with the free-body diagram. Also check the predictions of your solutions for extreme values of the variables. By doing so, you can often detect errors in your results.

ILLUSTRATION 6.7

A string is connected between surface and a block of mass 1 kg which is pulled by another string by applying force $F = 10 \text{ N}$ as shown in figure. Calculate

- tension in string (1)
- tension in string (2)



Sol. For calculating tension force in strings, we need to draw the F.B.D. of the block. The force acting on block are tension force at top and bottom side of the block and weight of the block.

Tension is a pulling force, it will act away from the block as shown in figure.

If we apply a force at the end of a light string, the force applied is equal to tension developed. Hence, the tension in string (2), $T_2 = F = 10 \text{ N}$

Now we apply Newton's second law, $\sum F_y = ma_y$

As $a_y = 0$, $\sum F_y = 0$.

From free body diagram of block,

$$T_2 = T_1 + mg \Rightarrow 10 = T_1 + 10 \Rightarrow T_1 = 0$$

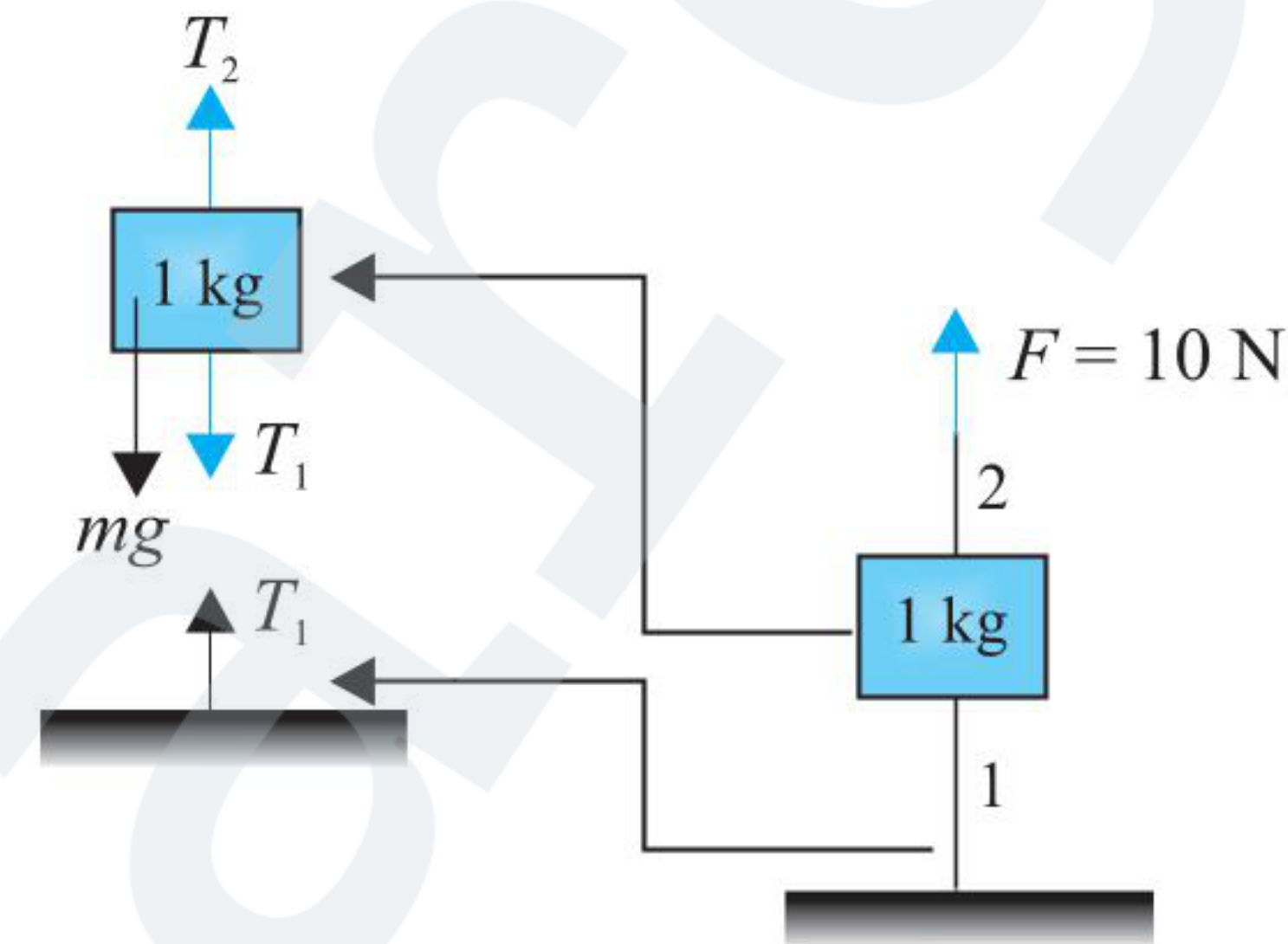
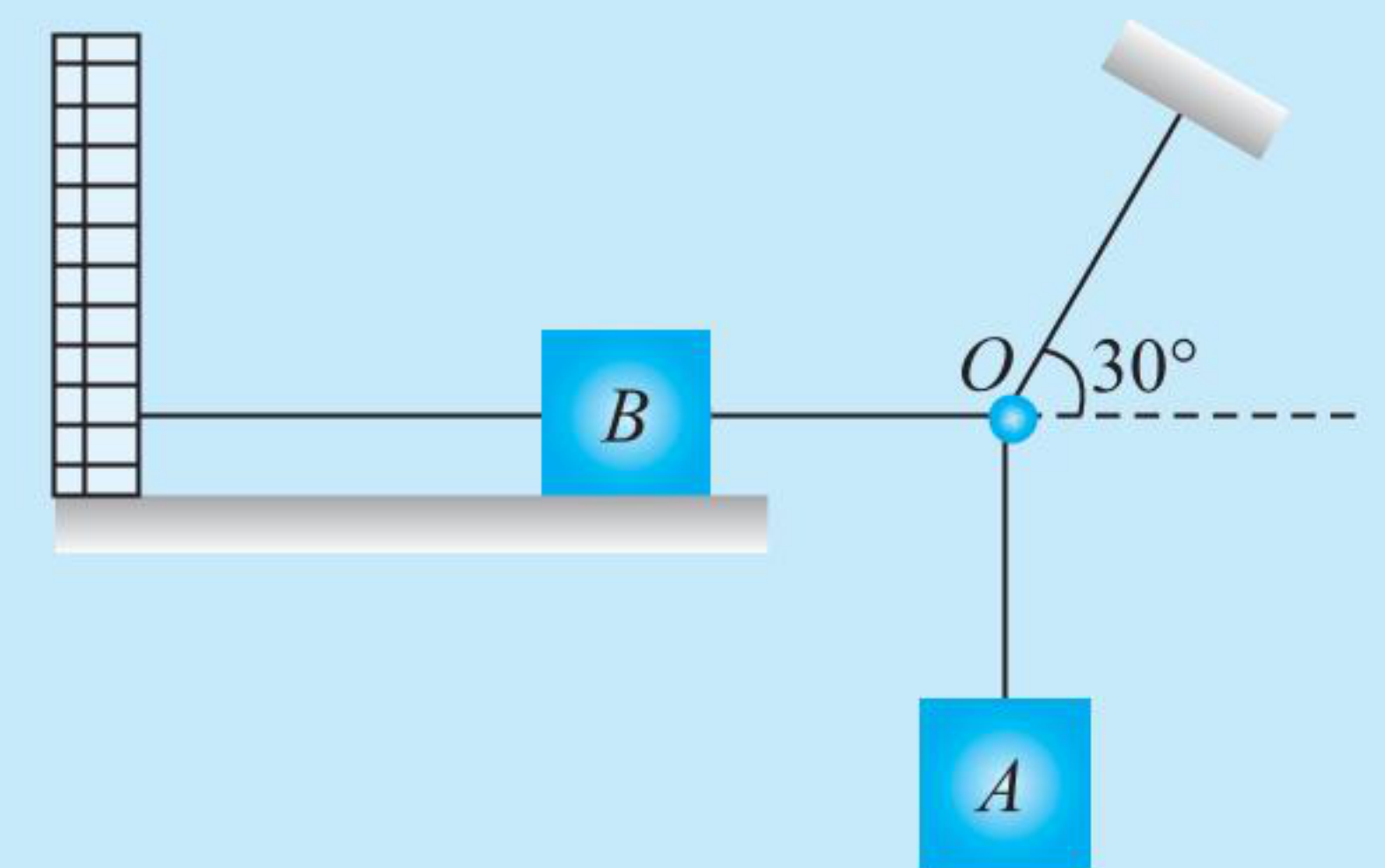


ILLUSTRATION 6.8

The breaking strength of the string connecting wall and block B is 175 N. Find the magnitude of weight of block A for which the system will be stationary. Block B weighs 700 N.



Sol. The breaking strength of the string means maximum possible tension in the string.

As system is at rest from Newton's second law of motion, $\sum F = ma$. As $a = 0$, $\sum F = 0$

From F.B.D. of A, $\sum F_y = 0$

$$T_A = W_A \quad \dots(i)$$

From F.B.D. of B, $\sum F_x = 0$

$$T_B = T_1 = 175 \text{ N} \quad \dots(ii)$$

From F.B.D. of junction O,

Balancing forces in horizontal direction, $T_2 \cos 30^\circ = T_B$

$$T_2 \left(\frac{\sqrt{3}}{2} \right) = 175 \Rightarrow T_2 = \frac{350}{\sqrt{3}} \text{ N}$$

Balancing forces in vertical direction, $T_2 \sin 30^\circ = T_A = W_A$

$$\left(\frac{350}{\sqrt{3}} \right) \times \left(\frac{1}{2} \right) = W_A \Rightarrow W_A = \frac{175}{\sqrt{3}} \text{ N}$$

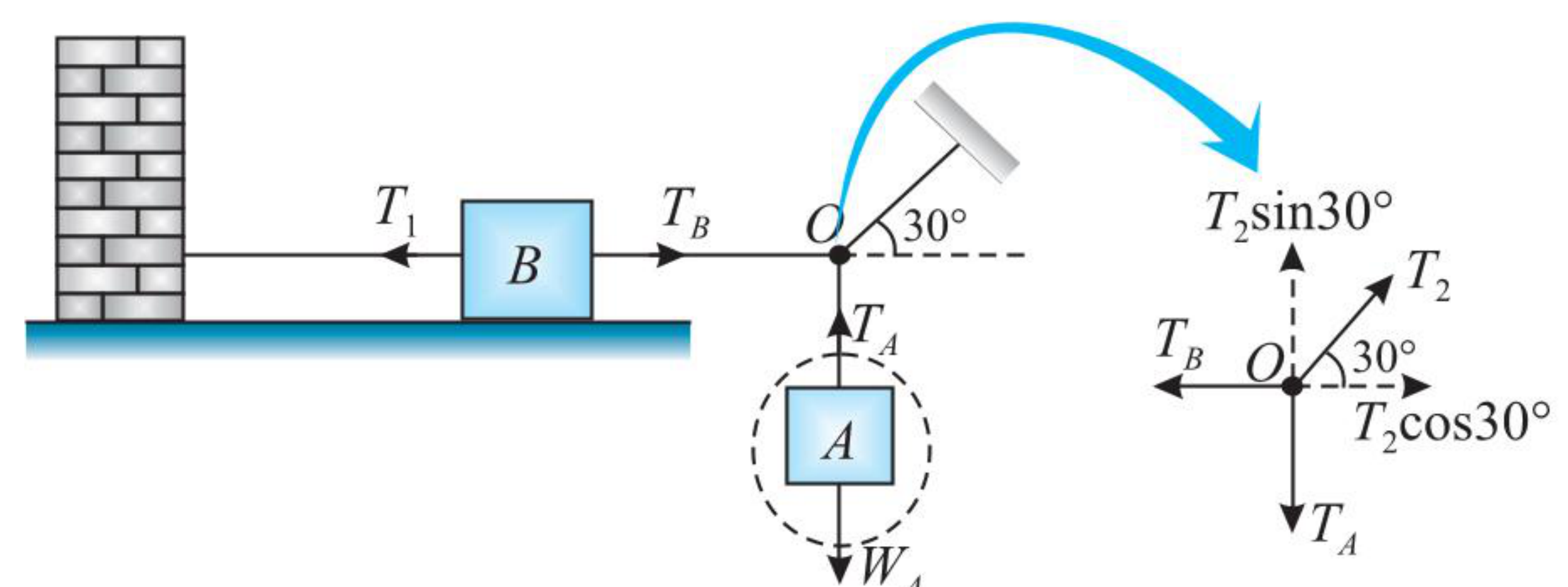
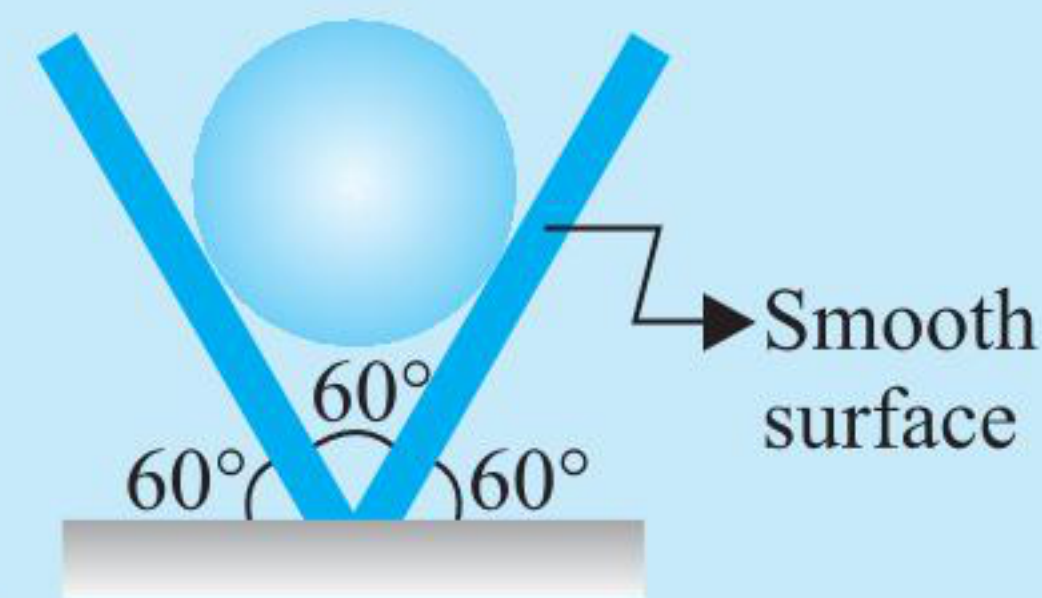


ILLUSTRATION 6.9

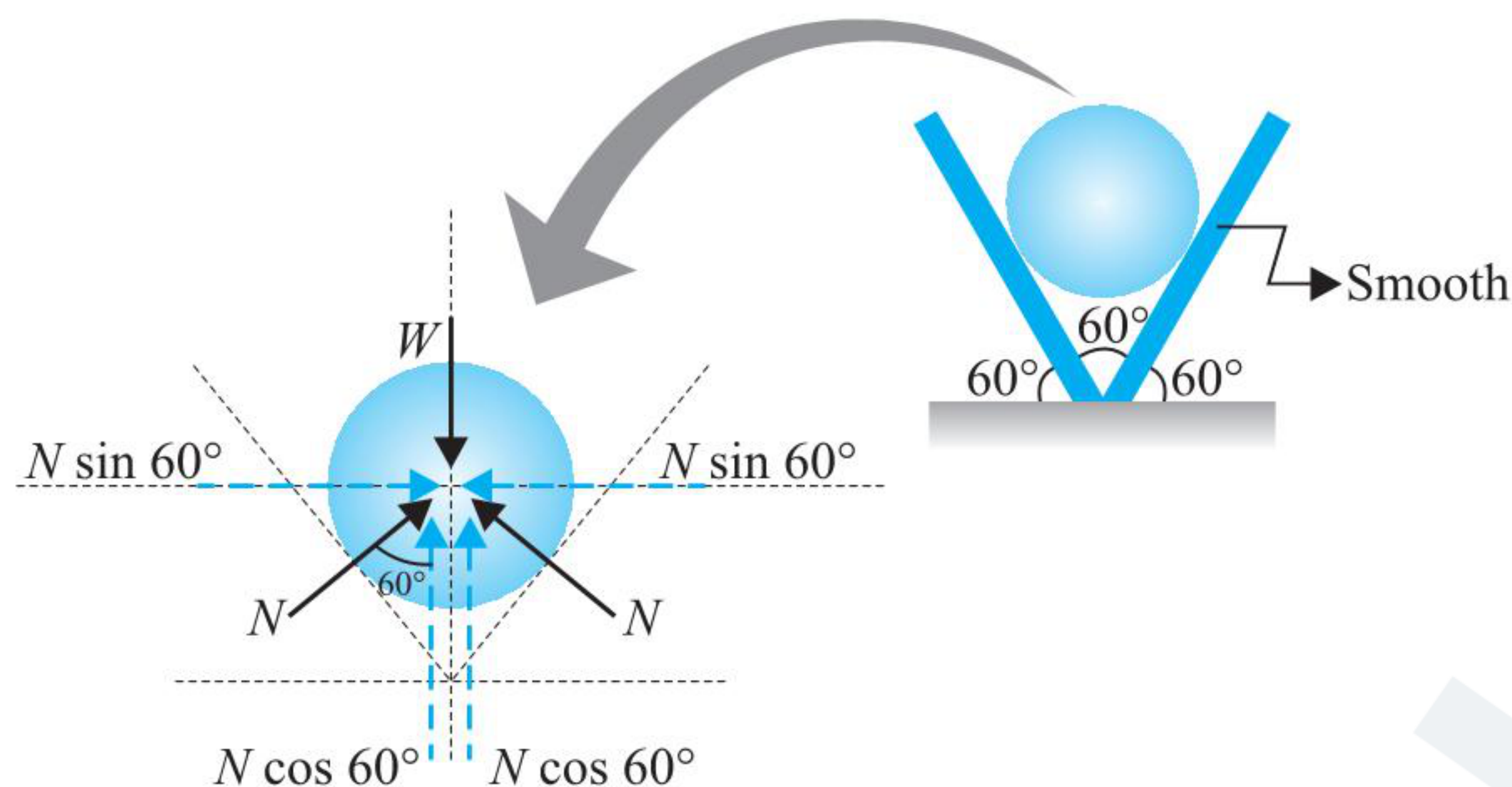
A cylinder of weight W is resting on two inclined planes forming a V-groove as shown in figure. Ignore friction everywhere.



(a) Draw its free body diagram of the sphere.

(b) Calculate normal reactions between the cylinder and two inclined walls.

Sol. Consider the cylinder as our system. On cylinder, there will be two contact forces and one field force, contact force in the form of normal reactions exerted by inclined planes and field force is the weight of the cylinder. As we know normal reaction is a pushing force, it will act towards and in normal to cylinder or in radial direction and weight will act vertically downwards through center of cylinder as shown in free body diagram of cylinder. As the cylinder is at rest, net force on the cylinder should be zero.



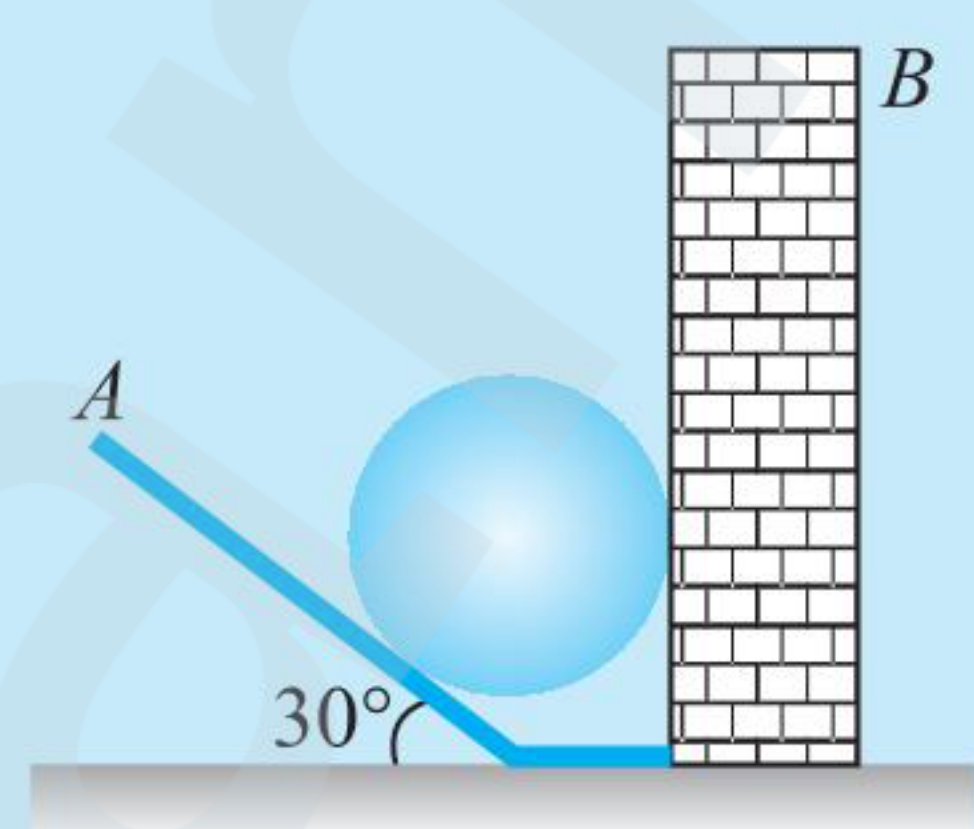
In horizontal direction, $\sum F_x = 0$

and in vertical direction $\sum F_y = 0$ or $2N \cos 60^\circ = W$

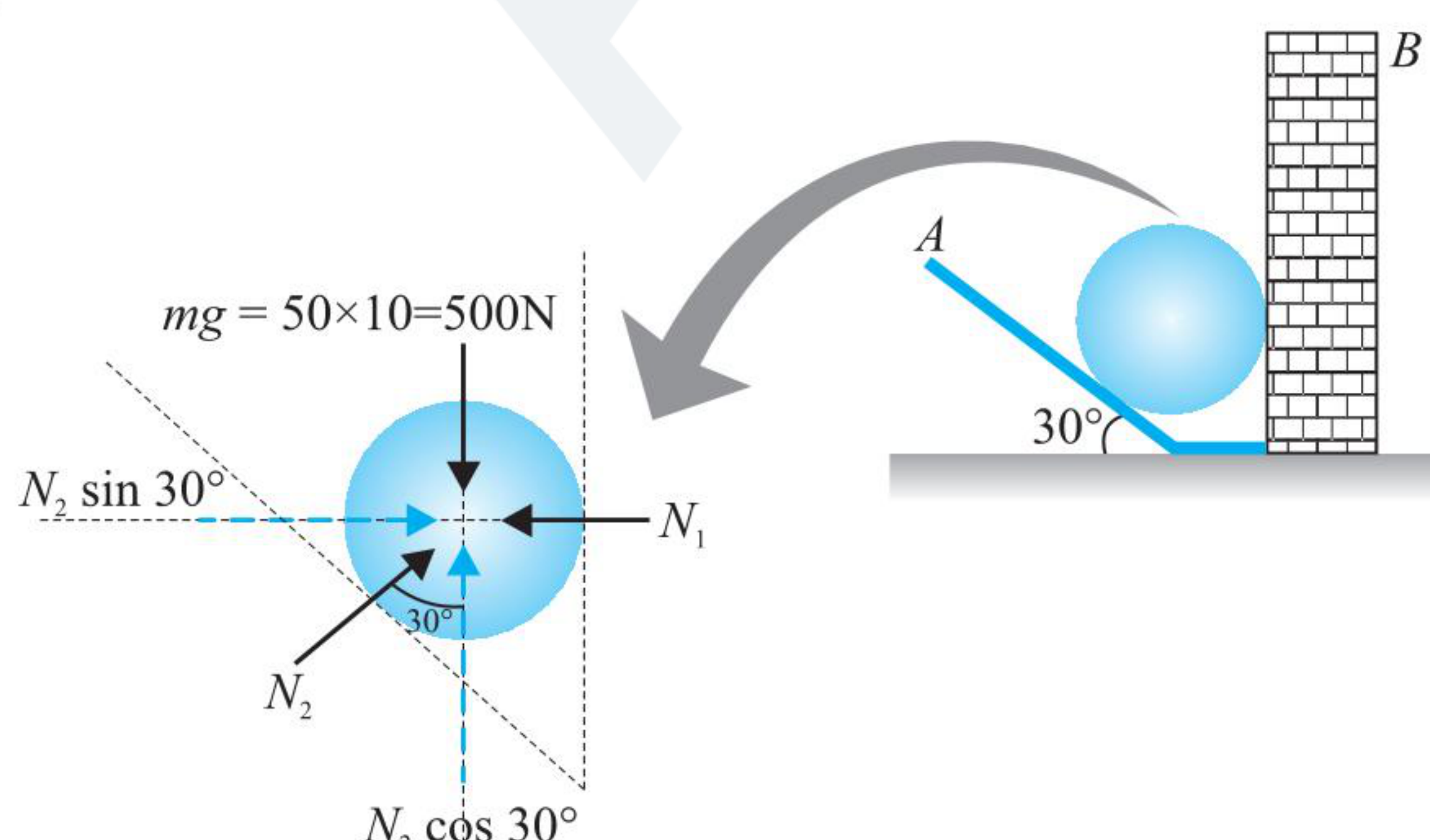
$$\Rightarrow 2N \times \left(\frac{1}{2}\right) = W \Rightarrow N = W$$

ILLUSTRATION 6.10

A 50-kg homogeneous smooth sphere rests on the 30° incline A and bears against the smooth vertical wall B . Calculate the contact forces at A and B .



Sol. Consider the sphere as our system. On sphere there will be two contact forces and one field force, contact force in the form of normal reactions exerted by inclined planes and vertical wall also field force is the weight of the sphere as shown in free body diagram of cylinder. Here the sphere is at rest, net force on the sphere should be zero.



In vertical direction : $\sum F_y = 0$ or $N_2 \cos 30^\circ = 500$

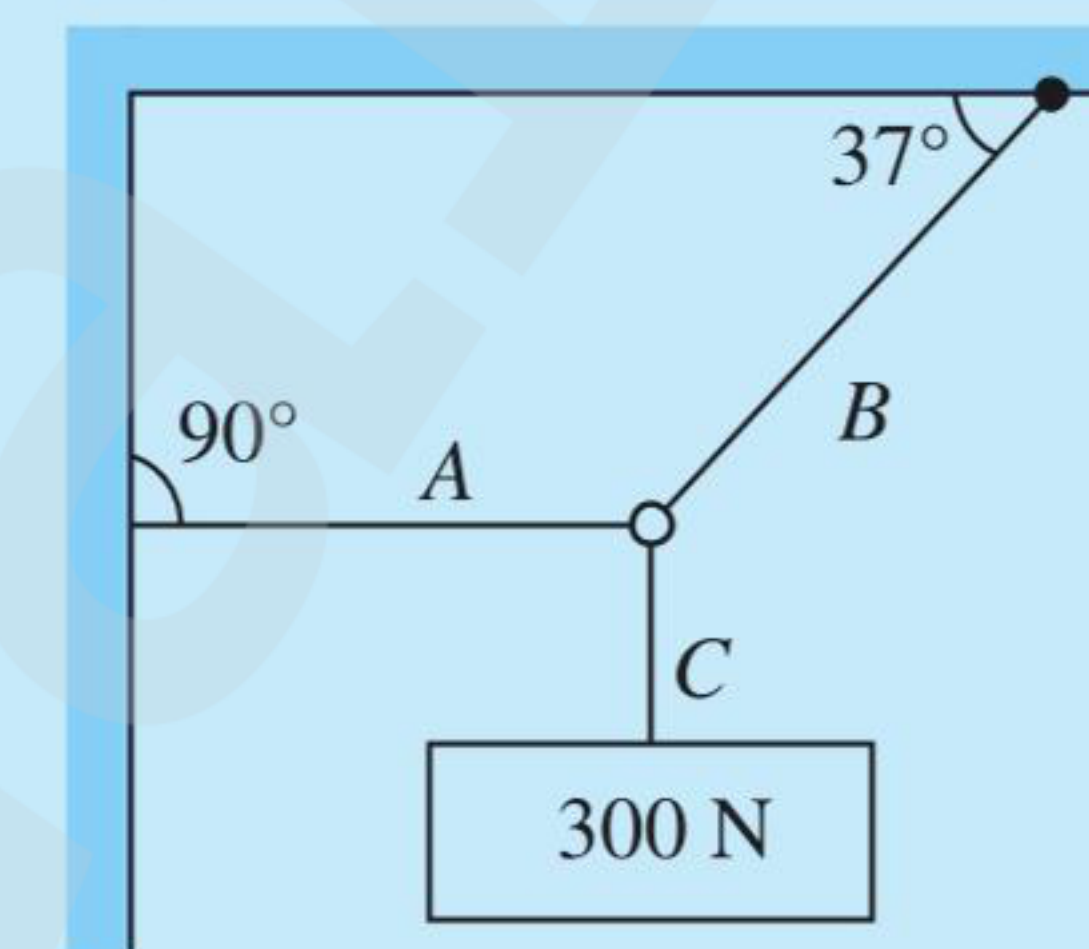
$$\Rightarrow N_2 \times \left(\frac{\sqrt{3}}{2}\right) = 500 \Rightarrow N_2 = \frac{1000}{\sqrt{3}} \text{ N}$$

In horizontal direction, $\sum F_x = 0$ or $N_1 = N_2 \sin 30^\circ$

$$\Rightarrow N_1 = \left(\frac{1000}{\sqrt{3}}\right) \times \left(\frac{1}{2}\right) = \frac{500}{\sqrt{3}} \text{ N}$$

ILLUSTRATION 6.11

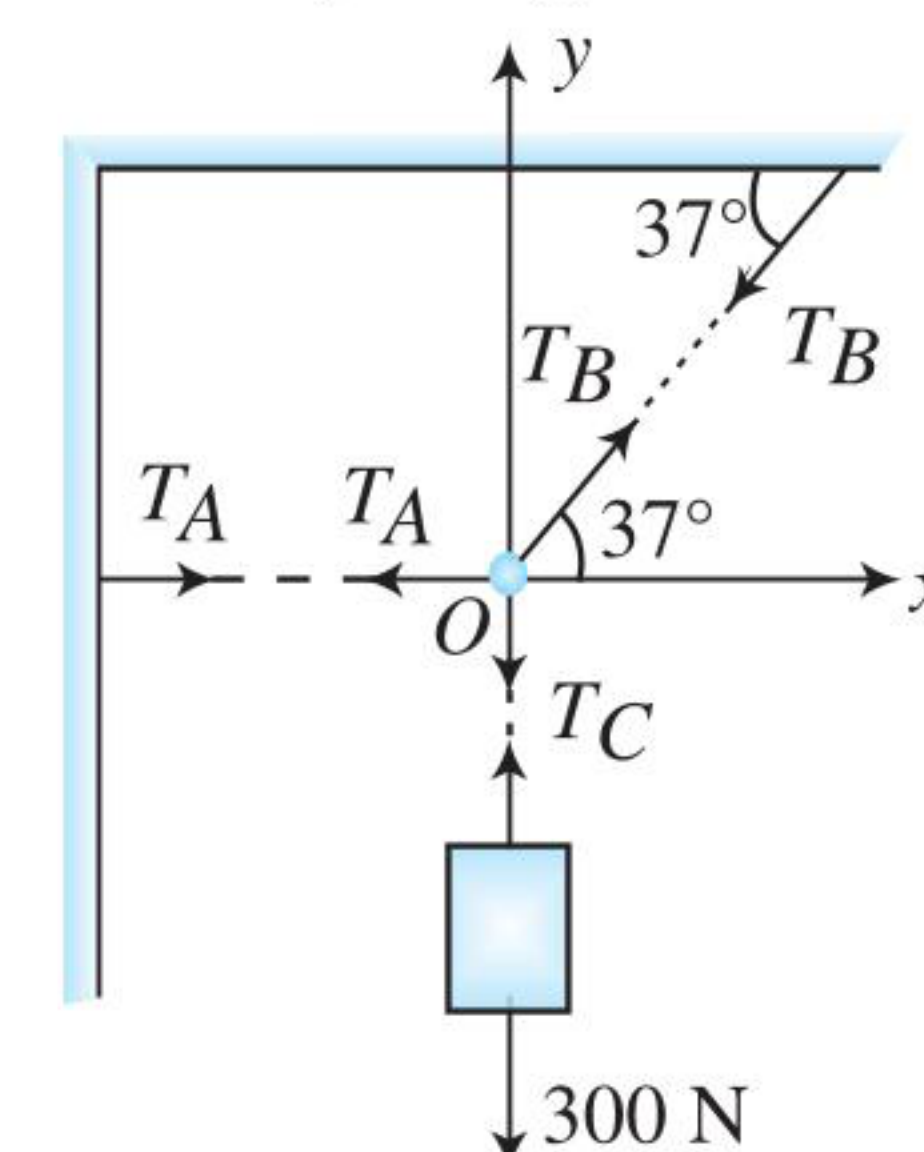
A block of mass 30 kg is suspended by three strings as shown in figure. Find the tension in each string.



Sol.

Method I: Considering equilibrium of each part of system

The whole system is in equilibrium; therefore, for each part $\sum \vec{F} = 0$. From the free-body diagram of block C, $T_C = 300 \text{ N}$.



Now consider the equilibrium of point O,

$$\sum F_x = 0 \text{ or } T_B \cos 37^\circ - T_A = 0$$

$$\therefore T_A = T_B \cos 37^\circ = T_B \cdot \frac{4}{5} \quad \dots(i)$$

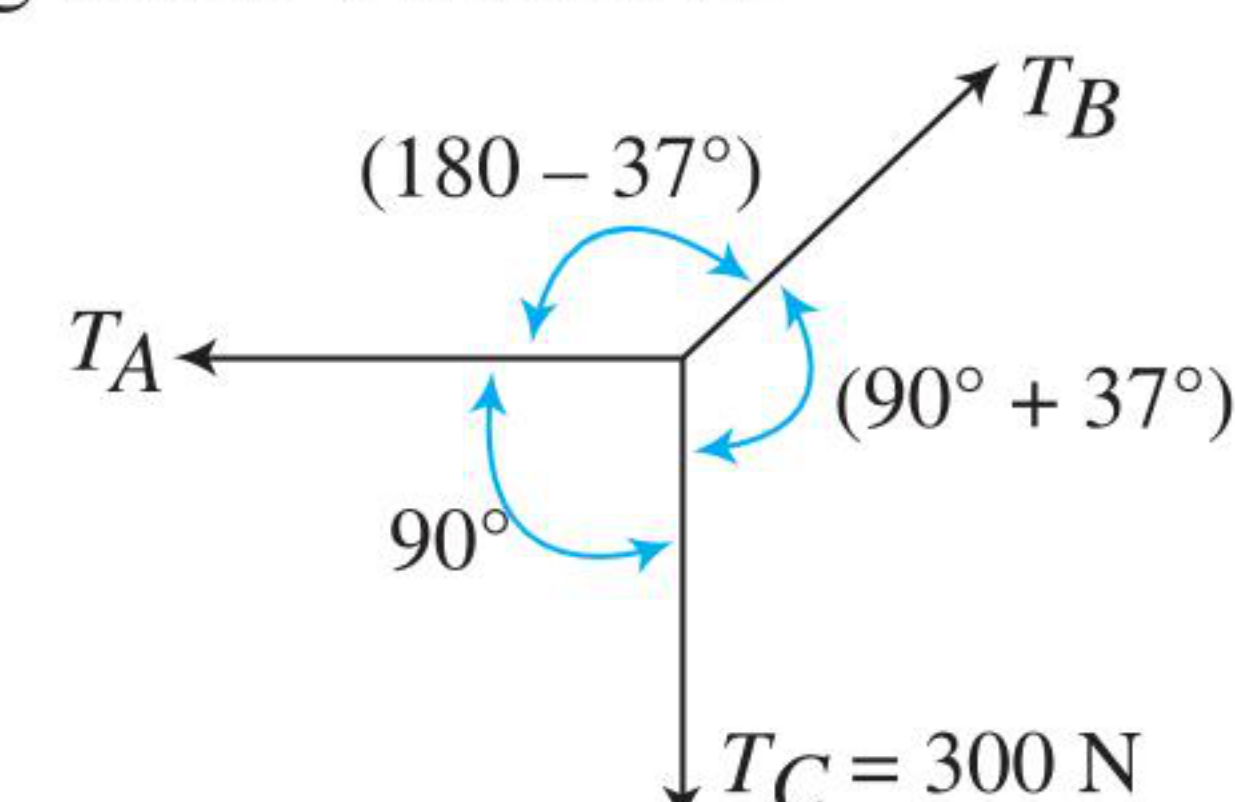
$$\text{and } \sum F_y = 0 \text{ or } T_B \sin 37^\circ - T_C = 0 \quad \dots(ii)$$

$$\therefore T_B = \frac{T_C}{\sin 37^\circ} = \frac{300}{3/5} = 500 \text{ N}$$

From Eq. (i), we get

$$T_A = \frac{4}{5} T_B = \frac{4}{5} \times 500 = 400 \text{ N}$$

Method II: Using Lami's theorem



By Lami's theorem, we have

$$\frac{T_A}{\sin(90 + 37^\circ)} = \frac{T_B}{\sin 90^\circ} = \frac{T_C}{\sin(180 - 37^\circ)}$$

But $T_C = 300 \text{ N}$

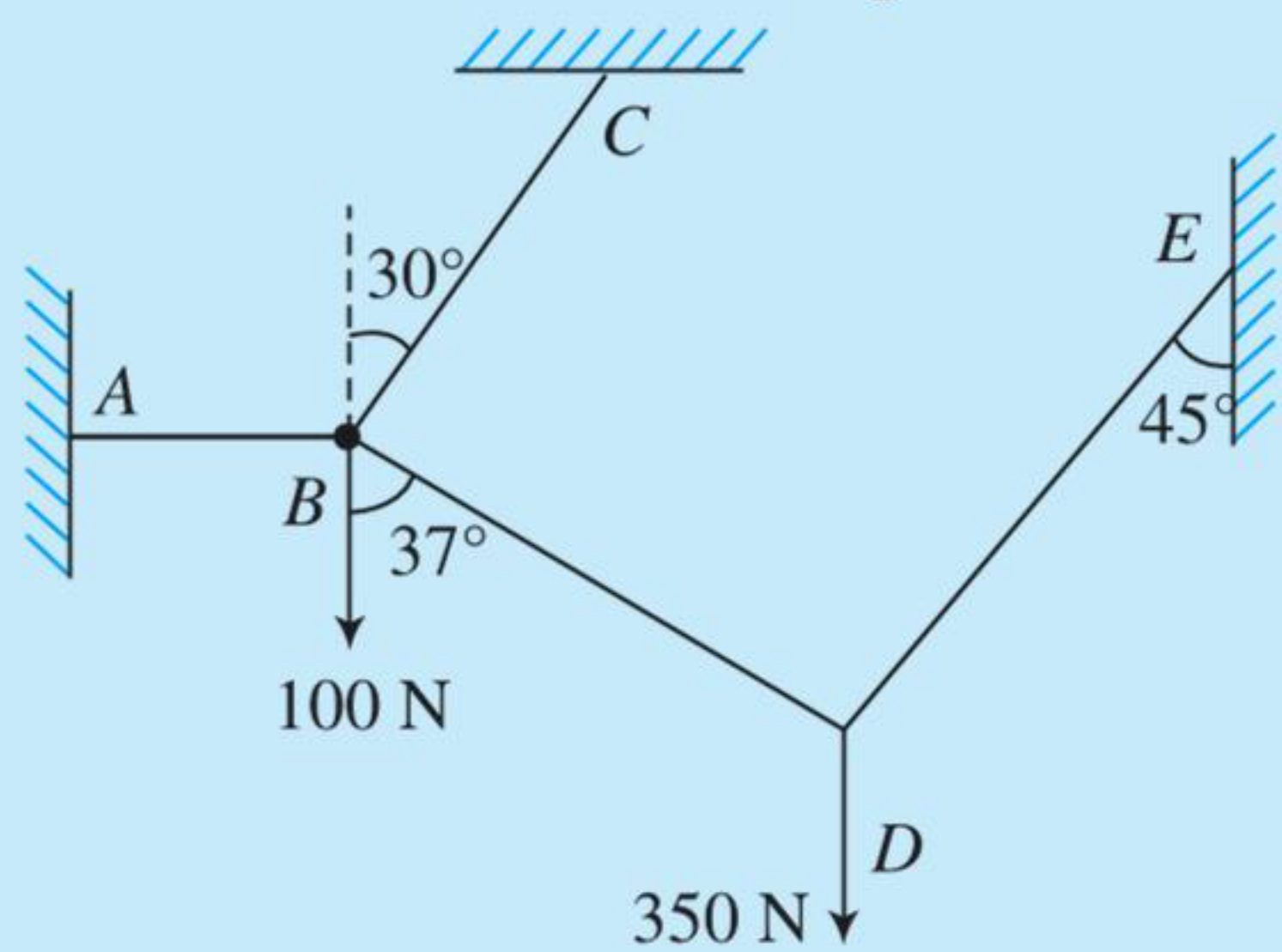
and $T_B = \frac{T_C}{\sin 37^\circ} = \frac{300}{3/5} = 500 \text{ N}$

$$T_A = T_C \left(\frac{\sin(90 + 37^\circ)}{\sin(180 - 37^\circ)} \right) = 300 \left(\frac{\cos 37^\circ}{\sin 37^\circ} \right) = 400 \text{ N}$$

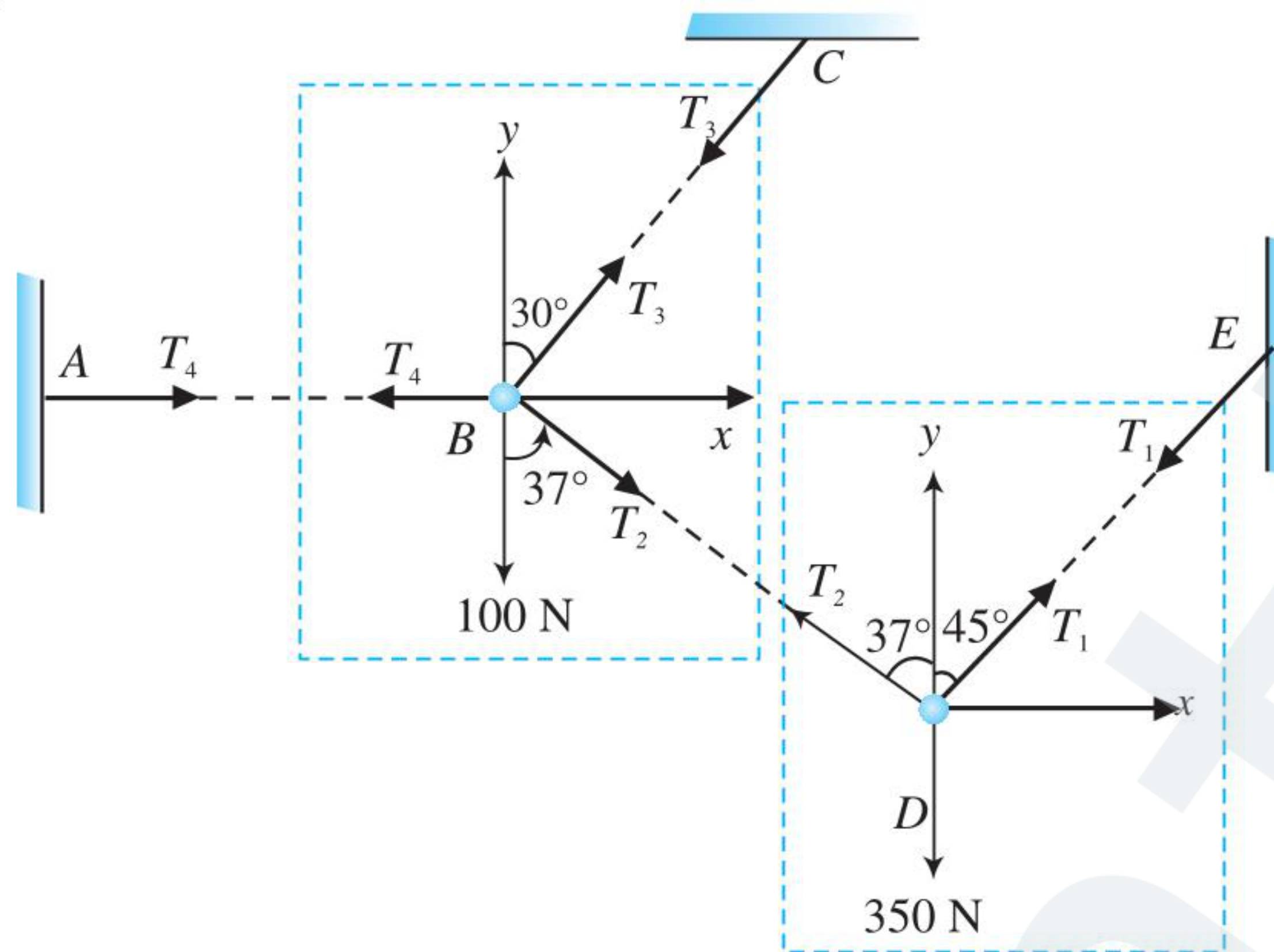
ILLUSTRATION 6.12

Two particles of masses 10 kg and 35 kg are connected with four strings at points B and D as shown in figure.

Determine the tensions in various segments of the string.



Sol. The free-body diagram of the whole system is shown in figure.



Analyzing the equilibrium of point D

$$\sum F_x = 0 \text{ or } T_1 \sin 45^\circ - T_2 \sin 37^\circ = 0 \quad \dots(i)$$

$$\text{and } \sum F_y = 0 \text{ or } T_1 \cos 45^\circ + T_2 \cos 37^\circ = 350 \quad \dots(ii)$$

From (i), we have $T_2 = \frac{T_1 \sin 45^\circ}{\sin 37^\circ}$

Now from (ii), $T_1 \cos 45^\circ + \frac{T_1 \sin 45^\circ}{\sin 37^\circ} \times \cos 37^\circ = 350$

$$\text{or } \frac{T_1}{\sqrt{2}} + \frac{T_1}{\sqrt{2}} \times \frac{4}{3} = 350$$

$$\text{or } \frac{T_1}{\sqrt{2}} \left[1 + \frac{4}{3} \right] = 350 \Rightarrow T_1 = 150\sqrt{2} \text{ N}$$

$$\text{and } T_2 = \frac{T_1 \sin 45^\circ}{\sin 37^\circ} = \frac{150\sqrt{2} \times \left(\frac{1}{\sqrt{2}} \right)}{3/5} = 250 \text{ N}$$

Analyzing the equilibrium of point B

$$\sum F_x = 0 \text{ or } T_2 \sin 37^\circ + T_3 \sin 30^\circ - T_4 = 0 \quad \dots(i)$$

$$\text{and } \sum F_y = 0 \text{ or } T_3 \cos 30^\circ - T_2 \cos 37^\circ - 100 = 0 \quad \dots(ii)$$

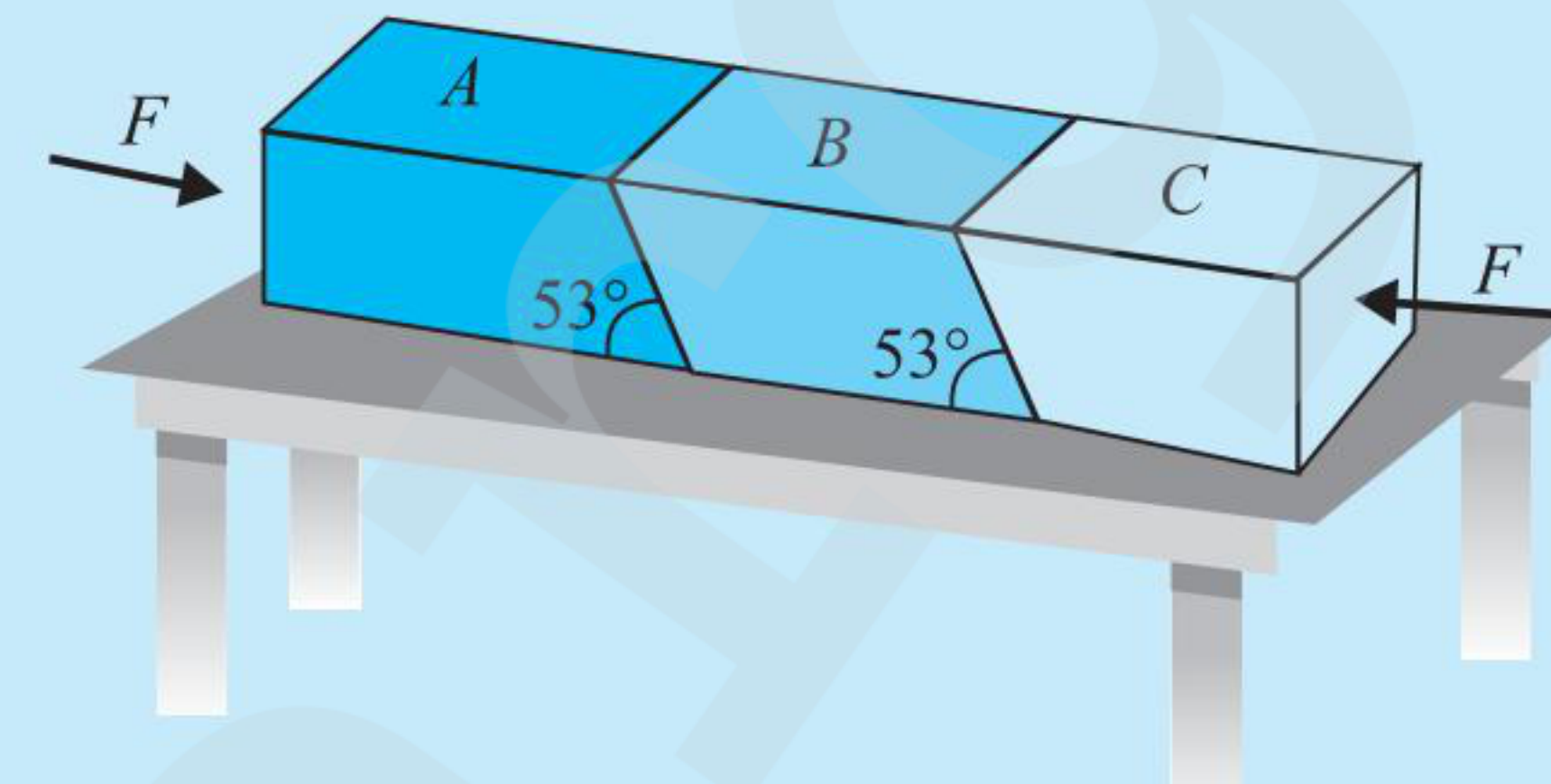
From (ii), $T_3 \cos 30^\circ - 250 \times \frac{4}{5} - 100 = 0$

$$\Rightarrow T_3 = 200\sqrt{3} \text{ N}$$

From (i), $T_4 = 150 + 100\sqrt{3} = 50(3 + 2\sqrt{3}) \text{ N}$

ILLUSTRATION 6.13

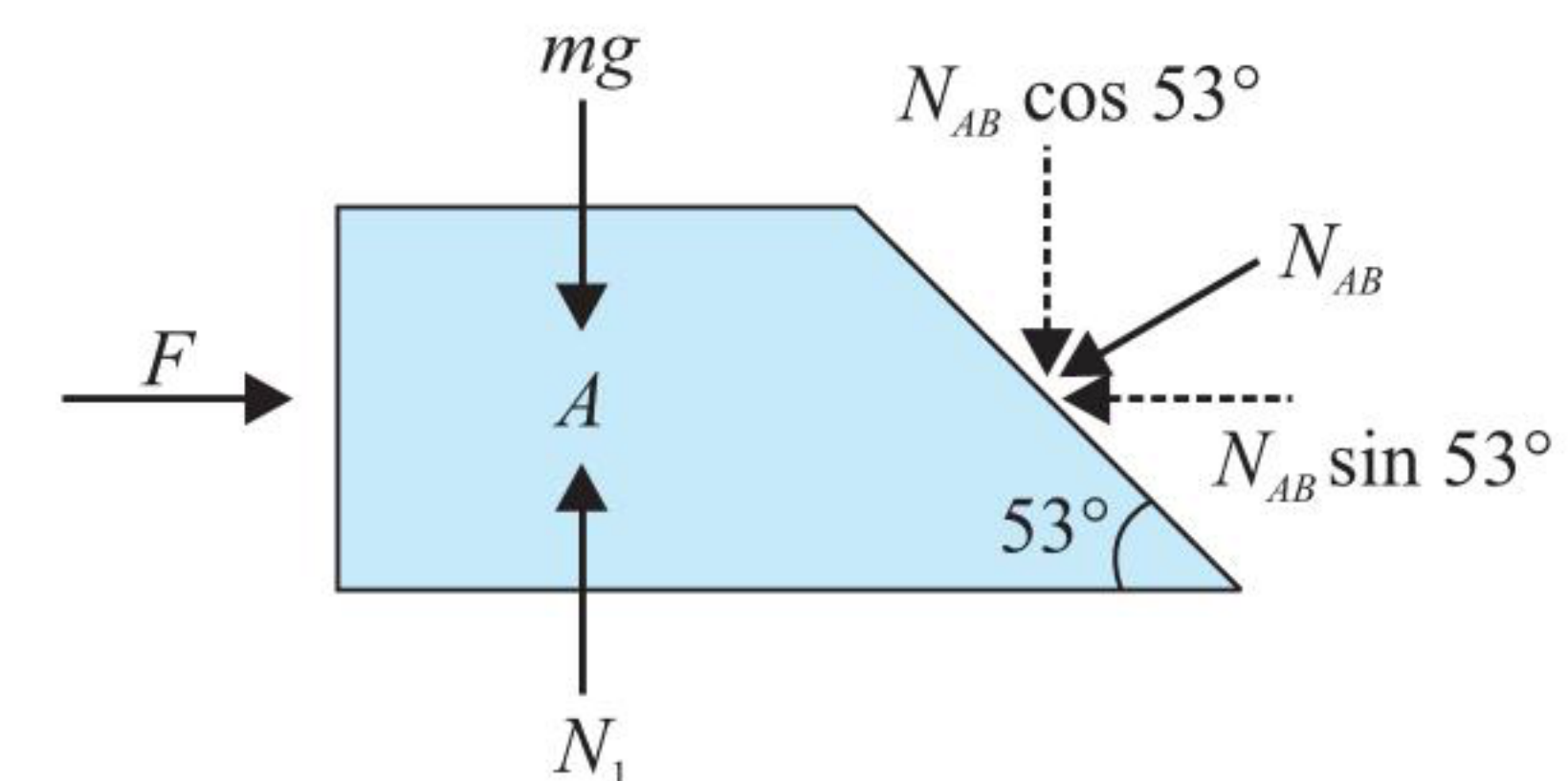
Three blocks A , B and C each of mass $m = 10 \text{ kg}$ are placed on a smooth horizontal table. There is no friction between the contact surfaces of the blocks as well. Horizontal force $F = 40 \text{ N}$ is applied on each of A and B as shown. Find the



- magnitude of normal reaction between blocks $A - B$ and $B - C$.
- normal reaction applied by ground on the blocks A , B and C .

Sol. As equal and opposite forces are acting on the system of the blocks, it means no net external force is acting on the system. Hence, the system will be in equilibrium. For finding normal reactions between the blocks, we need to draw F.B.D. of the blocks.

Block A :



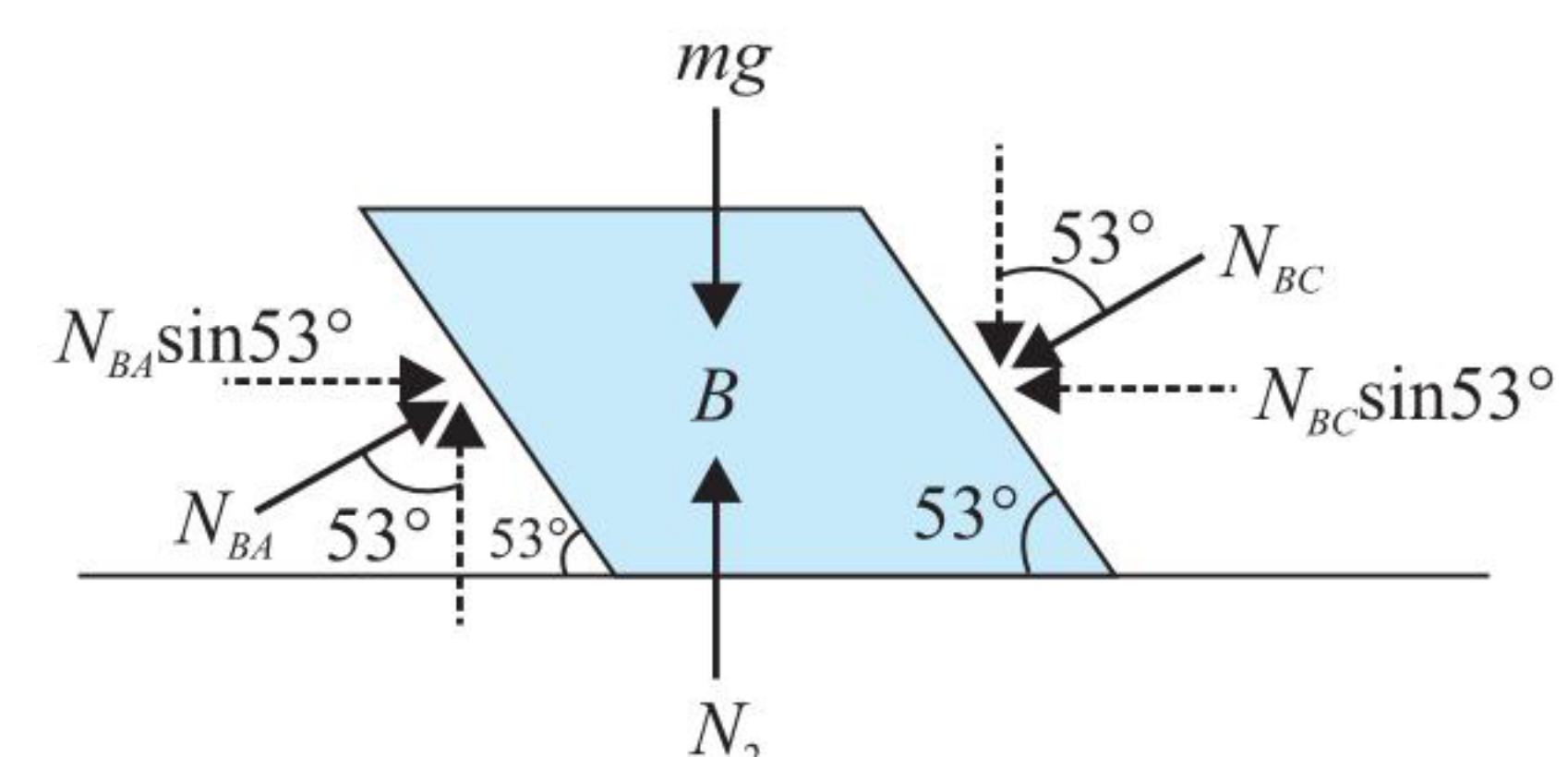
For equilibrium of block A ,

In horizontal direction: $N_{AB} \sin 53^\circ = F \Rightarrow N_{AB} = \frac{5}{4} F = 50 \text{ N}$

In vertical direction:

$$N_1 = N_{AB} \cos 53^\circ + mg = 50 \times \frac{3}{5} + 10 \times 10 = 130 \text{ N}$$

Block B :



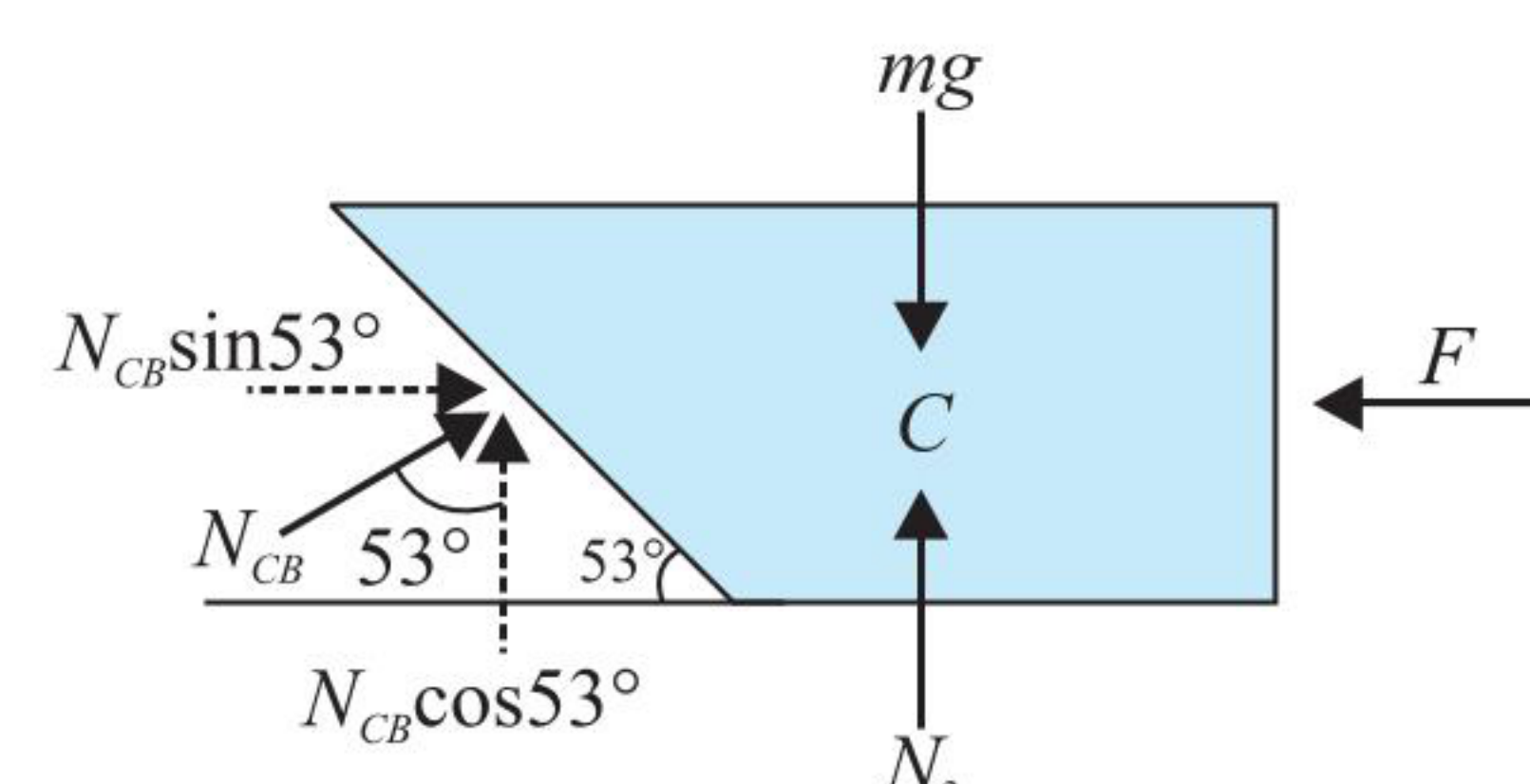
For equilibrium of block B ,

In horizontal direction:

$$N_{BC} \sin 53^\circ = N_{BA} \sin 53^\circ \Rightarrow N_{BC} = N_{BA} = N_{AB} = 50 \text{ N}$$

In vertical direction: $N_2 = mg = 10 \times 10 = 100 \text{ N}$

Block C :



For equilibrium of block C ,

In horizontal direction: $N_{CB} \sin 53^\circ = F$

$$\text{or } N_{CB} = \frac{5}{4}F = \frac{5}{4} \times 40 = 50 \text{ N}$$

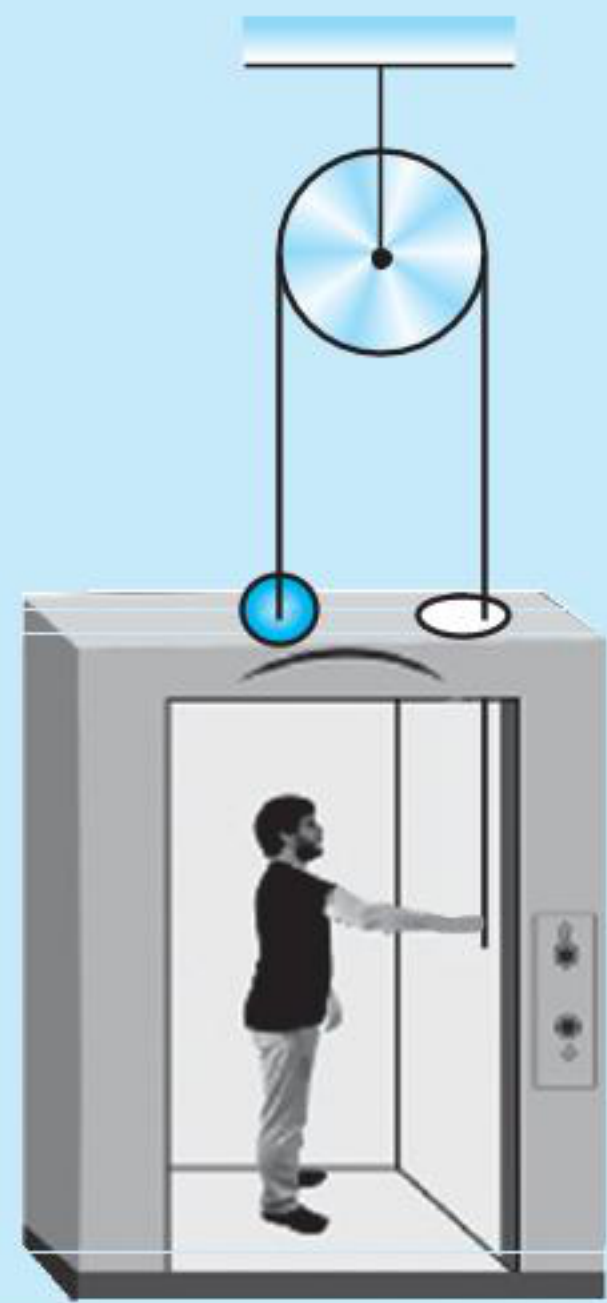
In vertical direction: $N_3 + N_{CB} \cos 53^\circ = mg$

$$N_3 + 50 \times \frac{3}{5} = 10 \times 10 \Rightarrow N_3 = 70 \text{ N}$$

ILLUSTRATION 6.14

A man of mass m in hanging cabin of mass M is able to keep himself and cabin at equilibrium.

- Draw the forces acting on man and cabin.
- Calculate force applied by the man on rope and normal reaction of the floor acting on man.

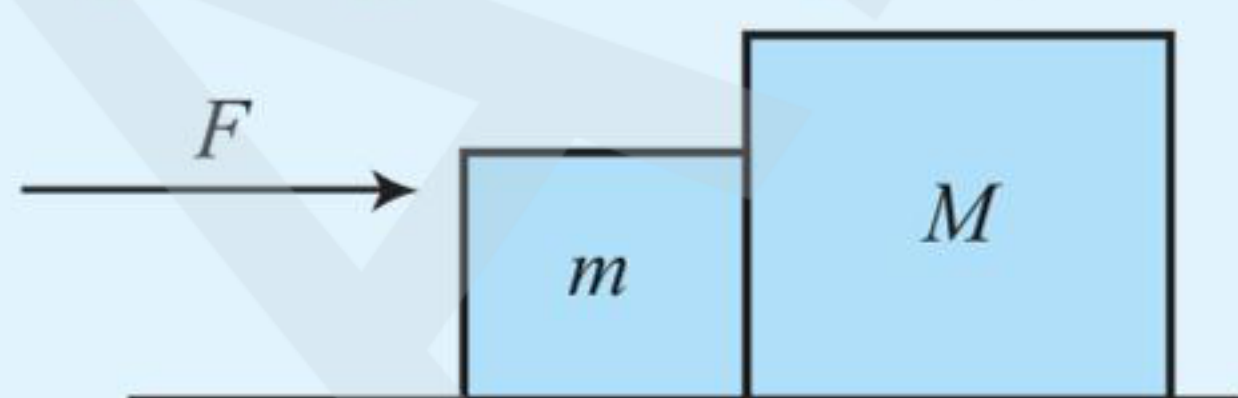


Sol. The force applied by man at the end of a light string is equal to tension in string. As the cabin is at equilibrium, it means net force on the system should be zero i.e., $\sum F = 0$.

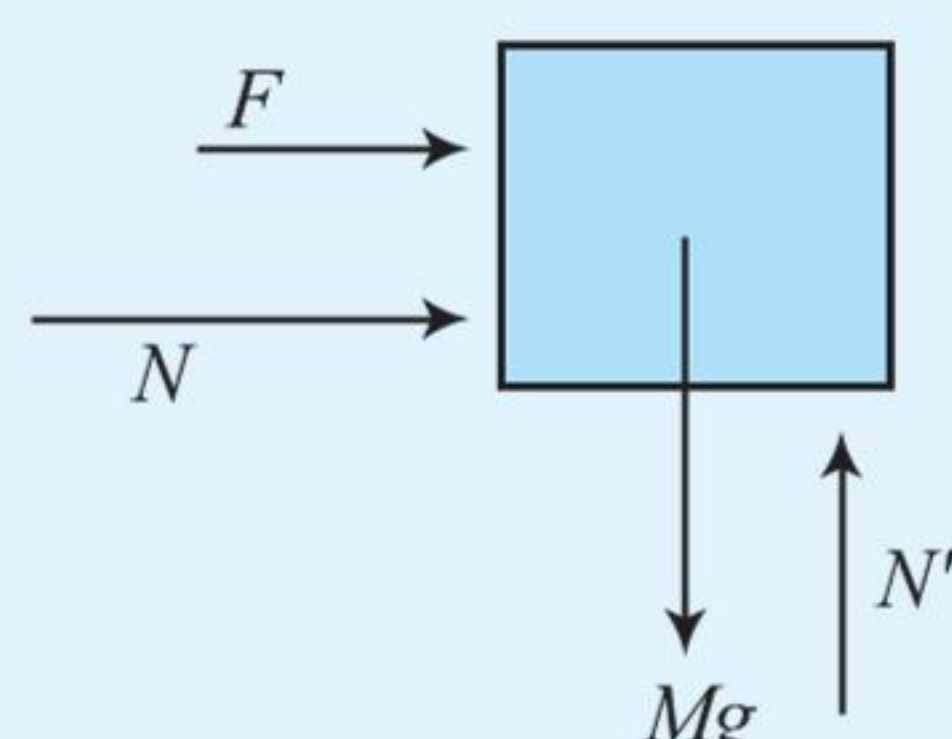
Free body diagrams	Equation of motion
	From F.B.D. of man and cabin $2T = (M + m)g$ $\Rightarrow T = \left(\frac{m + M}{2}\right)g \dots (i)$ From F.B.D. of man: $T + N = mg \dots (ii)$ Solving equations (i) and (ii), we have $\left(\frac{m + M}{2}\right)g + N = mg$ or $N = \left(\frac{m - M}{2}\right)g$

CONCEPT APPLICATION EXERCISE 6.2

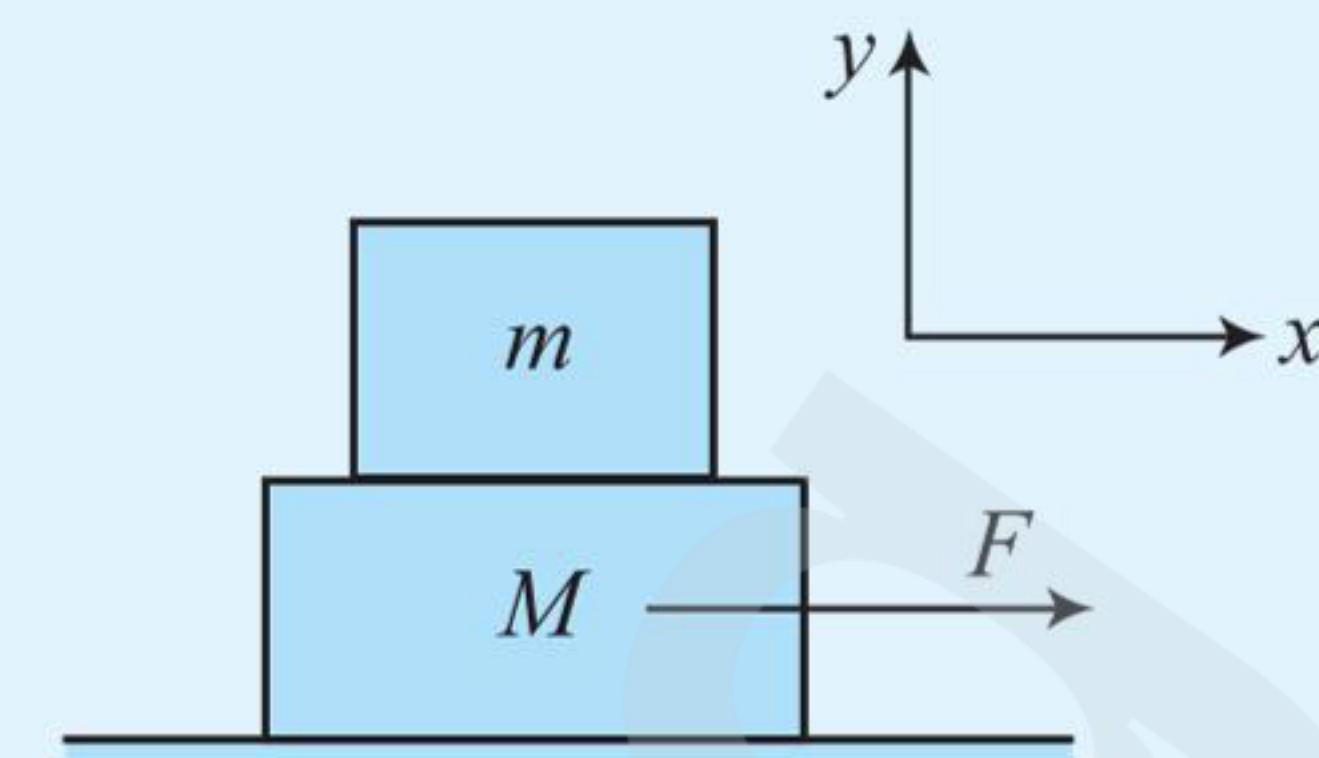
- As per the diagram given in figure,



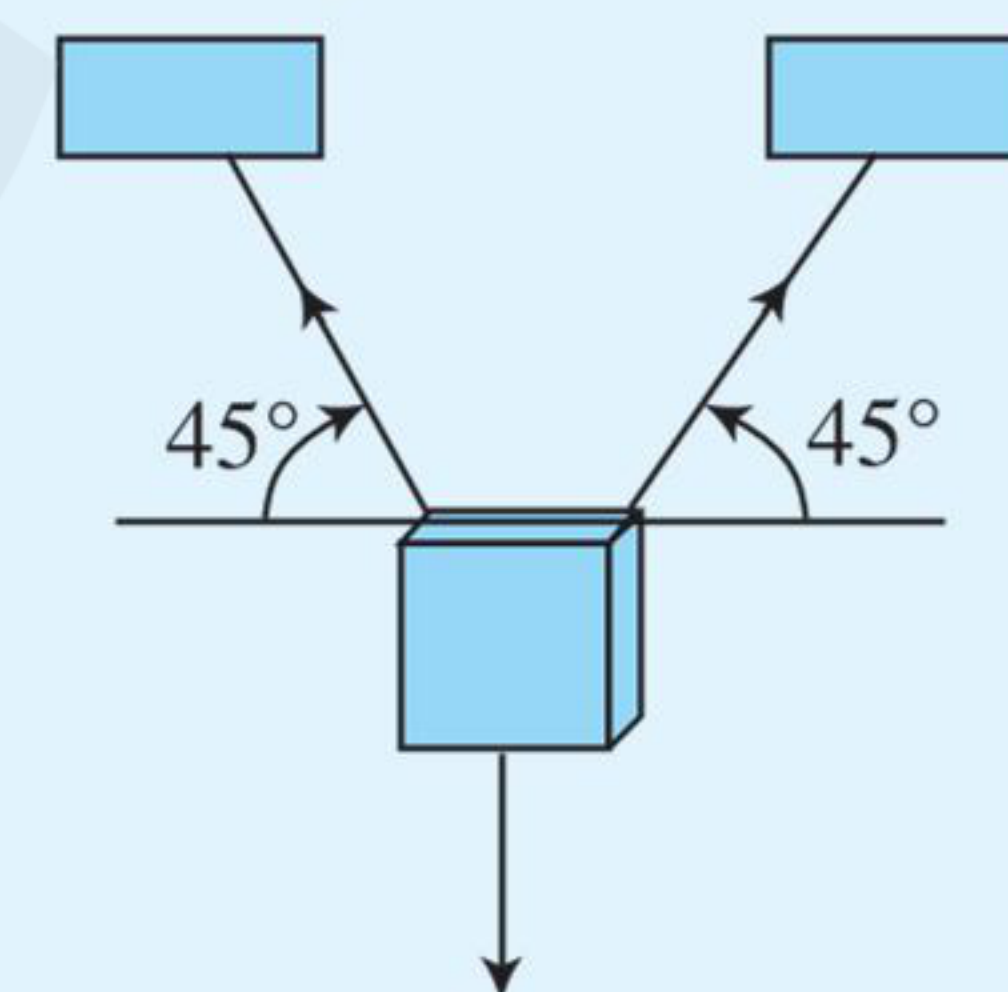
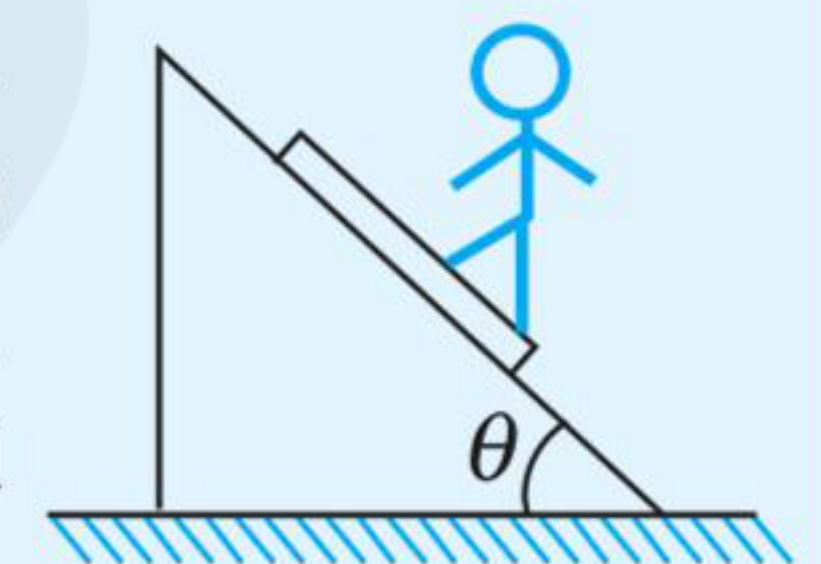
The free-body diagram of M is the figure shown below. Is it correct or incorrect? Assume all surfaces are frictionless.



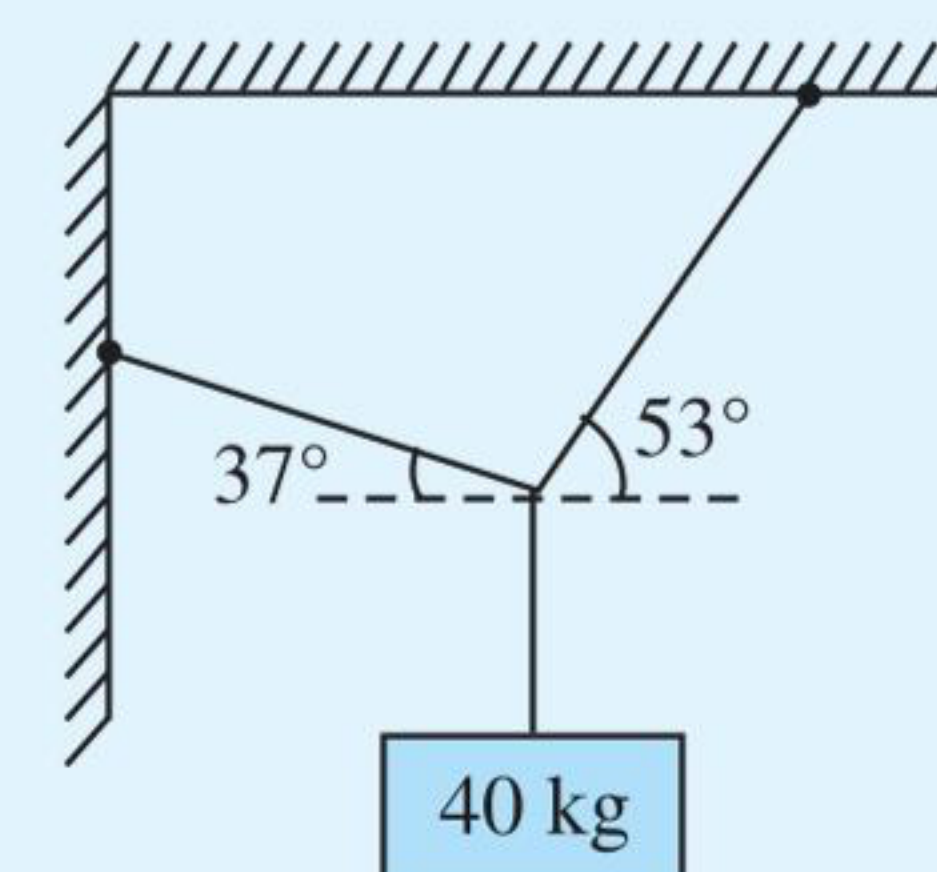
- A mass m is placed on a body of mass M . There is no friction anywhere. Force F is applied on M and it moves with acceleration a . Find the force (along x -axis) on the top body.



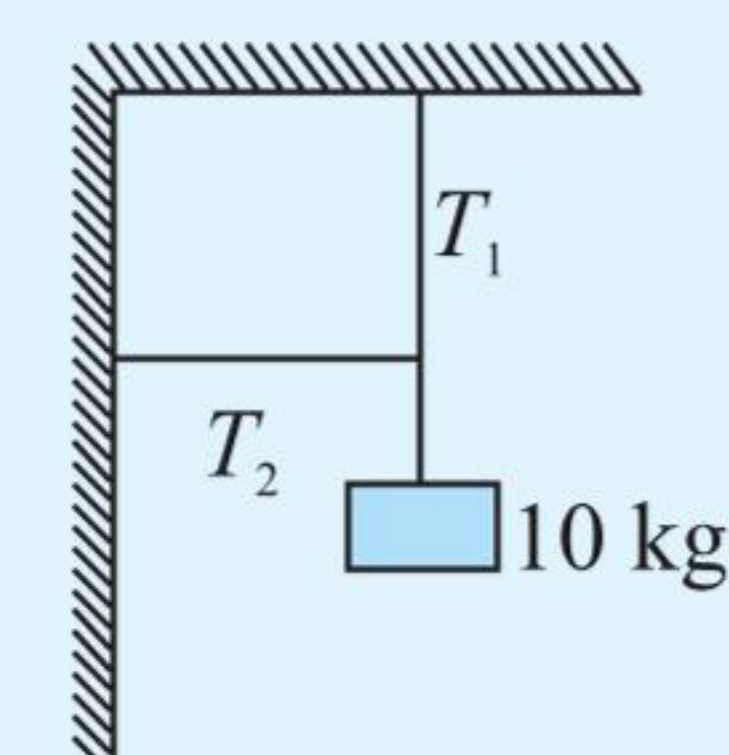
- In figure, the mass of the man is M . Calculate the mass of the man as registered by weighing machine. Assume weighing machine, man, and wedge all are stationary.
- A small object is suspended at rest from two strings as shown in figure. The magnitude of the force exerted by each string on the object is $10\sqrt{2}$ N. find the magnitude of the mass of the object.



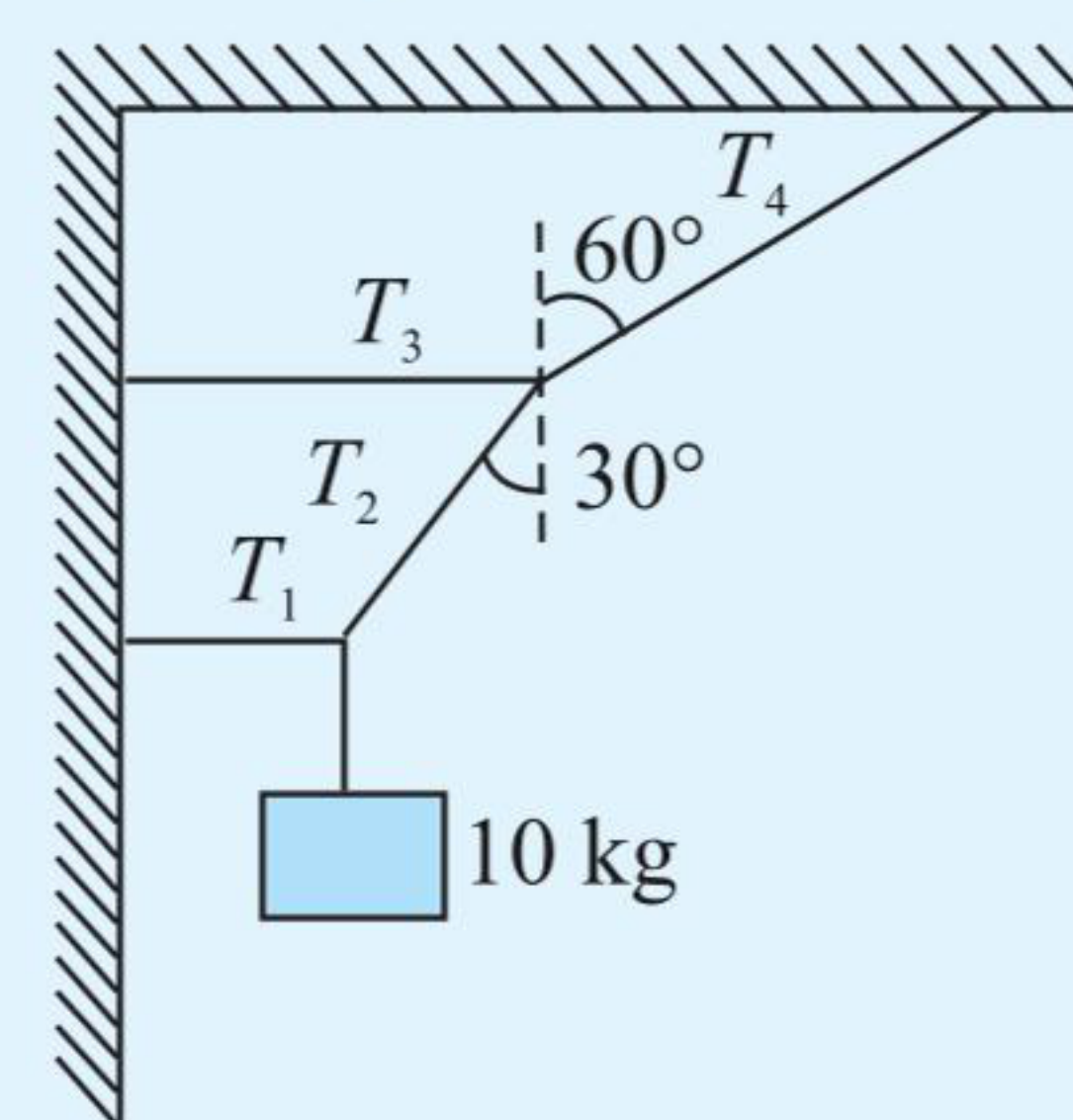
- The object in figure weighs 40 kg and hangs at rest. Find the tensions in the three cords that hold it.



- A block of mass $m = 10$ kg is suspended with the help of three strings as shown in figure. Find the tensions T_1 and T_2 .

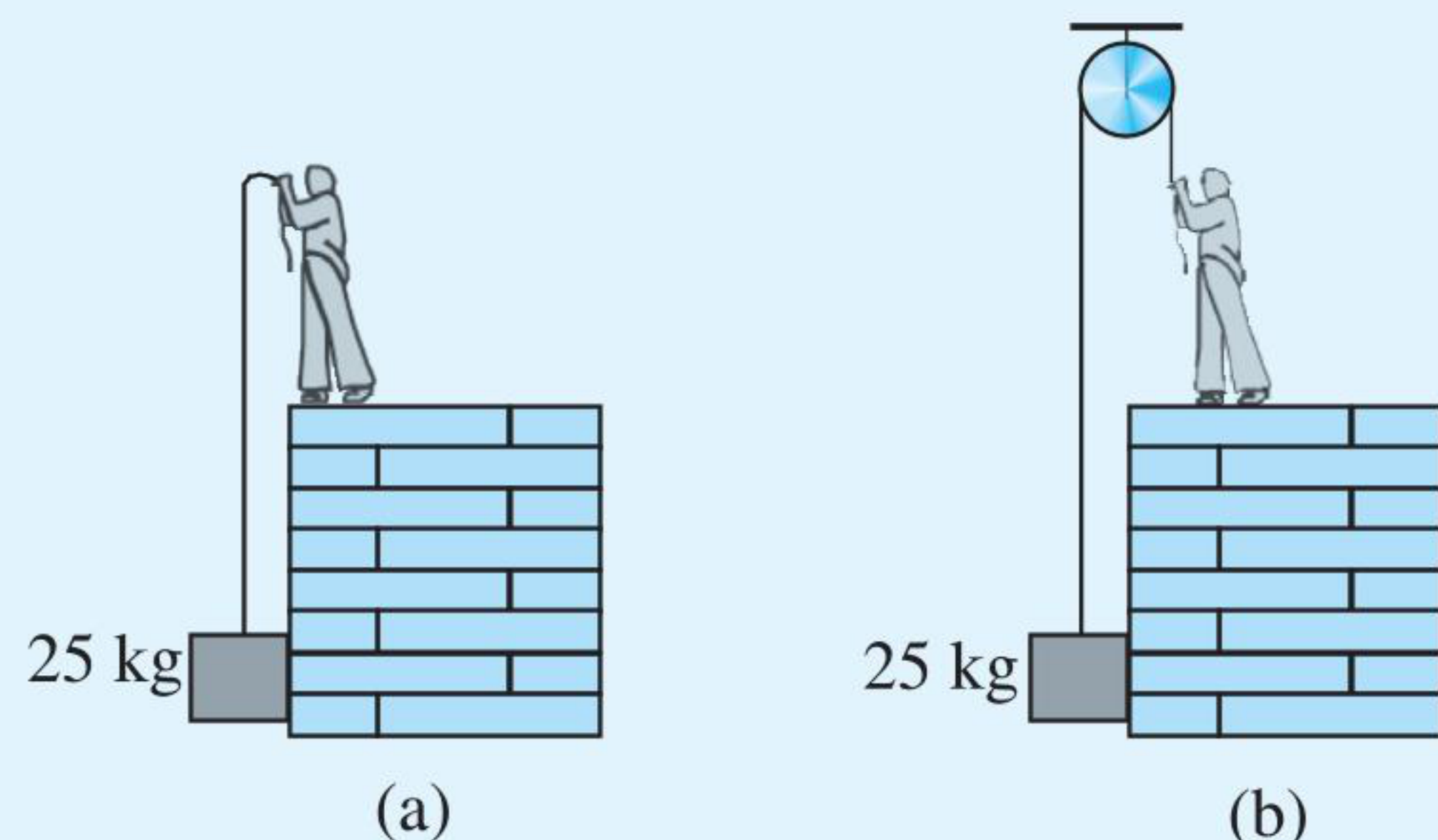


- Determine tension T_4 in figure

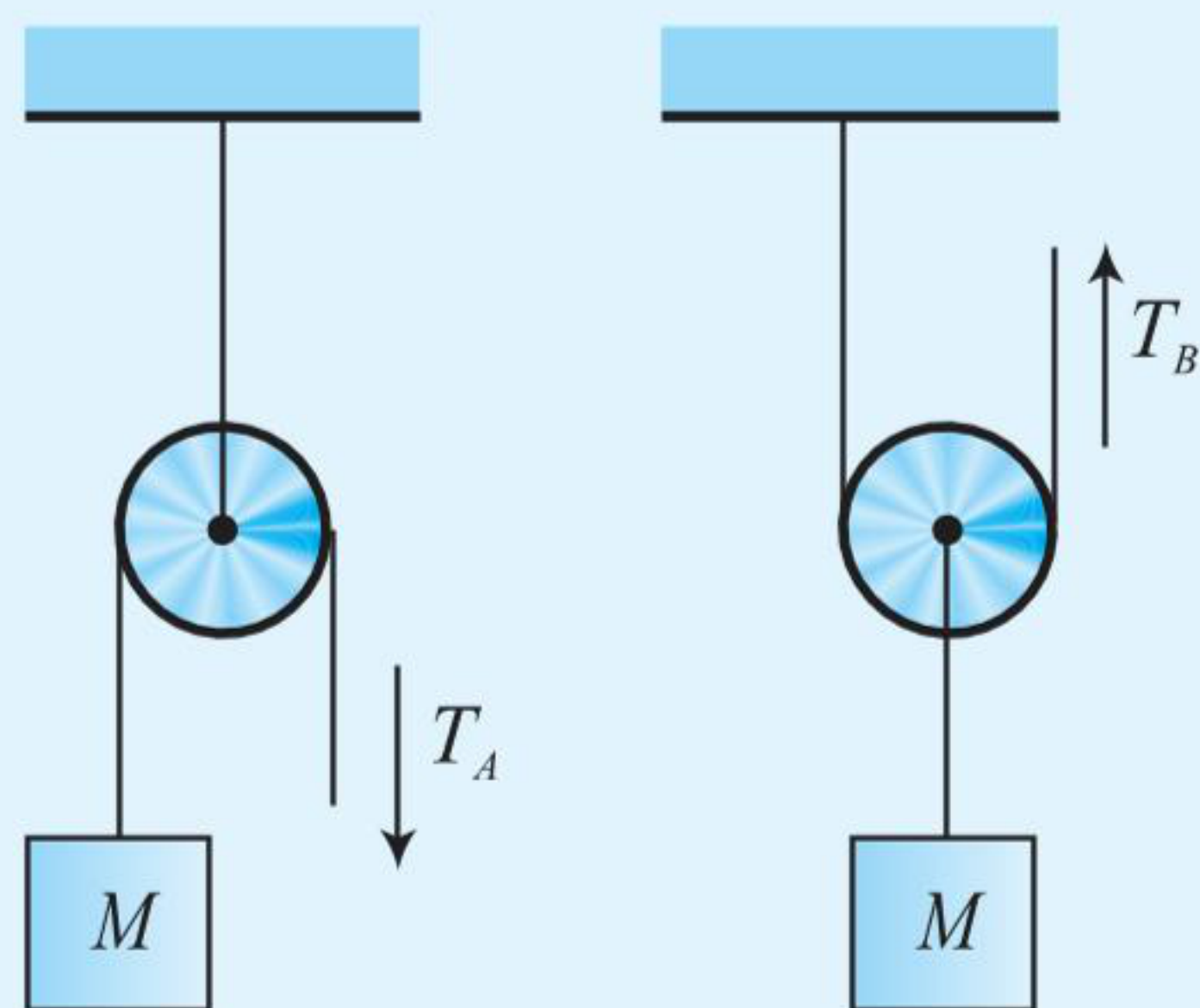


- A block of mass 25 kg is raised by a 50-kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases? If the floor yields

to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding?



9. Consider the two configurations shown in equilibrium. Find the ratio of T_A/T_B . (Ignore the mass of the rope and the pulley.)



ANSWERS

1. Incorrect
2. No force is acting in horizontal direction on m
3. $M \cos \theta$
4. 2 kg
5. $T_1 = 240$ N and $T_2 = 320$ N
6. $T_1 = 100$ N, $T_2 = 0$
7. 200 N
8. Case (b)
9. 2

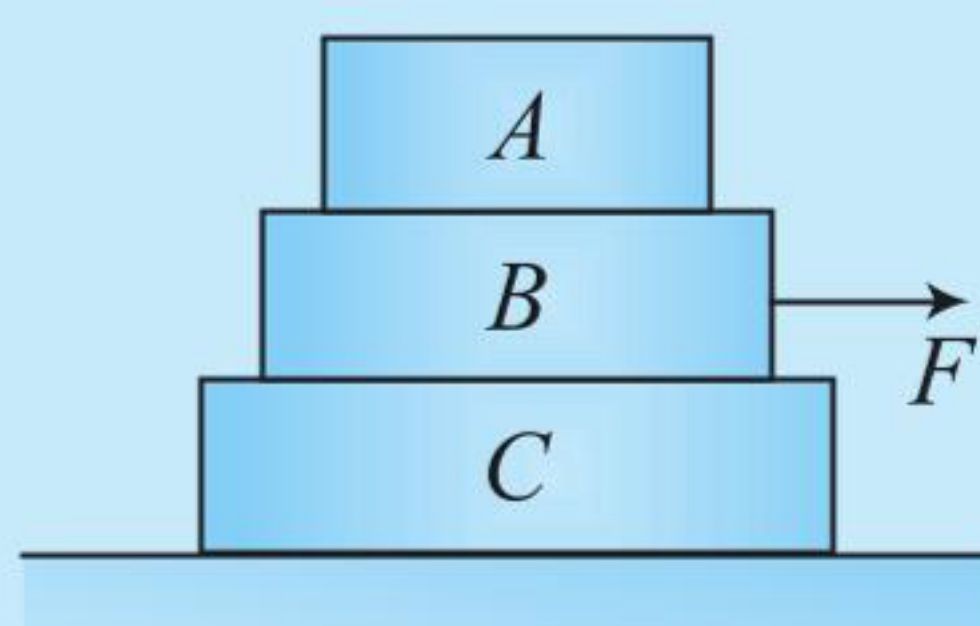
DYNAMICS OF PARTICLES: TRANSLATIONAL MOTION OF ACCELERATED BODIES

We can better understand the application of Newton's laws of motion through its application in different situations.

ILLUSTRATION 6.15

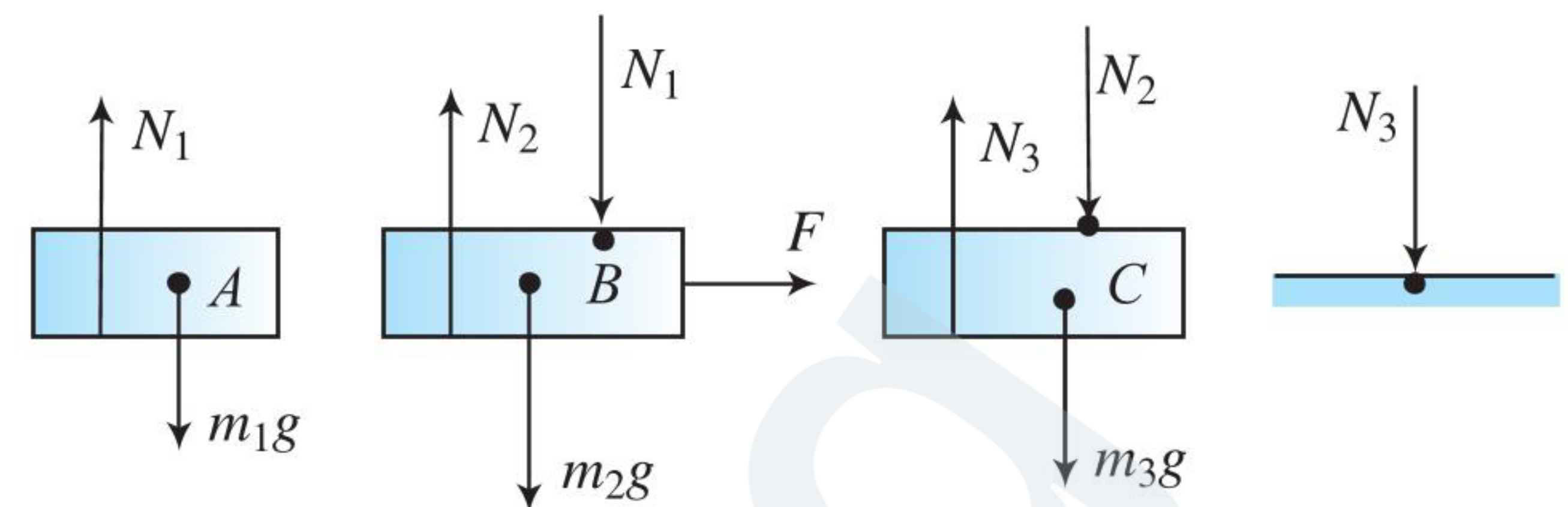
Three blocks A, B, and C of masses m_1 , m_2 , and m_3 , respectively, are resting one on top of the other as shown in figure.

A horizontal force F is applied on block B. Assuming all the surfaces are frictionless, calculate (1) acceleration of block A, block B, and block C; (2) normal reactions between A and B, B and C, and between C and ground.



Sol. This system cannot be in equilibrium because in the horizontal direction, the system has a net external force F . As far

as the vertical direction is concerned, all the forces are internal action and reaction forces. These forces are equal and opposite; hence, they will cancel each other out.



Body A: No external force is acting on it; hence, it will remain stationary in equilibrium. So acceleration at block A, $a_A = 0$ and

$$N_1 = m_1 g \quad \dots(i)$$

Body B: There is no external force acting on it vertically; hence, it will not have any acceleration in the vertical direction.

$$N_2 = N_1 + m_2 g = (m_1 + m_2)g \quad \dots(ii)$$

However, there is one external force F acting on it. So it will have some acceleration a in the horizontal direction such that

$$F = m_2 a_B \quad \dots(iii)$$

which gives acceleration of block B, $a_B = \frac{F}{m_2}$

Body C and earth: Same comments as in the case of body A above.

$$N_3 = N_2 + m_3 g \quad \dots(iv)$$

$$\text{and } N_3 = (m_1 + m_2 + m_3)g \quad \dots(v)$$

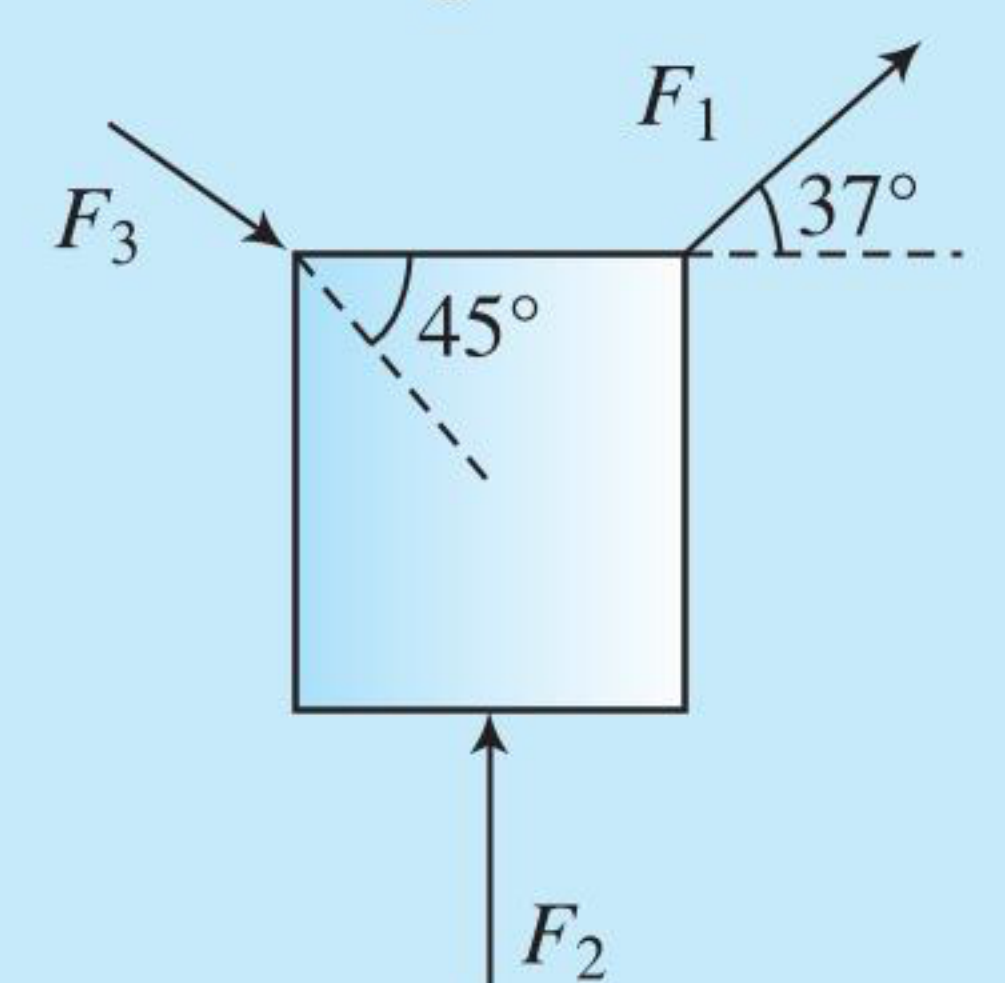
No external force acts on block C in the horizontal direction; hence, a_C is equal to 0.

ILLUSTRATION 6.16

Three boys, each of mass 45 kg, pull simultaneously a block on a smooth surface. The mass of block is 20.0 kg.

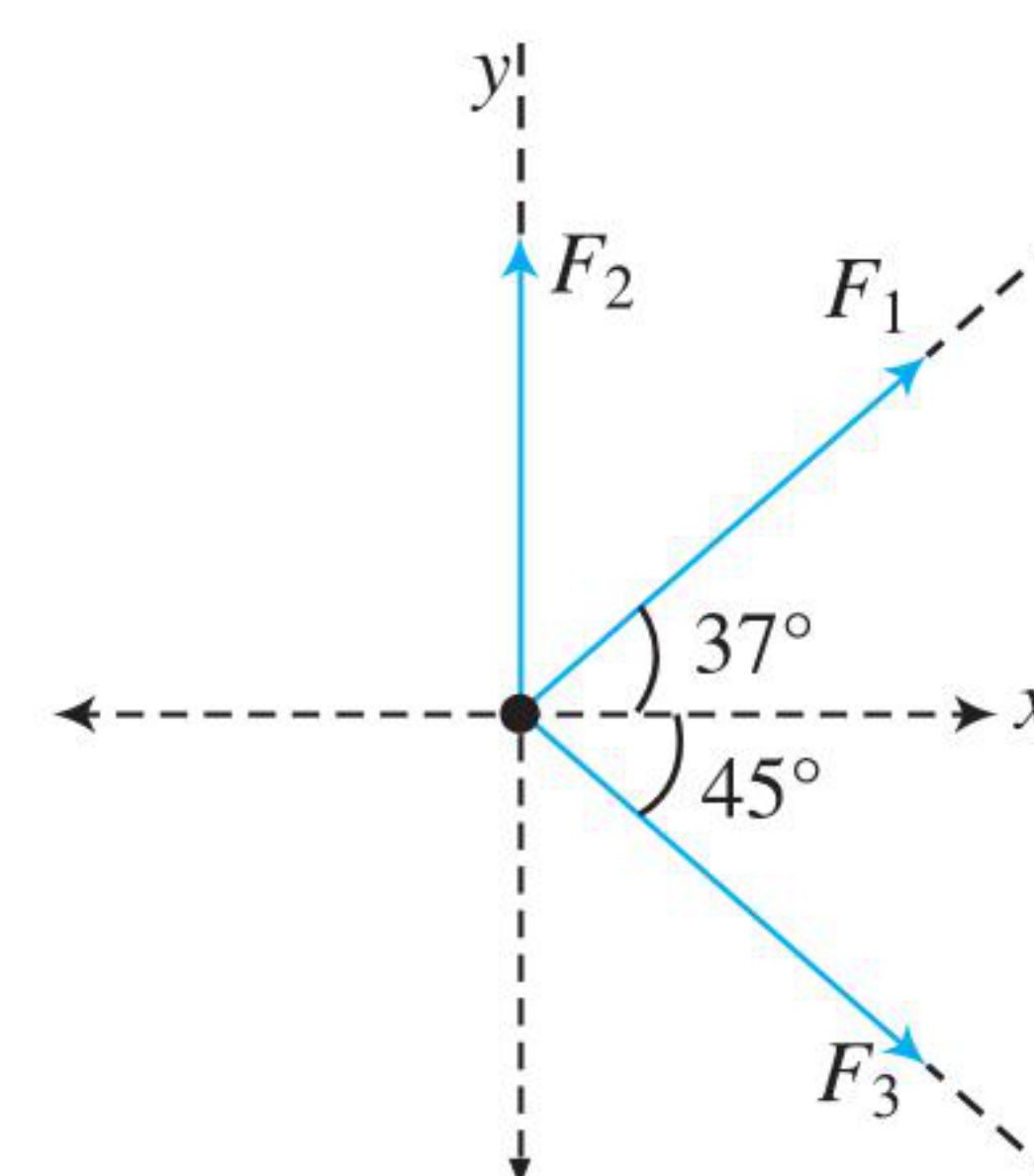
(a) Find the acceleration of the block.

(b) Find the acceleration of the boy exerting force F_1 . Assume no friction between the boy and the surface.



[Given $F_1 = 90$ N, $F_2 = 114$ N, and $F_3 = 128\sqrt{2}$ N]

Sol. First we will resolve all the forces acting on the block into x and y components.



$$\Sigma F_x = F_1 \cos 37^\circ + F_3 \cos 45^\circ$$

$$\Sigma F_y = F_1 \sin 37^\circ + F_2 - F_3 \sin 45^\circ$$

$$\text{Now, } a_x = \frac{\Sigma F_x}{m} \quad \text{and} \quad a_y = \frac{\Sigma F_y}{m}$$

$$a_x = \frac{(90.0) \cos 37^\circ + 128\sqrt{2} \cos 45.0^\circ}{20.0} = 10 \text{ m s}^{-2}$$

$$a_y = \frac{(90.0)(\sin 37^\circ) + 114 - (128\sqrt{2} \sin 45.0^\circ)}{20.0} = 2 \text{ m s}^{-2}$$

According to Newton's third law, the force exerted on the block by the boy must be equal to the force exerted on the boy by the block. Therefore,

$$a_1 = \frac{F'_1}{m} = \frac{90.0}{45} = 2 \text{ m s}^{-2}$$

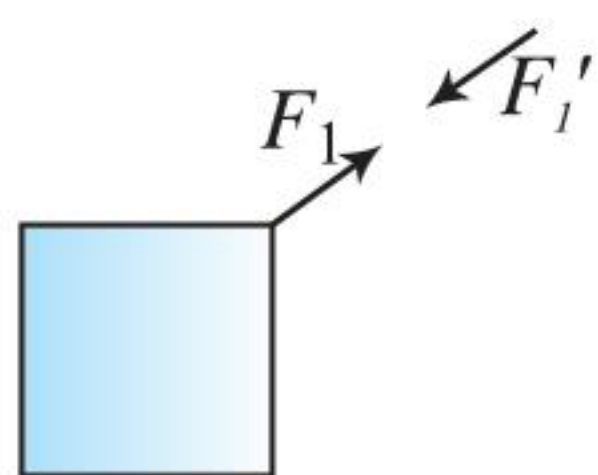
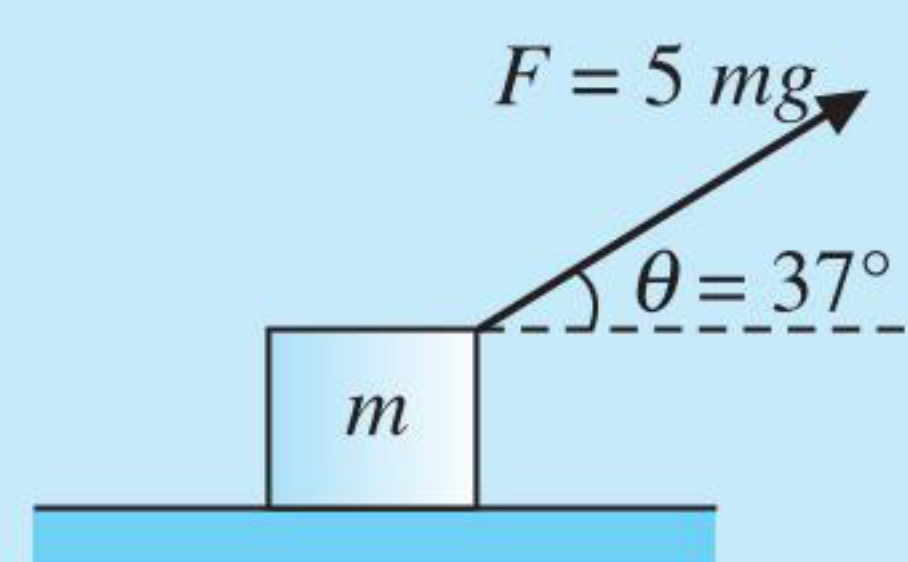
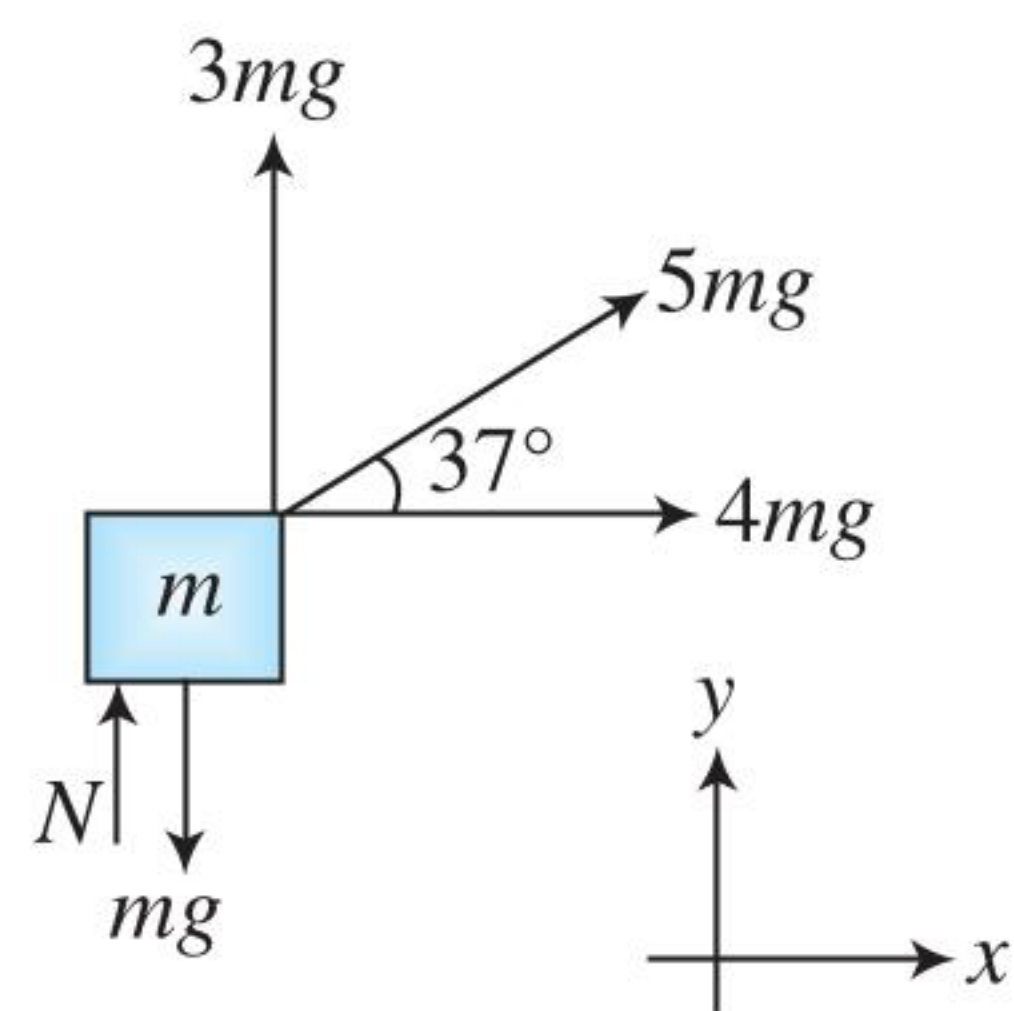


ILLUSTRATION 6.17

A block of mass m is placed on a horizontal surface. If the block is pulled by applying a force of magnitude $F = 5mg$ at an angle $\theta = 37^\circ$ with horizontal as shown in figure, find the acceleration of the block at the given instant.



Sol. Three forces act on the block (a) $mg \downarrow$ (b) $F = 5mg \nearrow$ (c) $N \uparrow$



Resolving the forces along x - and y -axis, we have equation of motion

$$\sum F_x = ma_x$$

$$4mg = ma_x$$

This gives $a_x = 4g$... (i)

$$\sum F_y = ma_y$$

$$3mg - mg + N = ma_y$$

$$2mg + N = ma_y \quad \dots \text{(iii)}$$

Let $a_y = 0$. Then $N = -2mg$

The negative result signifies that N will be directed down (opposite to the assumed direction), but the ground cannot pull the block down. Hence, the block will lose contact with the ground.

or $N = 0$

Hence, from (ii), $a_y = 2g \uparrow$

Hence, the net acceleration, $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

where $a_x = 4g$ from Eq. (i) and $a_y = 2g$ from Eq. (ii).

This gives $a = \sqrt{a_x^2 + a_y^2} = \sqrt{(4g)^2 + (2g)^2} = 2\sqrt{5}g$

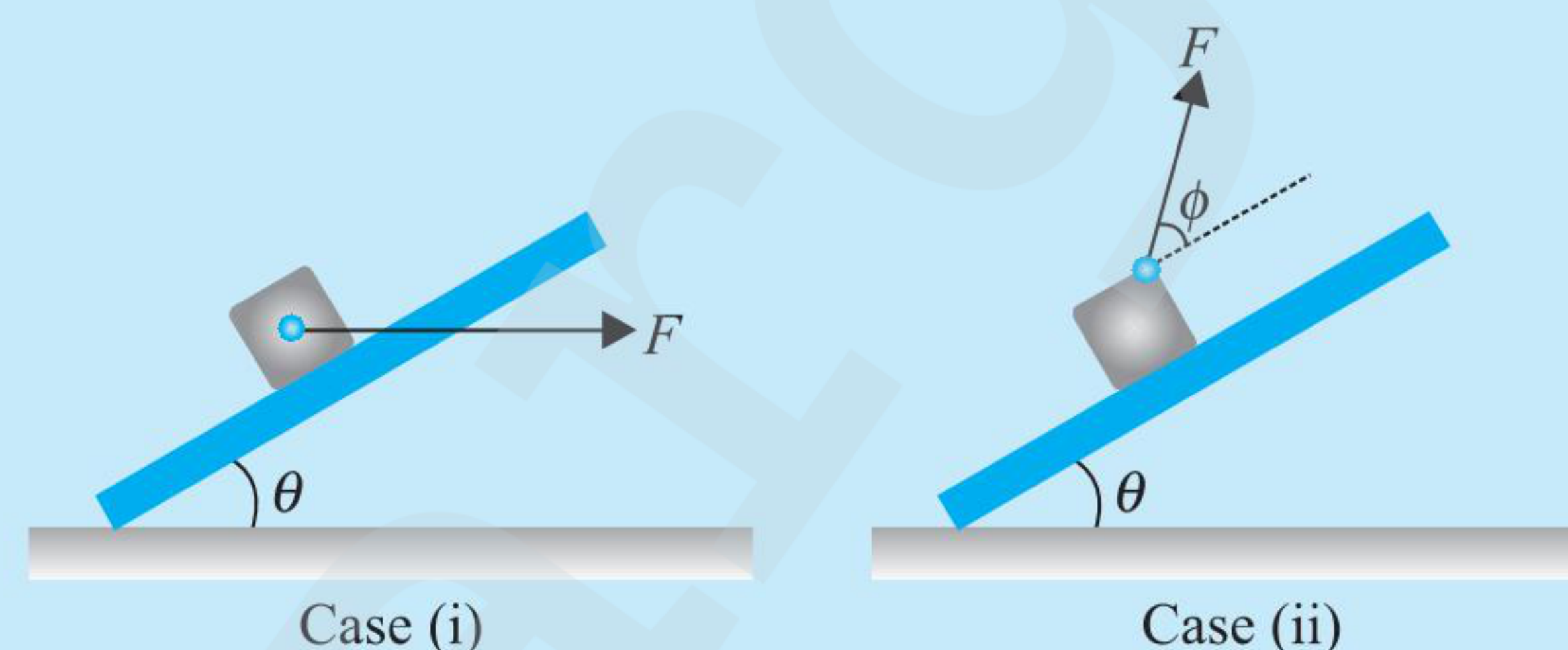
ILLUSTRATION 6.18

A smooth box of mass m is kept on an inclined plane of angle of inclination θ . A force F is applied on the box

- in horizontal direction as in figure (i).
- at an angle ϕ with the inclined plane as in figure (ii).

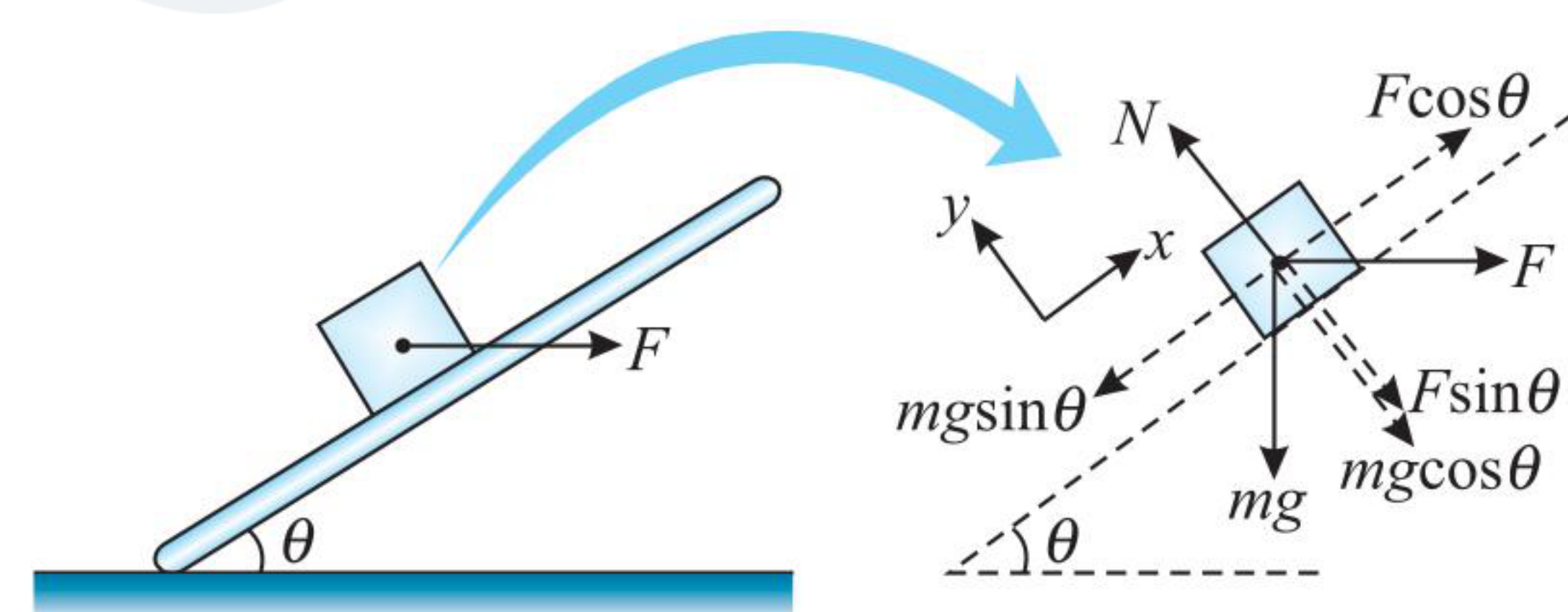
If the box does not lose contact, find

- acceleration of the block.
- normal reaction offered by the box on the inclined plane.



Sol. The motion of the block will be parallel to inclined surface, it means we should make the components of forces acting on the block parallel and perpendicular to inclined plane.

Case (i): Free body diagram



Now applying Newton's second law of motion

Force equation, $\sum F_x = ma_x$

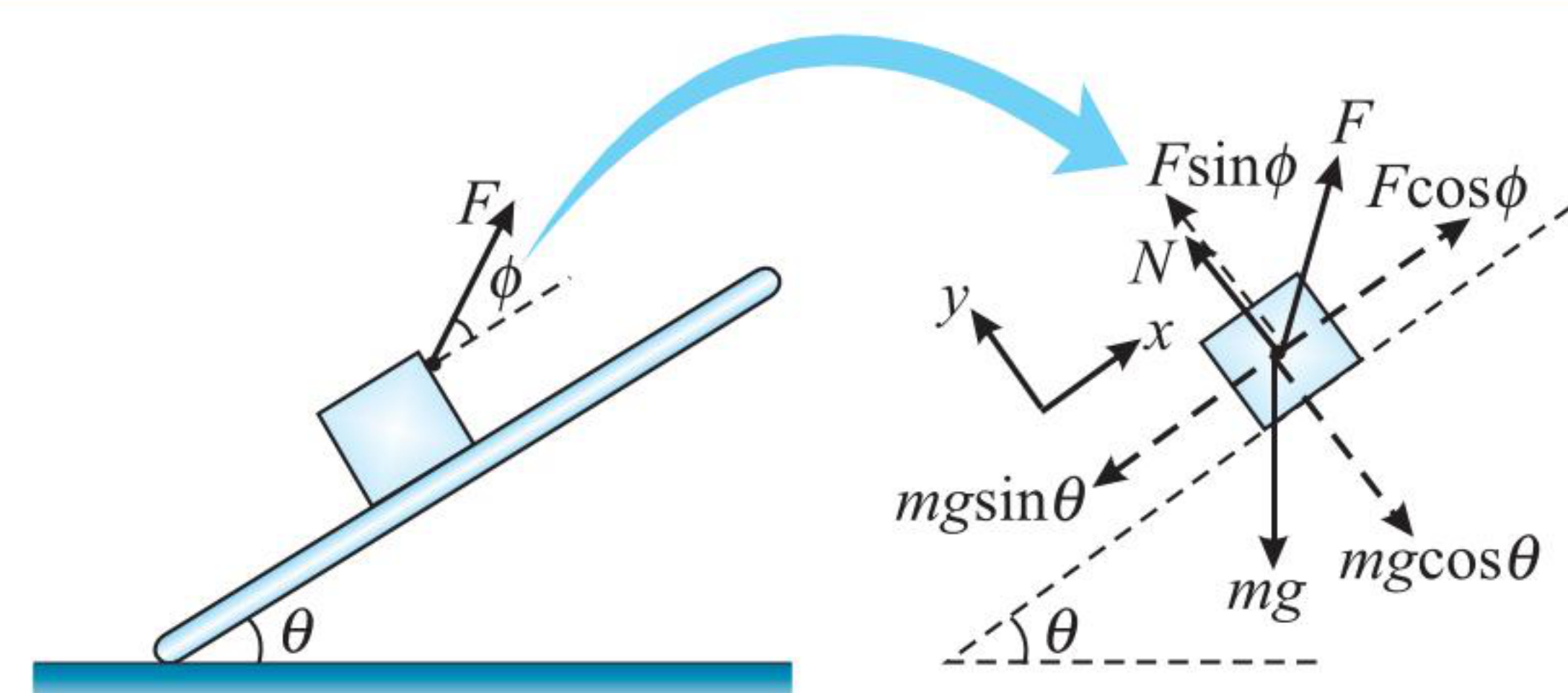
$$F \cos \theta - mg \sin \theta = ma_x$$

$$a_x = \frac{F \cos \theta - mg \sin \theta}{m} \quad \dots \text{(i)}$$

$\sum F_y = ma_y$. As $a_y = 0$, hence $\sum F_y = 0$

$$\Rightarrow N = (F \sin \theta + mg \cos \theta) \quad \dots \text{(ii)}$$

Case (ii): Free body diagram



Now applying Newton's second law of motion

Force equation, $\sum F_x = ma_x$

$$F \cos \phi - mg \sin \theta = ma_x$$

$$a_x = \frac{F \cos \phi - mg \sin \theta}{m} \quad \dots \text{(i)}$$

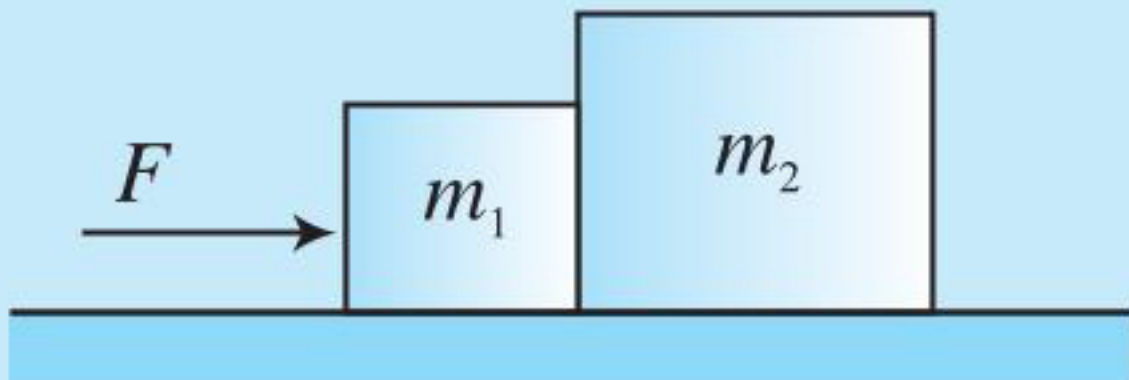
$\sum F_y = ma_y$. As $a_y = 0$, hence $\sum F_y = 0$

$$\Rightarrow N = mg \cos \theta - F \sin \phi \quad \dots \text{(ii)}$$

ANALYSIS OF NEWTON'S LAWS OF MOTION IN CONNECTED BODIES: PROBLEMS BASED ON NORMAL REACTION

ILLUSTRATION 6.19

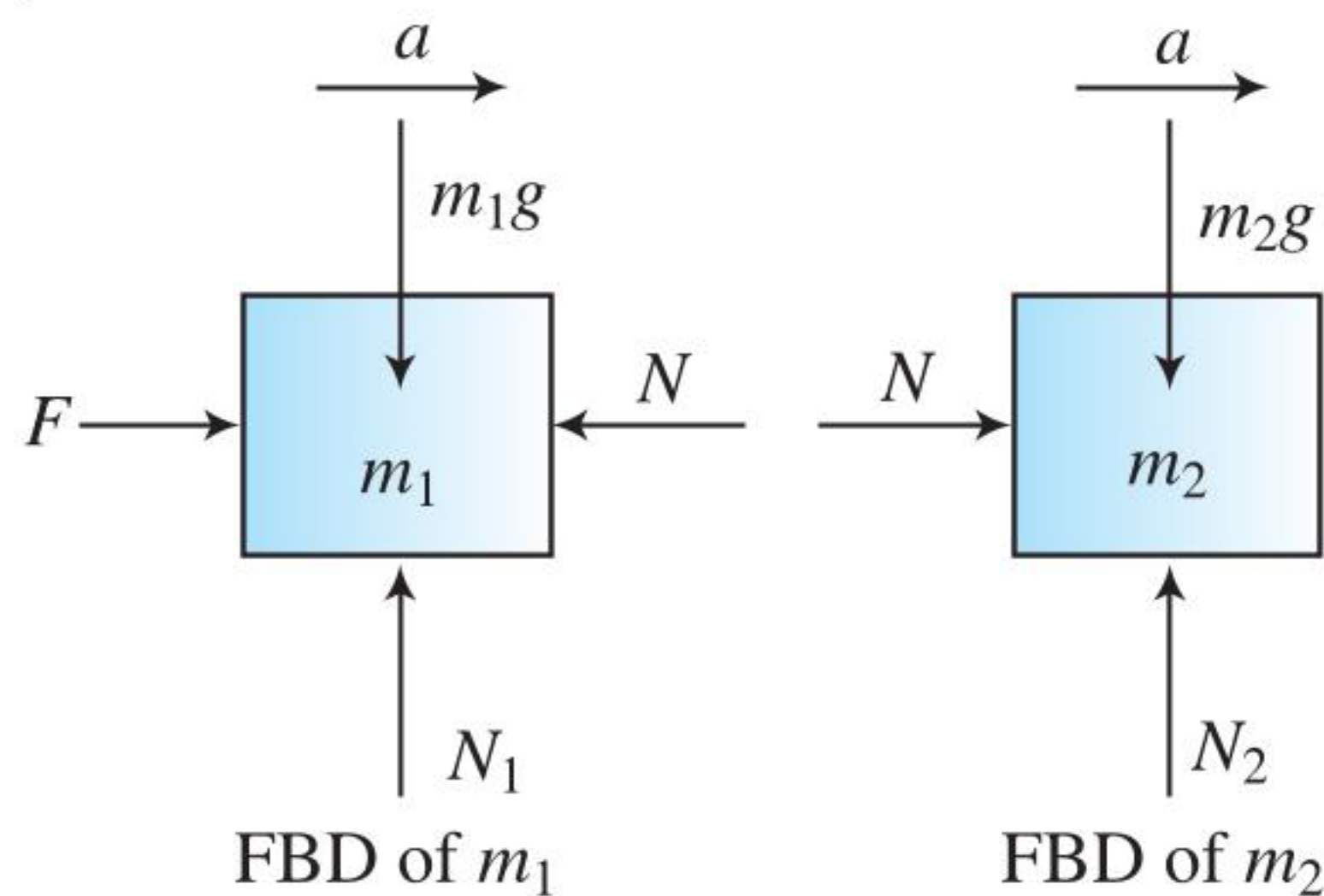
Two blocks of masses m_1 and m_2 are placed side by side on a smooth horizontal surface as shown in figure. A horizontal force F is applied on the block.



- (a) Find the acceleration of each block.
(b) Find the normal reaction between the two blocks.

Sol.

Method 1: Since the two blocks always remain in contact with each other, they must move with the same acceleration.



Using Newton's second law, we get

$$\text{For block } m_1: F - N = m_1 a \quad \dots(i)$$

$$\text{For block } m_2: N = m_2 a \quad \dots(ii)$$

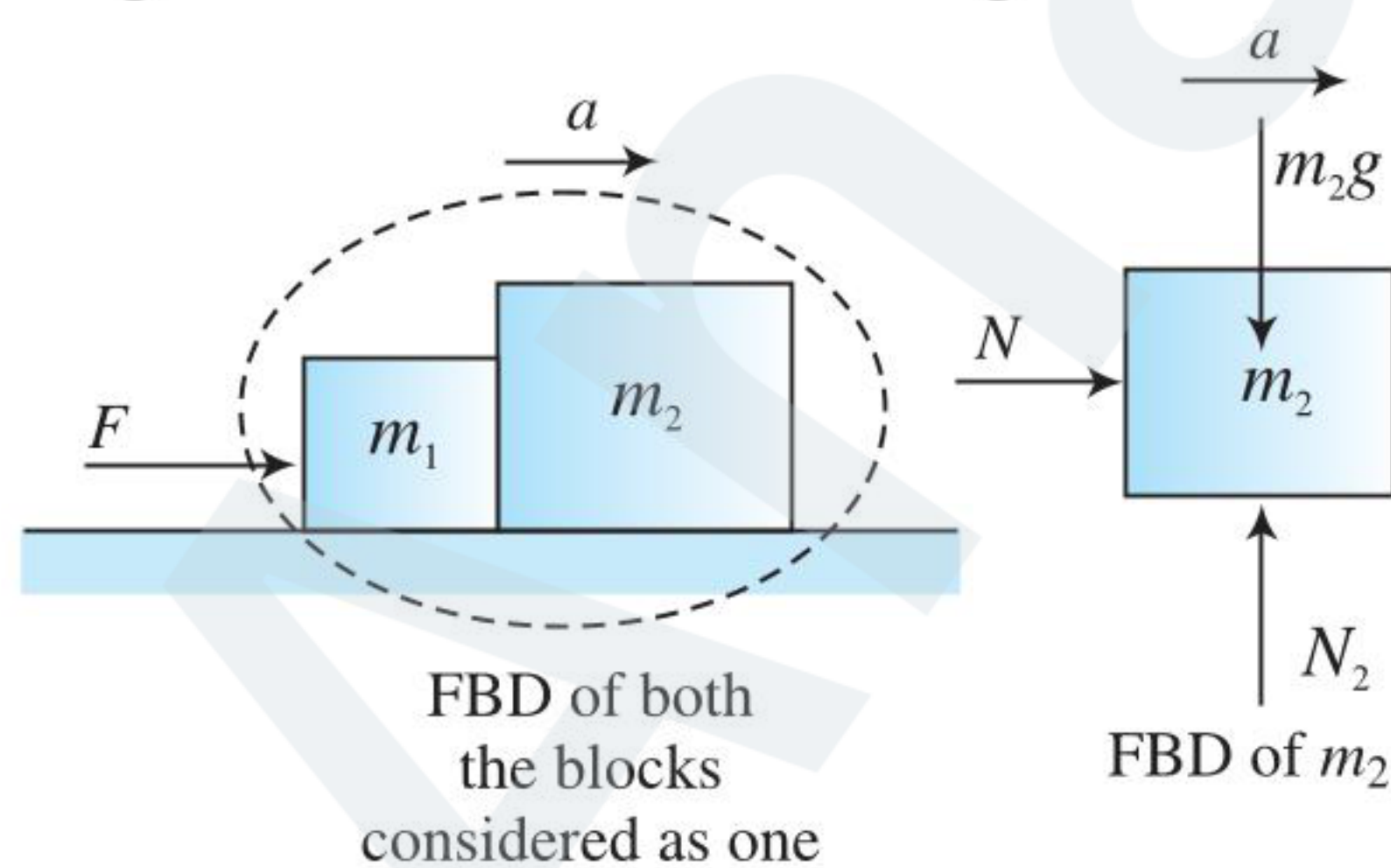
On adding the two equations, we get

$$F = (m_1 + m_2) a \Rightarrow a = \frac{F}{m_1 + m_2}$$

Substituting the value of a in (ii), we get

$$N = m_2 a = \frac{F m_2}{m_1 + m_2}$$

Method 2: The situation may be considered as follows: Instead of drawing the free-body diagrams of each block, we can draw the free-body diagram of both blocks together as shown in figure.



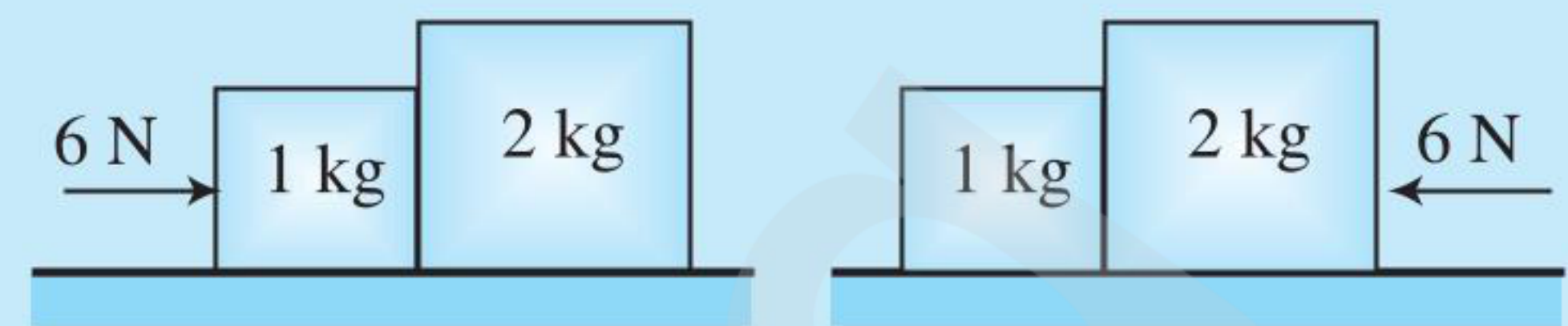
The net force acting on the system is F , and the total mass of the system is $m_1 + m_2$. Thus, $a = \frac{F}{m_1 + m_2}$

To find out the normal reaction N between the two blocks, we can imagine the following: Block m_2 is moving with an acceleration a ; therefore, the net force acting on it must be $m_2 a$, which is nothing but the normal reaction applied by the block m_1 . Thus,

$$N = m_2 a = \frac{m_2 F}{m_1 + m_2}$$

ILLUSTRATION 6.20

Two blocks of masses 1 kg and 2 kg are placed in contact on a smooth horizontal surface as shown in figures. A horizontal force of 6 N is applied (a) on a 1-kg block (b) a 2-kg block. Find the force of interaction of the blocks.



Sol.

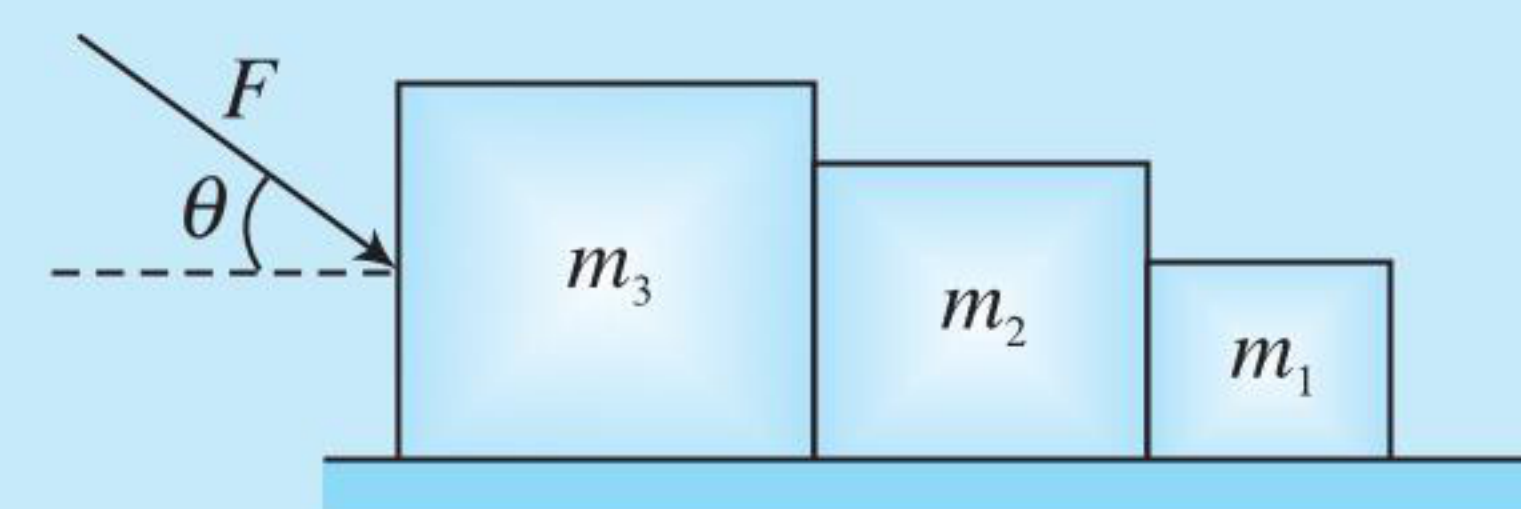
Since both the blocks are in contact, therefore, they will move together with an acceleration

$$a = \frac{F_{\text{net}}}{M_{\text{total}}} = \frac{6}{2+1} = 2 \text{ m s}^{-2}$$

Let the force of interaction between them is R_1. By Newton's second law for 2-kg block, Normal reaction on 2-kg block, $R_1 = 2a = 2 \times 2 = 4 \text{ N}$	Let the force of interaction between them is R_2. By Newton's second law for 1-kg block, normal reaction on 2-kg block, $R_2 = 1a = 1 \times 2 = 2 \text{ N}$

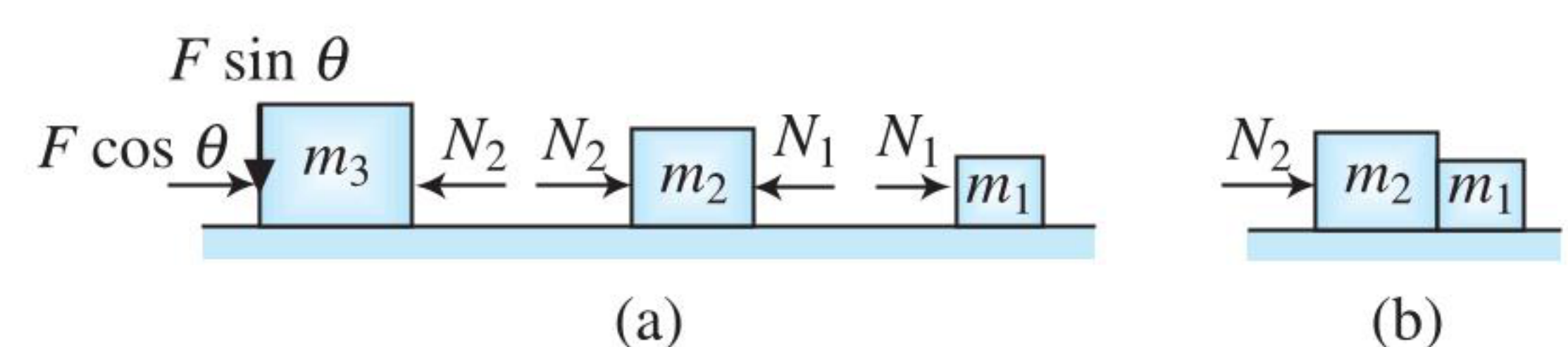
ILLUSTRATION 6.21

Find the force of interaction between the bodies as shown in figure. Blocks are in contact.



Sol. All the bodies move together along horizontal direction with an acceleration,

$$a = \frac{[F_{\text{horizontal}}]_{\text{net}}}{m_{\text{total}}} = \frac{F \cos \theta}{m_1 + m_2 + m_3}$$



By Newton's second law for block m_1 ,

$$N_1 = m_1 a = m_1 \left(\frac{F \cos \theta}{m_1 + m_2 + m_3} \right)$$

and for block m_2 , we have $N_2 - N_1 = m_2 a$

$$\text{or } N_2 = N_1 + m_2 a = N_1 + m_2 \left(\frac{F \cos \theta}{m_1 + m_2 + m_3} \right)$$

Here the force of interaction is of compressive nature.

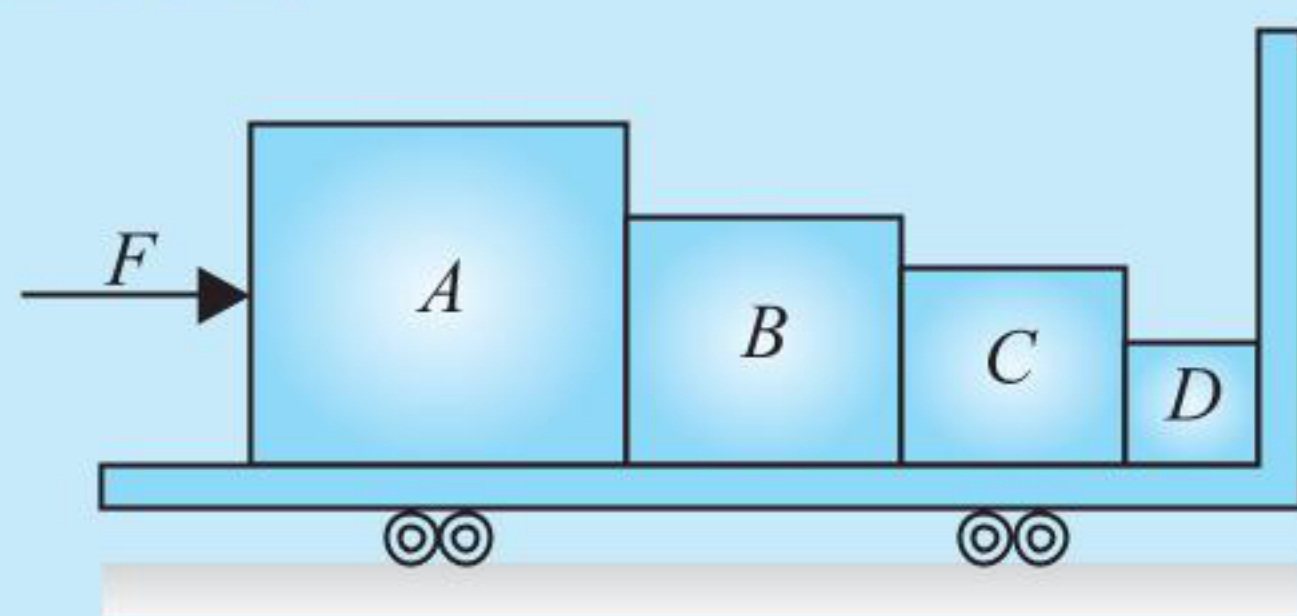
$$\text{Hence, } N_2 = \frac{(m_1 + m_2) F \cos \theta}{(m_1 + m_2 + m_3)}$$

We can calculate the value of N_2 by taking m_1 and m_2 together as system. From FBD of ' $m_1 + m_2$ ' as Fig. (b)

$$\text{or } N_2 = (m_1 + m_2)a = \frac{(m_1 + m_2) F \cos \theta}{(m_1 + m_2 + m_3)}$$

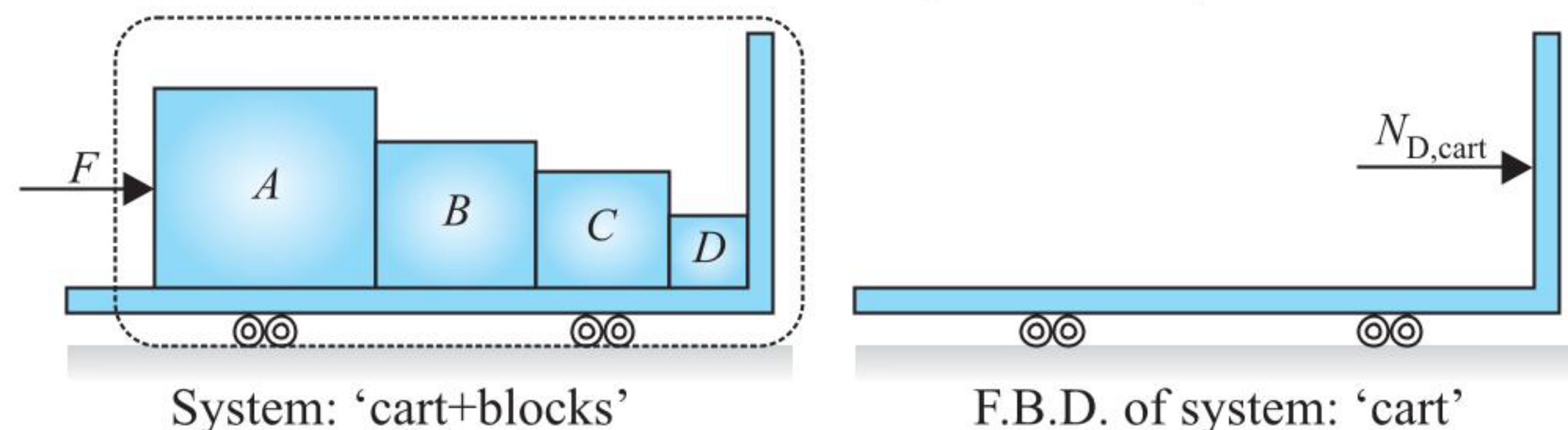
ILLUSTRATION 6.22

A toy cart has mass of 4 kg and is kept on a smooth horizontal surface. Four blocks A, B, C and D of masses 2 kg, 2 kg, 1 kg and 1 kg, respectively, have been placed on the cart. A horizontal force of $F = 60$ N is applied to the block A (see figure). Find the contact force between block D and the front vertical wall of the cart.



Sol.

Let us take the cart and the blocks together as system.



Acceleration of the system,

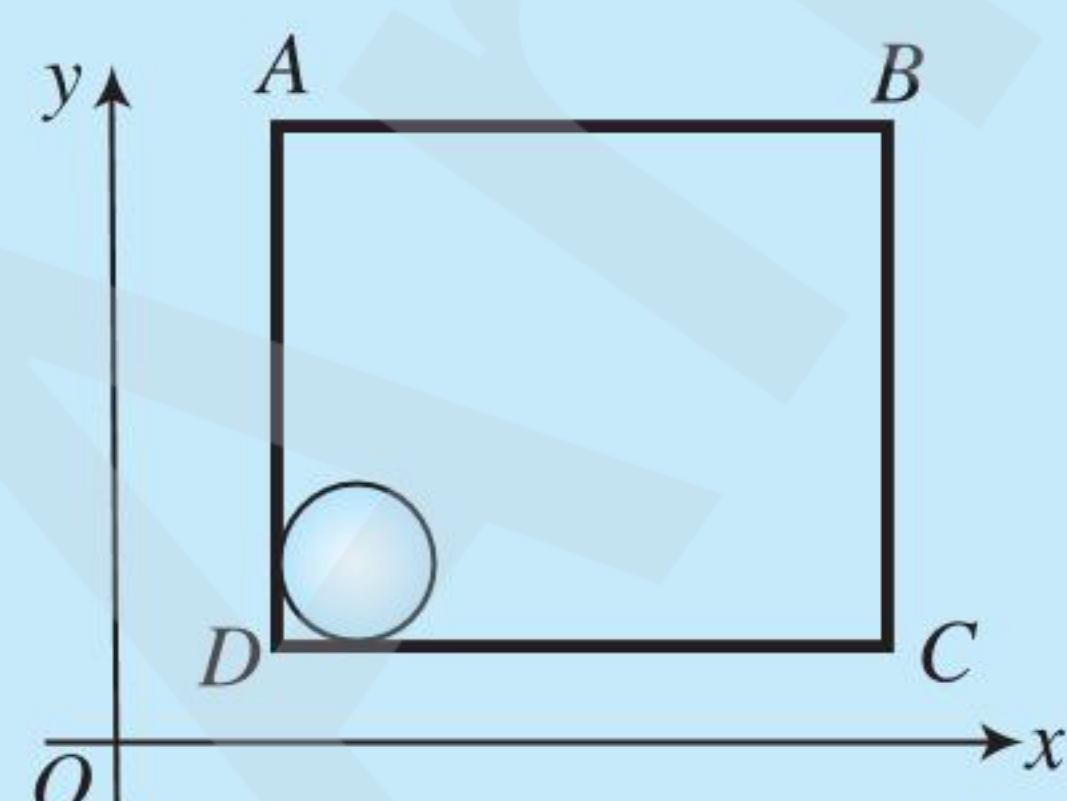
$$a = \frac{F}{m_A + m_B + m_C + m_D + m_{\text{cart}}} = \frac{60}{2 + 2 + 1 + 1 + 4} = 6 \text{ m/s}^2$$

The cart moves because of the normal reaction applied by the block 'D' on the cart. Now considering the F.B.D. of the cart (considering horizontal forces only)

$$\therefore N_{D, \text{cart}} = m_{\text{cart}} \cdot a = 4 \times 6 = 24 \text{ N}$$

ILLUSTRATION 6.23

A solid sphere of mass 2 kg is resting inside a cube as shown in figure. The cube is moving with a velocity $\vec{v} = (5t\hat{i} + 2t\hat{j}) \text{ m s}^{-1}$. Here t is time in seconds. All surfaces are smooth. The sphere is at rest with respect to the cube. What is the total force exerted by the sphere on the cube?



Sol. The velocity of the sphere is same as that of the cube, which is given as $\vec{v} = 5t\hat{i} + 2t\hat{j}$.

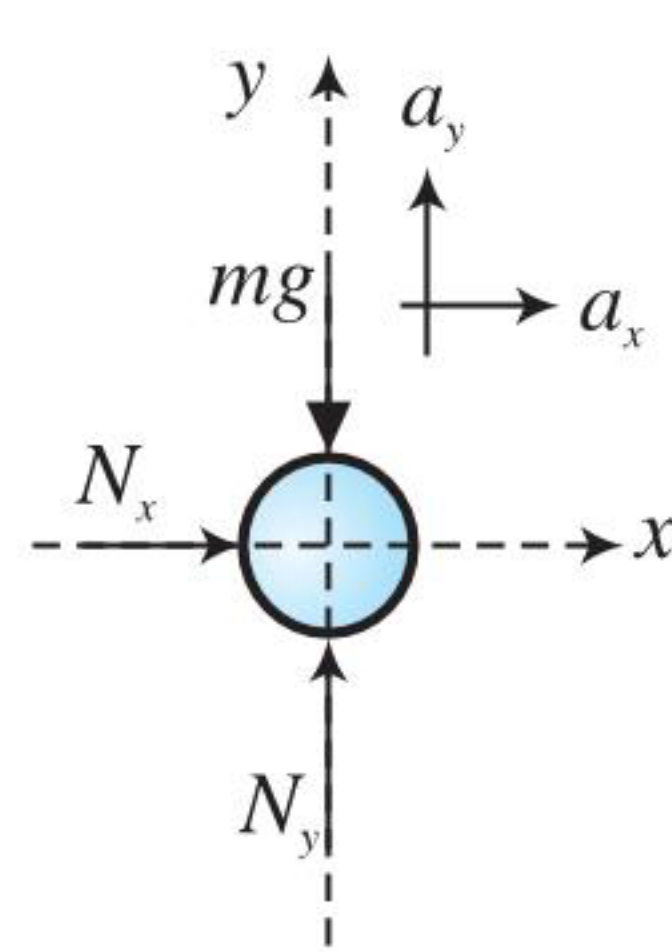
Hence, acceleration of the sphere: $\vec{a} = \frac{d\vec{v}}{dt}$

$$\text{or } \vec{a} = (5\hat{i} + 2\hat{j}) \text{ m s}^{-2}$$

Hence, $a_x = 5 \text{ m s}^{-2}$ and $a_y = 2 \text{ m s}^{-2}$

From FBD of sphere,

$$N_x = ma_x = 2 \times 5 = 10 \text{ N}$$

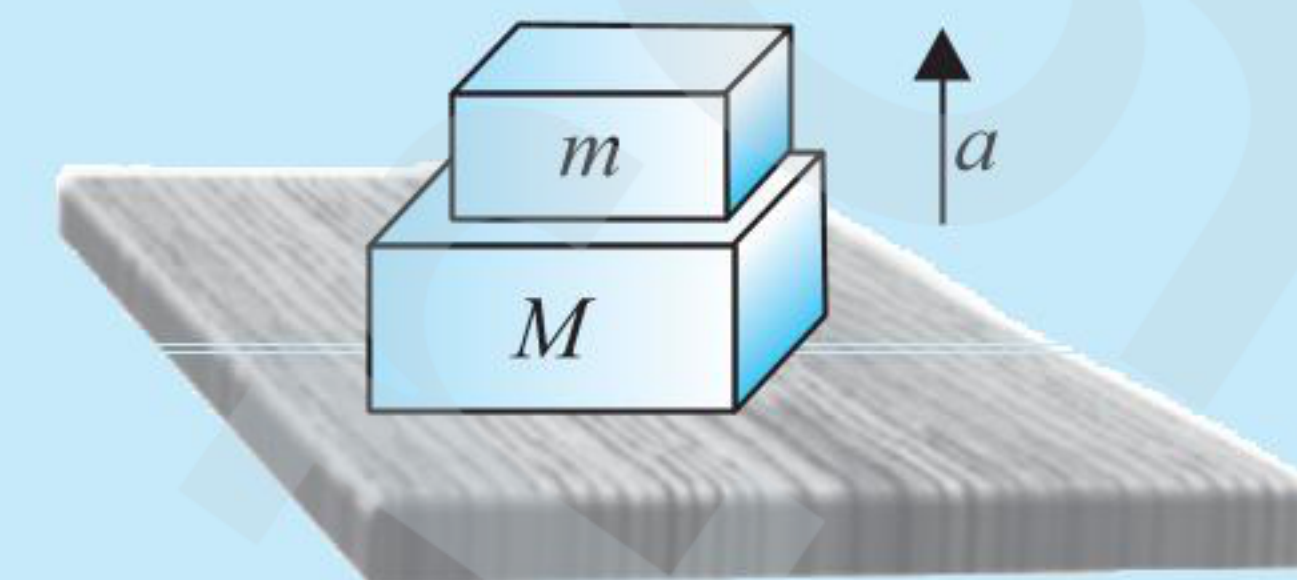


$$N_y - mg = ma_y \Rightarrow N_y = 2 \times 10 + 2 \times 2 = 24 \text{ N}$$

$$\text{Total force} = \sqrt{N_x^2 + N_y^2} = \sqrt{(10)^2 + (24)^2} = 26 \text{ N}$$

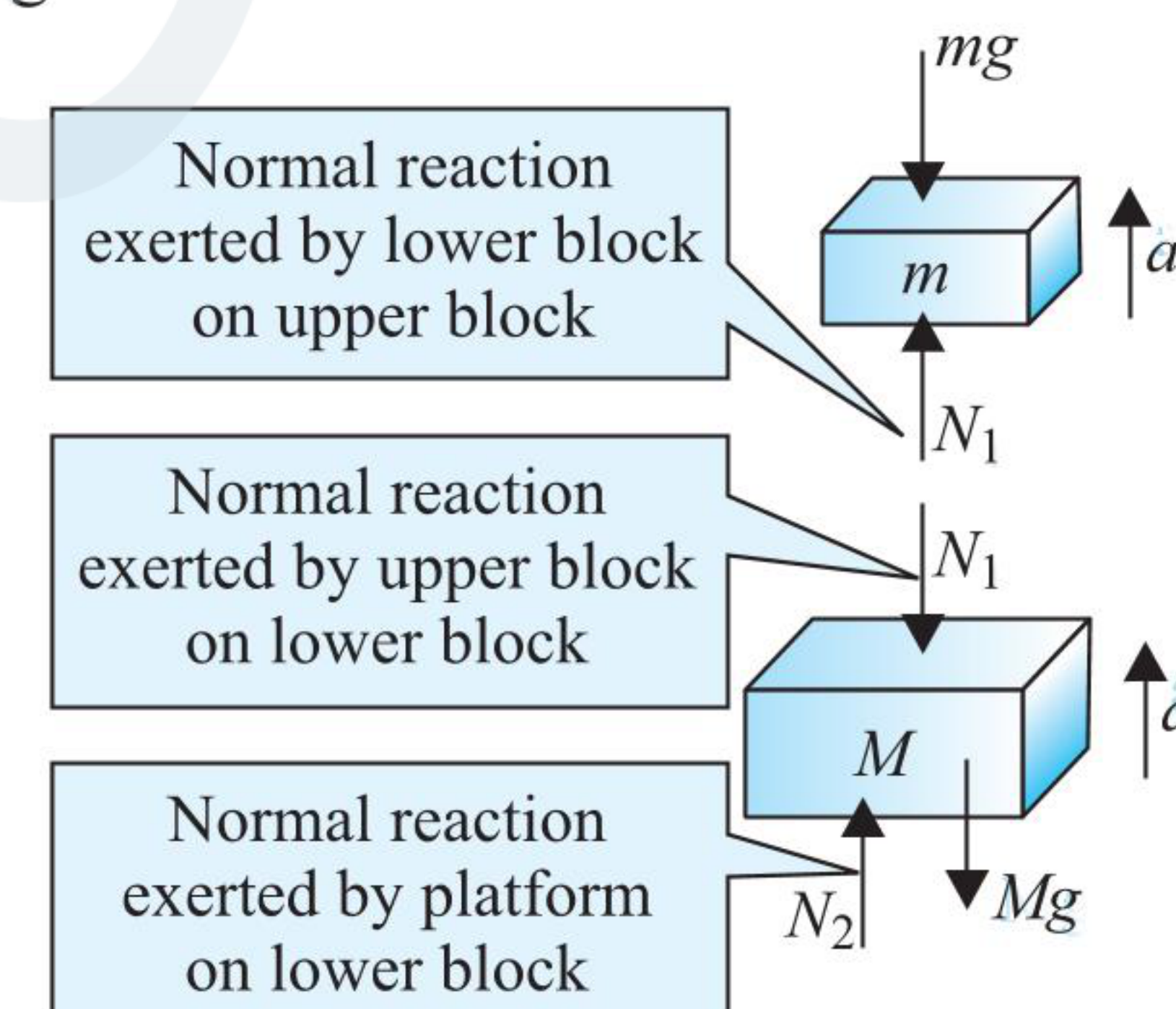
ILLUSTRATION 6.24

Two blocks m and M are placed on a platform which moves up with an upward acceleration a . Find the normal reaction between the blocks and normal reaction on block exerted by platform.



Sol. For finding normal reaction between the blocks we need to draw the F.B.D. of the blocks and for calculating the normal reactions between the blocks and normal reaction exerted by platform, we should analyze the F.B.D. of the blocks.

Now applying Newton's laws of motion



$$\text{For block } m, \quad \sum F_y = ma \Rightarrow N_1 - mg = ma \quad \dots(i)$$

$$N_1 = m(a + g)$$

$$\text{For block } M, \quad \sum F_y = Ma \Rightarrow N_2 - Mg = Ma \quad \dots(ii)$$

$$N_2 = (m + M)(a + g)$$

APPARENT WEIGHT

Usually, the weight of an object can be determined with the aid of a weighing machine. However, even though a weighing machine is working properly, there are situations in which it does not give the correct weight. In such situations, the reading on the weighing machine gives only the "apparent" weight, rather than the gravitational force or "true" weight. The apparent weight is the force that the object exerts on the weighing machine with which it is in contact.

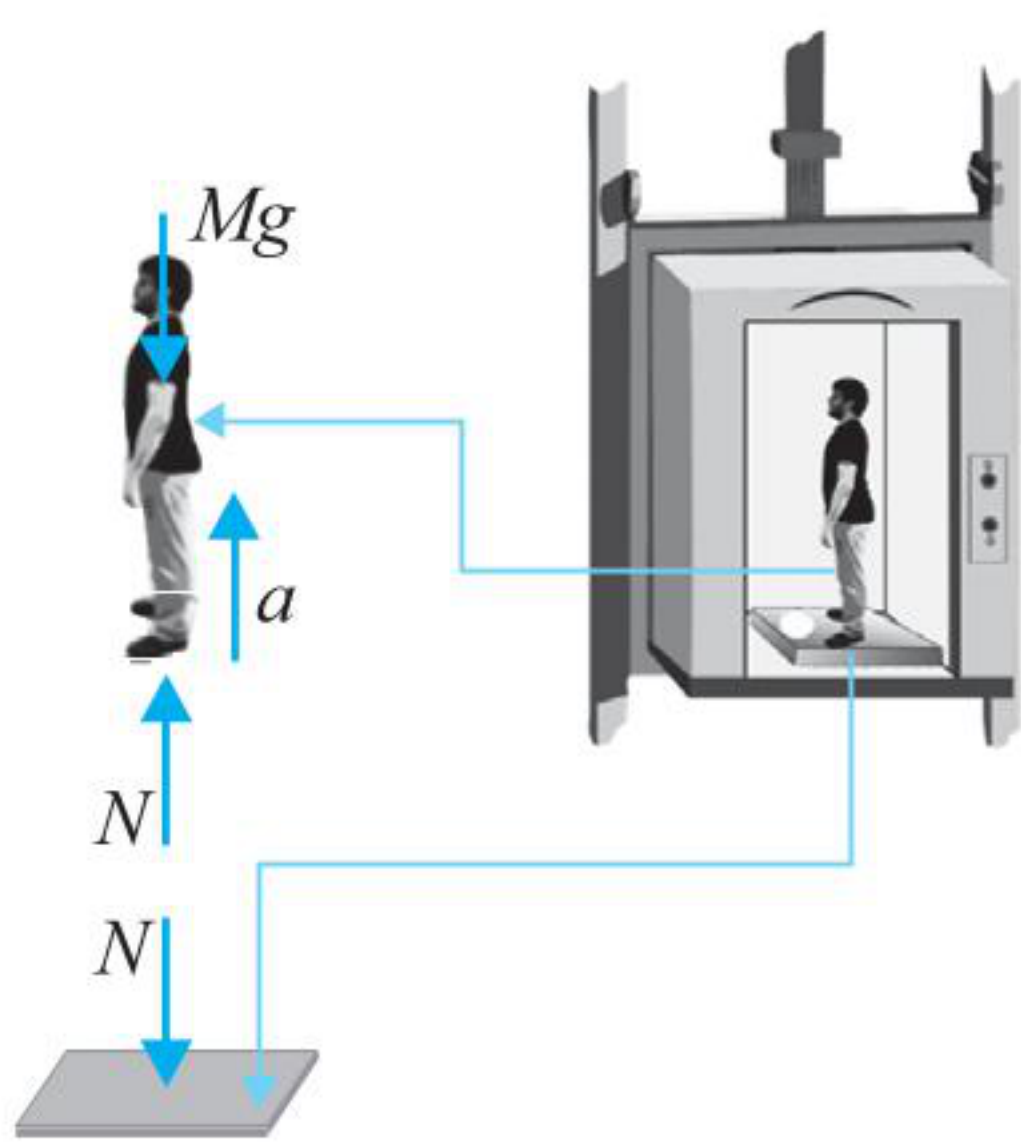
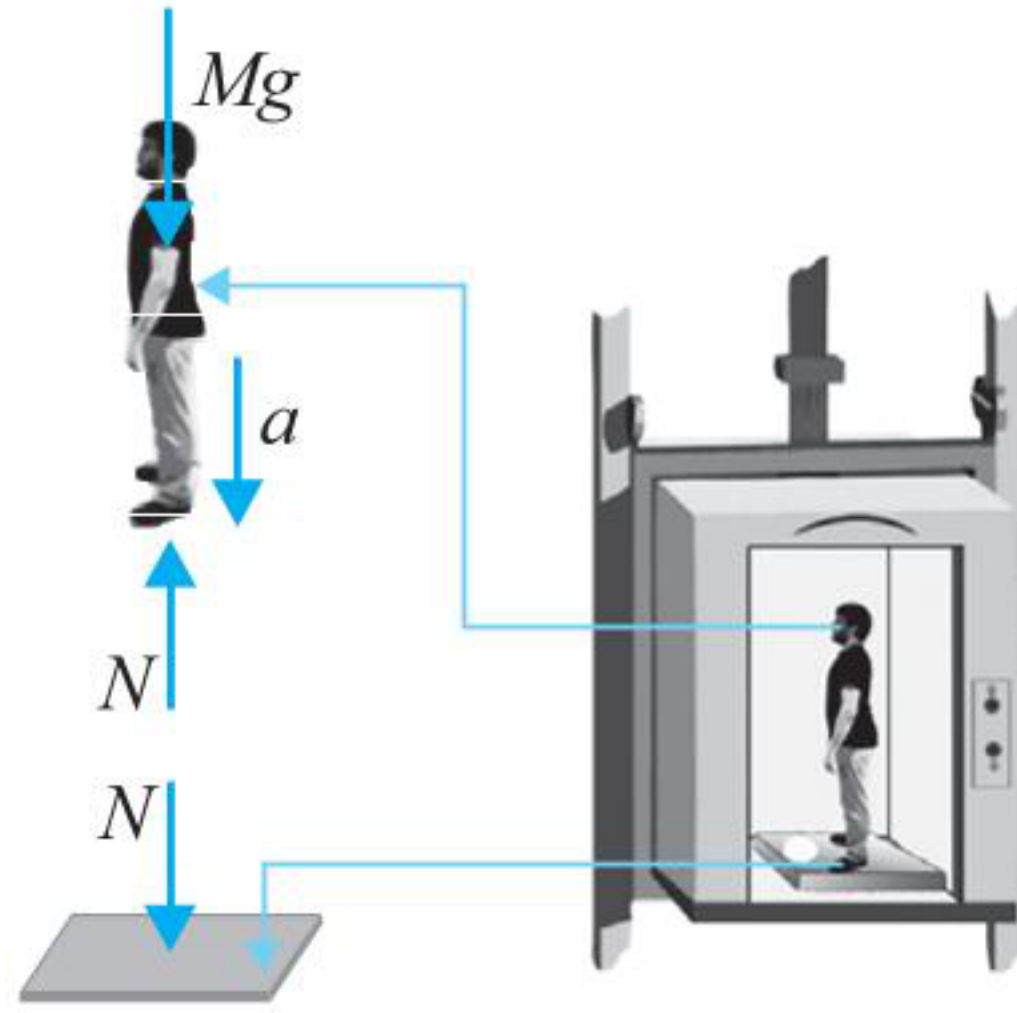
ILLUSTRATION 6.25

A man of mass M stands on a weighing machine in an elevator. Calculate the reading of the weighing machine, if the lift is

- accelerating upwards with an acceleration a .
- accelerating downwards with an acceleration a .



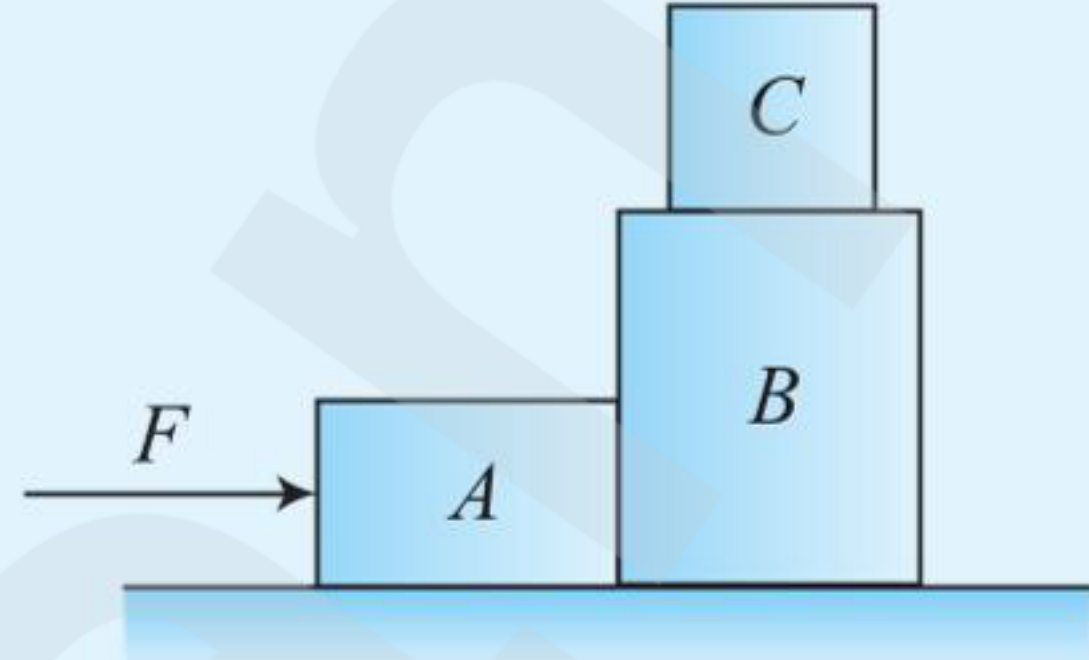
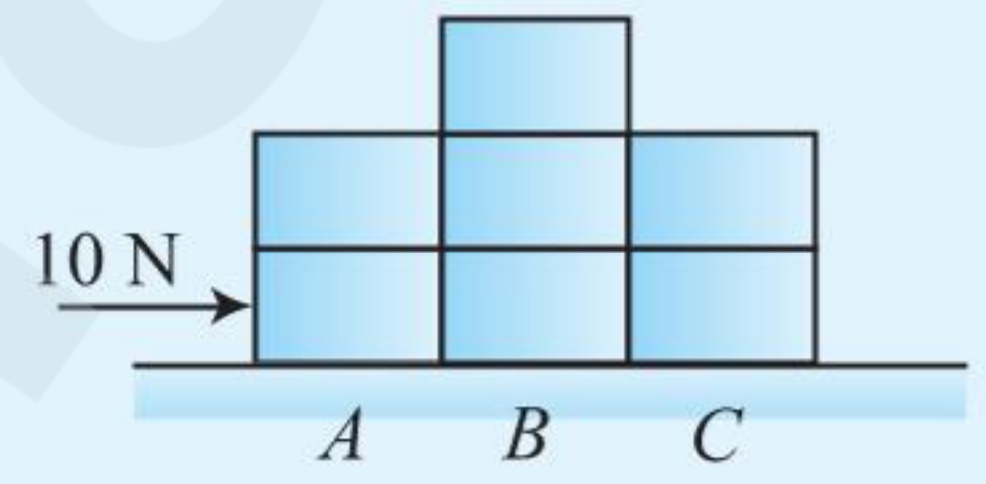
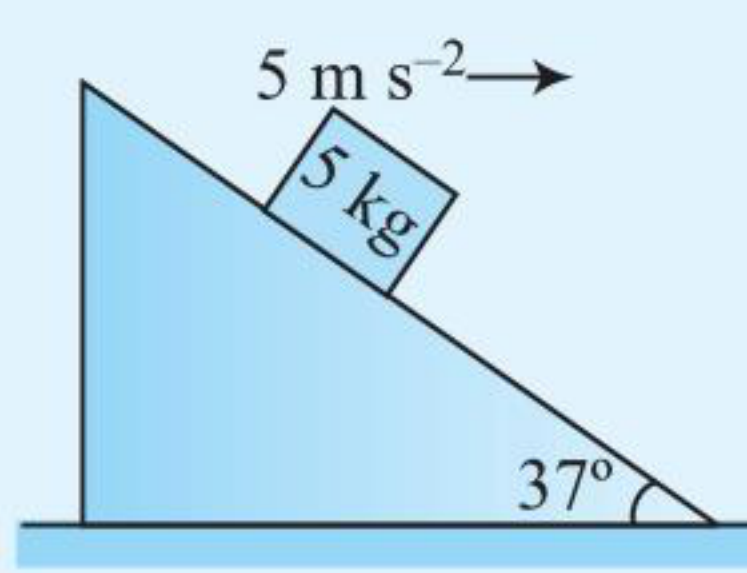
Sol. The weighing machine measures the normal reaction.

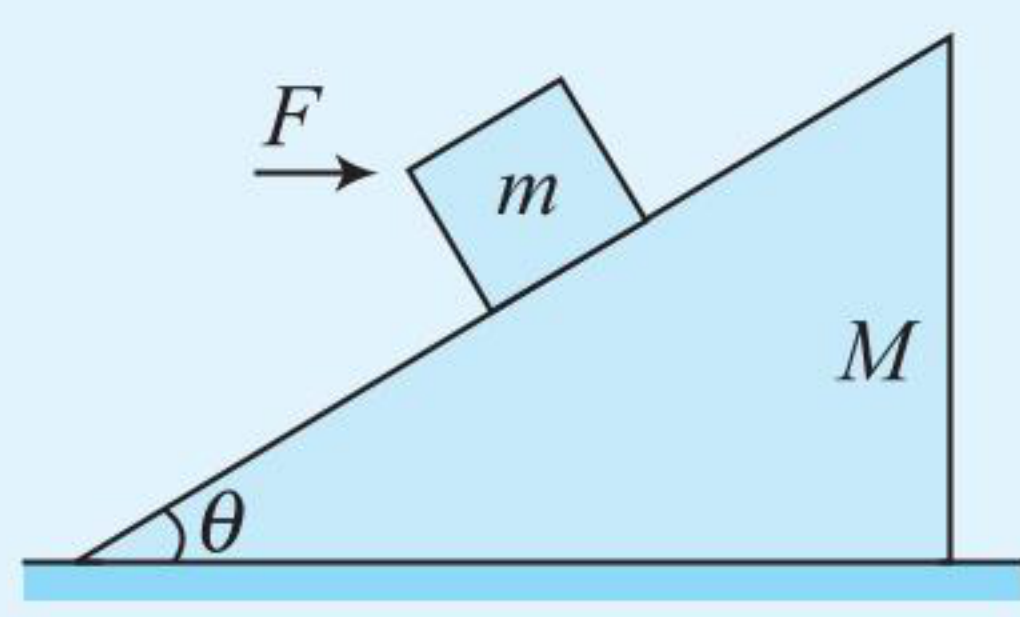
The lift accelerating upwards	The lift accelerating downwards
From FBD of man,  using Newton's second law, $N - Mg = Ma \Rightarrow N = M(g + a)$ N is the normal reaction on weighing machine. Hence, the reading of weighing machine will be $M(g + a)$.	From FBD of man,  using Newton's second law, $Mg - N = Ma \Rightarrow N = M(g - a)$ N is the normal reaction on weighing machine. Hence, the reading of weighing machine will be $M(g - a)$.

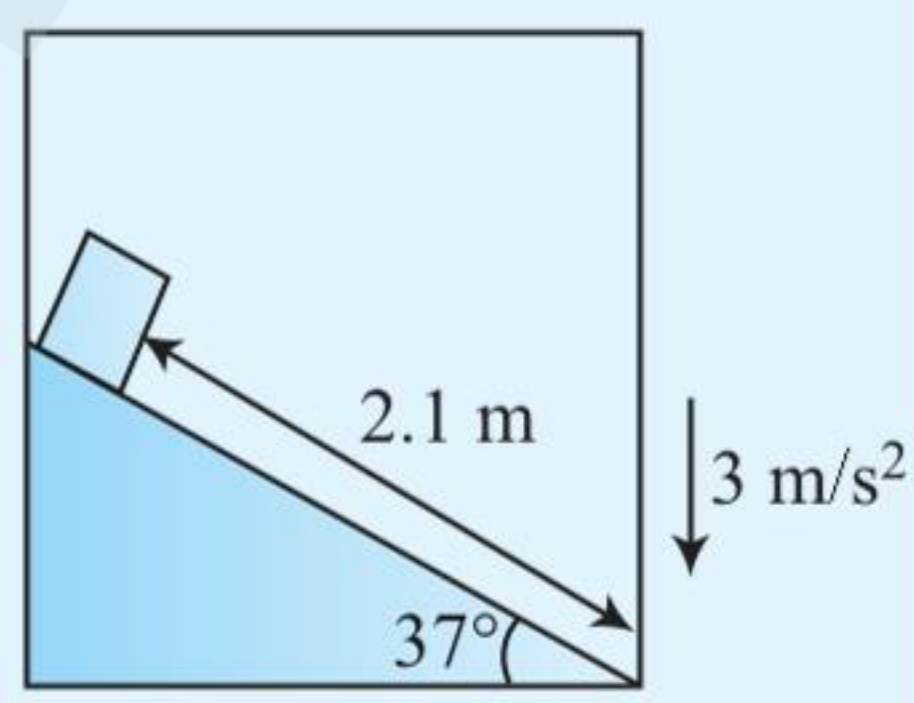
Important Points:

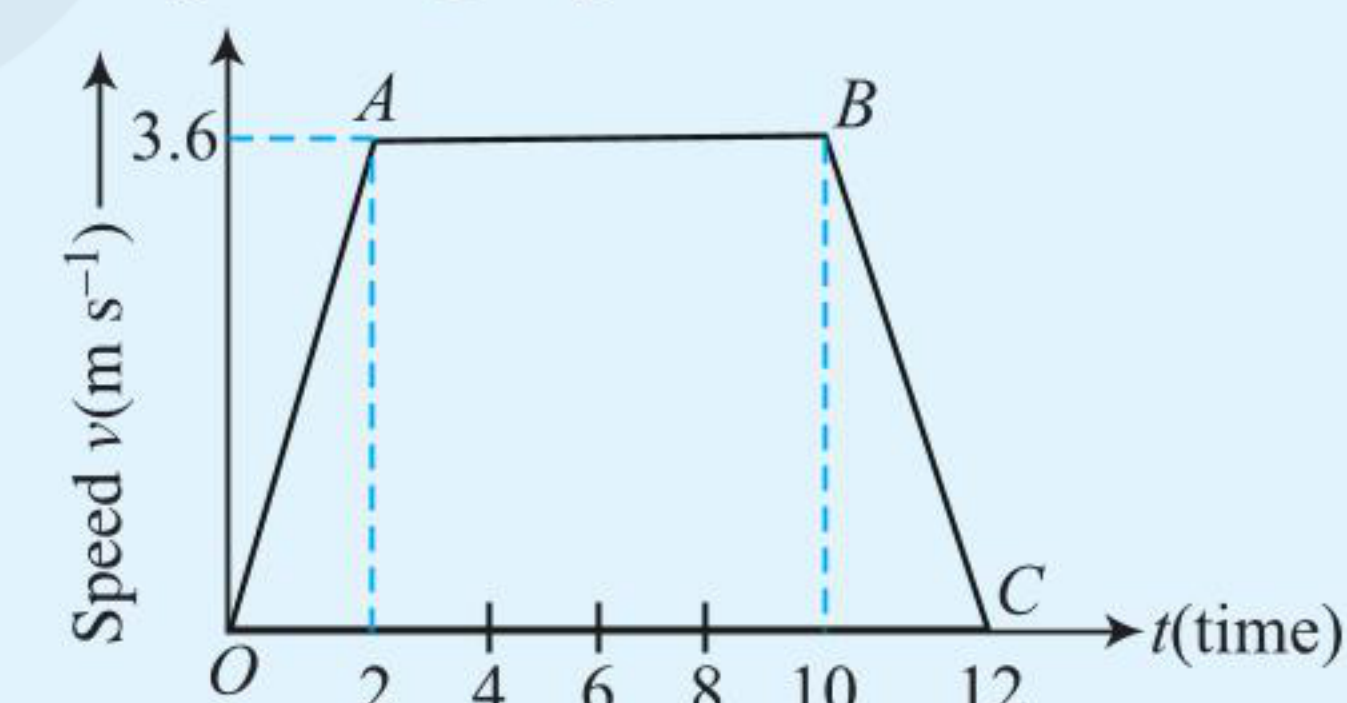
If the elevator is accelerating, the apparent weight and the true weight are not equal. When the elevator accelerates upward, the apparent weight is greater than the true weight. Conversely, if the elevator accelerates downward, the apparent weight is less than the true weight. In fact, if the elevator falls freely, so its acceleration is equal to the acceleration due to gravity, the apparent weight becomes zero. In a situation such as this, where the apparent weight is zero, the person is said to be "weightless." The apparent weight, then, does not equal the true weight if the scale and the person on it are accelerating.

CONCEPT APPLICATION EXERCISE 6.3

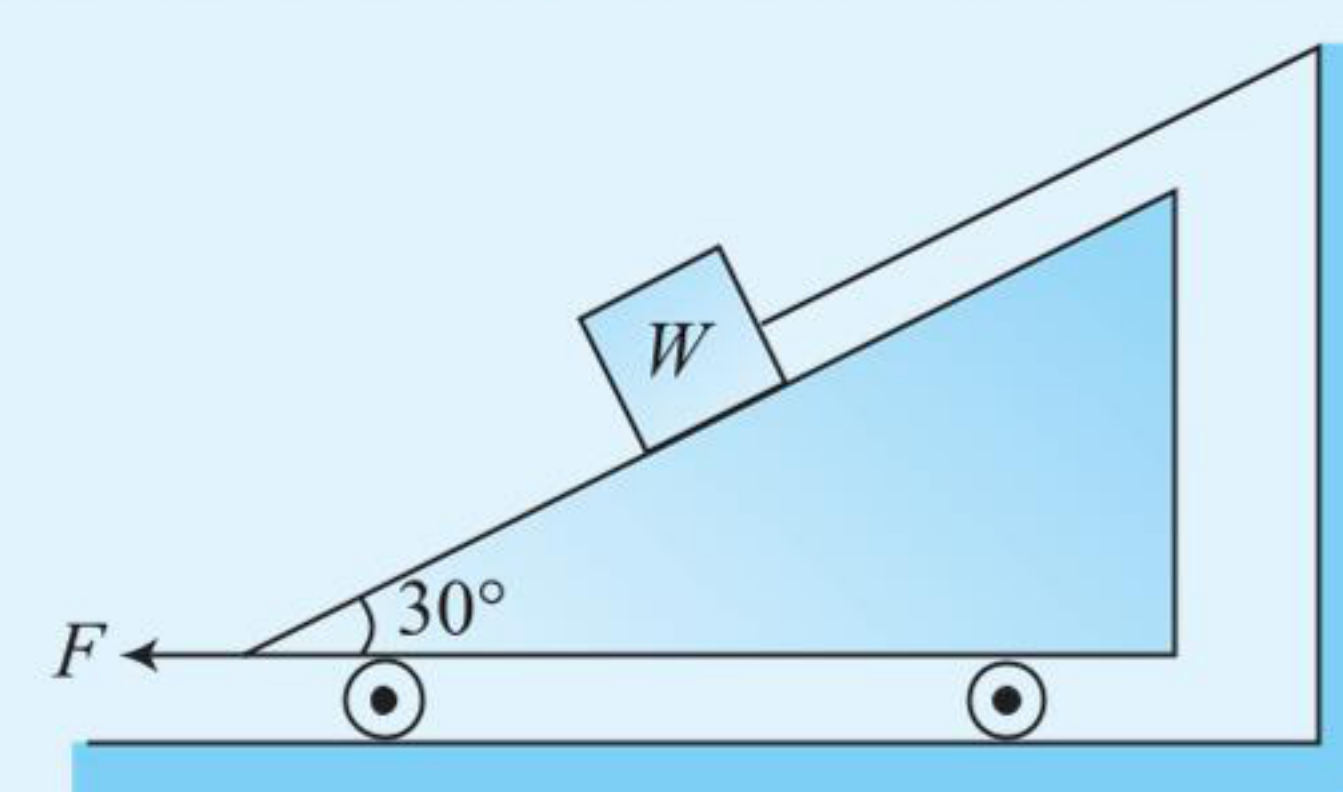
- The masses of blocks A , B , and C are 1 kg, 2 kg, and 0.5 kg, respectively. All surfaces are smooth. If force $F = 50$ N acts as shown in figure at the instant shown, find the force which A exerts on B and the acceleration of C .

- Seven identical dominoes (i.e., blocks) each of mass $m = 1$ kg are to be stacked in three columns (figure gives an example) and pushed across a frictionless ice rink by a horizontal 10-N force. Assume dominoes do not slip w.r.t. each other. How many dominoes should be in each column, with a minimum of one, (a) to maximize the acceleration of the dominoes, (b) to maximize the force on column C due to column B , (c) to maximize the net force on column B due to columns A and C , and (d) to maximize the force on column B due to column A ?

- An inclined plane is moved toward right with an acceleration of 5 m/s^2 as shown in figure. Find force in newton which block of mass 5 kg exerts on the incline plane. (All surfaces are smooth.)


- A small cubical block is placed on a triangular block M so that they touch each other along a smooth inclined contact plane as shown in figure. The inclined surface makes an angle θ with the horizontal. A force F is to be applied on the block m in horizontal direction so that the two bodies move without slipping against each other assuming the floor to be smooth also. Determine the
 (a) normal force with which m and M press against each other.
 (b) magnitude of external force F . Express your answers in terms of m , M , θ , and g .


- A block of mass 1 kg is kept on the tilted floor of a lift moving down with 3 m/s^2 . If the block is released from rest as shown, what will be the time taken by block to reach the bottom?

- A lift is going up. The total mass of the lift and the passengers is 1500 kg. The variation in the speed of the lift is given by the graph.



- What will be the tension in the rope pulling the lift at time t equal to
 i. 1 s ii. 6 s iii. 11 s
- What will be the average velocity and the average acceleration during the course of the entire motion?
- A block of weight W is placed on a wedge and arranged as shown in figure. Find the force F needed to hold the cart equilibrium if there is no friction.



ANSWERS

- $\frac{100}{3}$ N
- (a) Does not depend upon the number of blocks
 (b) $A(1), B(1), C(5)$
 (c) $A(1), B(5), C(1)$
 (d) $A(1)$, remaining six can be placed in any arrangement.
- 55 N
- (a) $(M + m)a$ (b) $\frac{mg}{M}(m + M) \tan \theta$
- 1 s
- (a) (i) 17700 N, (ii) 15000 N, (iii) 12300 N
 (b) $3 \text{ ms}^{-1}, 0 \text{ ms}^{-2}$
- $\frac{\sqrt{3}W}{4}$

PROBLEMS BASED ON BLOCKS CONNECTED WITH STRINGS

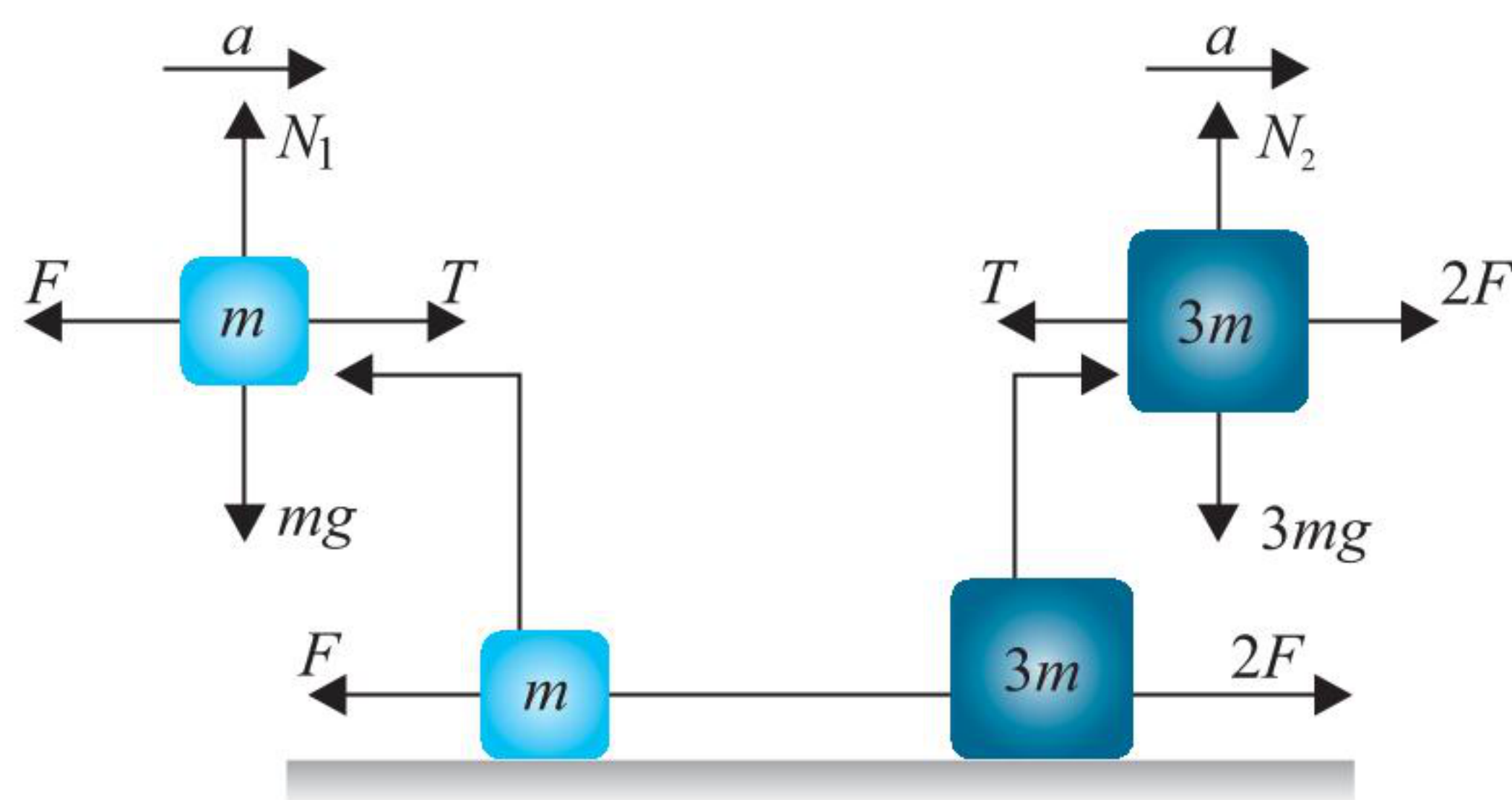
ILLUSTRATION 6.26

Two smooth blocks of masses m and $3m$ are connected by an inextensible light string. If the forces F and $2F$ act on the blocks, the find



- (a) tension in the string, and
(b) acceleration of the blocks.

Sol. Both the blocks will move in rightwards direction with same acceleration.



The free body diagrams of both the blocks and

Now using Newton's second law of motion, $\sum F_x = ma_x$

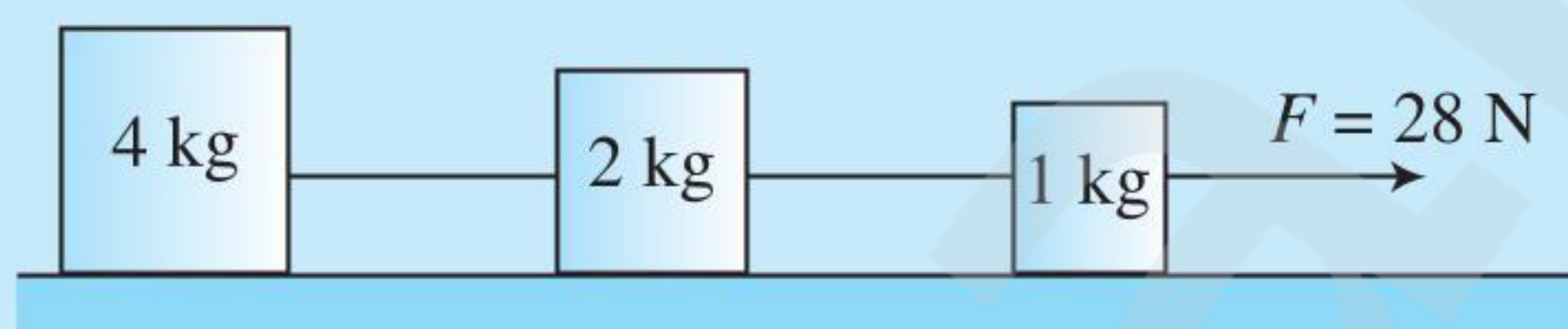
From F.B.D of m : $T - F = ma$... (i)

From F.B.D of $3m$: $2F - T = 3ma$... (ii)

Solving equations (i) and (ii), $T = \frac{5F}{4}$, $a = \frac{F}{4m}$

ILLUSTRATION 6.27

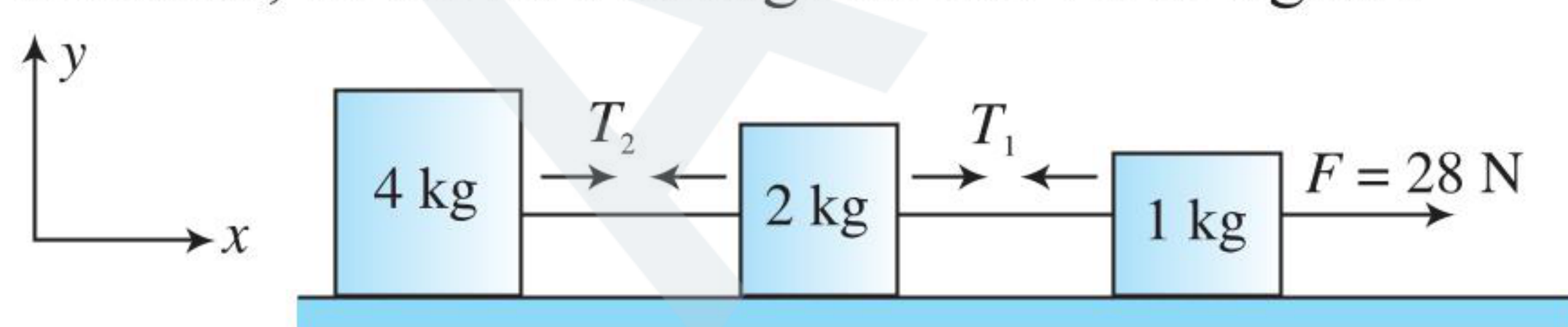
In the arrangement shown in figure, the strings are light and inextensible. The surface over which blocks are placed is smooth. Find:



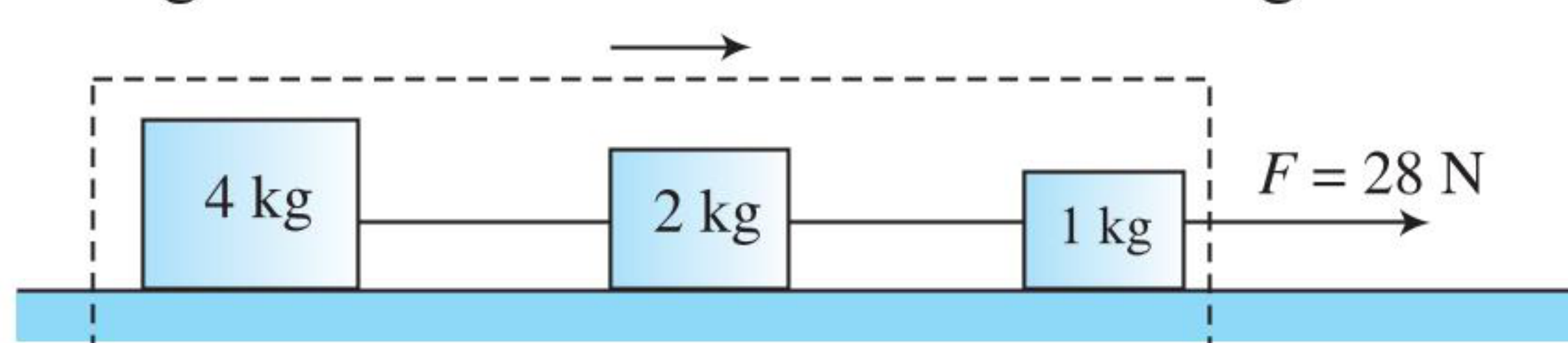
- (a) the acceleration of each block
(b) the tension in each string

Sol.

- (a) Let a be the acceleration of each block and T_1 and T_2 be the tensions, in the two strings as shown in figure.



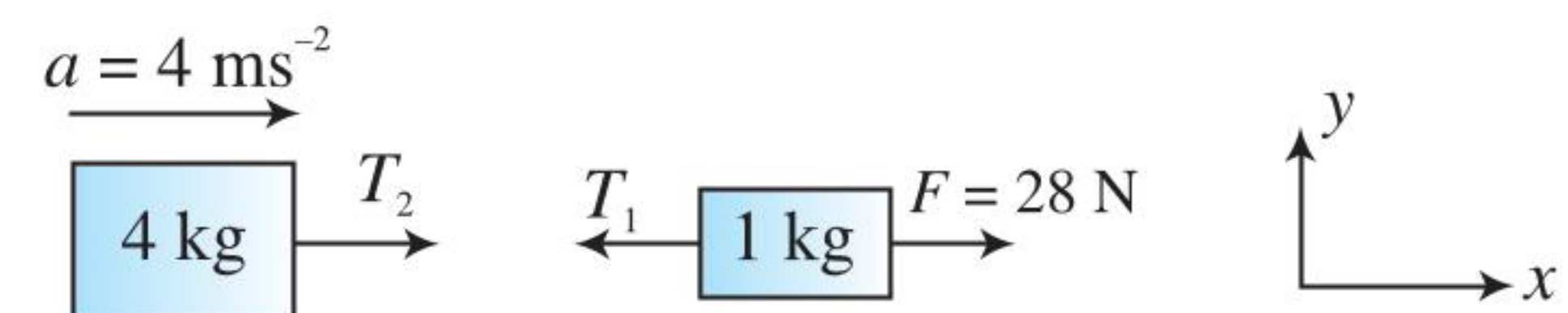
Taking the three blocks and the two strings as the system.



Using $\sum F_x = ma_x$

$$\text{or } 28 = (4 + 2 + 1)a \quad \text{or } a = \frac{28}{7} = 4 \text{ m s}^{-2}$$

- (b) Free-body diagrams (showing the forces in x -direction only) of 4-kg block and 1-kg block are shown in figure.



Using $\sum F_x = ma_x$

For 1-kg block: $F - T_1 = (1)a$

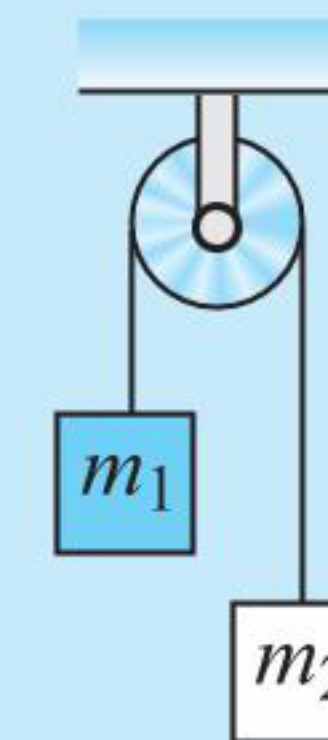
$$\text{or } 28 - T_1 = (1)(4) = 4 \Rightarrow T_1 = 28 - 4 = 24 \text{ N}$$

For 4-kg blocks: $T_2 = (4)a$

$$\Rightarrow T_2 = (4)(4) = 16 \text{ N}$$

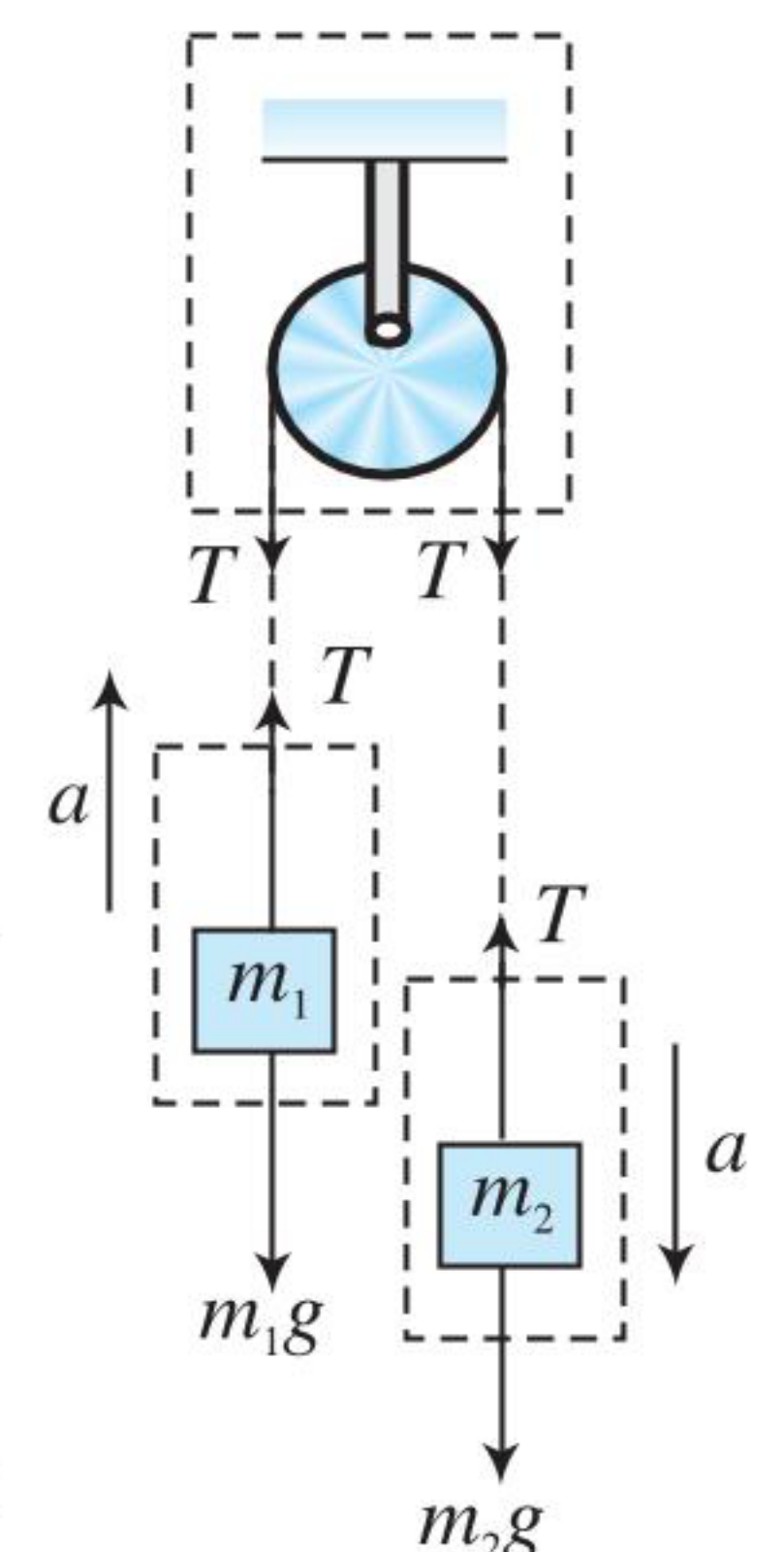
ILLUSTRATION 6.28

Two masses m_1 and m_2 are attached to a flexible inextensible massless rope, which passes over a frictionless and massless pulley. Find the accelerations of the masses and tension in the rope.



Sol. Let the tension in the rope be T (in fact, tension is the property of a point of the rope). In this case with ideal pulley (massless and frictionless) and ideal rope (inextensible, massless, and flexible), the tension will remain constant throughout the rope.

Let the acceleration of m_2 be vertically downwards, and acceleration of m_1 will also be a , vertically upwards. This is because the rope is inextensible; during motion, the length of the rope must not change, and the rope must not slacken either.



From the above statement, you must not conclude that the accelerations of the masses connected by a rope are always equal. The relationship between the accelerations of the masses depends on the configuration of the pulley-rope system, which can be obtained from the fact that the length of an ideal rope must not change and the rope must not slacken.

Using equation $\sum \vec{F} = m\vec{a}$ for the force diagrams of m_1 and m_2 ,

$$T - m_1g = m_1a \quad \dots (i)$$

$$m_2g - T = m_2a \quad \dots (ii)$$

Adding (i) and (ii), $m_2g - m_1g = m_1a + m_2a$

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1} \right) g \quad \dots (iii)$$

Substituting this value of a in (i), we get

$$T = m_1g + m_1 \left(\frac{m_2 - m_1}{m_2 + m_1} \right) g = \frac{2m_1m_2}{(m_1 + m_2)} g \quad \dots (iv)$$

Important Points:

These type of problems in which all connected bodies have same acceleration magnitude, can be solved by the following method:

$$\text{For calculating } a \text{ use } a = \frac{F_{\text{net}}}{M_{\text{total}}} = \frac{\text{Driving force}}{\text{Total mass}}$$

For driving force:

(a) If mass moves vertically, take mg

(b) If mass moves horizontally, take zero

If mass moves on inclined plane of inclination, take $mg \sin \theta$. The external forces acting parallel to string taken as positive if acting in the direction of motion and negative if they are acting in the direction opposite to motion.

In above illustration:

Driving force, $F_{\text{net}} = m_2g - m_1g$

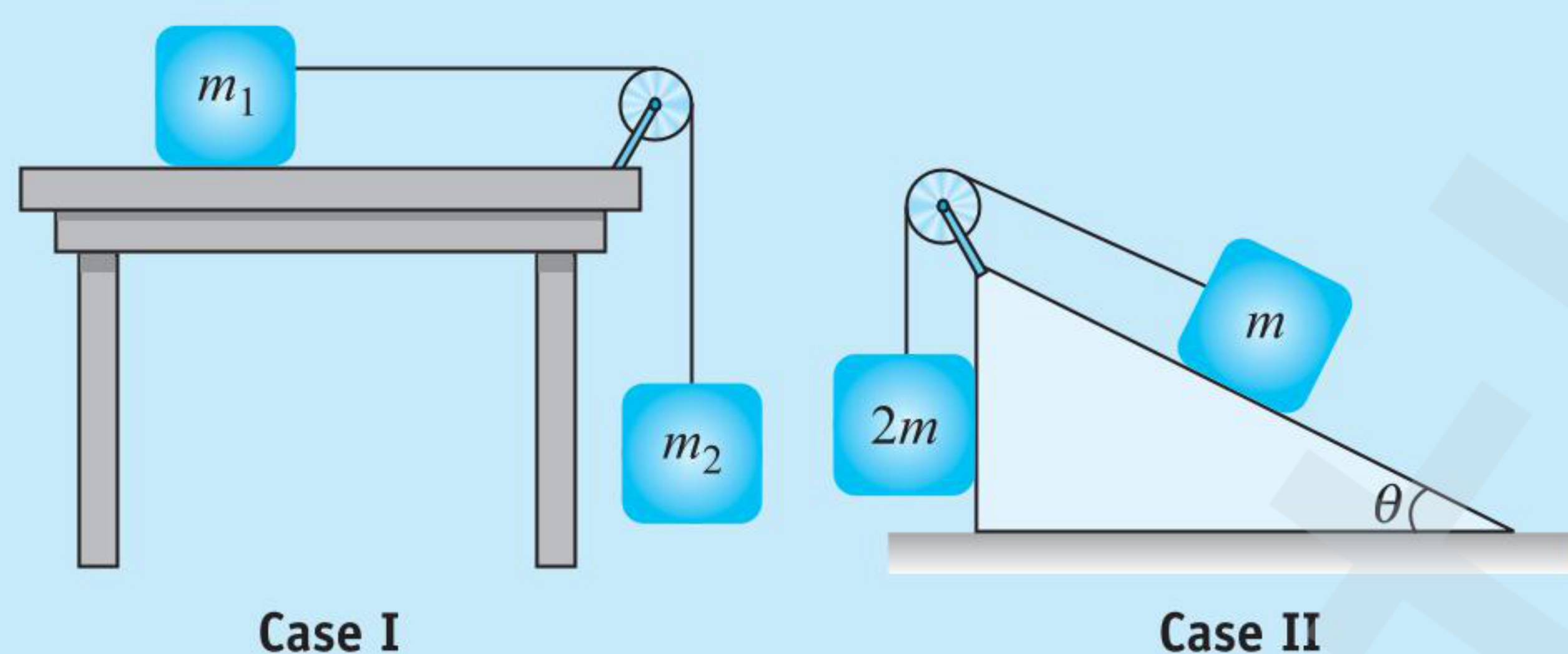
Here m_1g is taken negative as it is acting in the direction opposite in motion.

Acceleration of the system,

$$a = \frac{F_{\text{net}}}{M_{\text{total}}} = \frac{m_2g - m_1g}{(m_1 + m_2)} = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$$

ILLUSTRATION 6.29

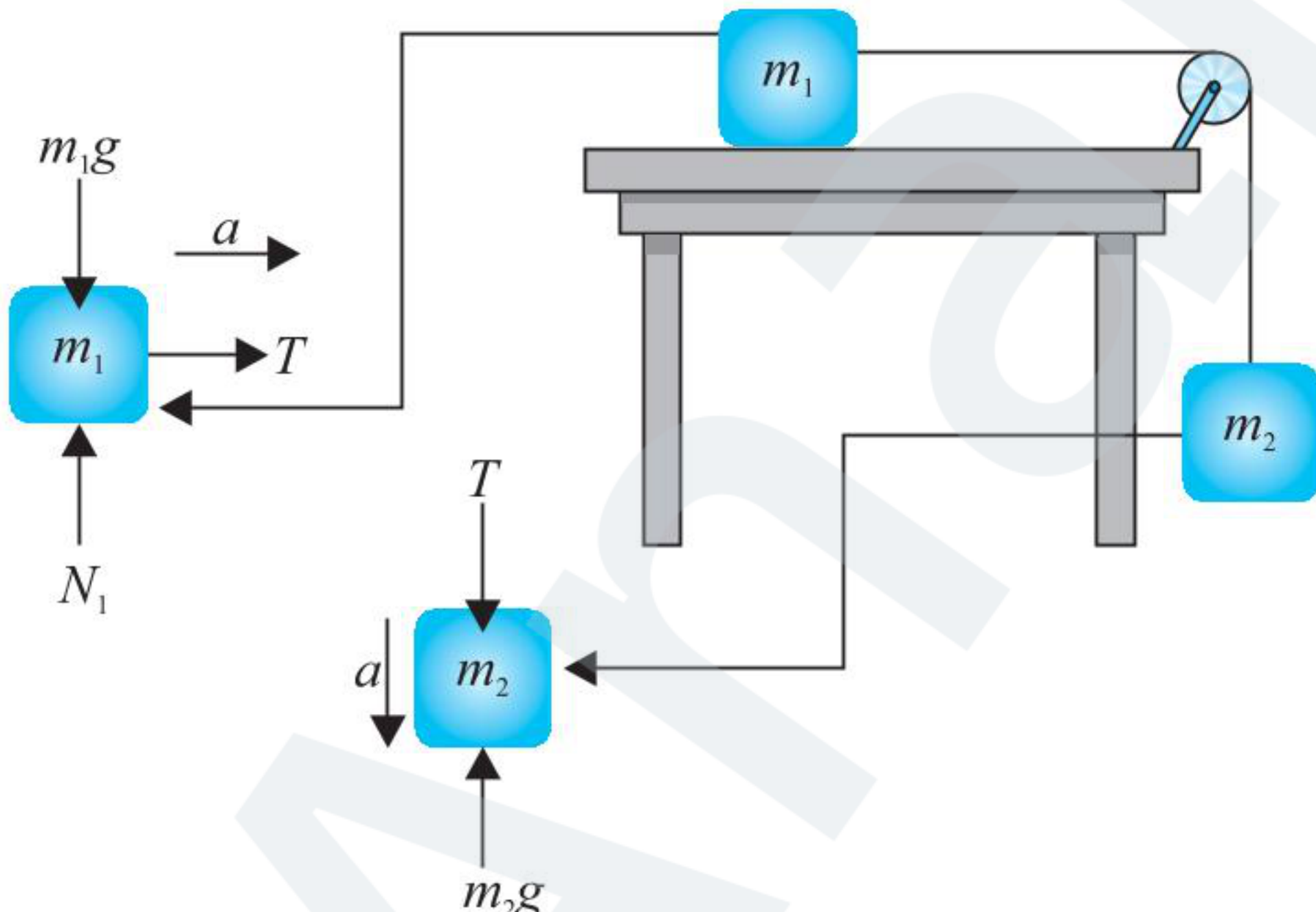
Calculate the acceleration of the system in case (1) and (2).



Sol.

Case I

Free body diagrams



Equation of motion

$$\text{For } m_1 \quad T = m_1 a \quad \dots(i)$$

$$\text{For } m_2 \quad m_2g - T = m_2 a \quad \dots(ii)$$

From (i) and (ii),

$$\Rightarrow a = \frac{m_2g}{(m_1 + m_2)}$$

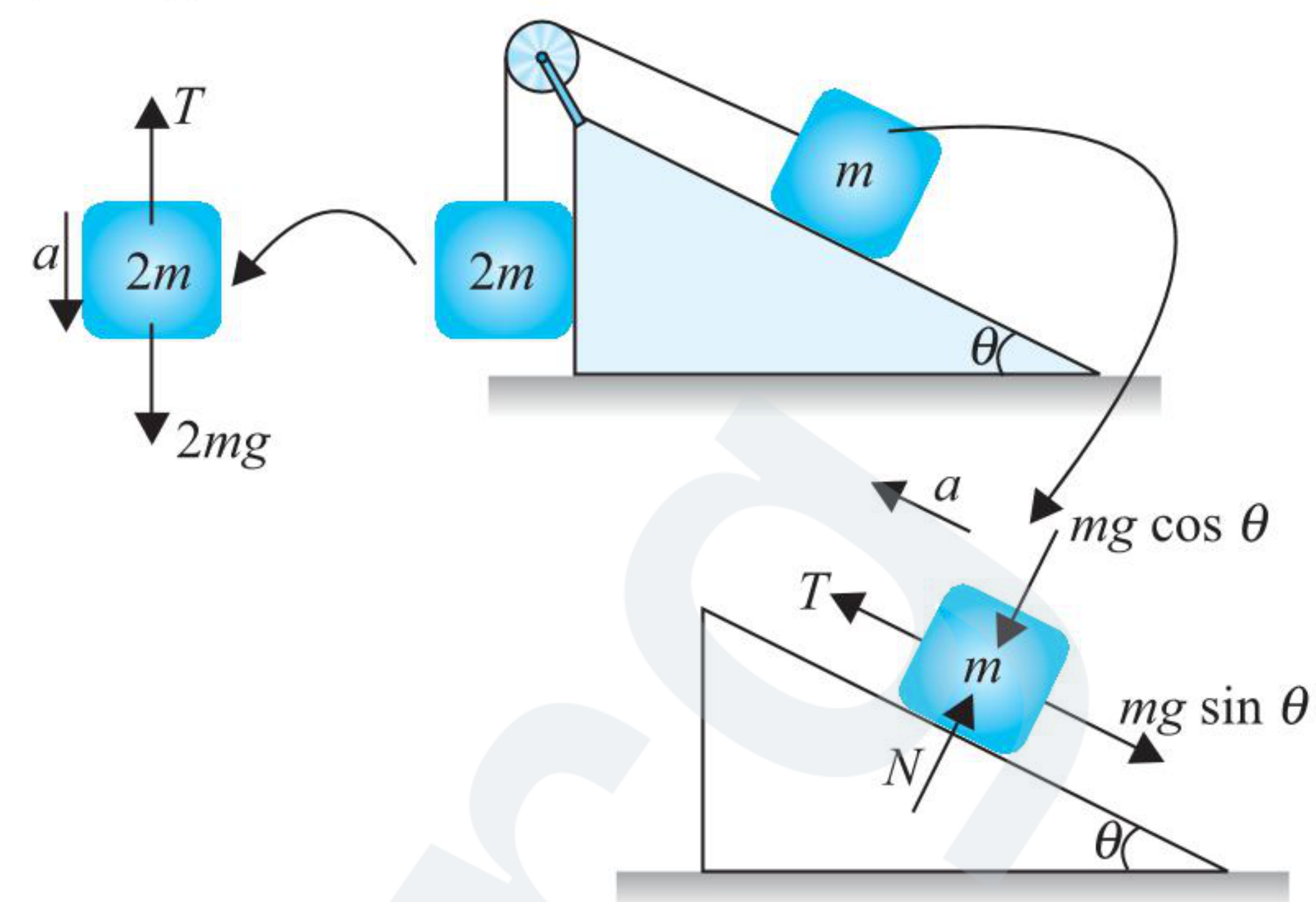
Approach 2

Driving force, $F_{\text{net}} = m_2g$

Acceleration of the system, $a = \frac{F_{\text{net}}}{M_{\text{total}}} = \frac{m_2g}{(m_1 + m_2)}$

Case II

Free body diagrams



Equation of motion

$$T - mg \sin \theta = ma \quad \dots(i)$$

$$2mg - T = 2ma \quad \dots(ii)$$

From equation (i) and (ii), $2mg - mg \sin \theta = (m + 2m)a$

$$a = \frac{(2 - \sin \theta)}{3} g$$

Approach 2

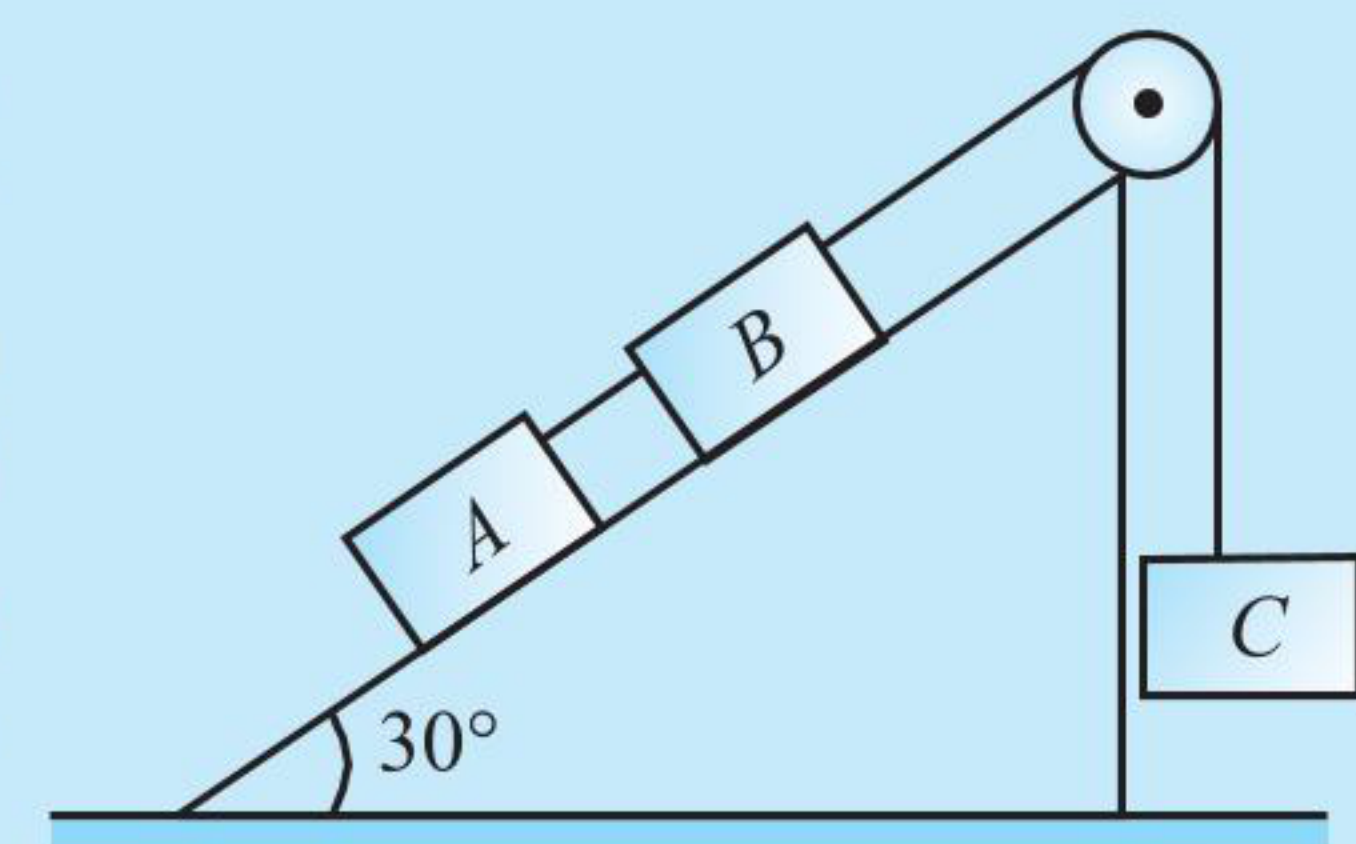
Driving force $F_{\text{net}} = 2mg - mg \sin \theta$

Acceleration of the system,

$$a = \frac{F_{\text{net}}}{M_{\text{total}}} = \frac{2mg - mg \sin \theta}{(2m + m)} = \frac{(2 - \sin \theta)}{3} g$$

ILLUSTRATION 6.30

In the given figure, blocks A and B are connected together by a string and placed on a smooth inclined plane. B is connected to C (which is suspended vertically) by another string which passes over a smooth pulley fixed to the plane. The mass of A is $m_A = 1 \text{ kg}$ and mass of B is $m_B = 2 \text{ kg}$.



(a) If the system is at rest, find the mass of C.

(b) If the mass of C is twice the mass calculated in (a), then find the acceleration of the system.

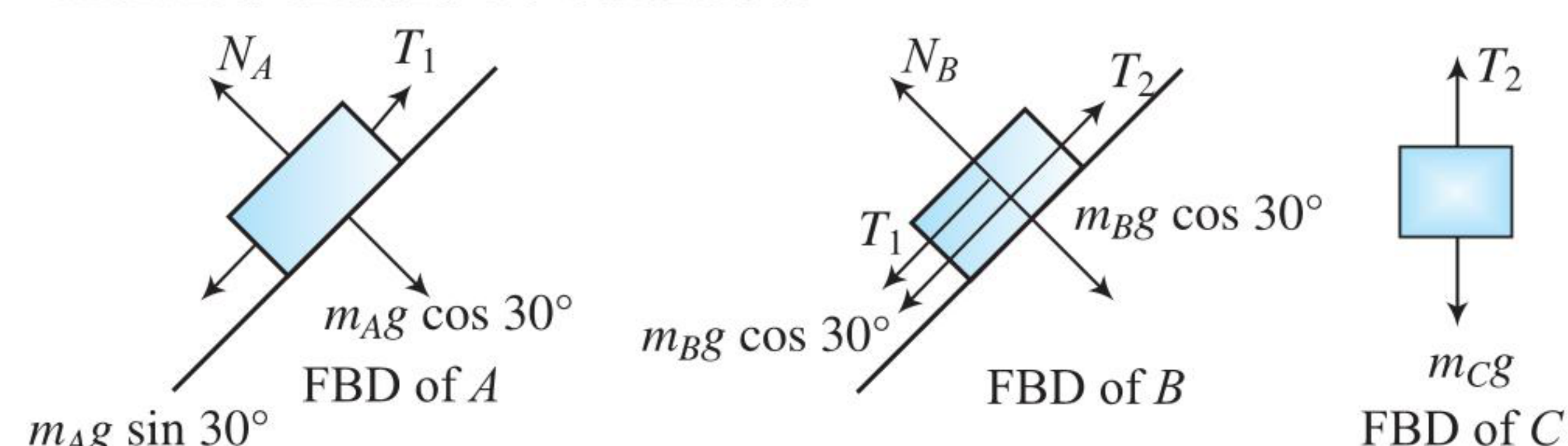
Sol.

(a) From the free-body diagram of A:

T_1 : Tension in string between A and B

N_A : Normal reaction between A and incline

As A is at rest, net force parallel and perpendicular to inclined should be balanced.



$$N_A = m_A g \cos \theta = 10 \cos 30^\circ = 5\sqrt{3} \text{ N}$$

$$T_1 = m_A g \sin \theta = 10 \sin 30^\circ = 5 \text{ N}$$

From the free-body diagram of B:

As B is connected to both strings, two tensions T_1 and T_2 will act on it.

T_2 is the tension (force) of string between B and C acting upwards

T_1 is the tension of string between A and B acting downwards

Balancing forces:

$$N_B = m_B g \cos 30^\circ = 20 \cos 30^\circ = 10\sqrt{3} \text{ N}$$

$$\begin{aligned} T_2 &= T_1 + m_B g \sin 30^\circ = 5 + 20 \sin 30^\circ \\ &= 5 + 20 \times \frac{1}{2} = 15 \text{ N} \end{aligned}$$

Force diagram of C:

T_2 is the pulling force of string on block C , therefore, $m_C g = T_2$, hence $m_C = 1.5 \text{ kg}$

(b) In this case, mass of $C = 3 \text{ kg}$

Let the acceleration of the system be a . We can assume an arbitrary direction of motion. Let the blocks move up the incline and block C move downward.

$$\text{From FBD of A: } T_1 - 10 \sin 30^\circ = 1a \quad \dots(i)$$

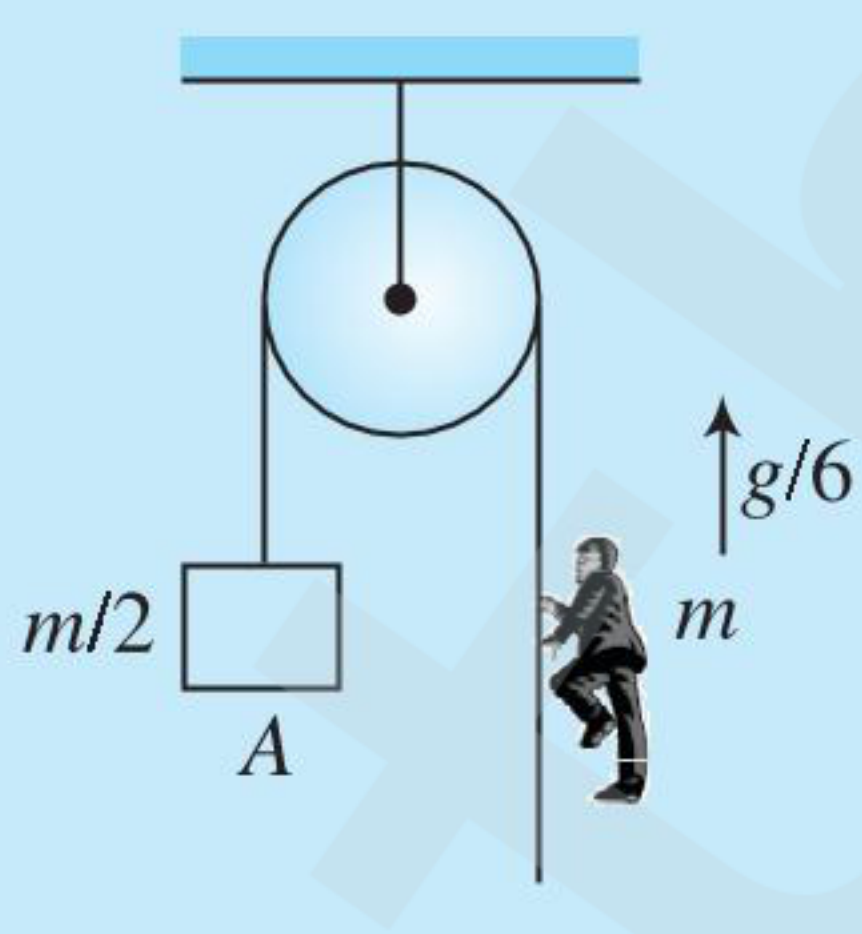
$$\text{From FBD of B: } T_2 - T_1 - 20 \sin 30^\circ = 2a \quad \dots(ii)$$

$$\text{From FBD of C: } 30 - T_2 = 3a \quad \dots(iii)$$

Solving (i), (ii), and (iii), $a = 2.5 \text{ m s}^{-2}$

ILLUSTRATION 6.31

Block A of mass $m/2$ is connected to one end of light rope which passes over a pulley as shown in figure. A man of mass m climbs the other end of rope with a relative acceleration of $g/6$ with respect to rope. Find the acceleration of block A and tension in the rope.



Sol. Let the acceleration of block be (a_0) in upward direction. As the block is tied with the rope, hence the acceleration of the rope connecting the block should also be a_0 (upward). It means the acceleration of the rope on right side of the man with respect to ground be a (upwards).

$$\vec{a}_{\text{man}} = \vec{a}_{\text{man,rope}} + \vec{a}_{\text{rope}} \Rightarrow a = \frac{g}{6} + a_0$$

From the force diagram of man,

$$T - mg = ma = m \left(\frac{g}{6} + a_0 \right) \quad \dots(i)$$

From force diagram of block A ,

$$T - \frac{m}{2}g = \frac{m}{2}a_0 \quad \dots(ii)$$

Subtracting (i) from (ii),

$$\frac{m}{2}g = \frac{m}{2}a_0 - \frac{mg}{6} + ma_0$$

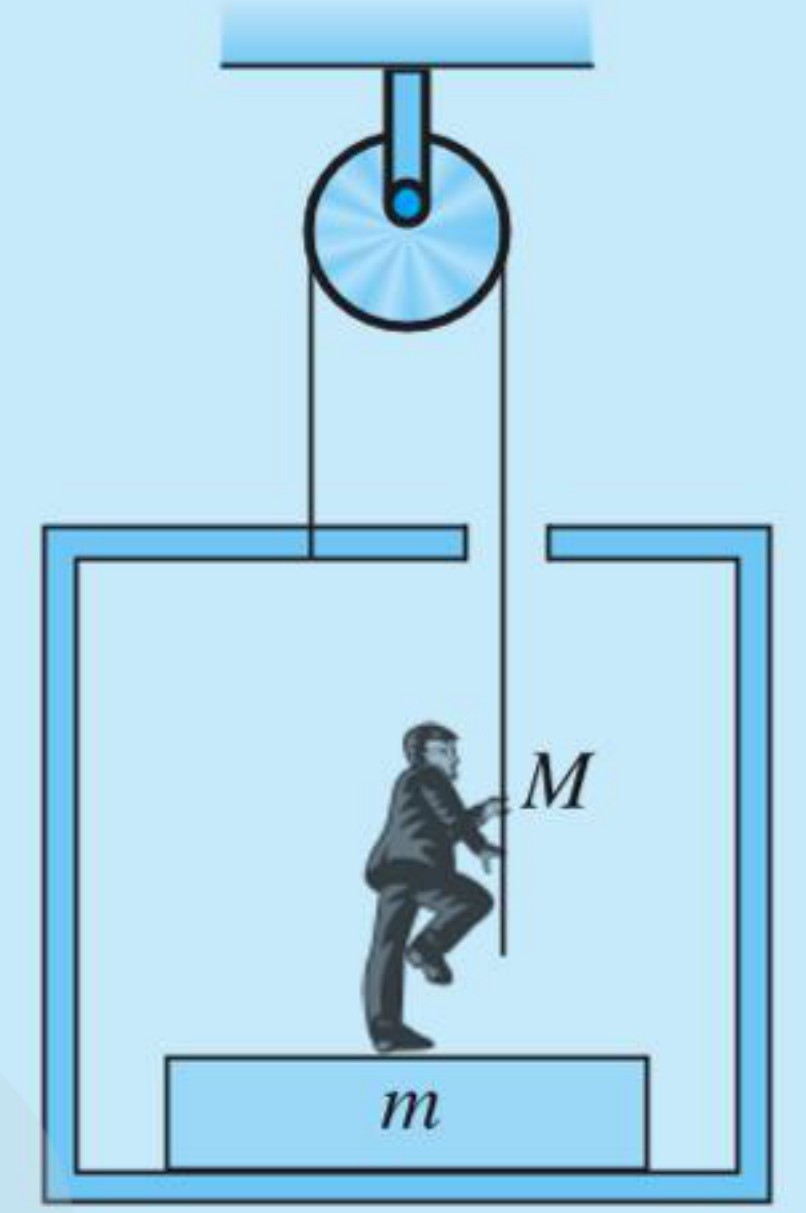
$$\text{or } \frac{3}{2}ma_0 = \frac{mg}{2} + \frac{mg}{6} \Rightarrow a_0 = \frac{4}{9}g$$

$$\text{From (ii), } T = \frac{m}{2}a_0 + \frac{m}{2}g$$

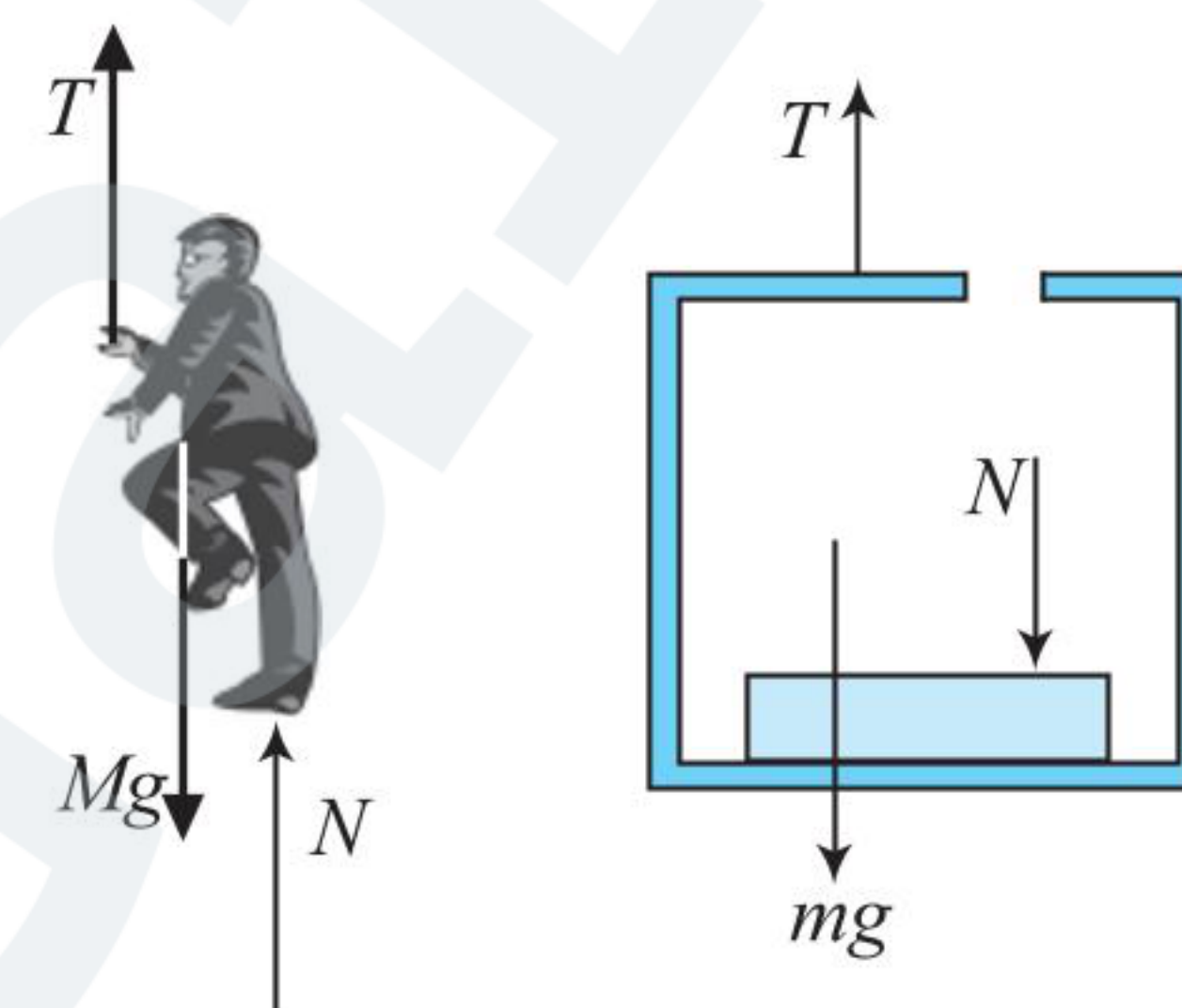
$$= \frac{m}{2} \times \frac{4}{9}g + \frac{m}{2}g \Rightarrow T = \frac{13mg}{18}$$

ILLUSTRATION 6.32

A man of mass M is standing on a plank kept in a box. The plank and box as a whole has mass m . A light string passing over a fixed smooth pulley connects the man and box. If the box remains stationary, find the tension in the string and force exerted by the man on the plank. (Given $M > m$)



Sol. The fixed pulley is taken as frame of reference. The forces on man and box with plank are shown in figure.



The forces are as follows:

(a) Weight of the man = Mg

(b) Tension in the string = T

(c) Normal contact force between the man and the plank = N

(d) The weight of the plank and box = mg

The box remains at rest, the man will have to be at rest

Referring to free-body diagram of man,

the equation of motion of the man is given as

$$T + N = Mg \quad \dots(i)$$

Referring to the free-body diagram of lift,

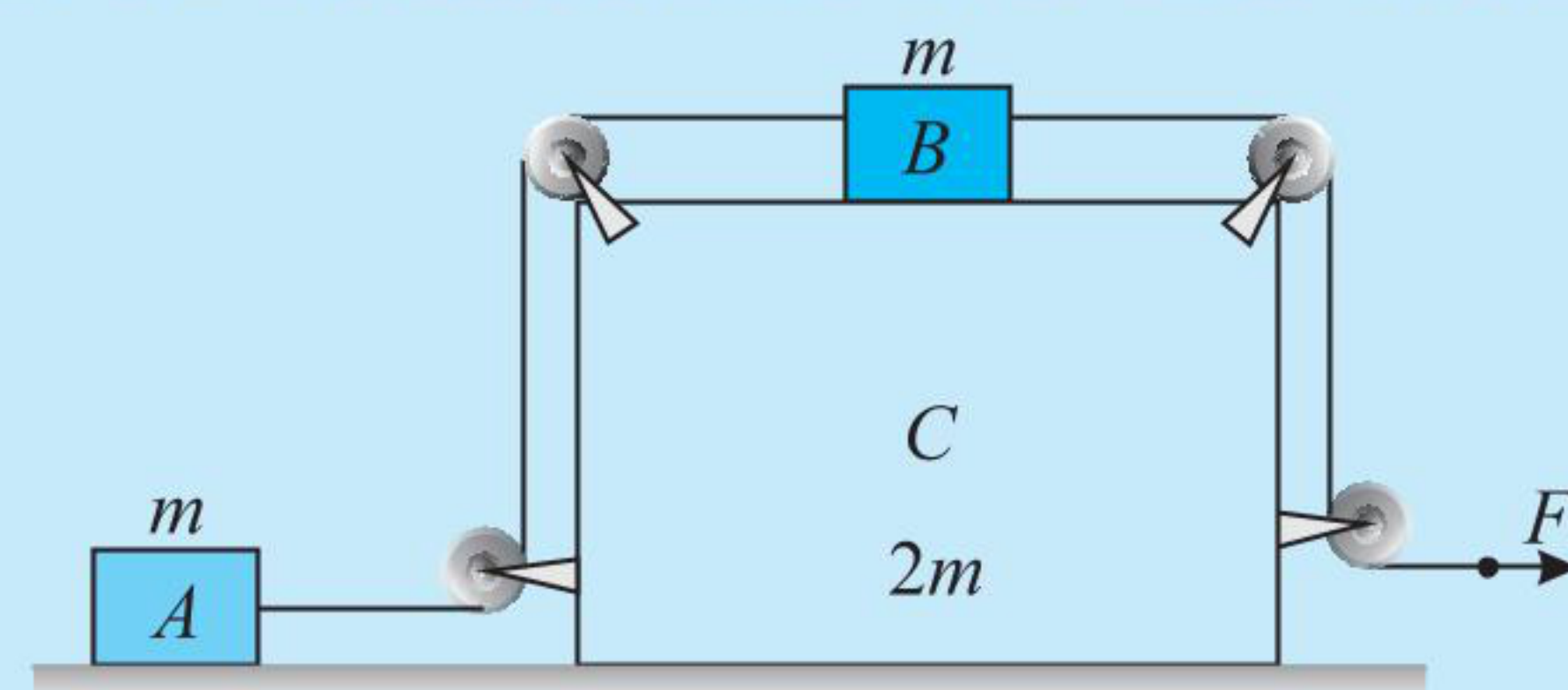
$$T = N + mg \quad \dots(ii)$$

Solving (i) and (ii), we obtain

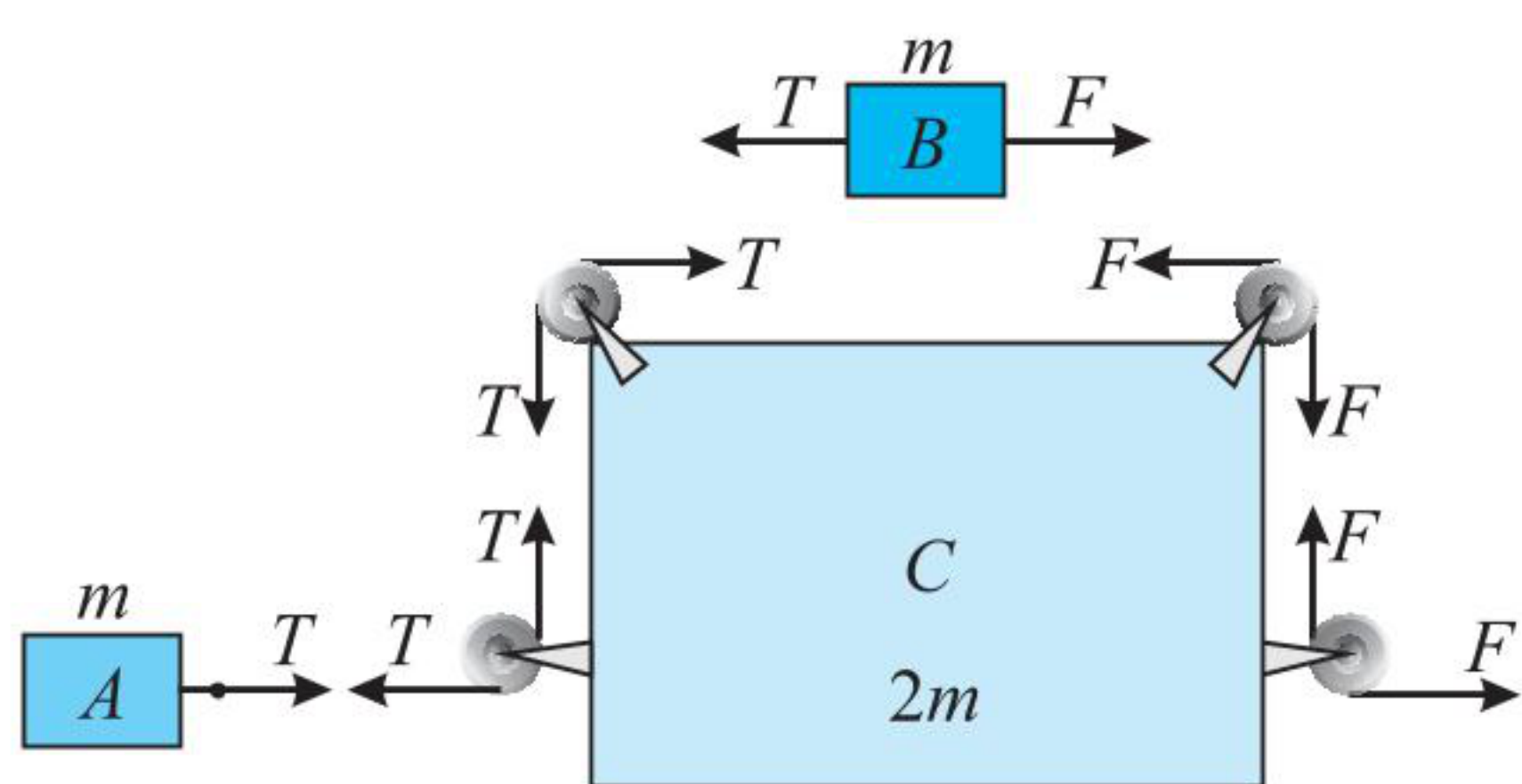
$$T = \frac{(M+m)g}{2} \text{ and } N = \frac{(M-m)g}{2}$$

ILLUSTRATION 6.33

In the system shown in the figure, all surfaces are smooth. Block A and B have mass m each and mass of block C is $2m$. All pulleys are massless and fixed to block C . Strings are light and the force F applied at the free end of the string is horizontal. Find the acceleration of all three blocks.



Sol. Let us draw the F.B.D of the blocks A , B and C . From F.B.D, we can observe the net force on the block ' C ' in horizontal direction is zero. Hence, block ' C ' will not move.



Blocks A and B are connected directly by a string. Hence, move with same acceleration say a

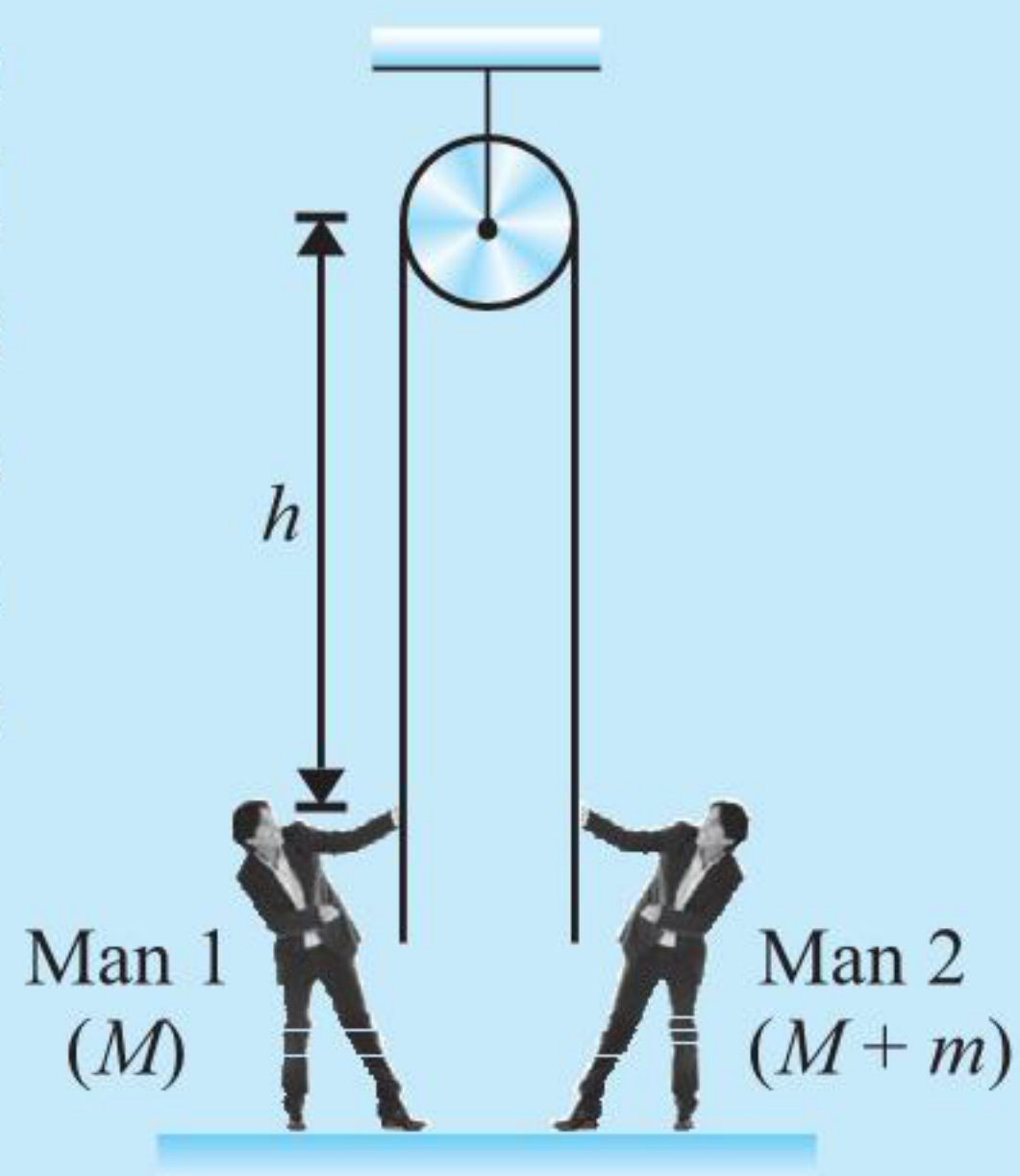
Equation of motion of the block A: $T = ma$... (i)

For block B: $F - T = ma$... (ii)

From (i) and (ii), we get $a = \frac{F}{2m}$

ILLUSTRATION 6.34

Two men of masses M and $M + m$ start simultaneously from the ground and climb with uniform accelerations up from the free ends of a massless inextensible rope which passes over a smooth pulley at a height h from the ground.



- Which man reaches the pulley first?
- If the man who reaches first takes time t to reach the pulley, then find the distance of the second man from the pulley at this instant.

Sol.

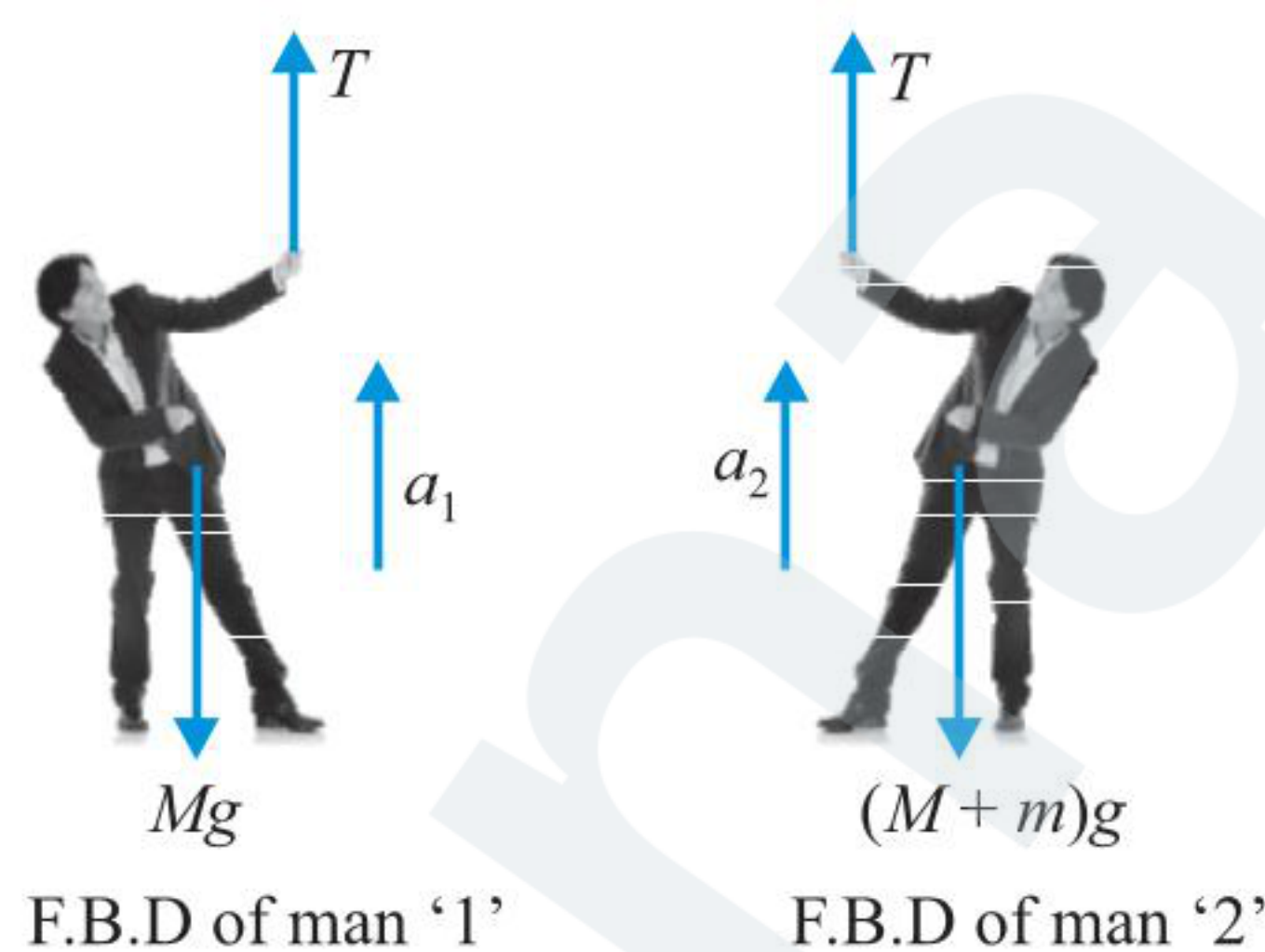
- Let a_1 and a_2 be the accelerations of the two men in upward direction, and T the tension in the rope. Then,

$$T - Mg = Ma_1 \Rightarrow a_1 = \frac{T}{M} - g \quad \dots (i)$$

$$\text{and } T - (M + m)g = (M + m)a_2 \Rightarrow a_2 = \frac{T}{M + m} - g \quad \dots (ii)$$

$$\therefore a_2 < a_1$$

Hence the lighter man will reach the pulley first.



- From (i) and (ii), we can find $a_2 = \frac{Ma_1 - mg}{M + m}$

The lighter man ascends a distance h in time t with acceleration a_1 .

$$\text{Hence, } h = \frac{1}{2} a_1 t^2 \Rightarrow a_1 = \frac{2h}{t^2} \quad \dots (iii)$$

Let s be the distance travelled by the heavier man in this time t . Then

$$\begin{aligned} s &= \frac{1}{2} a_2 t^2 = \frac{t^2}{2} \left[\frac{M}{M + m} a_1 - \frac{mg}{M + m} \right] \\ &= \frac{t^2}{2(M + m)} \left[M \left(\frac{2h}{t^2} \right) - mg \right] = \frac{1}{2(M + m)} [2Mh - mgt^2] \end{aligned}$$

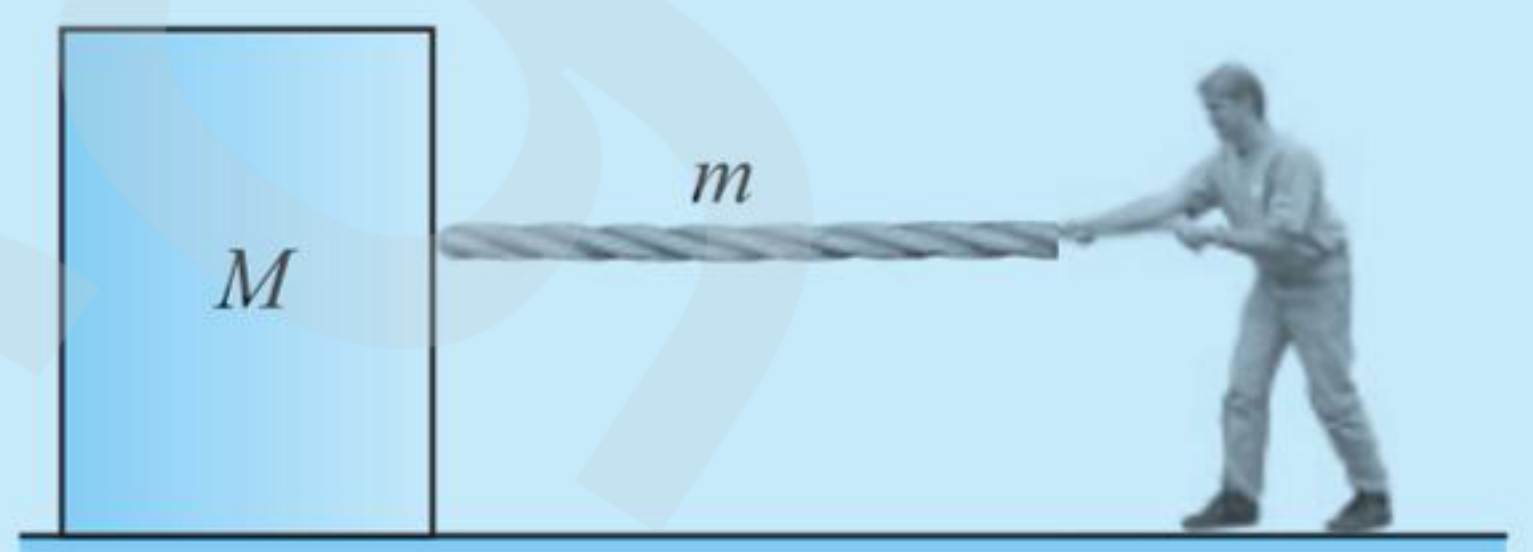
The distance of the second man from the pulley is

$$h - s = \frac{1}{(M + m)} \left[Mh + mh - Mh + \frac{mgt^2}{2} \right] = \frac{m}{(M + m)} \left[\frac{gt^2}{2} + h \right]$$

PROBLEMS OF STRING WITH MASS

ILLUSTRATION 6.35

A block of mass M is being pulled with the help of a string of mass m and length L . The horizontal force applied by the man on the string is F .



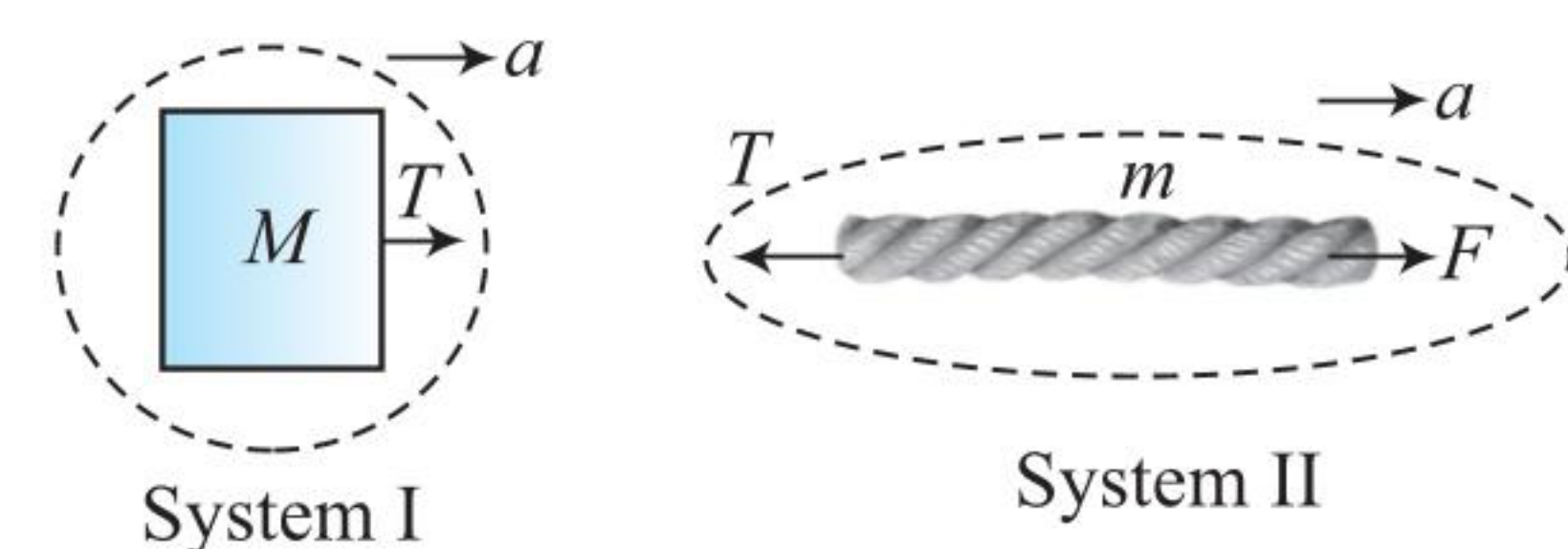
Determine

- Find the force exerted by the string on the block and acceleration of system.
- Find the tension at the mid point of the string.
- Find the tension at a distance x from the end at which force is applied.

Assume that the block is kept on a frictionless horizontal surface and the mass is uniformly distributed in the string.

Sol.

- Let the force applied by string to the block be T . For part (a), consider one system is block and other string. Let the acceleration of the system (block + string) be a .



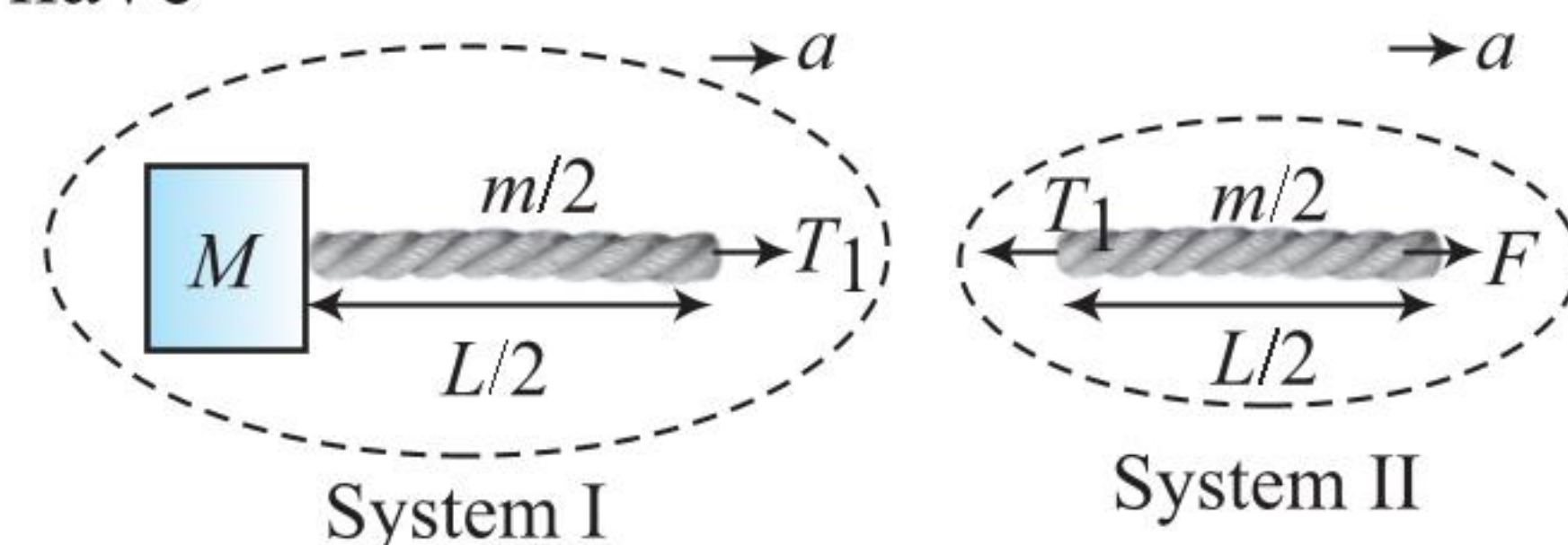
Now we apply Newton's law to each of them.

$$\text{For system II: } F - T = ma \quad \dots (i)$$

$$\text{For system I: } T = Ma \quad \dots (ii)$$

$$\text{After solving (i) and (ii), } a = \frac{F}{M + m}, \quad T = \frac{MF}{M + m}$$

- Now we have to redefine our system. Choose system 1 as block and half string and system 2 as the other half string. On applying Newton's second law to system 1 and system 2, we have



$$\text{System II: } F - T_1 = \frac{m}{2} \times a \quad \dots (iii)$$

Mass per unit length of string = m/L

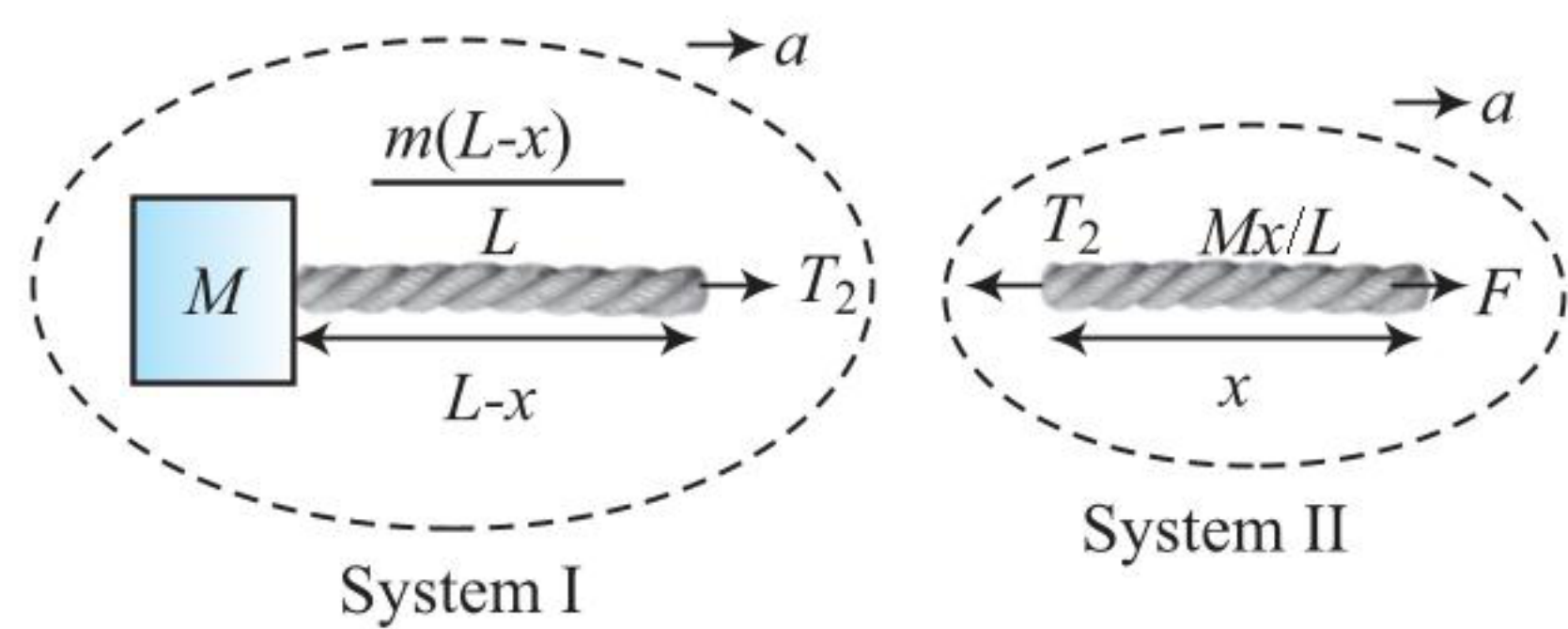
$$\text{Hence, mass of } L/2 \text{ length of string} = \frac{m}{L} \times \frac{L}{2} = \frac{m}{2}$$

$$\text{System I: } T_1 = \left(M + \frac{m}{2} \right) a \quad \dots (iv)$$

After solving, (iii) and (iv), we get

$$a = \frac{F}{M + m}, \quad T_1 = \frac{(M + m/2)F}{(M + m)}$$

- (c) Now we can redefine our system in the block and string of length $L - x$ (system I) and string of length x (system II)



$$\text{System II: } F - T_2 = \left(\frac{m}{L} \times x \right) a \quad \dots(v)$$

$$\text{System I: } T_2 = \left\{ \frac{m}{L} (L - x) + M \right\} a \quad \dots(vi)$$

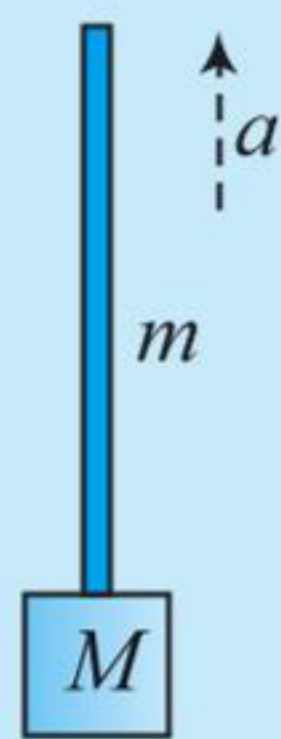
Solving, (v) and (vi), we get

$$a = \frac{F}{M + m} \quad \text{and} \quad T_2 = \frac{\{m(L - x)/L + M\}F}{M + m}$$

Here we see that acceleration in each part is same, but tension changes along the string.

ILLUSTRATION 6.36

A body of mass M is hanging by an inextensible string of mass m . If the free end of the string accelerates up with constant acceleration a , find the variation of tension in the string as a function of the distance measured from the mass M (bottom of the string).



Sol. Let T be the tension in the string at a distance x from its bottom. Referring to the FBD, we have the following equations:

Force equation for m_1 :

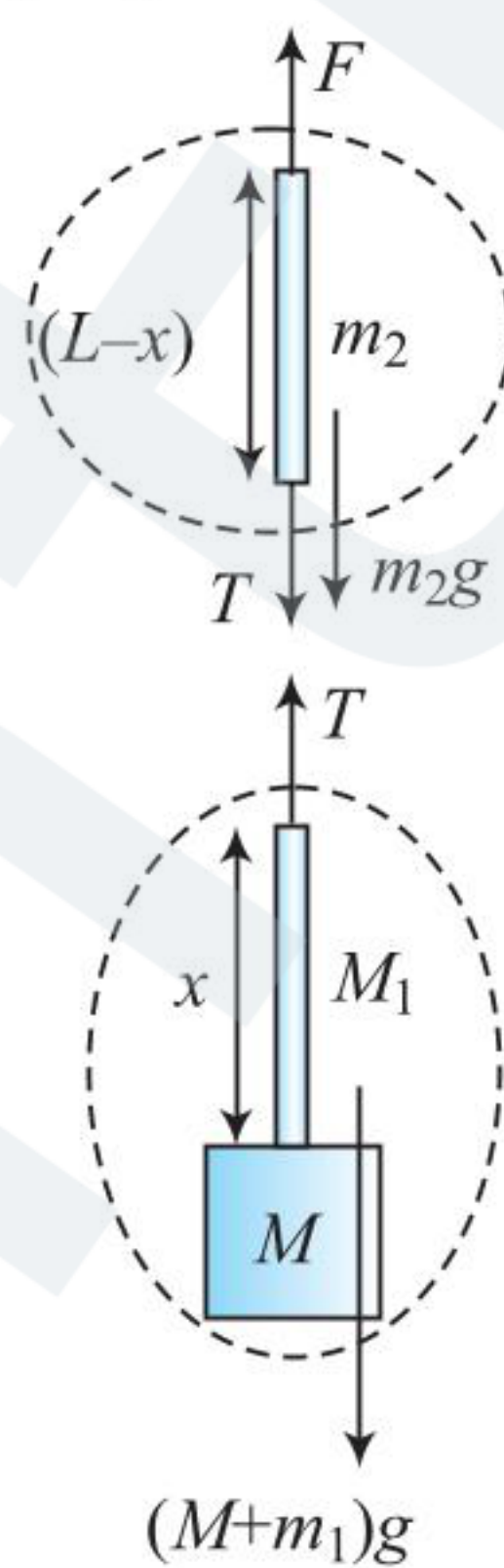
$$T - m_1 g - Mg = (M + m_1)a \quad \dots(i)$$

We are writing equation of motion for lower part as we are not given the force acting on top of the string

Substituting $m_1 = \frac{m}{l}x$ in equation (i), we have

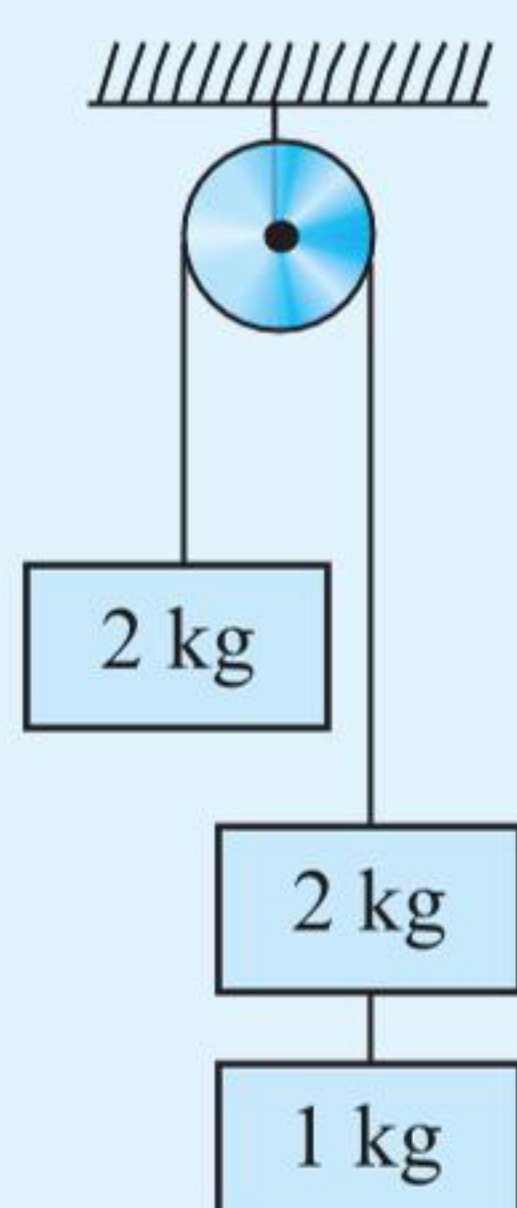
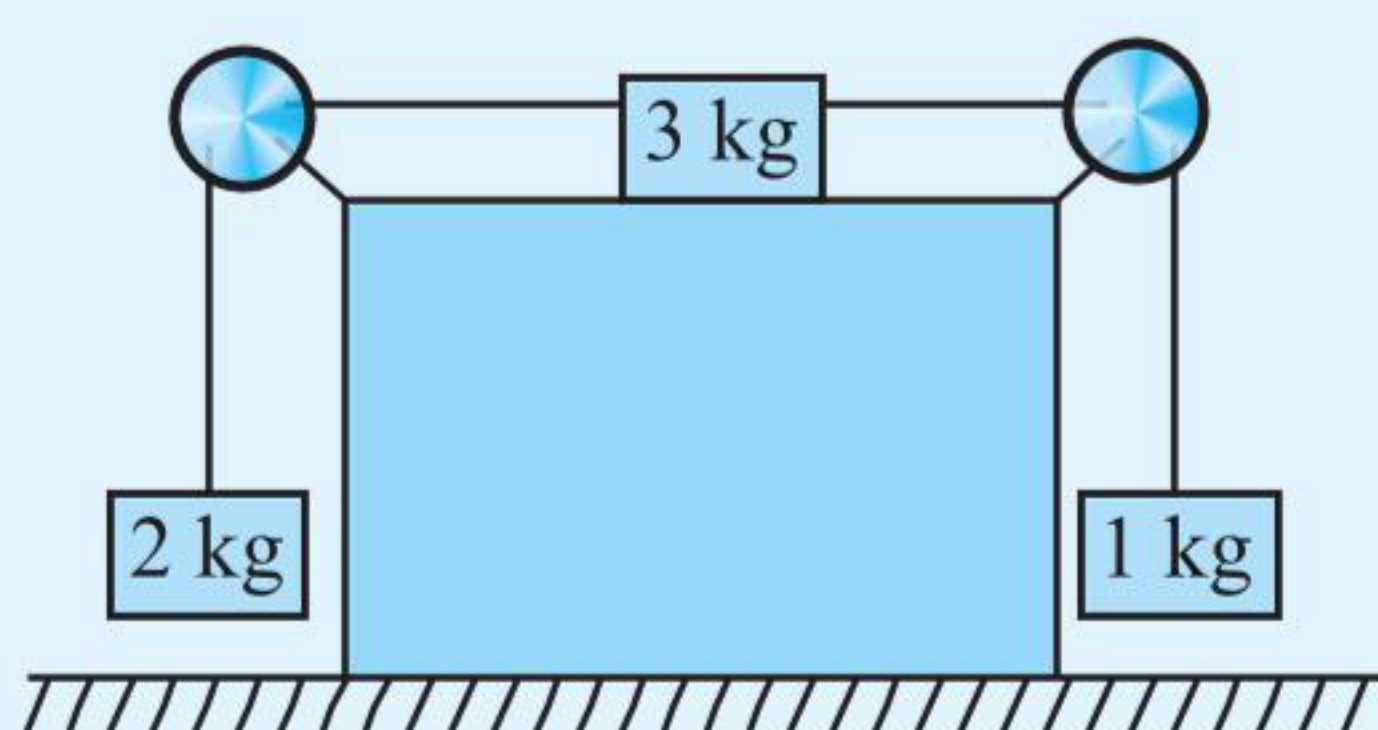
$$T = \left(M + \frac{mx}{l} \right) (g + a).$$

At $x = l$, $T = (M + m)(g + a) = F(\text{say})$

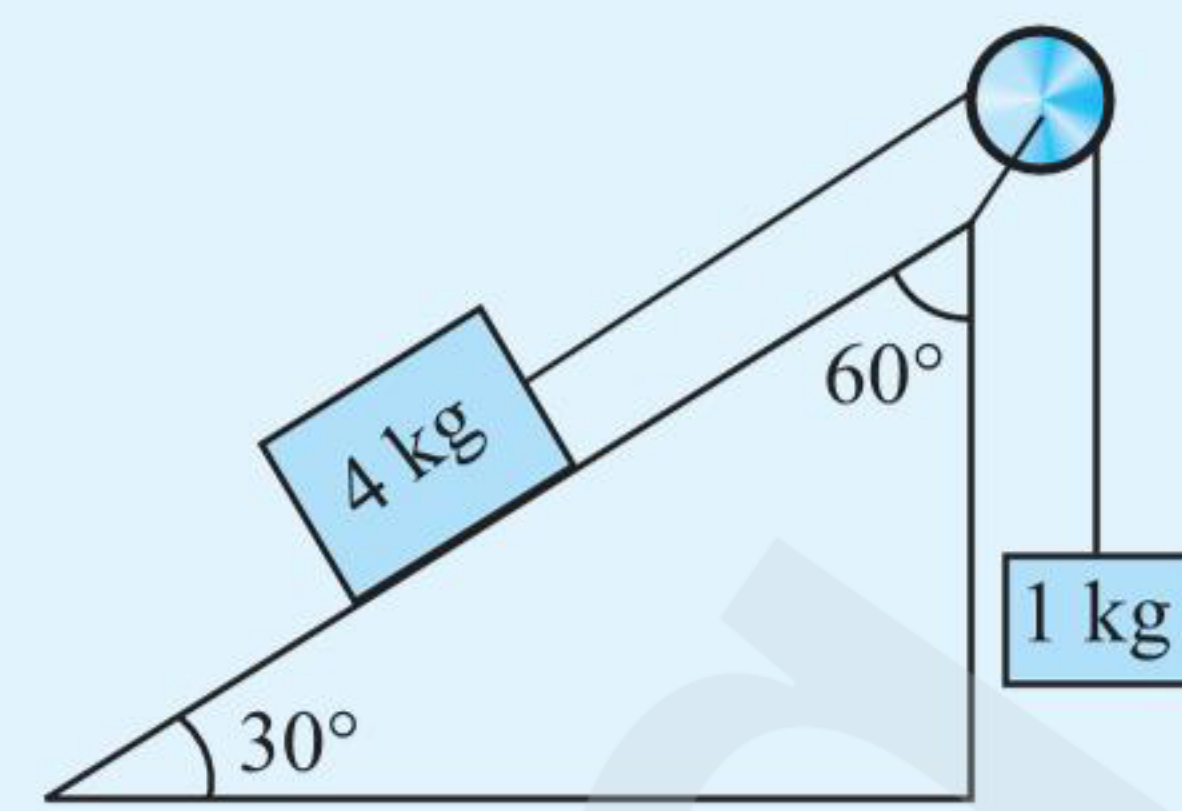


CONCEPT APPLICATION EXERCISE 6.4

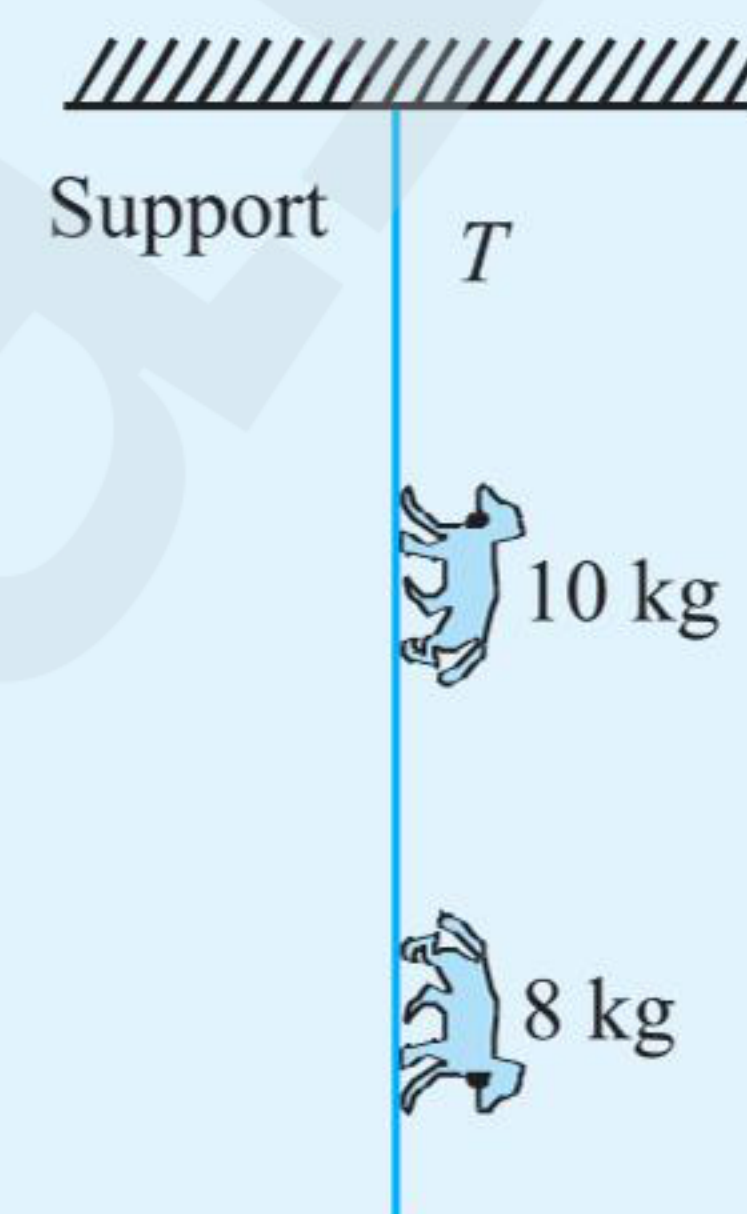
- Consider the system shown in figure. The system is released from rest, find the tension in the cord connected between 1 kg and 2 kg blocks.
- The system shown in figure is released from rest. Calculate the tension in the strings and the force exerted by the strings on the pulleys, assuming pulleys and strings are massless.



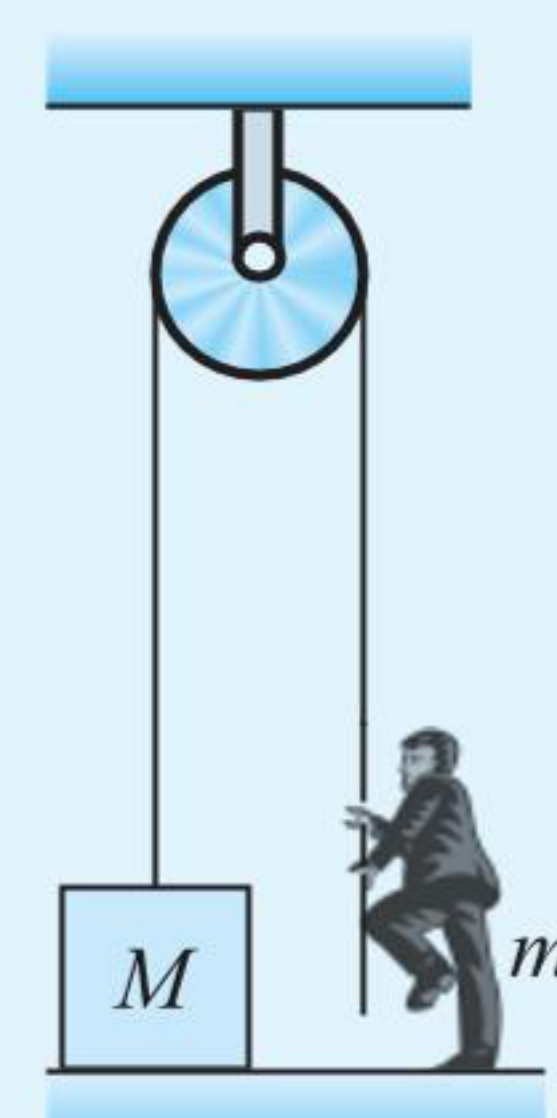
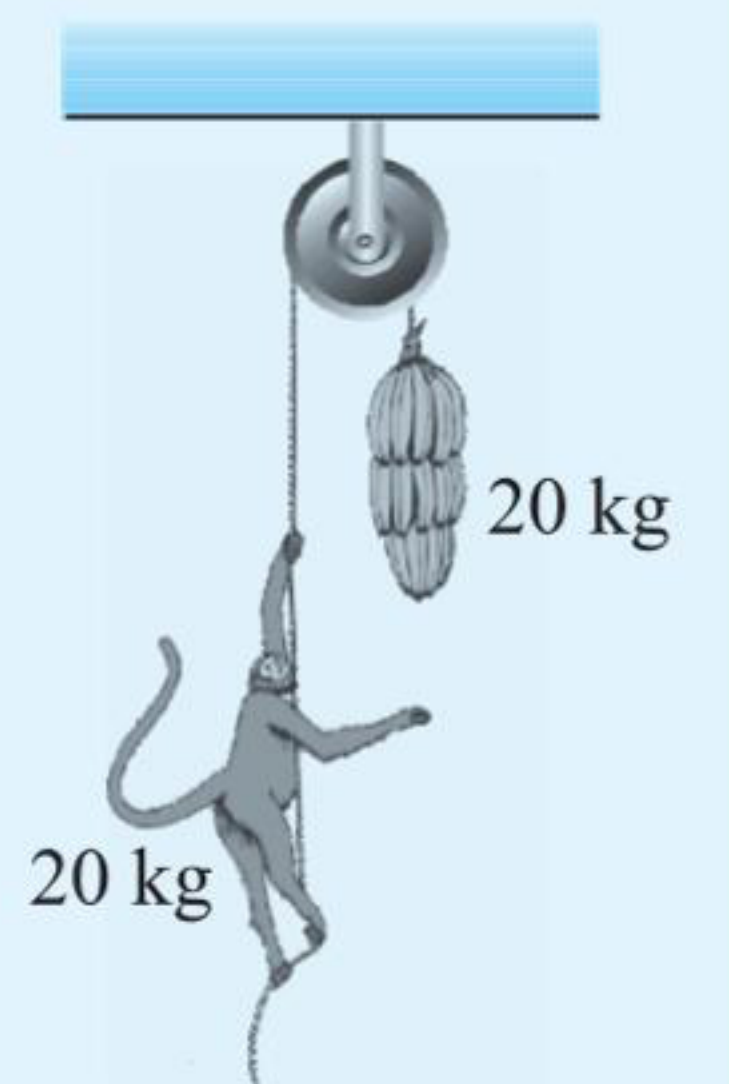
- Find the acceleration of blocks and tension in the cord in the device shown in figure. Assume no friction anywhere.



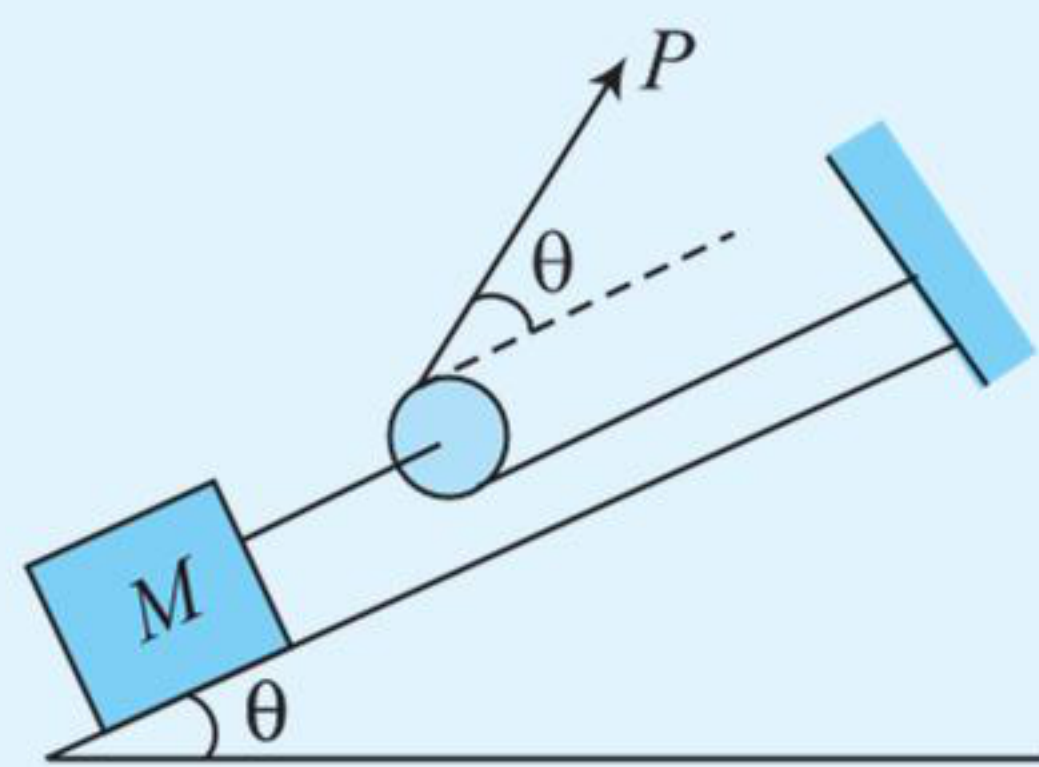
- Two monkeys of masses 10 kg and 8 kg are moving along a vertical rope as shown in figure. The former climbing up with an acceleration of 2 m s^{-2} , while the latter coming down with a uniform velocity of 2 m s^{-1} . Find the tension in the rope at the fixed support.



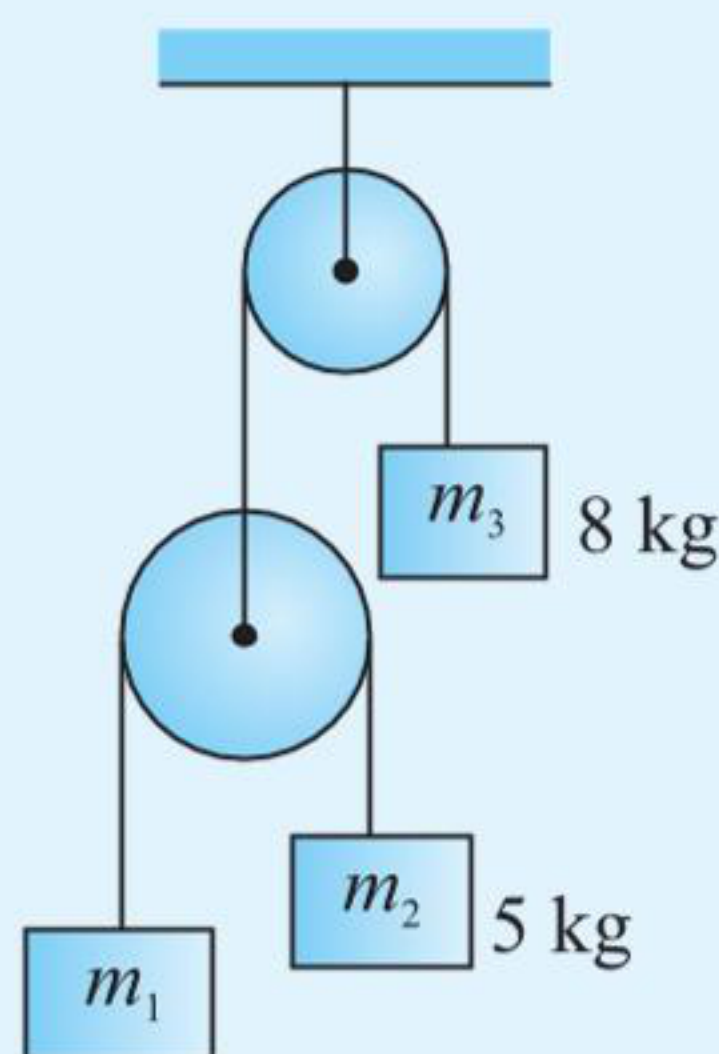
- A homogeneous rod of length L is acted upon by two forces F_1 and F_2 applied to its ends and directed opposite to each other. With what force F will the rod be stretched at the cross section at a distance l from the end where F_1 is applied?
- A 20-kg monkey has a firm hold on a light rope that passes over a frictionless pulley and is attached to a 20-kg bunch of bananas (as shown in figure). The monkey looks up, sees the bananas, and starts to climb the rope to get them.
 - As the monkey climbs, do the bananas move up, down, or remain at rest?
 - As the monkey climbs, does the distance between the monkey and the bananas decrease, increase, or remain constant?
 - The monkey releases its hold on the rope. What happens to the distance between the monkey and the bananas while it is falling?
 - Before reaching the ground, the monkey grabs the rope to stop its fall. What do the bananas do?
- In the given figure, the block of mass M is at rest on the floor. At what acceleration with which should a boy of mass m climb along the rope of negligible mass so as to lift the block from the floor?



8. What should be the minimum force P to be applied to the string so that block of mass m just begins to move up the frictionless plane?



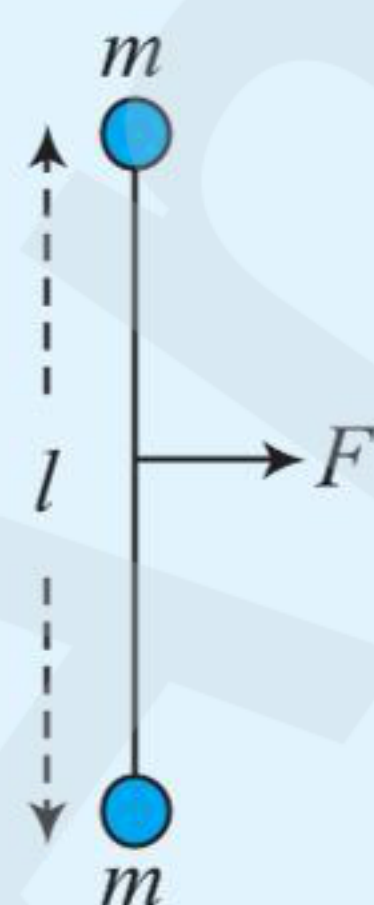
9. Three blocks m_1 , m_2 , and m_3 are arranged as shown in figure. If $m_2 = 5$ kg and $m_3 = 8$ kg, at what value of m_1 will 8 kg mass be at rest?



10. In the given figure, the man and the platform together weigh 950 N. The pulley can be treated as frictionless. Determine how hard the man has to pull on the rope to lift himself upward above the ground with constant velocity. If the weight of man is 550 N, what is the normal reaction between them?



11. A locomotive accelerates a train of identical railway carts. The carts are numbered consecutively with the cart next to locomotive having the number 1. The tension in the connection between the carts with numbers 4 and 5 is three times bigger than the tension in the connection between the carts with numbers 14 and 15. What is the number of the last cart? There is no resistance to the train's motion.
12. Two identical small masses each of mass m are connected by a light inextensible string on a smooth horizontal floor. A constant force F is applied at the mid point of the string as shown in figure. Find the acceleration of each mass towards each other.



13. A body hangs from a spring balance supported from the roof of an elevator.
- If the elevator has an upward acceleration of 2.45 m s^{-2} and the balance reads 50 N, what is the true weight of the body?
 - Under what circumstances will the balance read 30 N?
 - What will be the balance reading if the elevator cable breaks?

ANSWERS

1. 8 N

2. $T_1 = \frac{7g}{6} \text{ N}; T_2 = \frac{5g}{3} \text{ N}$

Force on pulley $P_1 = \sqrt{2}T_1$

Force on pulley $P_2 = \sqrt{2}T_2$

3. 2 ms^{-2} , 12 N

4. 200 N

5. $\frac{(F_2 - F_1)l}{L} + F_1$

6. (a) The bananas move up.

(b) Distance between them stays the same.

(c) Distance between them does not change.

(d) The bananas will slow down at the same rate as the monkey; if the monkey comes to a stop, so will the bananas.

7. $a > \left(\frac{M}{m} - 1\right)g$

8. $\frac{mg \sin \theta}{1 + \cos \theta}$

9. $\frac{10}{3} \text{ kg}$

10. 950 N, 1500 N

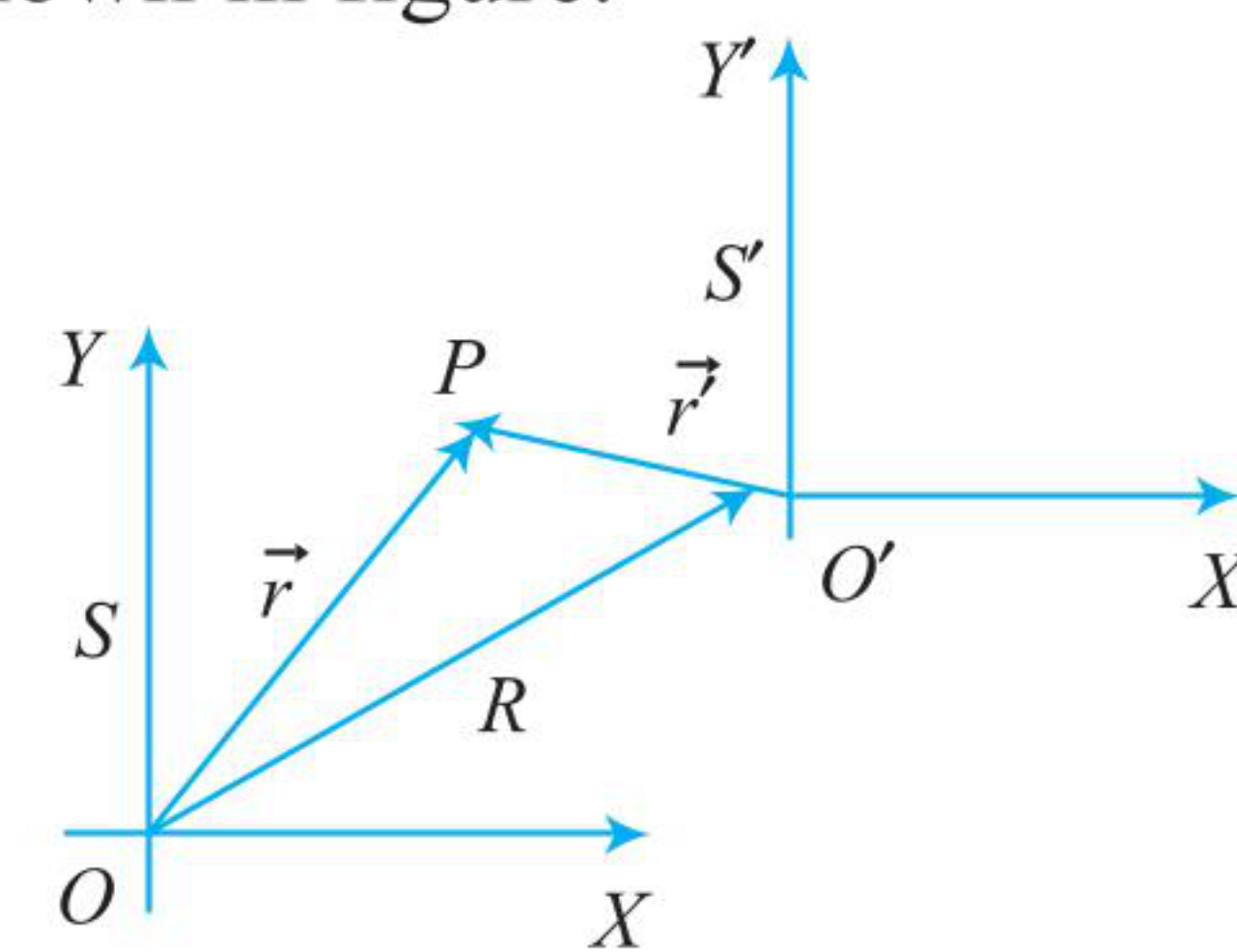
11. 19

12. $\frac{F}{2m} \tan \theta$

13. (a) 40 N (b) $a = -\frac{g}{4}$ (c) 0 N

NON-INERTIAL FRAME OF REFERENCE AND PSEUDO (FICTITIOUS) FORCE

A coordinate system whose origin either accelerates or rotates is called a non-inertial frame of reference. The motion of a particle (P) is studied from two frames of references, S and S' . S is an inertial frame of reference, and S' is a non-inertial frame of reference. At any time, the position vectors of the particle with respect to those two frames are $\vec{r}$ and $\vec{r}'$, respectively. At the same moment, the position vector of the origin of S' is $\vec{R}$ with respect to S as shown in figure.



From the vector triangle $OO'P$, we get $\vec{r}' = \vec{r} - \vec{R}$

Differentiating this equation twice with respect to time, we get

$$\frac{d^2 \vec{r}'}{dt^2} = \frac{d^2 (\vec{r})}{dt^2} - \frac{d^2 (\vec{R})}{dt^2} \Rightarrow \vec{a}' = \vec{a} - \vec{A}$$

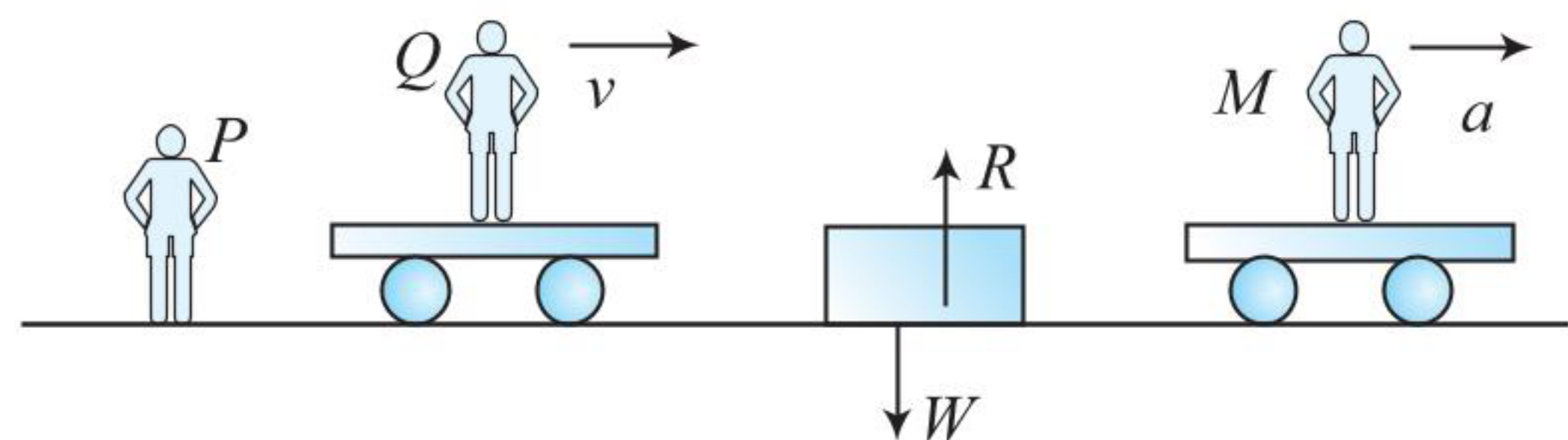
Here $\vec{a}'$ is the acceleration of the particle P relative to S' , $\vec{a}$ is the acceleration of the particle relative to S , and $\vec{A}$ is the acceleration of S' relative to S .

Multiplying the above equation by m (mass of the particle), we get $m\vec{a}' = m\vec{a} - m\vec{A}$

$$\Rightarrow \vec{F}' = \vec{F}_{(\text{real})} - m\vec{A} \Rightarrow \vec{F}' = \vec{F}_{(\text{real})} + (-m\vec{A})$$

We can observe an additional force $(-m\vec{A})$ apart from real force, that is acting on the particle as seen from an observer observing from reference frame S' . Let us learn about this additional force through an example

Suppose a box is placed on a railway platform. The only forces acting on it are its weight W acting downwards and the normal reaction R acting upwards. W and R are thus equal and opposite. Thus, the net external force acting on the box is zero. The box is at rest.



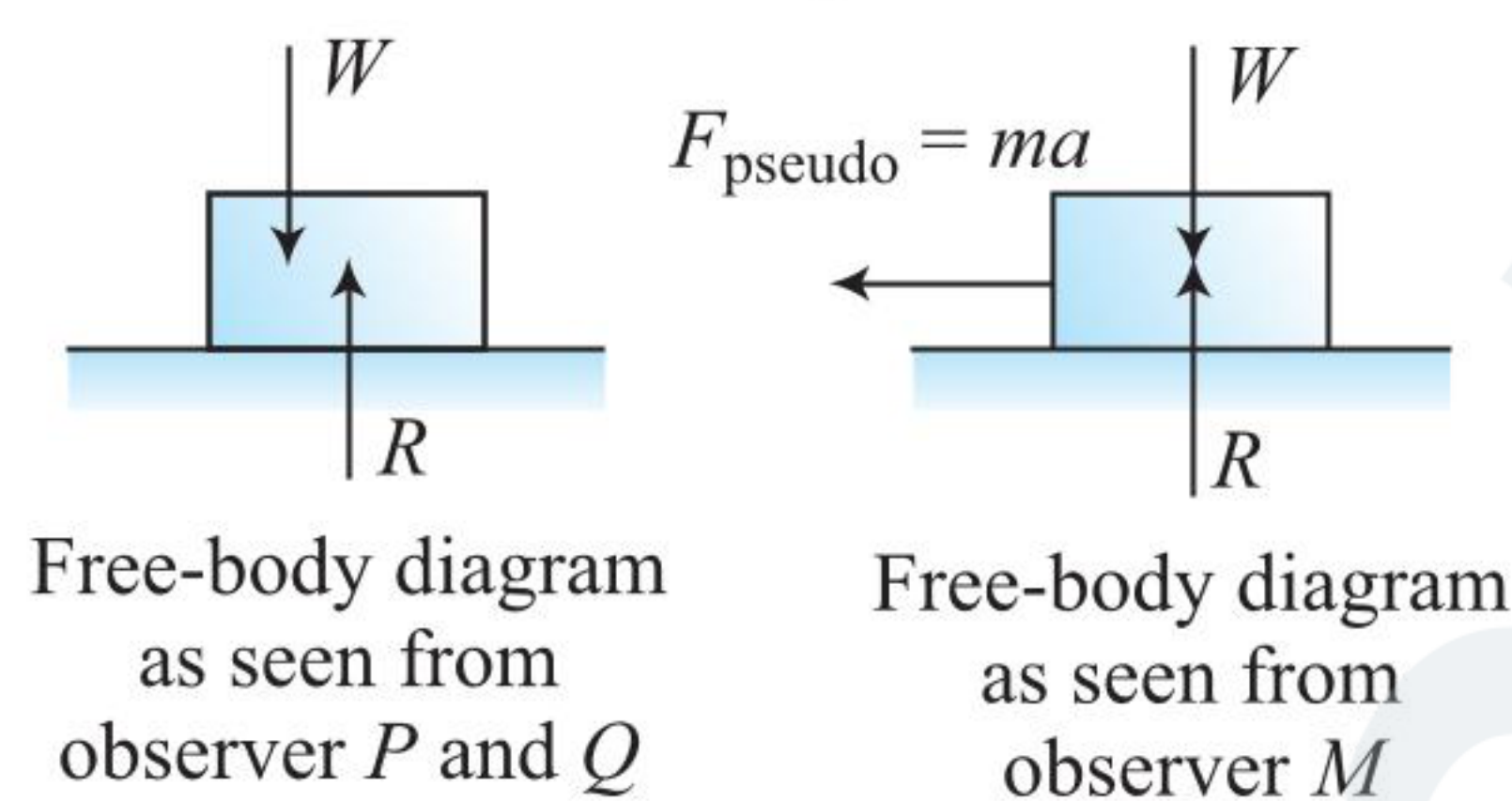
Let us consider the forces acting on this box and its state with respect to three observers, namely,

- A person P standing on the platform.
- A person Q sitting in a train moving with uniform velocity in a straight line parallel to the platform.
- A person M sitting in a train moving parallel to the platform but with some acceleration.

The box is at rest w.r.t. observer P . The box is in uniform motion in a straight line w.r.t. observer Q (moving with $-v$ velocity).

The box is having (negative) acceleration w.r.t. the observer M . For all the three observers (or frames of reference), the force acting on the box are the same: W and R . As W and R are equal and opposite, the net external force acting on the box is zero.

Let us reconsider the example of the box lying at rest on a railway platform. We saw that it has $-\vec{a}$ acceleration w.r.t. a train which is moving with an acceleration $\vec{a}$ even though no net external material force is acting on it. Neither first nor second law remains valid on the box with respect to the accelerating train.

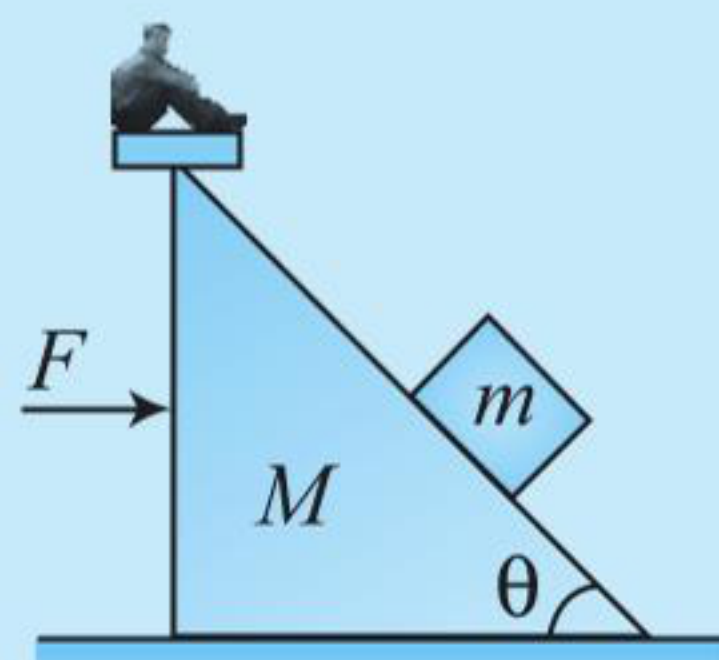


But still, if we want to apply Newton's laws of motion in a non-inertial reference frame, we are allowed to do so, provided we include an additional force, the pseudo force, in the free-body diagram.

If we apply an imaginary force ma in the direction opposite to the direction of motion of observer M , the observer M can write the equation of motion of box. Thus, Newton's laws can also be applied with respect to a non-inertial reference frame, provided we include an extra force $-ma$ on the system. This force has no existence in reality but has been included only to suit the calculations involved, by Newton's second law, while working out a problem w.r.t. a non-inertial reference frame. This imaginary force is known as "pseudo force" or "fictitious force" or "inertial force." (The term "pseudo" means something which is not real.)

ILLUSTRATION 6.37

The block of mass m is in equilibrium relative to the smooth wedge of mass M which is pushed by a horizontal force F . Find pseudo force acting on (a) m , (b) M as viewed by the observer sitting on the wedge. Will these pseudo forces (c) equal and opposite, action-reaction pairs? Explain.



Sol. Taking block and wedge as system.

Net force on the system horizontally = F

Total mass = $M + m$

Therefore, acceleration of the system in +ve x -direction

$$a = \frac{F}{M + m} \Rightarrow \vec{a} = \frac{F}{M + m} \hat{i}$$

- (a) Pseudo force acting on block m is in opposite direction of observer acceleration and magnitude equal to multiplication of mass of the object and the acceleration of the observer

$$\vec{F}_{PS} = -ma\hat{i} = -\frac{mF}{M + m} \hat{i}$$

- (b) Pseudo force acting on block M $\vec{F}'_{PS} = -Ma\hat{i}$

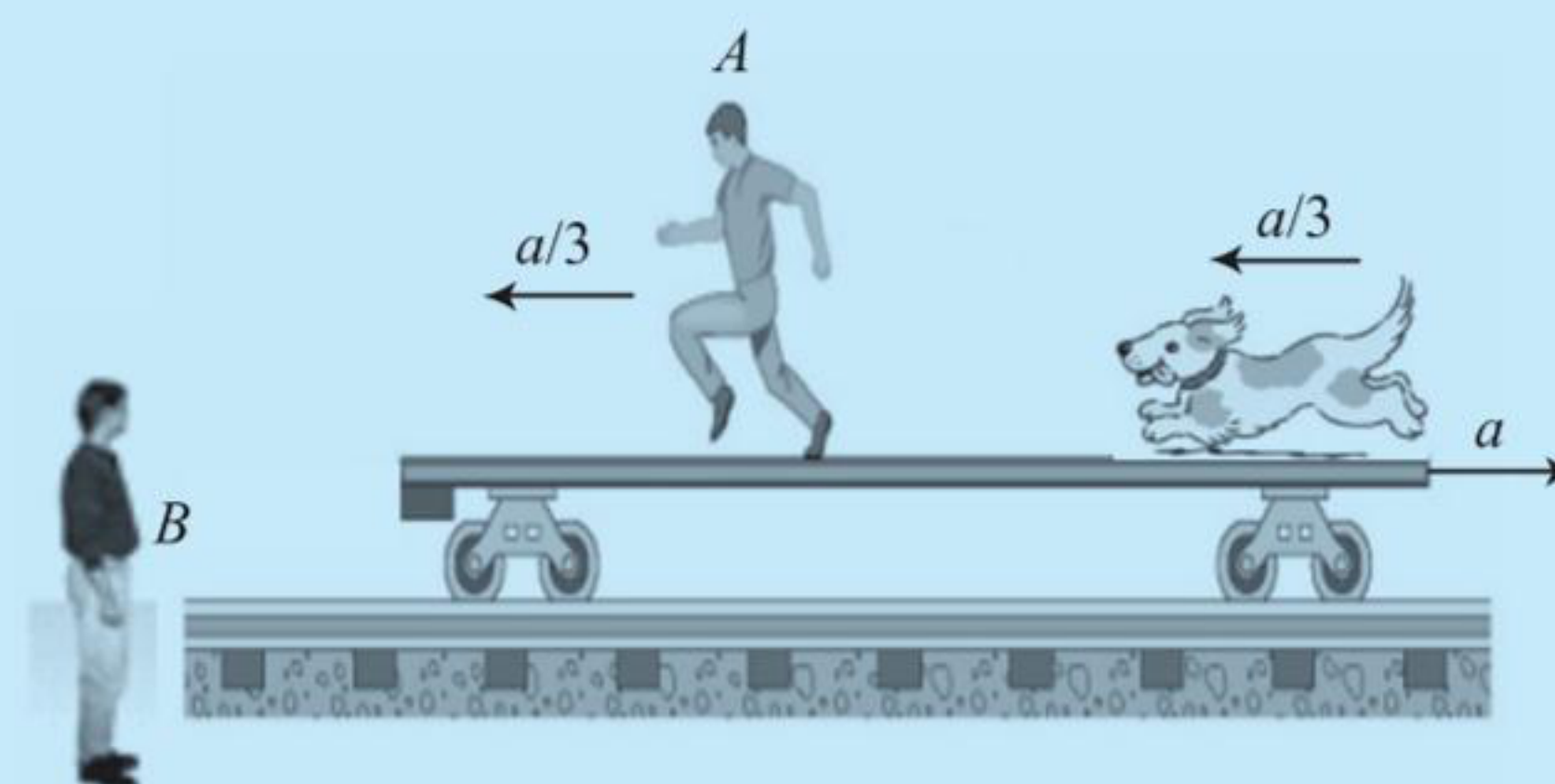
$$\vec{F}'_{PS} = -\frac{MF}{M + m} \hat{i}$$

- (c) Pseudo force is not a real force. It is applied to solve the problems of non-inertial frame. It does not have action-reaction pairs.

ILLUSTRATION 6.38

A rail-road car is moving towards right with acceleration a . A man A accelerating toward left with an acceleration of magnitude $a/3$ w.r.t. to car. A dog of mass m is following man A with an acceleration $a/3$ relative to the car. Observer B on ground is observing the dog and man A . Find the

- net force experienced by the dog as seen by observer B standing on ground.
- rate of change of linear momentum of the dog relative to the man A moving on trolley.
- pseudo-force on the dog as seen from man A .



Sol.

- (a) The net force acting on dog, as seen from observer 'B' on ground is

$$\vec{F} = m \vec{a}_{\text{dog, ground}} = m \vec{a}_d \quad \dots(i)$$

Acceleration of dog = Acceleration of dog w.r.t. car + Acceleration of car

$$\Rightarrow \vec{a}_d = \vec{a}_{d,c} + \vec{a}_c = -\frac{a}{3} \hat{i} + a \hat{i} = \frac{2a}{3} \hat{i}$$

$$\text{Hence, from (i), } \vec{F} = \frac{2ma}{3} \hat{i}$$

- (b) The rate of change of momentum of the dog relative to the man A $\frac{d\vec{p}}{dt} = m\vec{a}_{d,A}$ (ii)

$$\text{where } \vec{a}_{d,A} = \vec{a}_{d,c} - \vec{a}_{A,c} = -\frac{a}{3} \hat{i} - \left(-\frac{a}{3} \hat{i}\right) = 0$$

$$\text{Hence, from (ii) } \frac{d\vec{p}}{dt} = 0$$

(c) Pseudo-force acting on dog viewed by the man A is

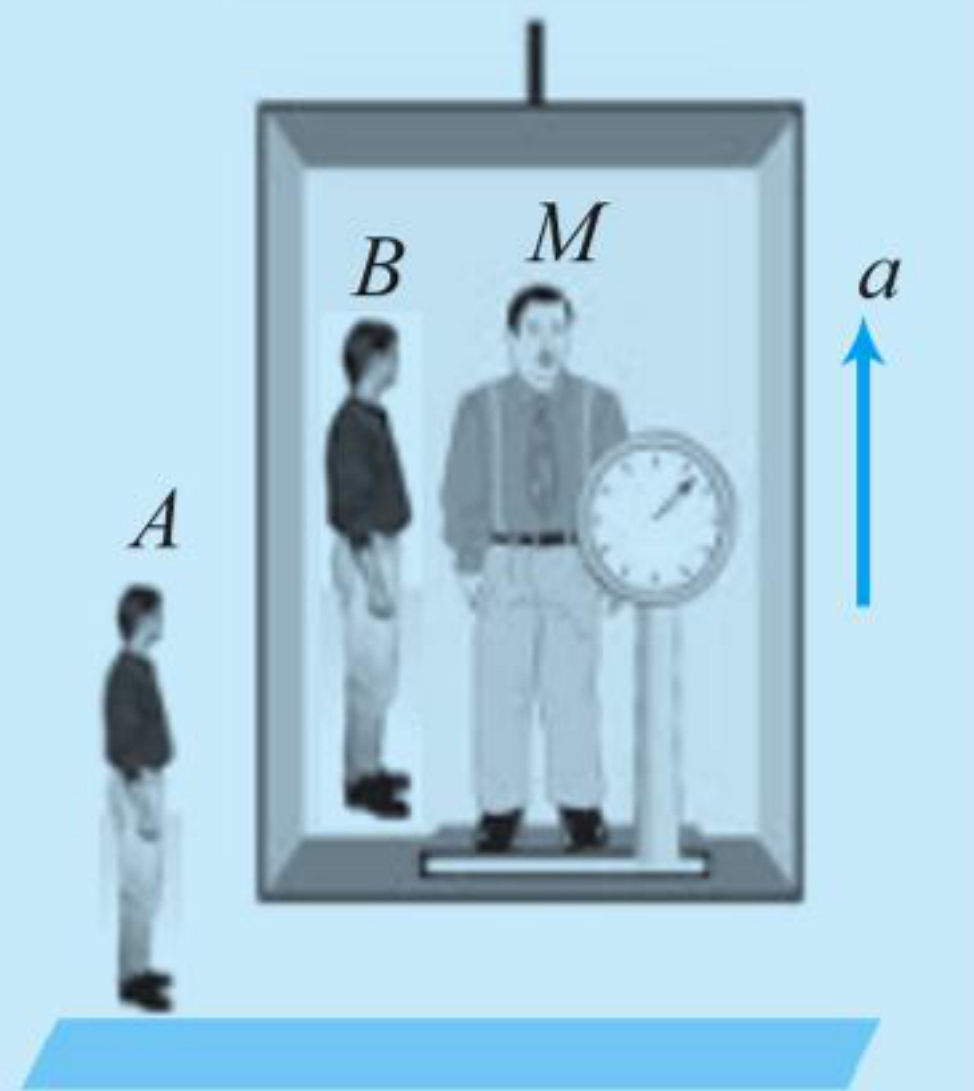
$$\vec{F}_{\text{pseudo}} = -m\vec{a}_{A, \text{ground}}$$

$$\text{where } \vec{a}_{A, \text{ground}} = \vec{a}_{A,C} + \vec{a}_C = -\frac{a}{3}\hat{i} + a\hat{i} = \frac{2a}{3}\hat{i}$$

$$\text{This gives } \vec{F}_{\text{pseudo}} = -\frac{2ma}{3}\hat{i}$$

ILLUSTRATION 6.39

A man of mass M stands on a weighing machine in an elevator accelerating upwards with an acceleration a . Draw the free-body diagram of the man as observed by the observer A (stationary on the ground) and observer B (stationary on the elevator). Also, calculate the reading of the weighing machine.



Sol.

Method 1: Observation from observer A on ground.

From FBD of man w.r.t. A , using Newton's second law,

$$N - Mg = Ma \Rightarrow N = M(g + a)$$

N is the reading of weighing machine.

Method 2: Observation from observer B on elevator. The observer ' B ' is moving in accelerated frame of reference. If he observes the man on weighing machine he will observe the man at rest w.r.t. elevator. If he is asked to draw a free-body diagram of M he will apply pseudo force on M apart from real forces acting on him.

From free-body diagram:

$$N = Mg + ma \Rightarrow N = M(g + a)$$

The value of N calculated from both the methods is same.

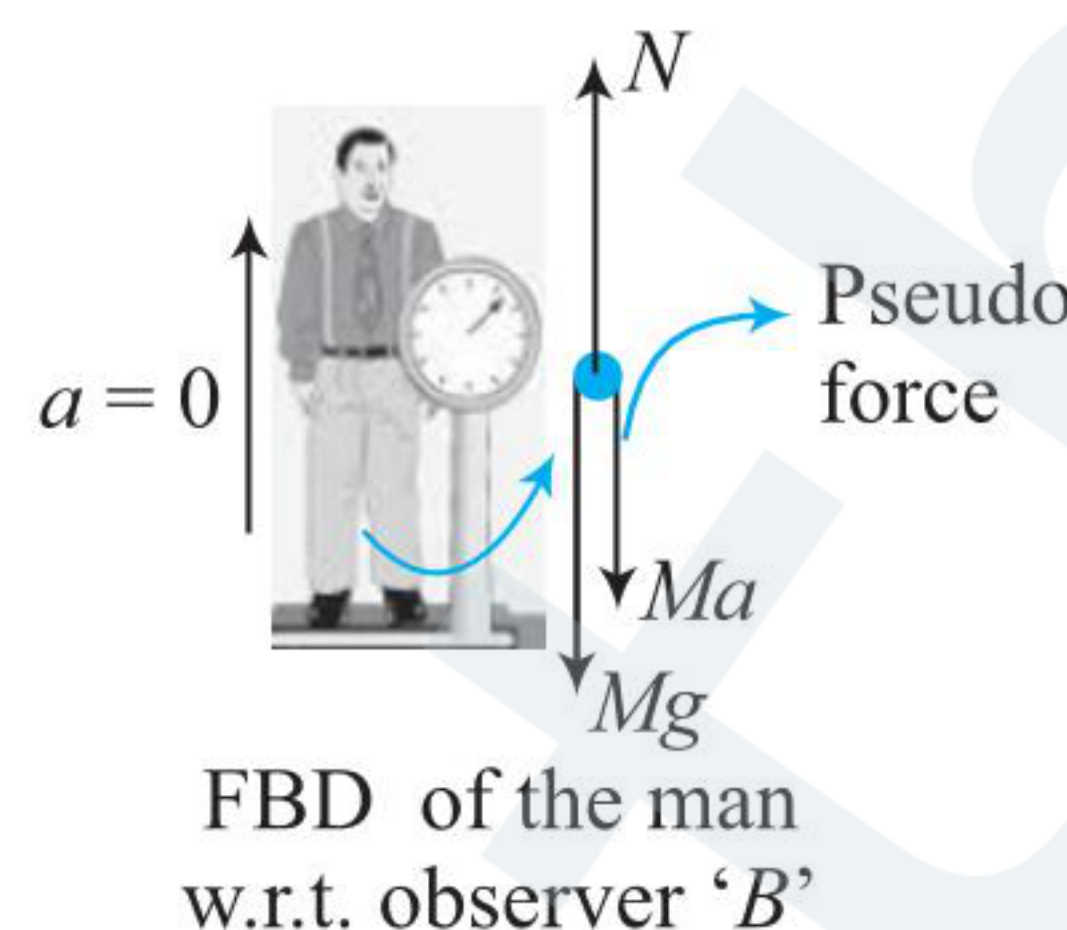
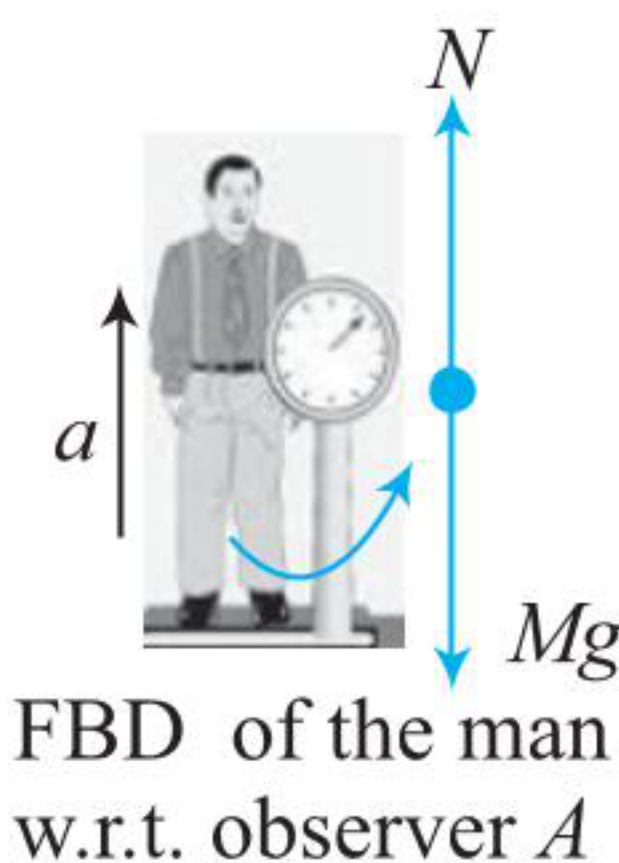
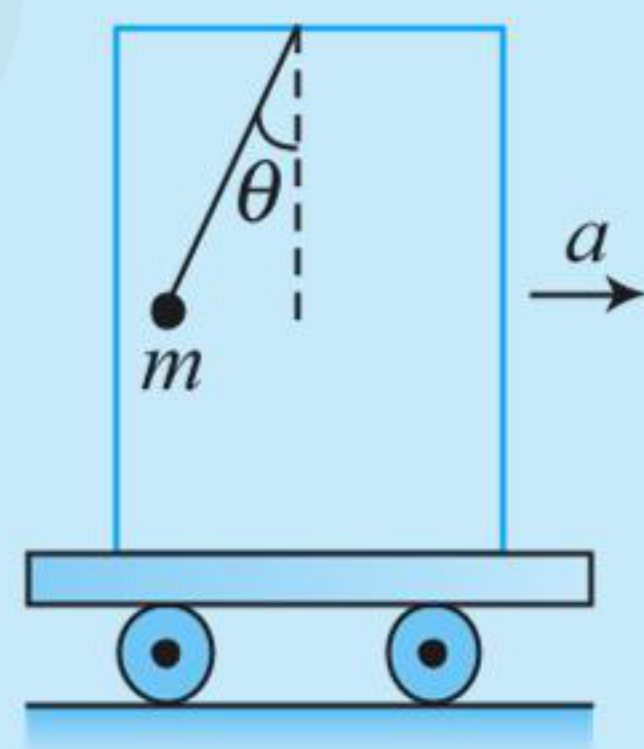


ILLUSTRATION 6.40

A bob of mass $m = 50$ g is suspended from the ceiling of a trolley by a light inextensible string. If the trolley accelerates horizontally, the string makes an angle $\theta = 37^\circ$ with the vertical. Find the acceleration of the trolley.

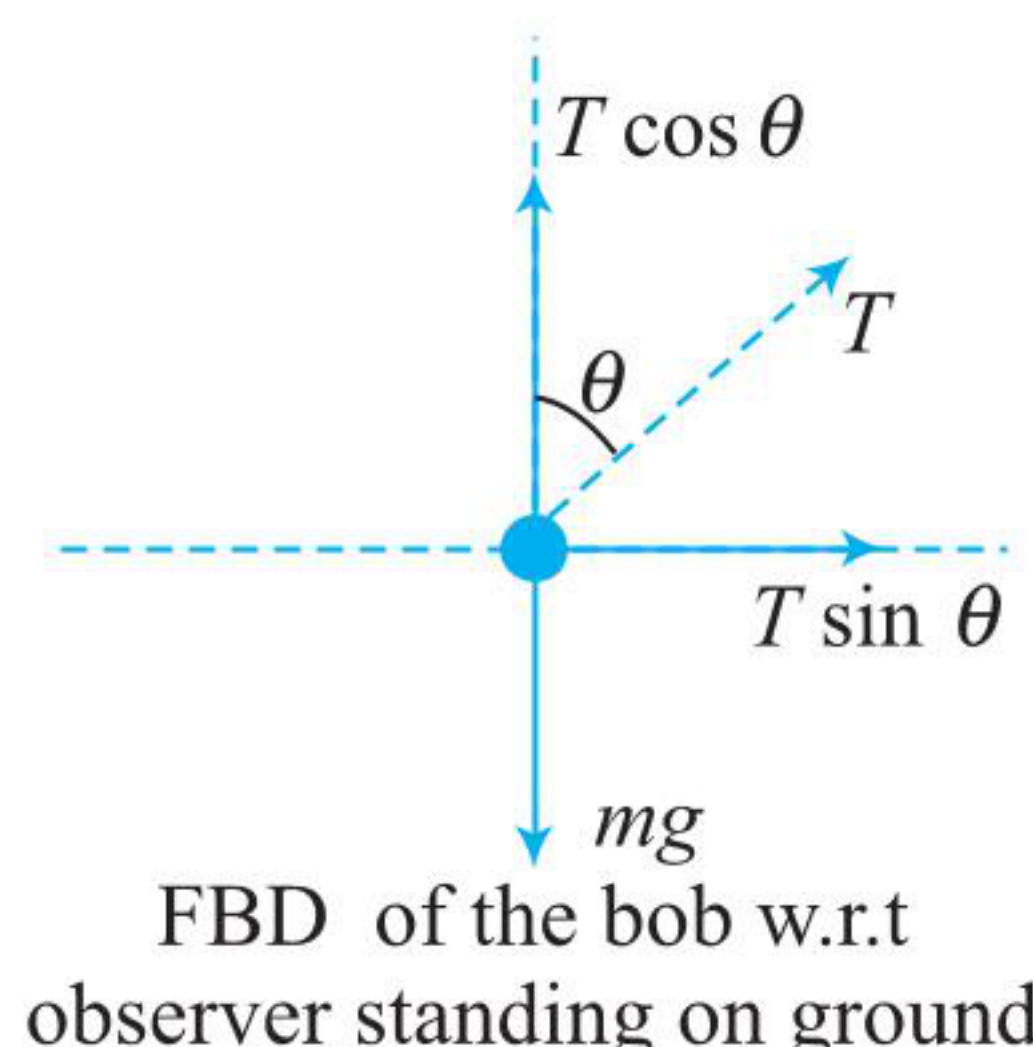


Sol.

Method 1: Problem solving from ground frame. The observer on the ground will observe the bob moving with acceleration a same as trolley. In FBD, he will observe only two real forces, weight and tension force.

The bob is at equilibrium in vertical direction, hence writing equation of motion in vertical direction

$$T \cos \theta = mg \quad \dots(i)$$



Along horizontal direction, the bob is accelerating with acceleration a .

$$T \sin \theta = ma \quad \dots(ii)$$

Using (i) and (ii), $\tan \theta = a/g$

$$\therefore a = g \tan \theta = 10 \tan 37^\circ = 7.5 \text{ m s}^{-2}$$

Therefore, acceleration of the trolley is 7.5 m s^{-2}

Method 2: Problem solving from non-inertial frame. If an observer sitting in the trolley and observes the bob, he will observe the bob at rest. If he asks to draw free-body diagram of bob he will apply pseudo force apart from real forces acting on the bob.

The bob is at equilibrium in vertical direction, hence writing equation of motion in vertical direction

$$T \cos \theta = mg \quad \dots(iii)$$

Along horizontal direction the bob is at rest.

$$T \sin \theta = ma \quad \dots(iv)$$

Using (iii) and (iv) $\tan \theta = a/g$

$$\therefore a = g \tan \theta = 10 \tan 37^\circ = 7.5 \text{ m s}^{-2}$$

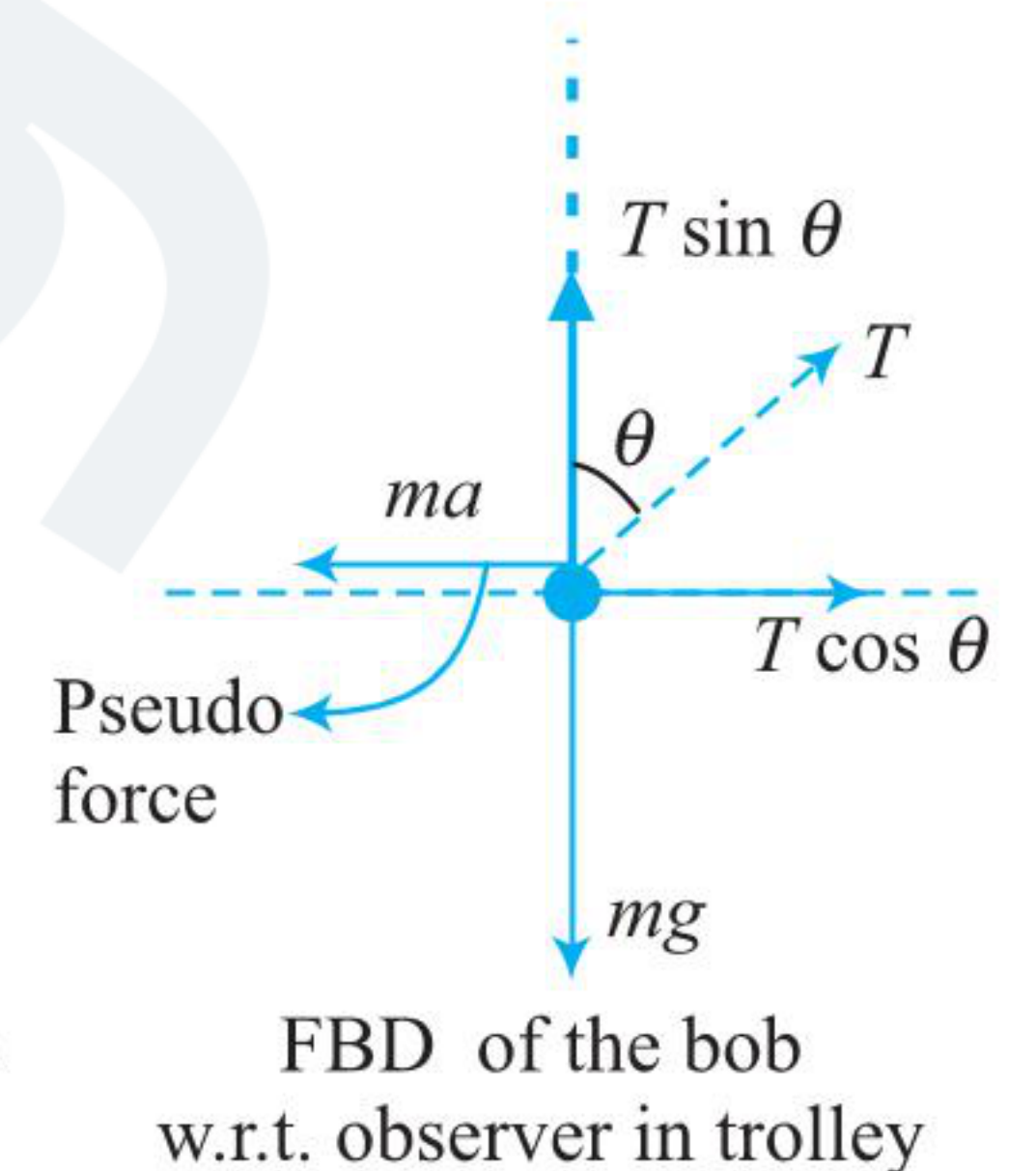
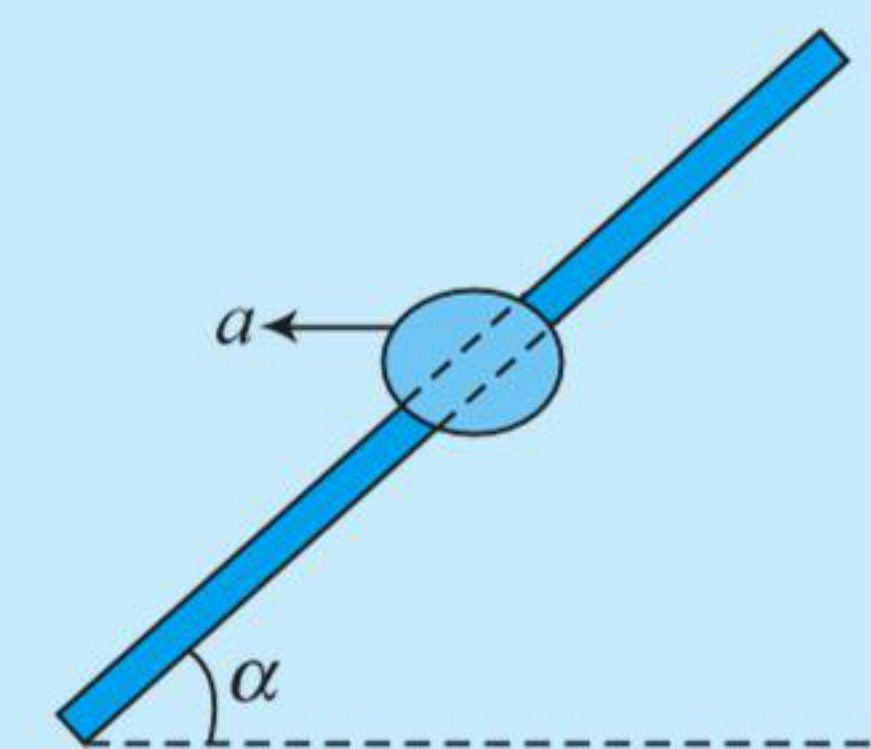


ILLUSTRATION 6.41

A bead of mass m is fitted on to a rod and can move on it without friction. At the initial moment the bead is in the middle of the rod.



The rod moves translationally in a horizontal plane with an acceleration a in a direction forming an angle α with the rod. Find the acceleration of the bead relative to the rod.

Sol.

Method 1: Observation from ground frame. Let a_r be the acceleration of the bead relative to the rod. Then $a_r \cos \alpha$ is the leftward acceleration of the bead relative to the rod and $a_r \sin \alpha$ is downward relative acceleration of the rod. If a_y and a_x be the absolute leftward horizontal and downward vertical acceleration of the bead, then

$$(\vec{a}_{\text{bead}})_x = (\vec{a}_{\text{bead,rod}})_x + (\vec{a}_{\text{rod}})_x \quad \dots(i)$$

$$\text{or } a_x = a_r \cos \alpha + a$$

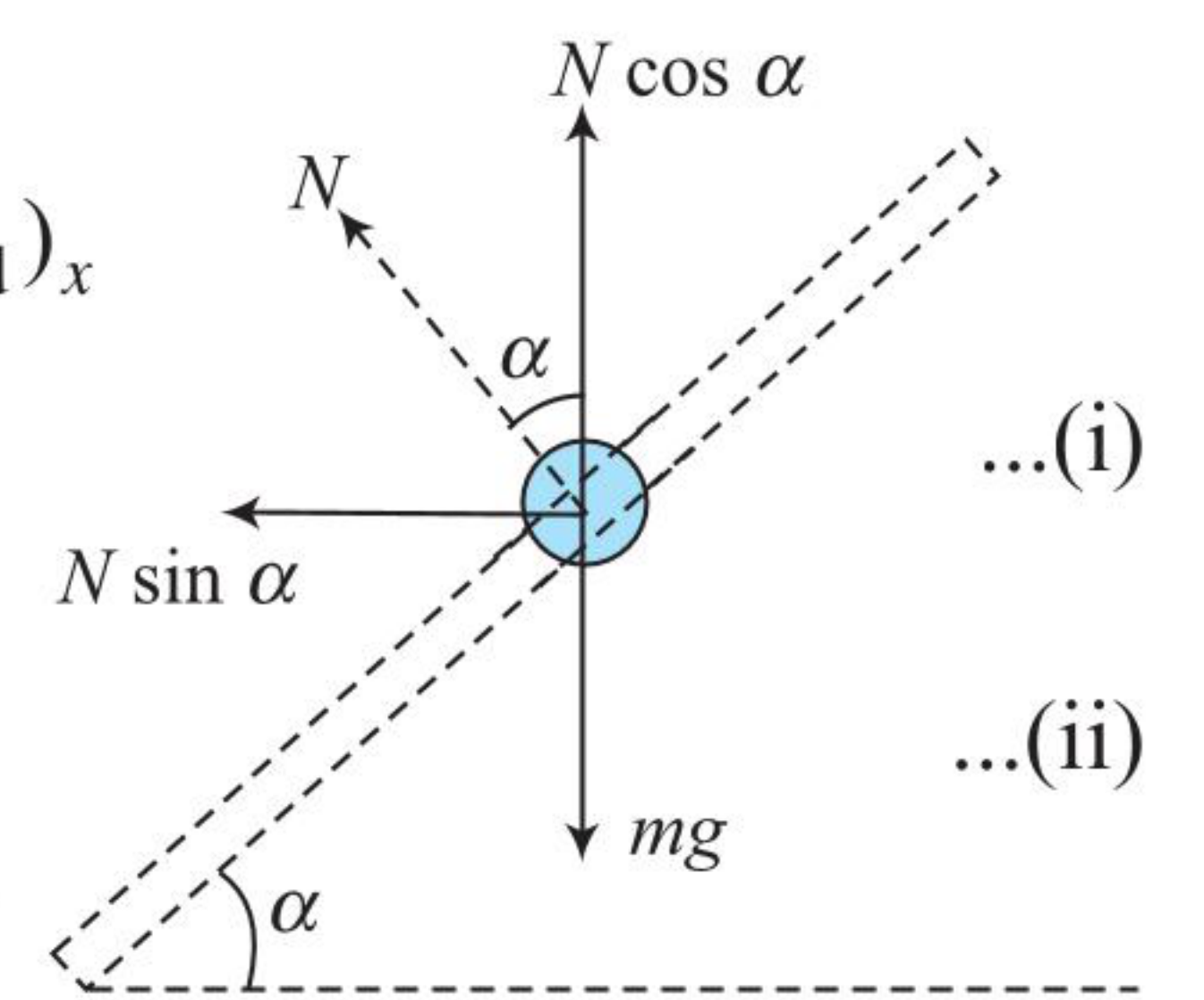
$$\text{and } a_r \sin \alpha = a_y - 0$$

$$\text{or } a_y = a_r \sin \alpha \quad \dots(ii)$$

From FBD of the bead (projecting forces vertically and horizontally)

$$mg - N \cos \alpha = ma_r \sin \alpha \quad \dots(i)$$

$$\text{and } N \sin \alpha = m(a_r \cos \alpha + a) \quad \dots(ii)$$

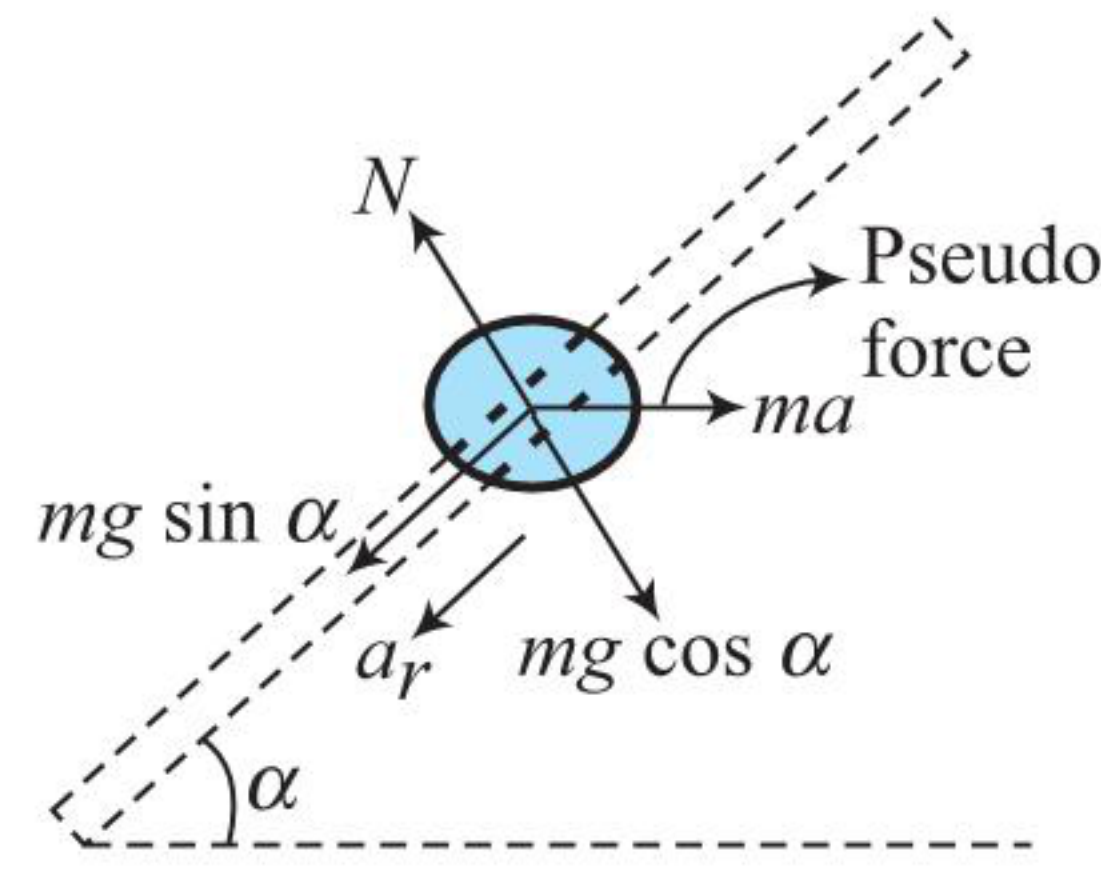


Eliminating N between (i) and (ii)

$$mg \sin \alpha = ma_r + ma \cos \alpha$$

or $a_r = g \sin \alpha - a \cos \alpha$

Method 2: Observation from an observer moving with rod. Considering bead w.r.t. rod, i.e., from non-inertial frame. A pseudo force of magnitude ma will act on the bead in the direction opposite to accelerator of rod, i.e., in right direction.



The bead is not moving perpendicular to rod. Hence,

$$N = mg \cos \alpha + ma \sin \alpha$$

Also in the direction along the rod, let acceleration of the bead w.r.t rod is a_r .

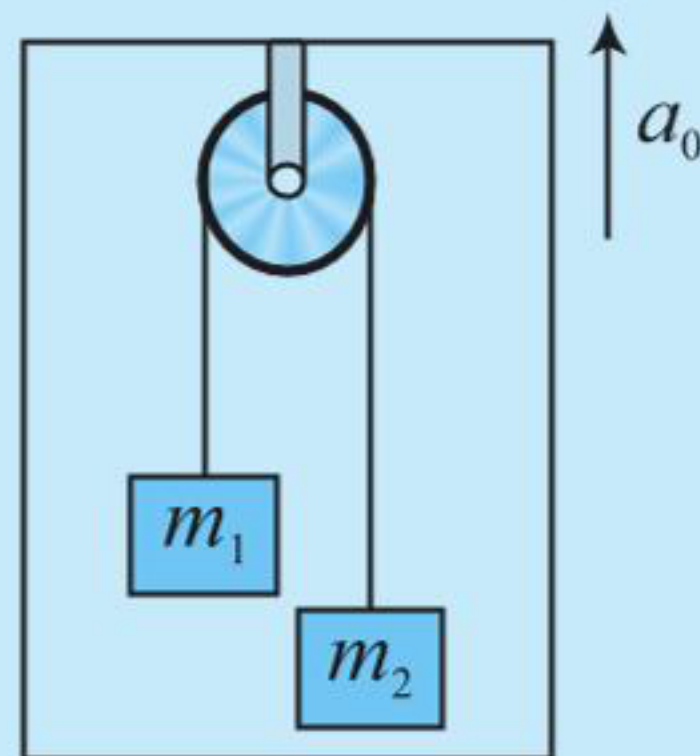
Equation of motion of bead with rod,

$$mg \sin \alpha - ma \cos \alpha = ma_r$$

$$\Rightarrow a_r = g \sin \alpha - a \cos \alpha$$

ILLUSTRATION 6.42

If the pulley is massless and moves with an upward acceleration a_0 , find the acceleration of m_1 and m_2 w.r.t. to elevator.



Sol. Method 1: (Ground frame):

Let acceleration of block m_1 with respect to pulley be a (upward) and the acceleration of m_2 w.r.t. pulley is a (downward)

Equation of motion for ' m_1 '

$$T - m_1 g = m_1 a_1 \quad \dots(i)$$

$$T - m_2 g = m_2 a_2 \quad \dots(ii)$$

$$\vec{a}_1 = \vec{a}_{1,p} + \vec{a}_p = a + a_0 \quad \dots(iii)$$

$$\vec{a}_2 = \vec{a}_{2,p} + \vec{a}_p = -a + a_0 \quad \dots(iv)$$

Substituting a_1 from (iii) in (i),

$$T - m_1 g = m_1 (a + a_0) \quad \dots(v)$$

Substituting a_2 from (iv) in (ii),

$$T - m_2 g = m_2 (-a + a_0) \quad \dots(vi)$$

Solving (v) and (iv),

$$T = \frac{2m_1 m_2}{m_1 + m_2} (g + a_0) \quad \text{and} \quad a = \frac{m_2 - m_1}{m_1 + m_2} (g + a_0)$$

Method 2: Solving problem from non-inertial frame of reference

Let us build the equations by using Newton's second law sitting on the accelerating pulley. Hence, we impose pseudo force $m_1 a_0 \downarrow$ and $m_2 a_0 \downarrow$ on both m_1 and m_2 , respectively, in addition to the upward tension and their weights $m_1 g \downarrow$ $m_2 g \downarrow$, respectively. If m_1 accelerates up relative to the pulley, m_2 must accelerate down relative to the pulley with acceleration a .

Force equation:

$$\text{For } m_1: T - m_1 g - m_1 a_0 = m_1 a \quad \dots(i)$$

$$\text{For } m_2: m_2 g + m_2 a_0 - T = m_2 a \quad \dots(ii)$$

Solving (i) and (ii), we have

$$a = \frac{m_2 - m_1}{m_1 + m_2} (g + a_0)$$

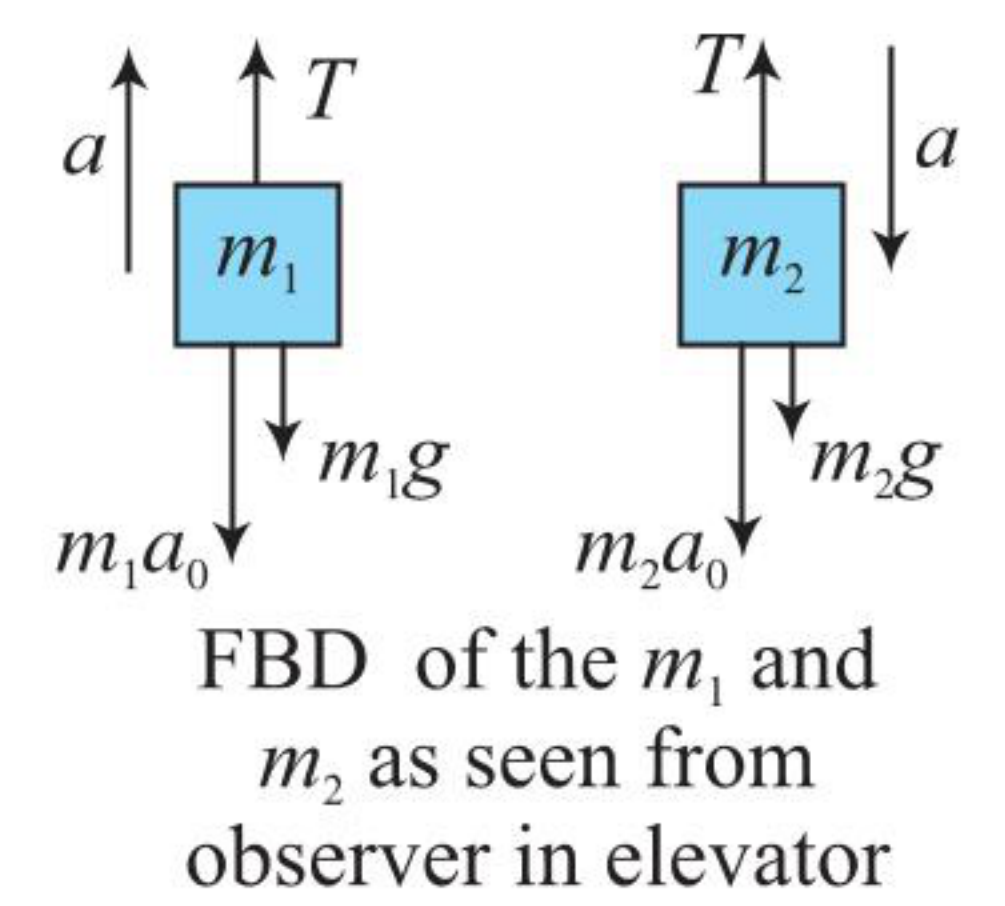
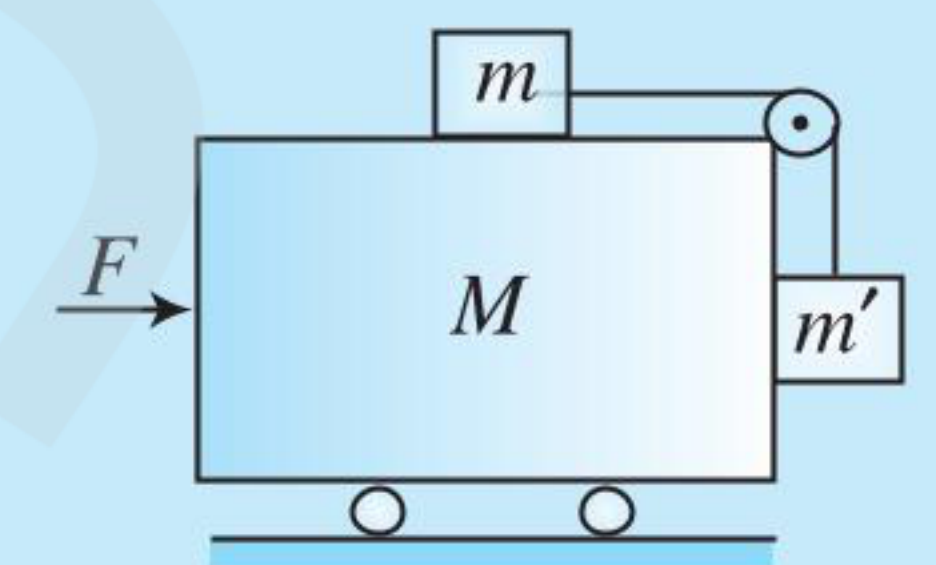


ILLUSTRATION 6.43

Two smooth blocks of masses m and m' connected by a light inextensible strings are moving on a smooth wedge of mass M . If a force F acts on the wedge the blocks do not slide relative to the wedge. Find the (a) acceleration of the wedge and (b) value of F .



Sol.

(a) The force acting on the blocks m and m' parallel to the surface in contact are given as $mA \leftarrow$, $T \rightarrow$, $m'g \downarrow$, and $T \uparrow$.

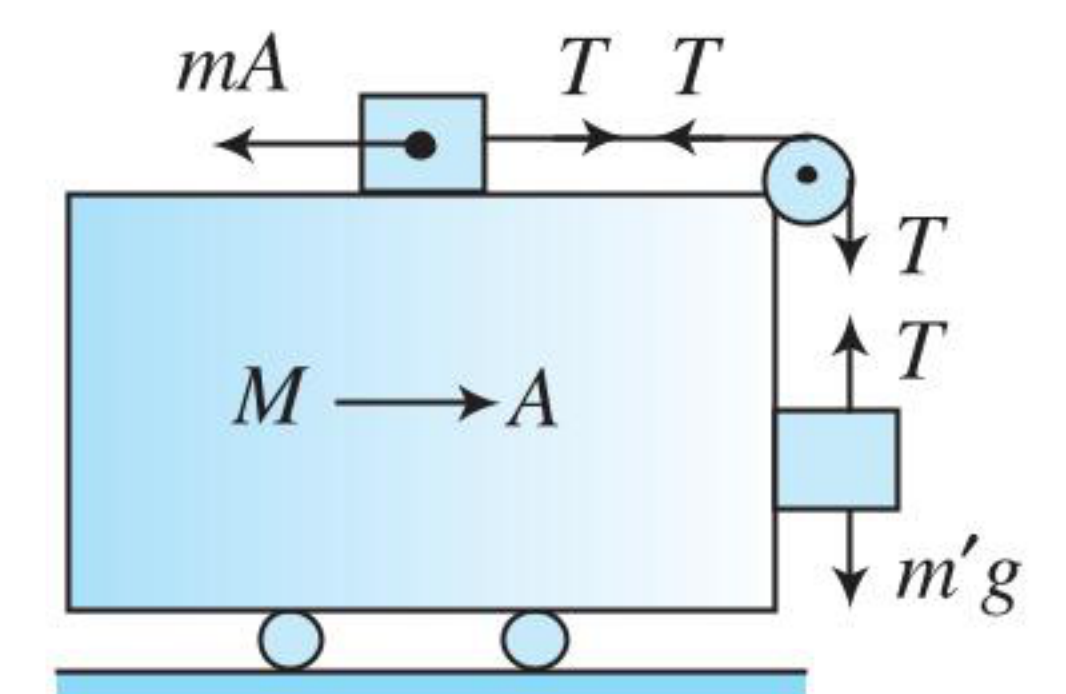
Force equation relative to M :

$$\text{Since } a_{mM} = a_0 = 0$$

$$\text{For } m: T = mA \quad \dots(i)$$

$$\text{For } m': T = m'g \quad \dots(ii)$$

$$\text{From Eq. (i) and (ii) } A = \frac{m'}{m} g$$



(b) Force equation on system ' $M + m + m'$ '

$$\sum F_x = ma_x$$

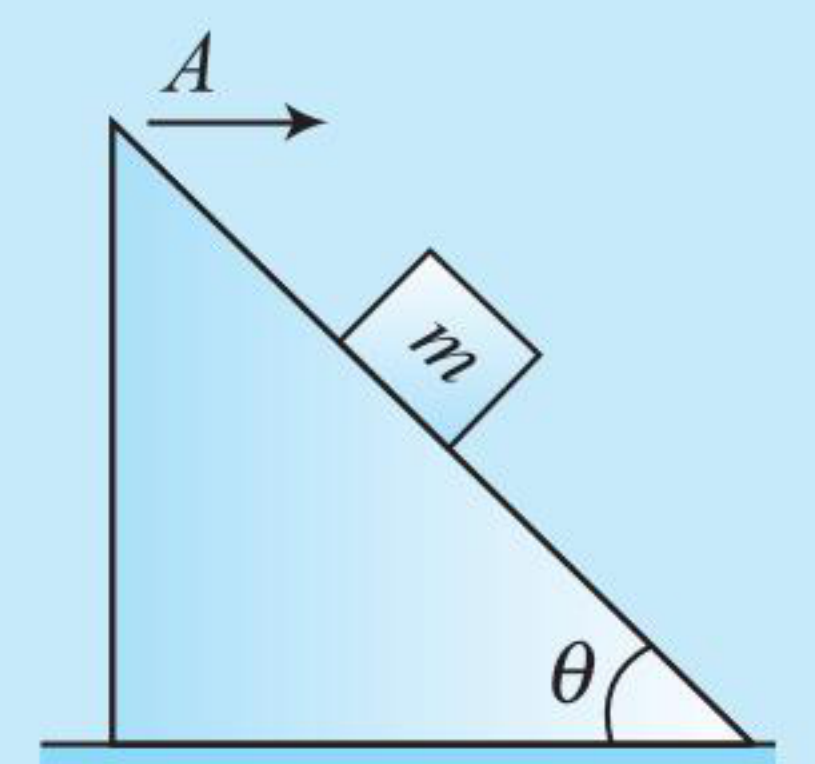
$$F = MA + mA + m'A$$

$$= (M + m + m') A$$

$$= (M + m + m') \frac{m'}{m} g$$

ILLUSTRATION 6.44

A block of mass m is placed on an inclined plane. With what acceleration A towards right should the system move on a horizontal surface so that m does not slide on the surface of inclined plane? Also calculate the force supplied by wedge on the block. Assume all surfaces are smooth.



Sol. Method 1: Analysis of forces on m relative to ground

If the motion of m is analyzed from ground, its acceleration is A and the forces acting on it are its weight mg and normal reaction N .

As m is at rest, moving with same acceleration as wedge in horizontal direction but in vertical direction, the block is at rest.

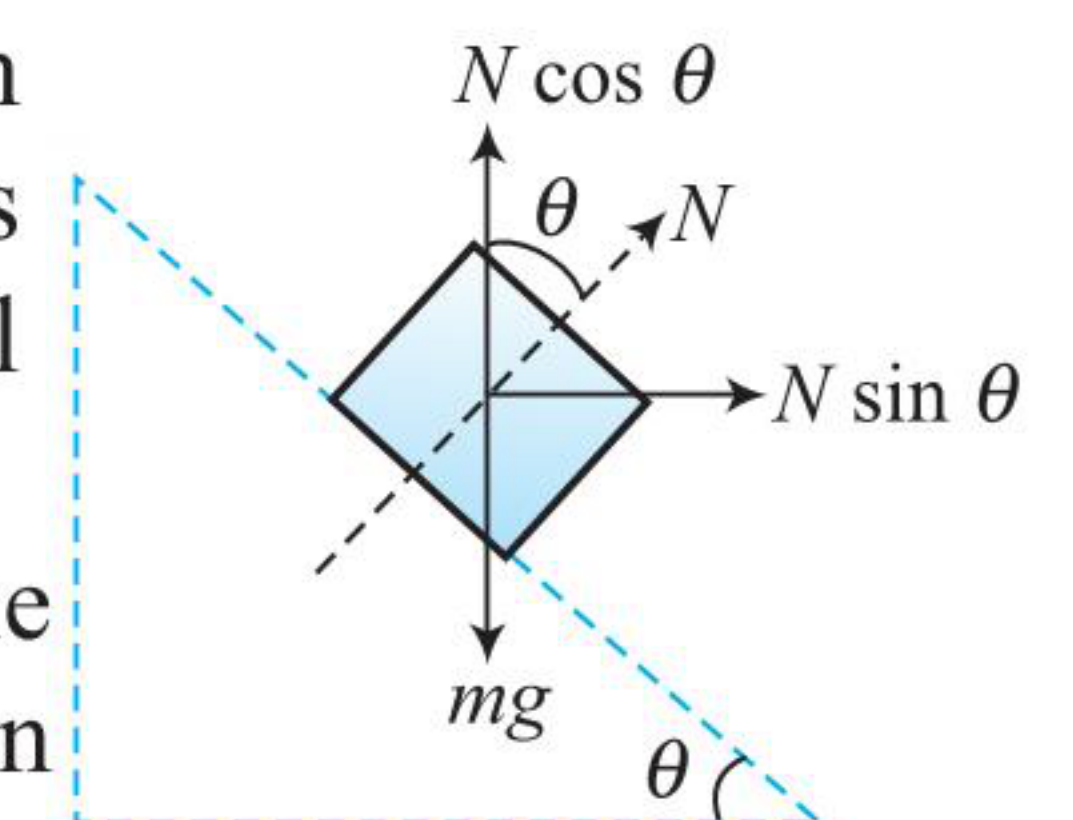
$$\sum \vec{F}_y = 0 \Rightarrow N \cos \theta = mg \quad \dots(i)$$

$$\sum \vec{F}_x = \sum m_i \vec{a}_i \Rightarrow N \sin \theta = mA \quad \dots(ii)$$

On solving (i) and (ii), we get $A = g \tan \theta$ and $N = \frac{mg}{\cos \theta}$

Method 2: Analysis of forces on m relative to the inclined plane

If the motion of m is analyzed from the view point of an observer standing on the inclined plane (i.e., relative to the plane), its



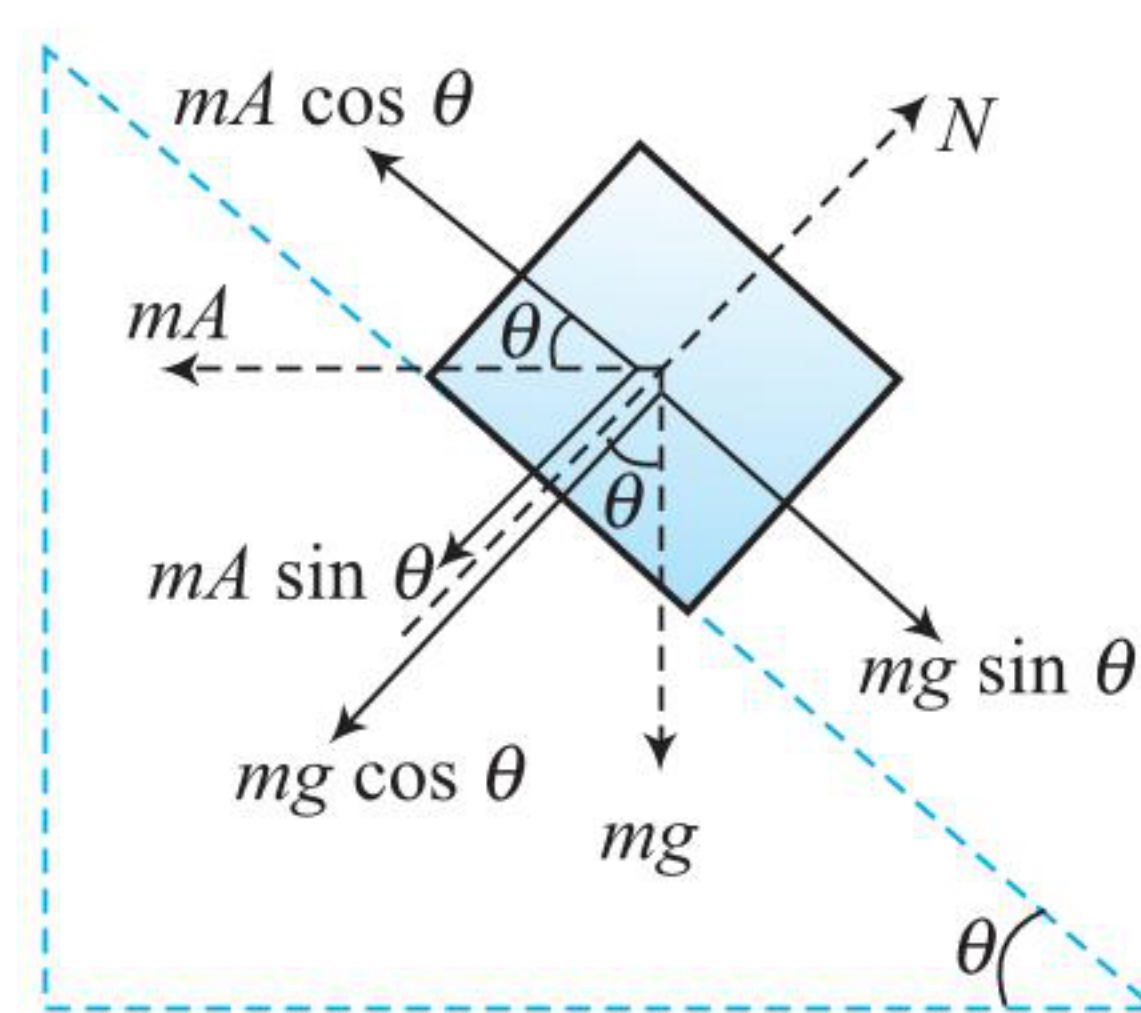
acceleration is 0 and the forces acting on it are: its weight, the normal reaction, and a pseudo force of magnitude mA towards left.

$$mA \sin \theta = mA \cos \theta$$

$$\Rightarrow A = g \tan \theta$$

$$\text{Also: } N = mg \cos \theta + mA \sin \theta$$

$$= mg \cos \theta + mg \tan \theta \sin \theta = \frac{mg}{\cos \theta}$$



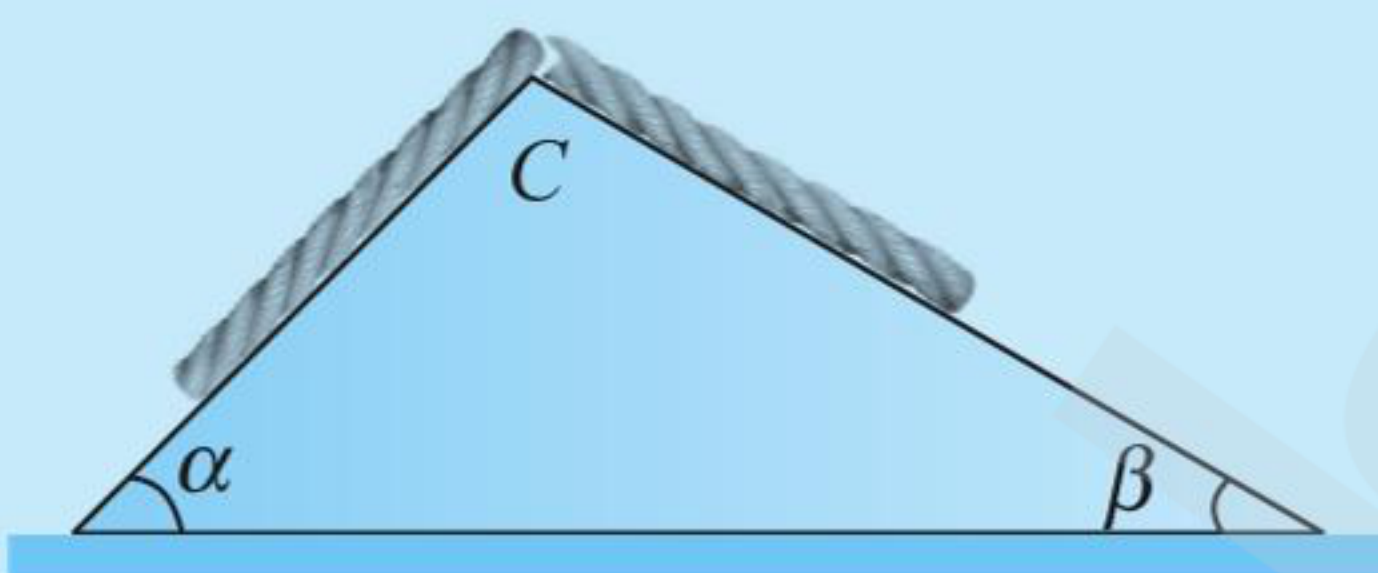
Important Points:

If we resolve the forces along the sloping side, we have net force along sloping direction $F = mA \cos \theta - mg \sin \theta$

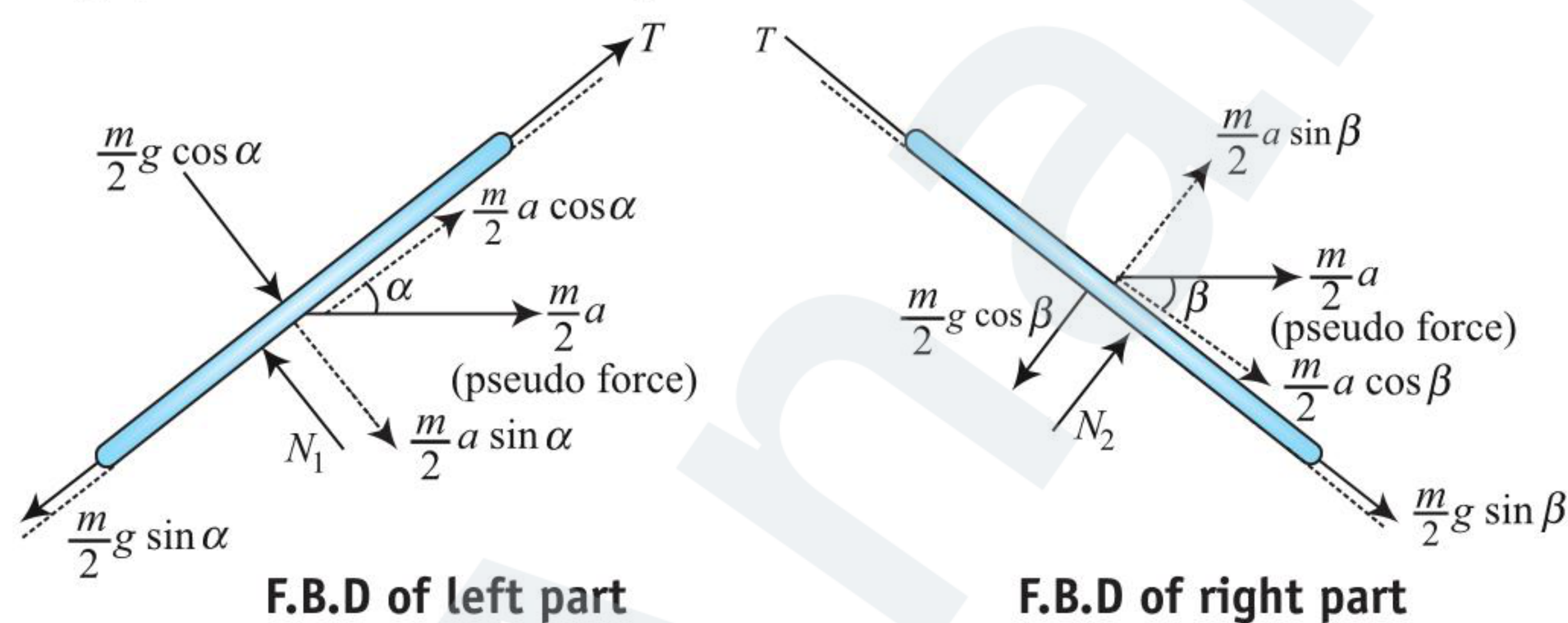
- If $mA \cos \theta > mg \sin \theta$, F is directed up along the slant. Then, the block will accelerate relative to the wedge with acceleration $a = A \cos \theta - g \sin \theta$
- If $mA \cos \theta = mg \sin \theta$, $F = 0$. Hence the block remains stationary relative to the wedge (or more with constant velocity relative to the wedge).
- If $mA \cos \theta < mg \sin \theta$, F is directed down along the slant. Hence, the block accelerates down the slant with acceleration $a = g \sin \theta - A \cos \theta$

ILLUSTRATION 6.45

A homogeneous flexible rope rests on a wedge whose side edges make angles α and β with the horizontal (refer figure). The central part of the rope lies on the upper rib C of the wedge. With what acceleration should the wedge be pulled to the left along the horizontal plane in order to prevent the displacement of the rope with respect to the wedge? [Consider all surfaces to be smooth]



Sol. Let us assume the wedge is moving with acceleration a , considering the parts of the rope on either side of incline as two different elements and analyzing each part of the rope w.r.t wedge, the F.B.D of each part.



As rope is not sliding w.r.t wedge, hence

$$\text{For left part: } T + \frac{m}{2} a \cos \alpha = \frac{m}{2} g \sin \alpha \quad \dots(i)$$

$$\text{For right part: } T = \frac{m}{2} a \cos \beta + \frac{m}{2} g \sin \beta \quad \dots(ii)$$

From (i) and (ii), we get

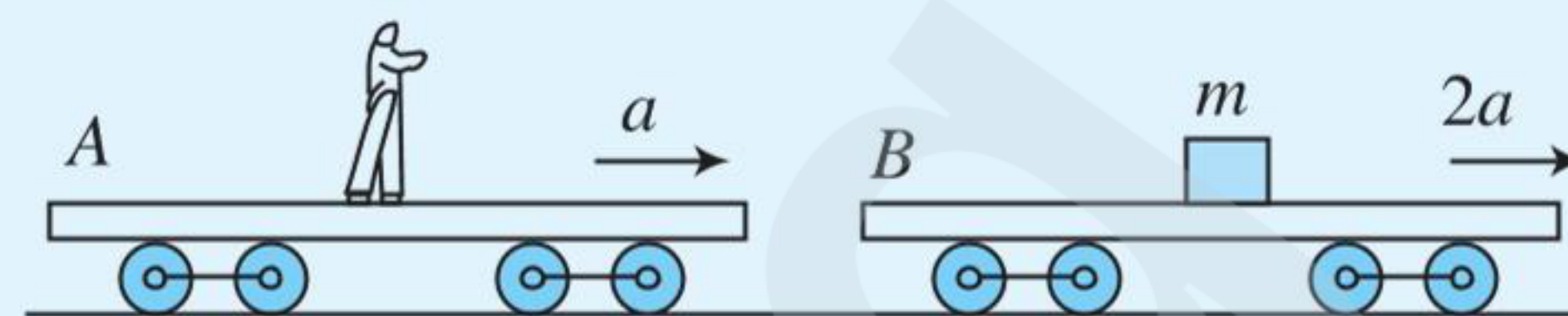
$$\frac{m}{2} a \cos \beta + \frac{m}{2} g \sin \beta + \frac{m}{2} a \cos \alpha = \frac{m}{2} g \sin \alpha$$

$$a[\cos \beta + \cos \alpha] = g[\sin \alpha - \sin \beta]$$

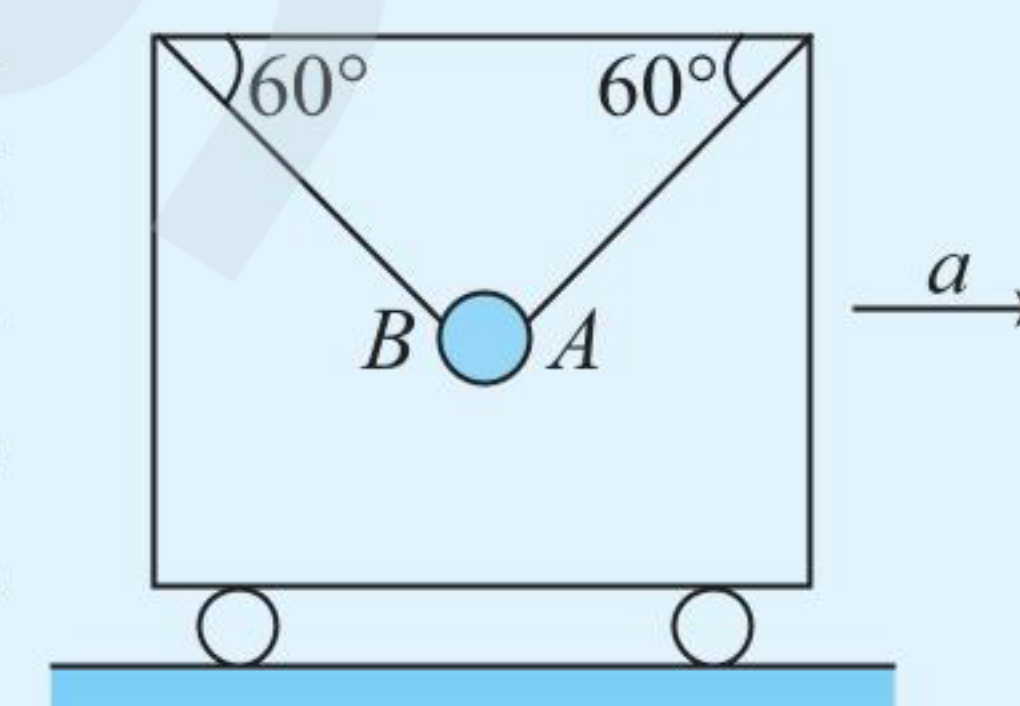
$$\Rightarrow a = \frac{g[\sin \alpha - \sin \beta]}{(\cos \alpha + \cos \beta)}$$

CONCEPT APPLICATION EXERCISE 6.5

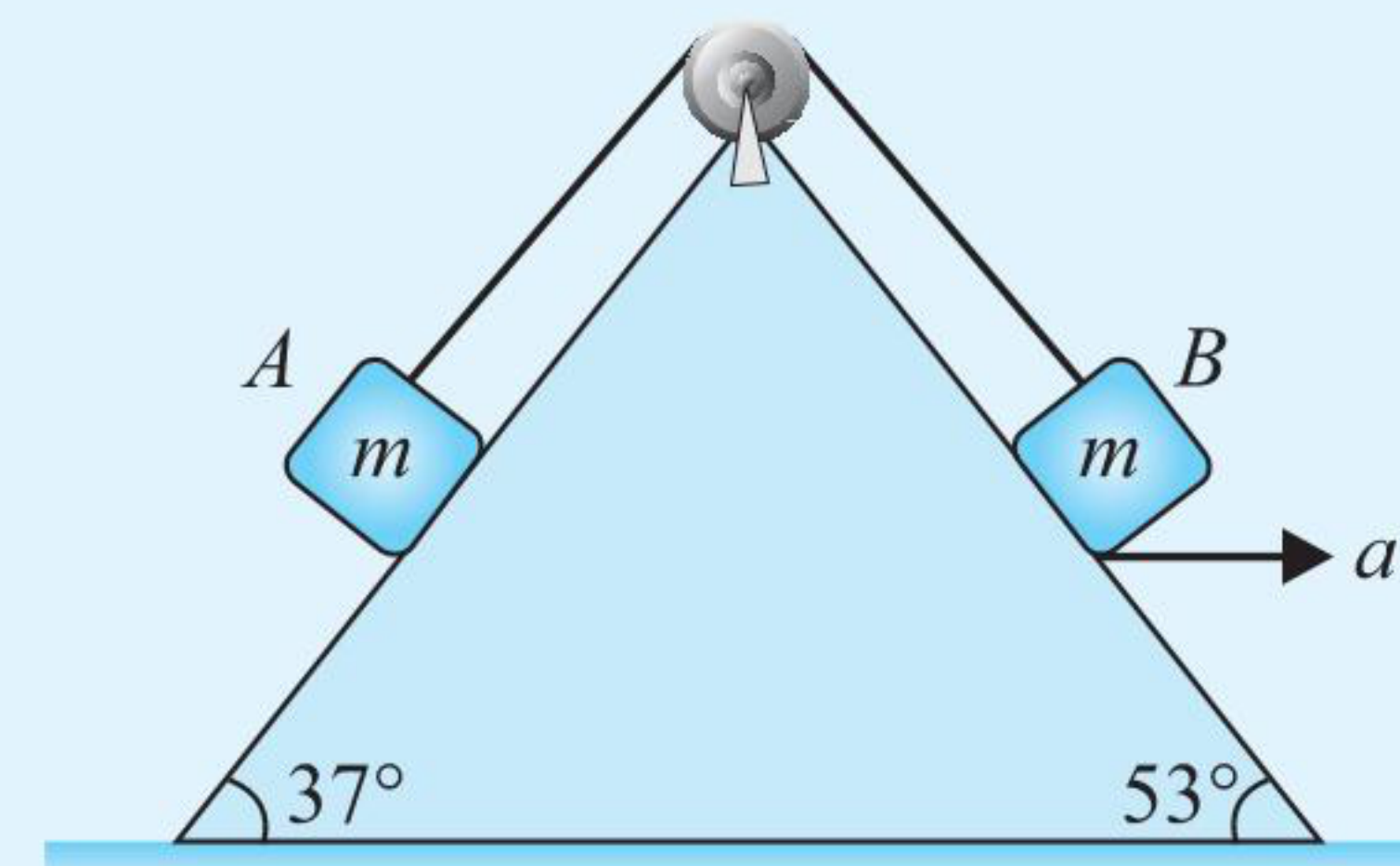
- Two trolleys A and B are moving with accelerations a and $2a$, respectively, in the same direction. To an observer in trolley A , Find the magnitude of the pseudo force acting on a block of mass m on trolley B .



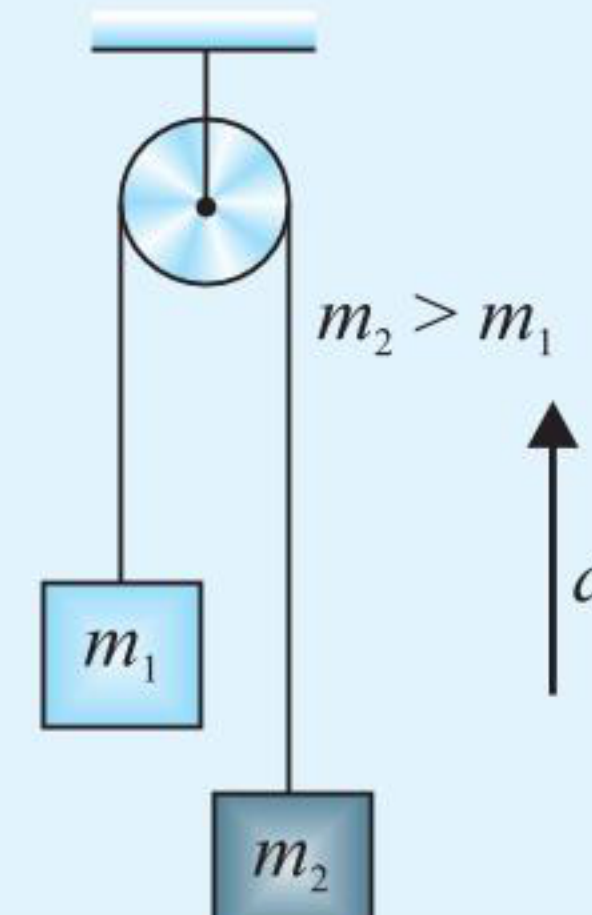
- A steel ball is suspended from the ceiling of an accelerating carriage by means of two cords A and B . Determine the acceleration a of the carriage which will cause the tension in A to be twice that in B .



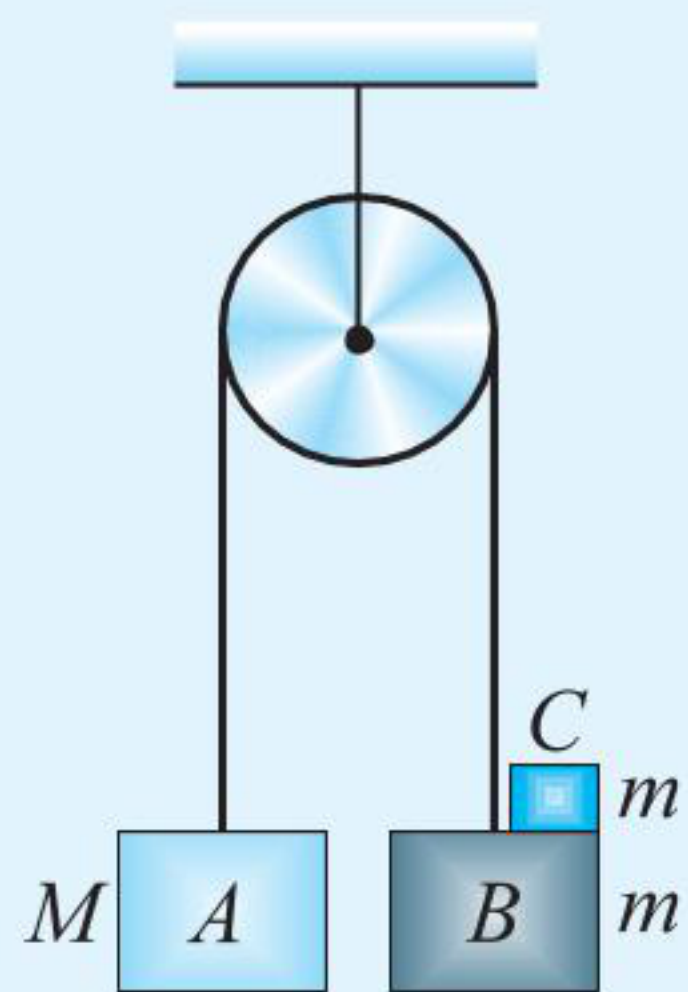
- Two blocks A and B of equal masses m kg each are connected by a light thread, which passes over a massless pulley as shown. Both the blocks lie on wedge of mass m . Assume friction to be absent everywhere and both the blocks to be always in contact with the wedge. The wedge lying over smooth horizontal surface is pulled towards right with constant acceleration a . (g is acceleration due to gravity).



- Find the normal reaction acting on block A
 - Find the normal reaction acting on block B
 - Find the accelerations of block A and block B w.r.t wedge.
 - Find the maximum value of acceleration a for which normal reactions acting on the block A and block B are non-zero.
- A person is standing on a weighing machine placed on the floor of an elevator. The elevator starts going up with some acceleration, moves with uniform velocity for a while and finally decelerates to stop. The maximum and the minimum weights recorded are 72 kg and 60 kg. Assuming that the magnitudes of the acceleration and the deceleration are the same, find (a) the true weight of the person and (b) the magnitude of the acceleration. Take $g = 10 \text{ m/s}^2$.
 - In the given figure, the pulley and strings are light. The pulley is suspended in a elevator moving up with acceleration a_0 . Find the accelerations of the blocks with respect to elevator.



6. In the given system, the pulley is light and frictionless; string is also massless. Initially the system is in equilibrium. Now a small block C of mass m is placed on the block B of mass M . If block C always remains on block B , find the normal reaction on C due to B .



ANSWERS

1. ma in backward direction.
2. $\frac{g}{3\sqrt{3}}$
3. (i) $\frac{m}{5}(4g - 3a)$ (ii) $\frac{m}{5}(3g + 4a)$
- (iii) $\frac{1}{10}(7a - g)$ (iv) $\frac{4}{3}g$
4. $66 \text{ kg}, \frac{10}{11} \text{ m/s}^2$
5. $\frac{(m_2 - m_1)}{(m_2 + m_1)}(g - a_0)$
6. $\frac{2Mmg}{(2M + m)}$

CONSTRAINT RELATIONS

Constraints mean that two bodies (in this case the bodies which are attached to the pulley) are not free to move the way they want. The accelerations between them are dependent on each other. We need to find out the relationship to solve the problems of Newton's laws of motion.

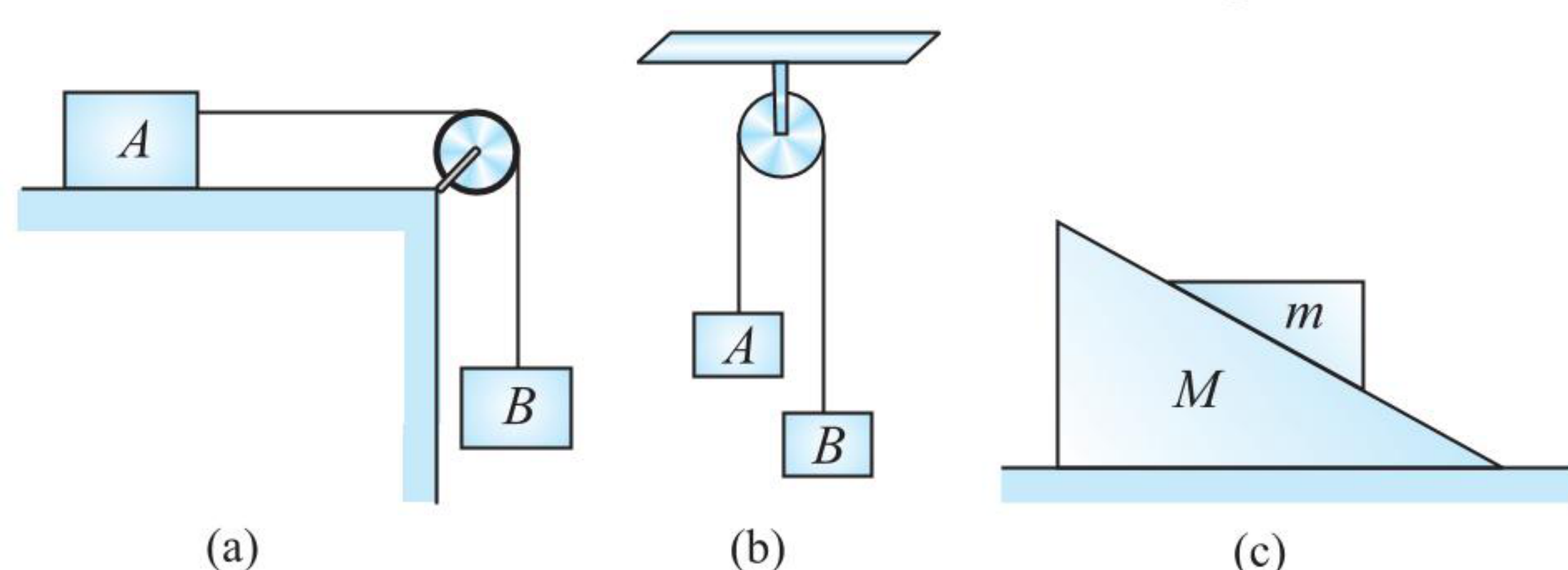
The equations showing the relation of the motions of a system of bodies, in which one motion is constrained by the other motions, are called *constrained relations*.

Constraints are the geometrical restrictions imposed on the motion of a body, which also govern the trajectory of the body. For example, a block placed on the table cannot move normal to the surface; it is bound to move parallel to the surface.

We have to use the method of constraint equations to relate the accelerations between the bodies.

First, we start our analysis with simple cases of pulleys. Consider the situation shown in figure. Two bodies are connected with a string which passes over a pulley at the corner of a table. Here if the string is inextensible, we can directly state that the displacement of A in the downward direction is equal to the displacement of B in the horizontal direction on table, and if displacements of A and B are equal in equal time, their speeds and acceleration magnitude must also be equal.

In figure, if the wedge and the block are free to move, it is obvious that the accelerations of the block and the wedge are related.



Look at figure. In this case, let us say that you have to find the acceleration of the masses. The number of unknowns will be

1. Tension, T
2. Acceleration, a_1 , of the mass
3. Acceleration of the other body, a_2

There are three unknowns. However, we will get only two equations—one for one mass and another for the other mass.

Clearly, you can see that Newton's laws are not sufficient to solve the problem. In such cases, we need additional equations. Therefore, constraints provide additional equations. These are provided by what are called as *constraint equations*.

In many cases, you can write down the relation of acceleration by just looking at the situation. In other cases, for complex relationships, we can think of four types of constraints. (1) pulley constraints, (2) wedge constraints, (3) combination of wedge and pulleys, and (4) general constraints.

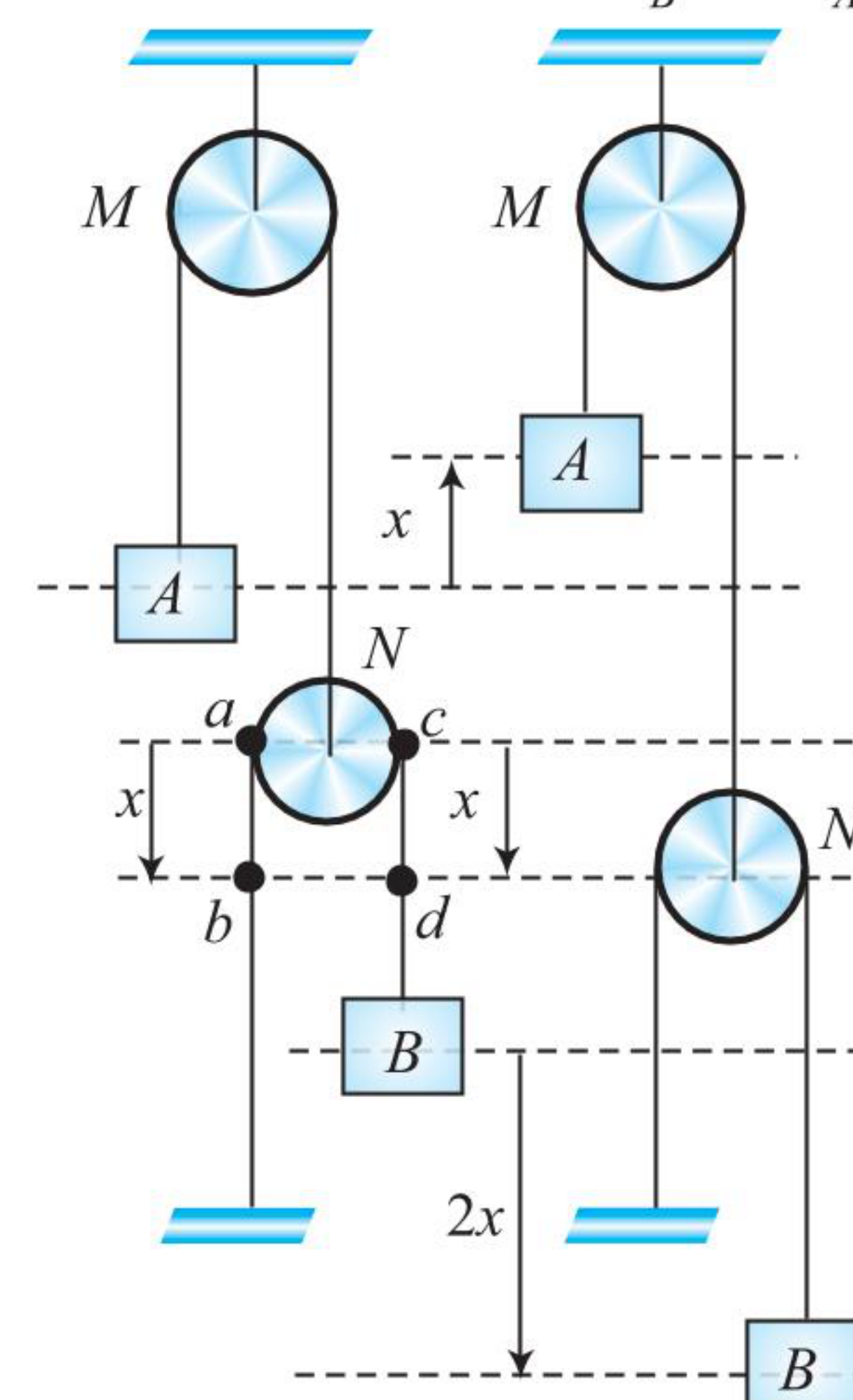
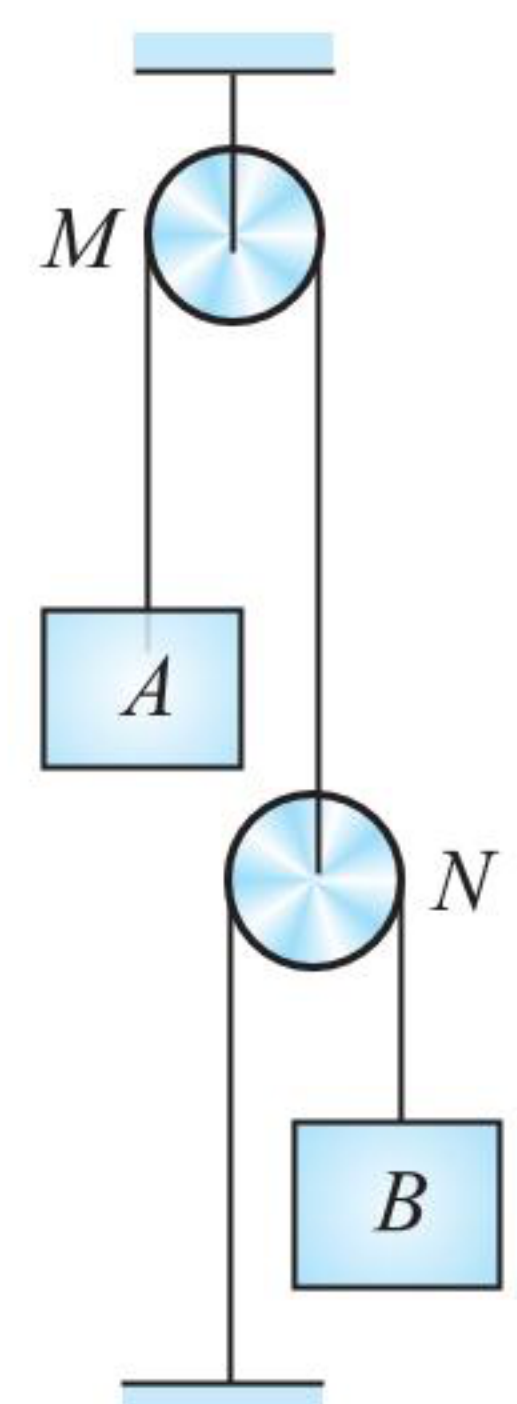
PULLEY CONSTRAINT

Pulley constraints are applicable when the bodies concerned are connected through pulleys and the rope connecting them is inextensible.

Let us learn the application of pulley constraint through the following cases through different methods:

Case I: Mass A is connected with a string which passes through a fixed pulley. The other end of the string is connected with a movable pulley N . Block B is connected with another string which passes through the pulley N as shown in figure. Find the reaction between the accelerations of A and B .

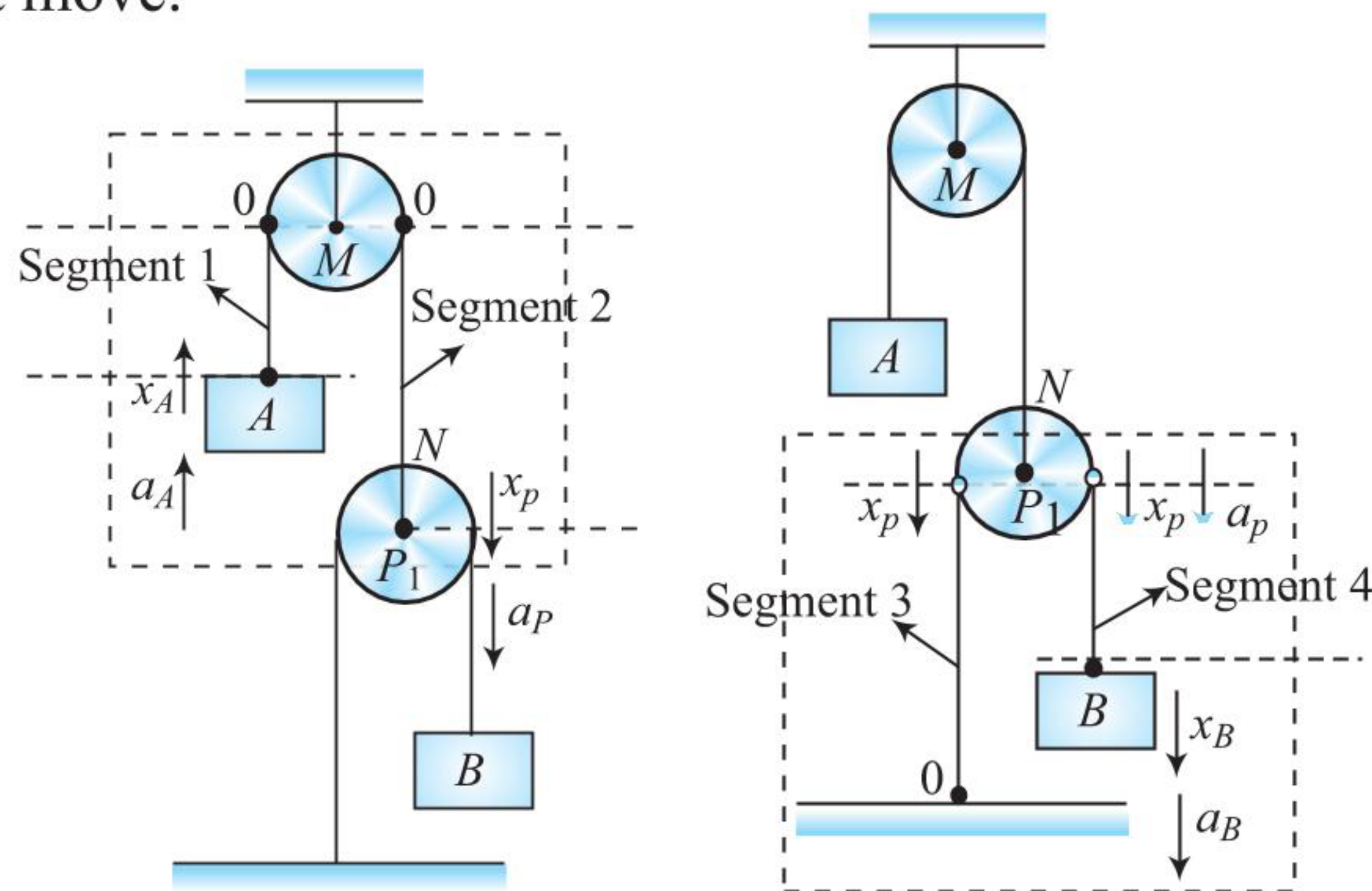
Method 1: Consider the situation shown in figure. If we consider that mass A is going up by distance x , pulley N , which is attached to the same string, will go down by the same distance x . Due to this, the string which is connected to mass B will now have free lengths ab and cd ($ab = cd = x$) which will go on the side of mass B due to its weight as the other end is fixed. Thus, mass B will go down by $2x$. Hence, its speed and acceleration will be twice that of block B . Hence, $v_B = 2v_A$ and $a_B = 2a_A$.



Method 2: The above case can be understood by another approach in which we will consider that the total length of the string will always be constant.

To relate the acceleration of the bodies, assume that various bodies move by a distance $x_1, x_2, \dots$. Calculate the number of segments in the string.

The segments in the first string are marked 1 and 2. The distance moved by the various elements are also marked (as shown in Figure). Note that the pulley, which is connected to the ceiling, cannot move.



(a) Considering segments in string 1 (b) Considering segments in string 2

Relate the distance moved. (Total change in the length of the string must be zero.) To do these, calculate the change in the length of each segment of the string. Then add these changes to get the total change in the length of the string should be zero.

Change in the length of segment 1

$$\Delta l_1 = (-x_A) + 0 = -x_A$$

(Why negative? Because as block A moves up, the length of segment of string comes closer to the fixed pulley M and the length of the segment decreases.)

Change in the length of segment 2

$$\Delta l_2 = (+x_P) + 0 = +x_P$$

Total change in length of string 1

$$-x_A + x_P = 0 \Rightarrow x_A = x_P$$

Once we have the relation between the distances, the relation between accelerations is simple. For the first string, it is $a_A = a_P$.

For second string, the distances moved by pulley N and block B are x_P and x_B , respectively, are shown in figure. The ground is at rest. Therefore, the end of the string that is connected to the ground will not move, its displacement can be taken zero.

Change in the length of segment 3

$$\Delta l_3 = (-x_P) + 0 = -x_P$$

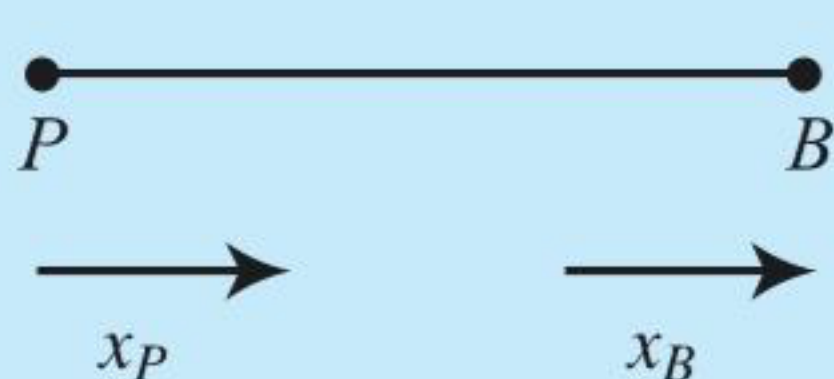
(Why negative? Because as pulley moves down, the string comes closer to the ground and the length of the segment decreases.)

Change in the length of segment 4

$$\Delta l_4 = (+x_B) + (-x_P) = x_B - x_P$$

Note: To see why this is so, let us consider a string with either points moving by a distance x_P and x_B . This is shown in figure.

Because the right end moves by x_B , the length of the string increases by x_B . When the left end moves by x_P , the length reduces by x_P . The change in length is, therefore, $x_B - x_P$.

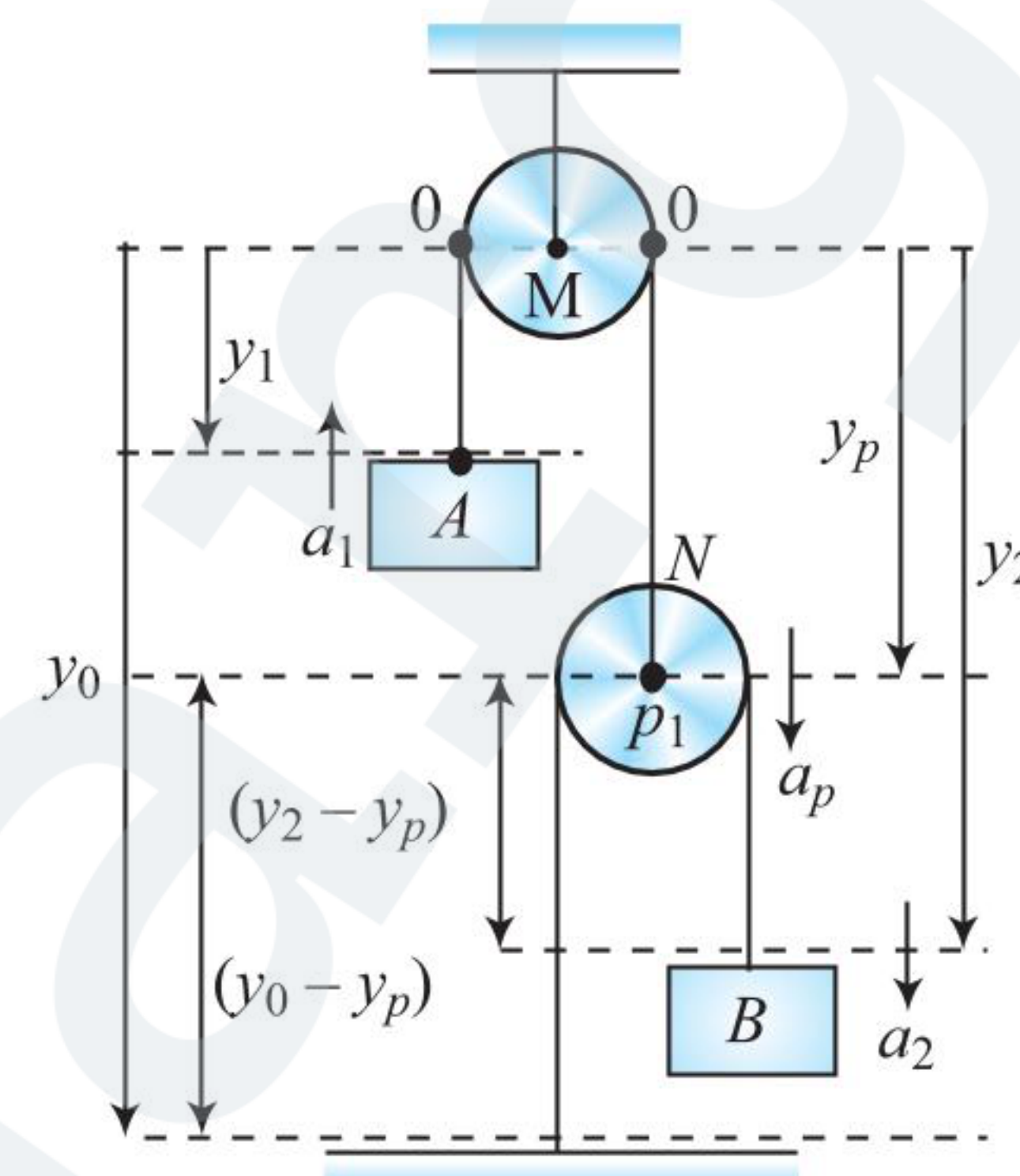


The total change in the length of the string is $-x_P + x_B - x_P = 0$

$$\Rightarrow x_B - 2x_P = 0 \quad \text{or} \quad x_B = 2x_P \quad \text{or} \quad x_B = 2x_A$$

Once we get the relation between the distances moved, the acceleration relation will be the same. The acceleration relation is also $a_B = 2a_A$.

Method 3: The positions of blocks and movable pulley are taken from fixed pulley as shown in figure. The length of string is constant.



For string 1:

$$y_1 + y_2 + l_0 = \text{constant} \quad \dots(i)$$

l_0 is the length of part of string on pulley M.

Differentiating (i) w.r.t. time,

$$\frac{dy_1}{dt} + \frac{dy_P}{dt} + \frac{dl_0}{dt} = 0$$

$$\text{or} \quad (-v_A) + v_P = 0$$

$$\text{or} \quad v_A = v_P$$

$$\text{Also} \quad a_A = a_P$$

$[dy_1/dt$ is negative as y_1 is decreasing with time and $dl_0/dt = 0$

as part of string on pulley is constant.

For string 2: $(y_0 - y_P) + (y_2 - y_P) + l'_0 = \text{constant}$

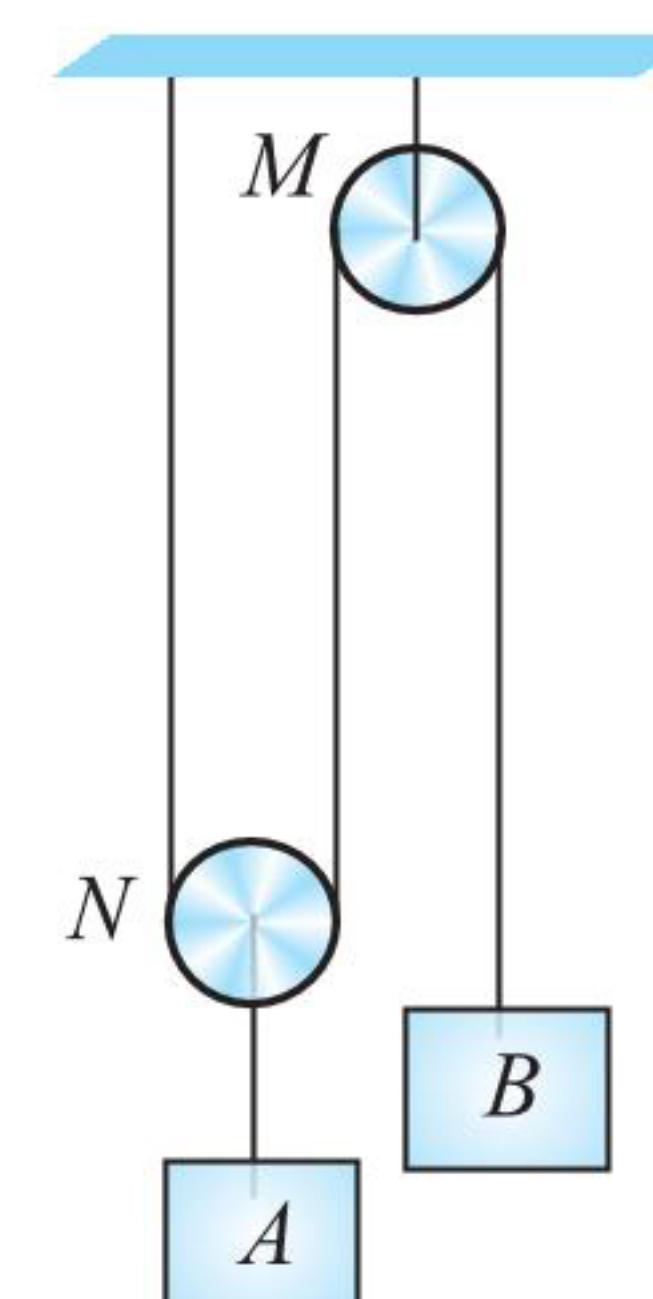
$$y_0 + y_2 - 2y_P + l'_0 = \text{Constant} \quad \dots(ii)$$

Differentiating (ii) w.r.t. time,

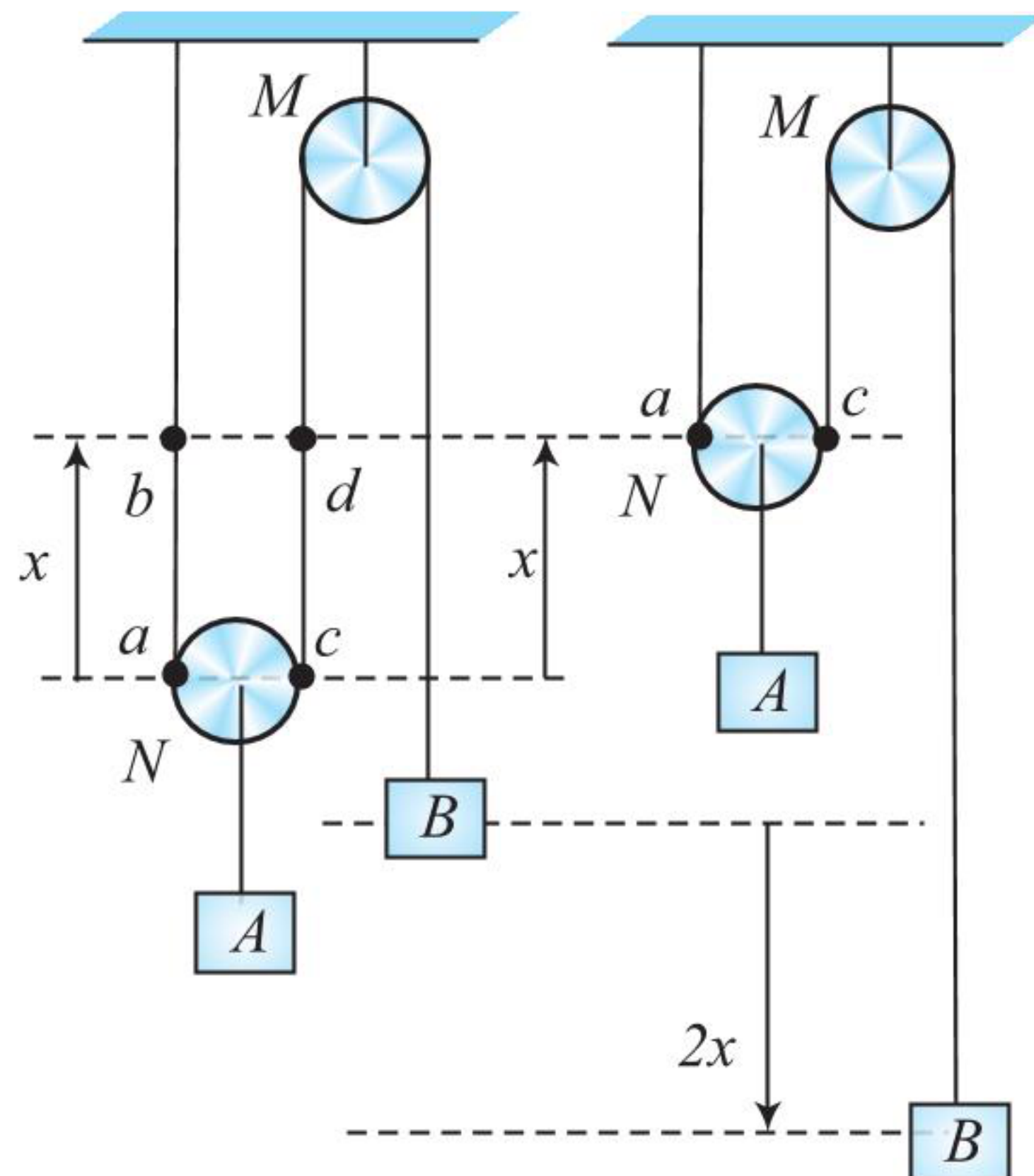
$$0 + (v_B) - 2(v_P) + 0 = 0 \Rightarrow v_B = 2v_P$$

$$\text{or} \quad v_B = 2v_A \Rightarrow a_B = 2a_A$$

Case II: In the given situation, two blocks A and B are arranged as shown in figure. M is a fixed pulley and N is a movable pulley. The system is released from rest. We need to find the relationship between displacements, velocities, and accelerations of the blocks. We will analyze this case through various methods:



Method 1: If mass A goes up by a distance x , we can observe that the string lengths ab and cd are slack. Due to the weight of block B , this length ($ab + cd = 2x$) will go on this side and block B will descend by a distance $2x$. As in equal time duration, B has travelled a distance twice that of A . So, $v_B = 2v_A$ and $a_B = 2a_A$.



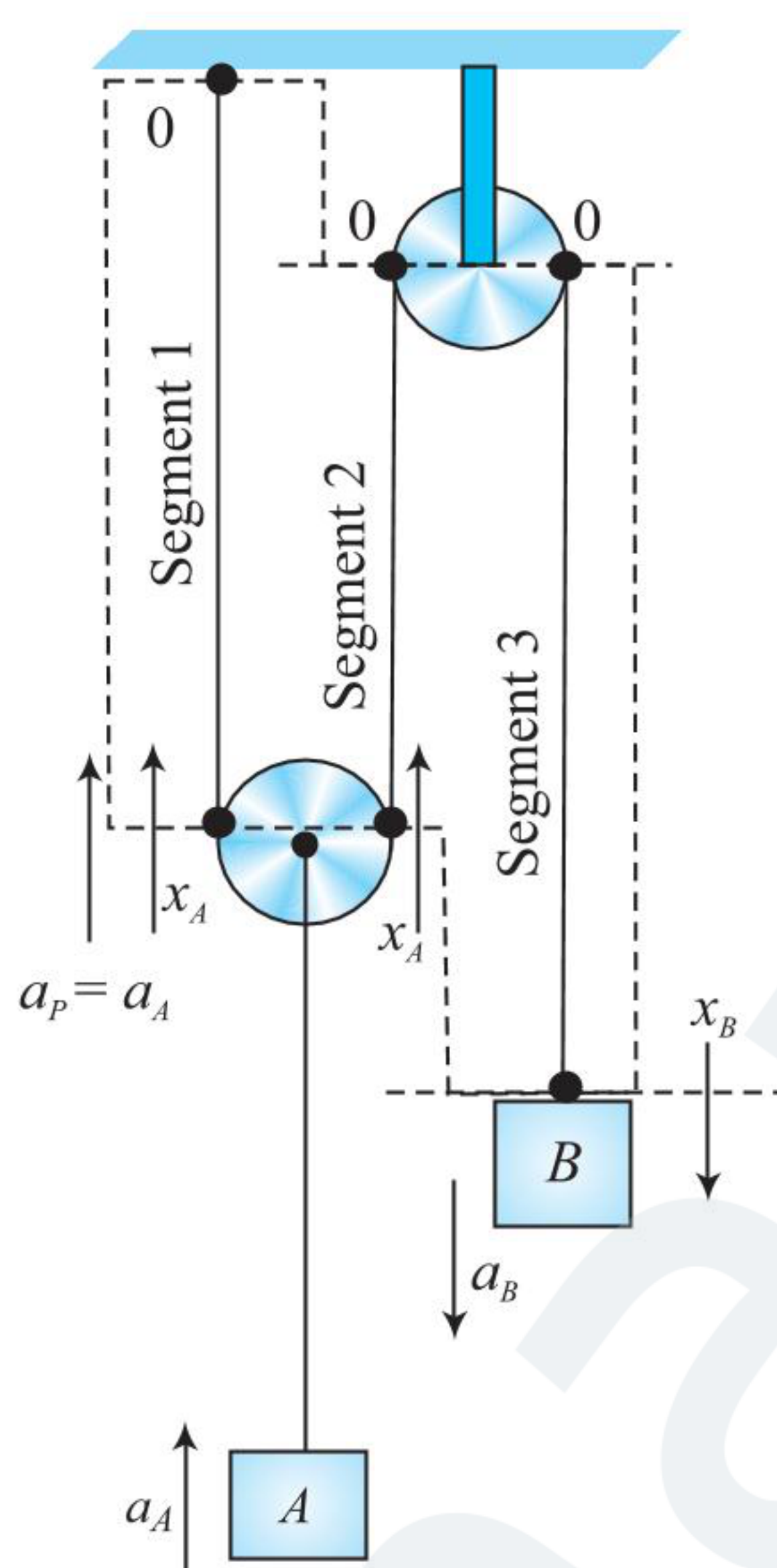
Method 2: Segment method

Change in the length of segment 1

$$\Delta l_1 = (0) + (-x_A) = -x_A$$

Change in the length of segment 2

$$\Delta l_2 = 0 + (-x_A) = -x_A$$



Change in the length of segment 3

$$\Delta l_3 = 0 + (+x_B) = +x_B$$

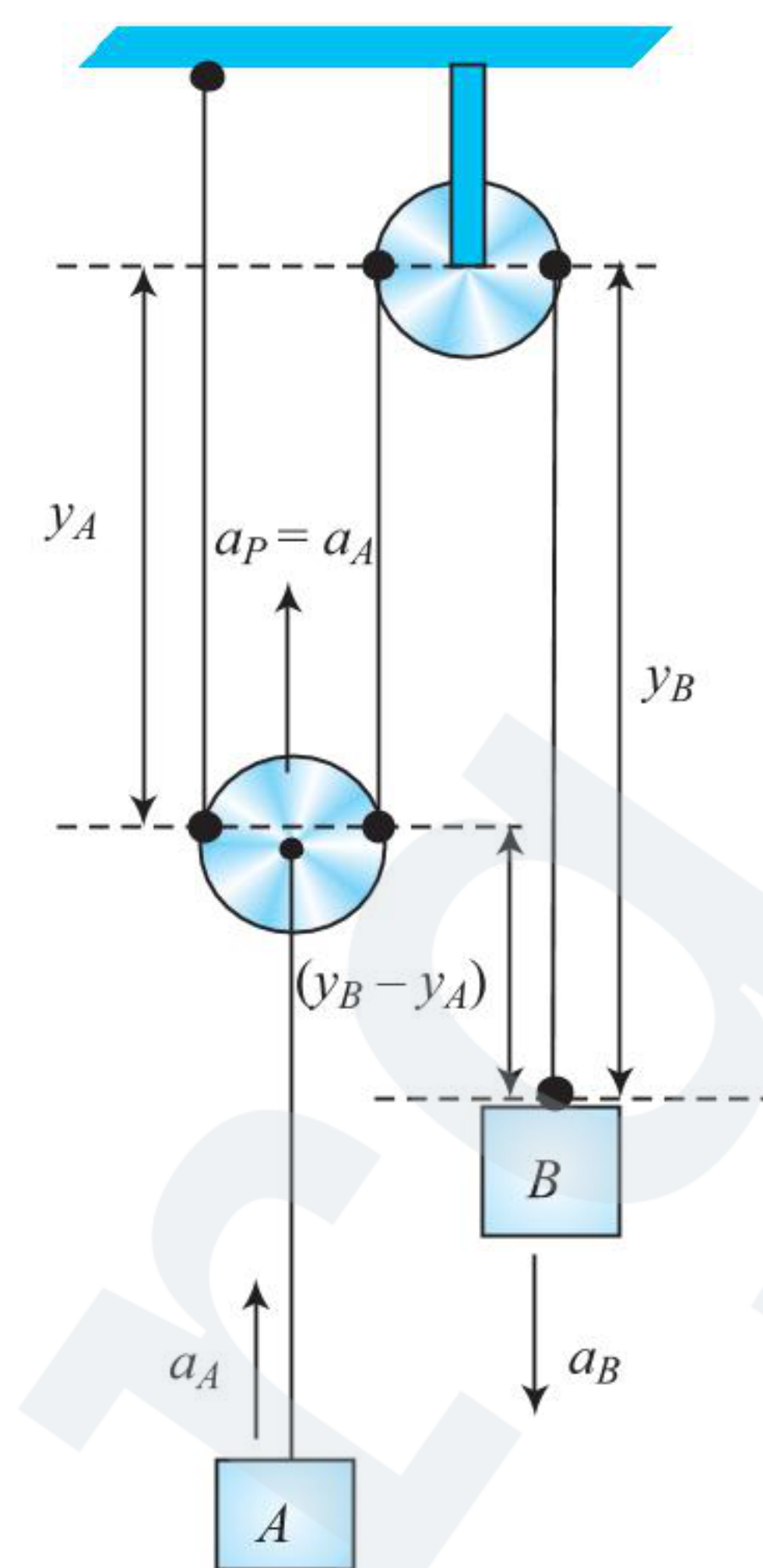
Total change in the length of string should be zero. Hence, the total sum of change of lengths of segments should be zero, i.e.,

$$\Delta l_1 + \Delta l_2 + \Delta l_3 = 0$$

$$(-x_A) + (-x_A) + (+x_B) = 0$$

$$\Rightarrow x_B - 2x_A = 0 \text{ or } a_B = 2a_A$$

Method 3: Let us consider the system shown in figure. It is clear from the figure that the position of A is governed by the position of the center of movable pulley. Let at any instant the block B is at y_B and the center of movable pulley is y_A from the reference line (dotted line). The total length of the cord:



$$y_B + 2y_A + l_0 = \text{constant} = l \text{ (say)} \quad \dots(i)$$

l_0 is the part of the cord which is over the pulley (remain constant). Differentiating (i) w.r.t. time, we get

$$\frac{dy_B}{dt} + \frac{2dy_A}{dt} + \frac{dl_0}{dt} = \frac{dl}{dt} \quad \dots(ii)$$

As l_0 and l are constant, therefore,

$$\frac{dl_0}{dt} = 0 \text{ and } \frac{dl}{dt} = 0$$

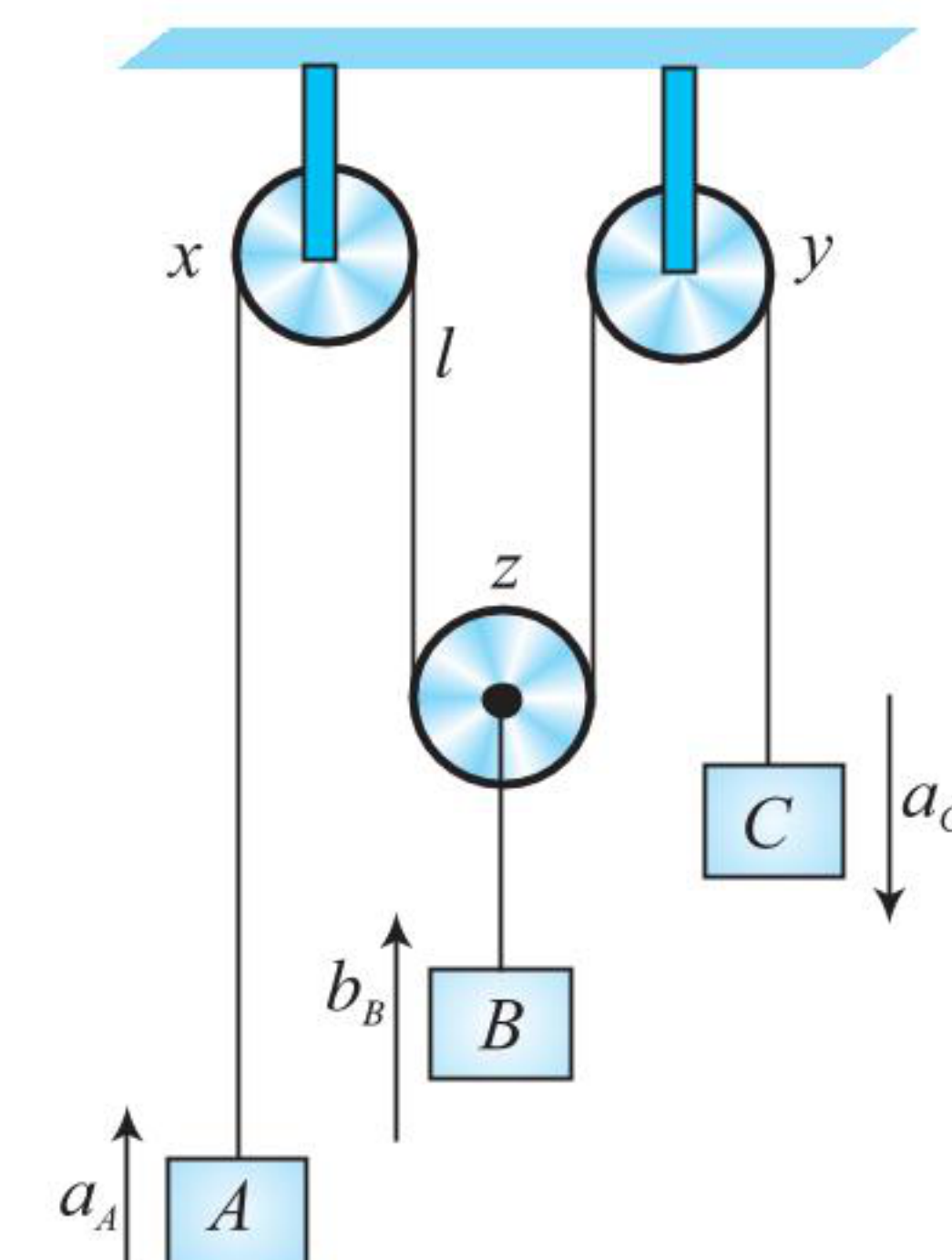
$$\text{and } \frac{dy_B}{dt} = v_B \text{ and } \frac{dy_A}{dt} = -v_A$$

$$\text{Equation (ii) becomes } v_B - 2v_A = 0 \quad \dots(iii)$$

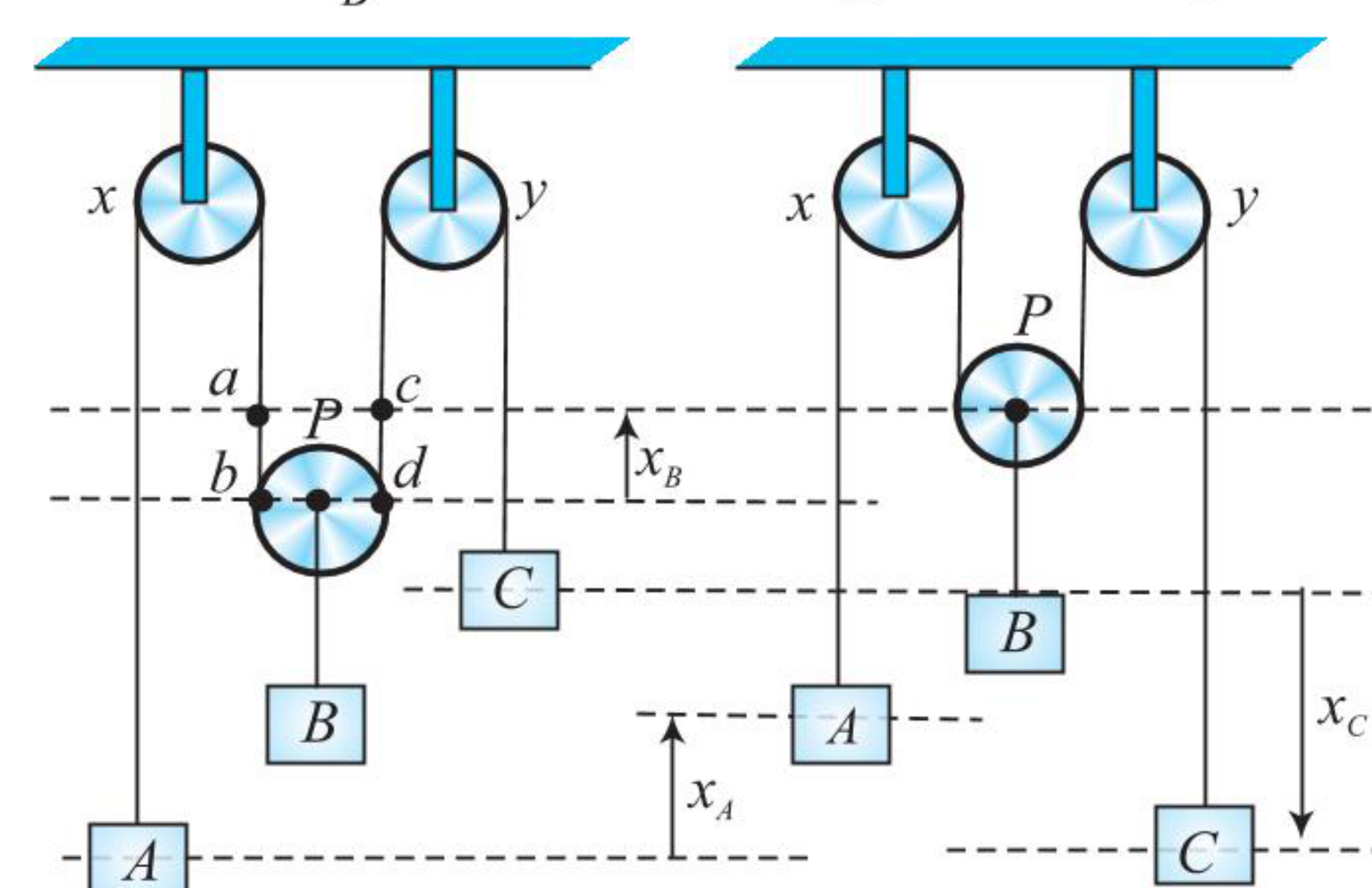
Differentiating once more, we get

$$a_B - 2a_A = 0 \text{ or } a_B = 2a_A$$

Case III: Here three blocks A , B , and C are connected with strings and pulleys as shown in figure. Here we develop constraint relation between the motion of masses A , B , and C .



Method 1: Let us assume that masses A and B would go up by distance x_A and x_B , respectively. These lengths of the string will slack as length $(ab + cd)$ above the pulley P . Thus, block B will go up by a distance x_B as shown in figure. Thus, we have



$$(ab + cd) + x_A = x_C$$

$$\text{or } 2x_B + x_A = x_C$$

Differentiating w.r.t. time, we get

$$2v_B + v_A = v_C \quad \dots(i)$$

Differentiating again w.r.t. time,

$$2a_B = a_C - a_A \quad \dots(ii)$$

Equations (i) and (ii) are the constraint relations for motion of masses A , B , and C .

Method 2:

Change in the length of segment 1

$$\Delta l_1 = (-x_A) + (0) = -x_A$$

Change in the length of segment 2

$$\Delta l_2 = 0 + (-x_B) = -x_B$$

Change in the length of segment 3

$$\Delta l_3 = 0 + (-x_B) = -x_B$$

Change in the length of segment 4

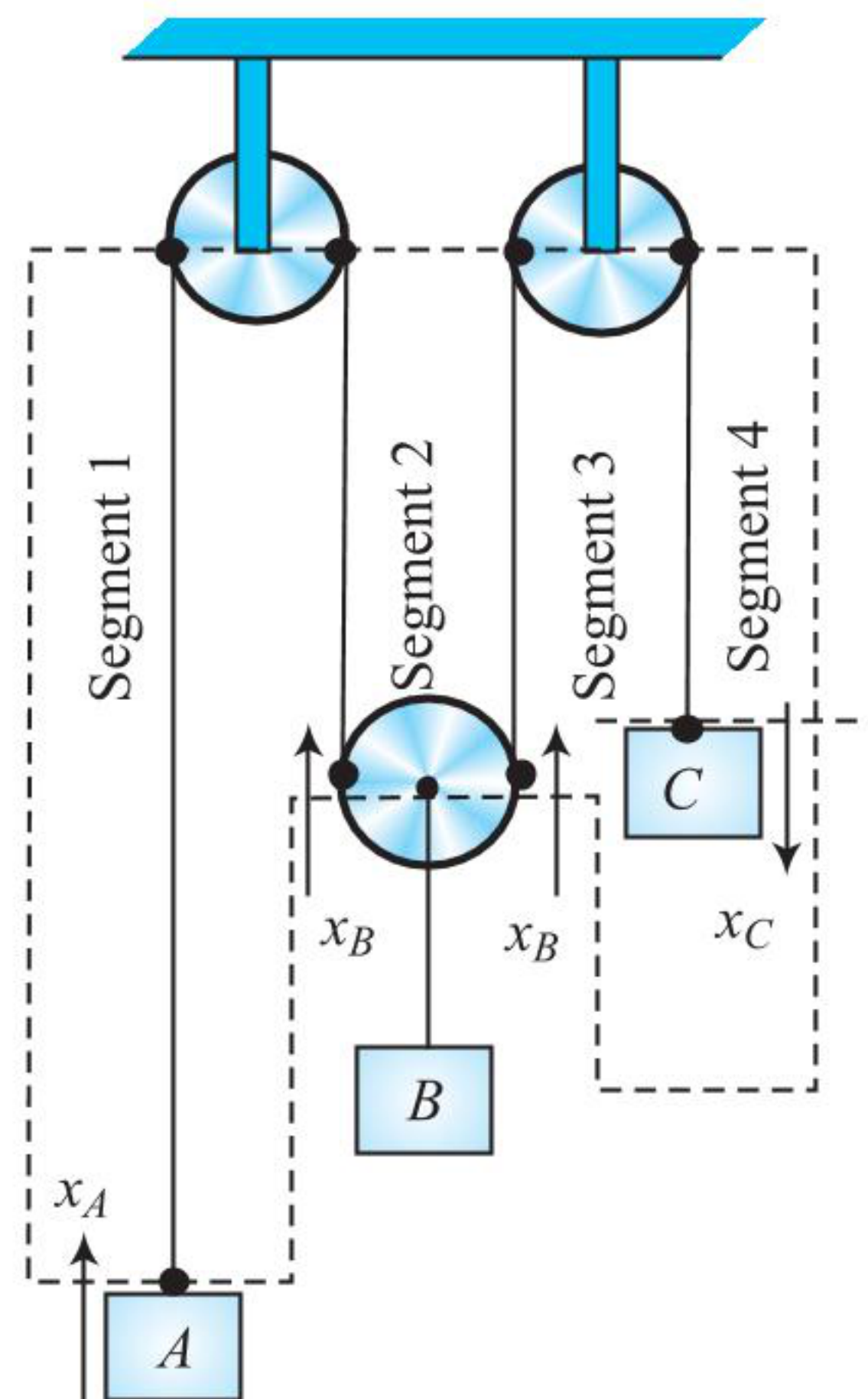
$$\Delta l_4 = (0) + (x_C) = x_C$$

Total change in the segment length should be zero.

$$\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4 = 0$$

$$(-x_A) + (-x_B) + (-x_B) + (x_C) = 0$$

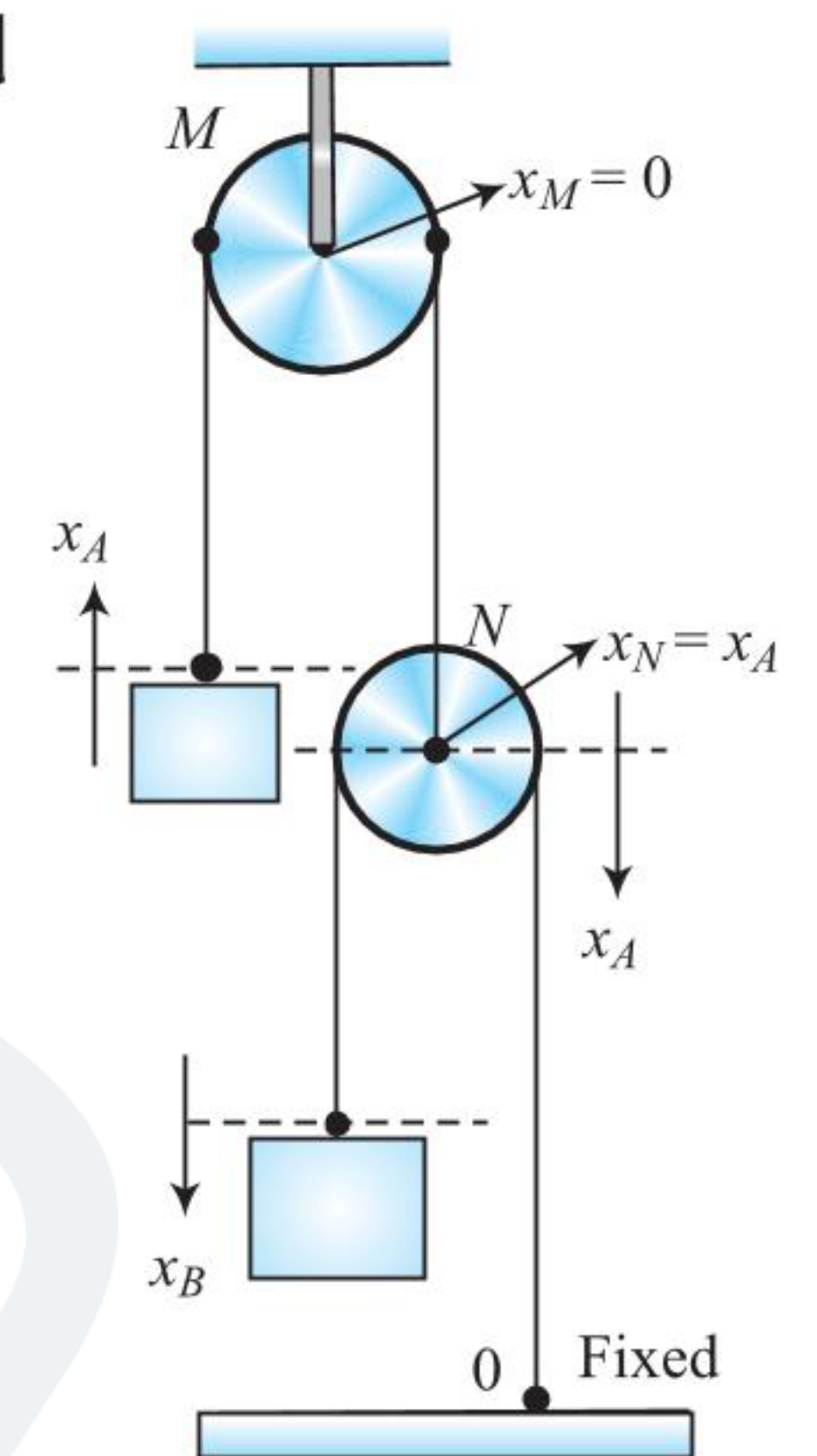
$$2x_B = x_C - x_A \Rightarrow 2a_B = a_C - a_A$$



Analysis of case II using shortcut method

- As pulley M is fixed, $|x_{P,2}| = |x_A|$.
- If block A moves up by x_A , pulley N should move x_A in downward direction as shown in figure.
- For pulley 2,

$$x_{P,2} = x_A = \frac{x_B + 0}{2} \Rightarrow x_B = 2x_A$$



Analysis of case III using shortcut method

As pulley M is fixed, $x_M = 0$.

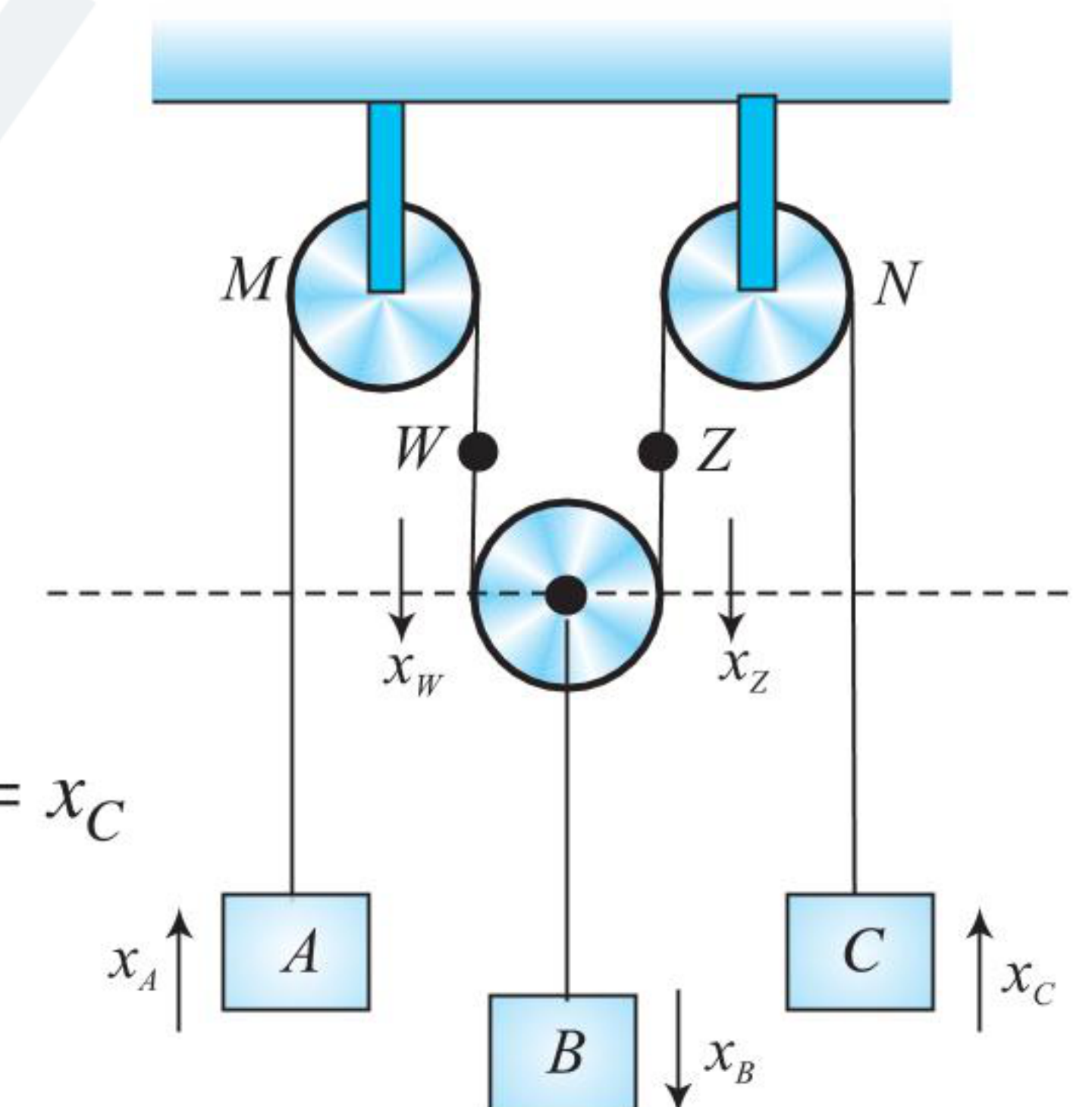
$$0 = \frac{x_A - x_W}{2} \Rightarrow x_W = x_A$$

Pulley N is also fixed.

$$x_N = 0 = \frac{x_C - x_Z}{2} \Rightarrow x_Z = x_C$$

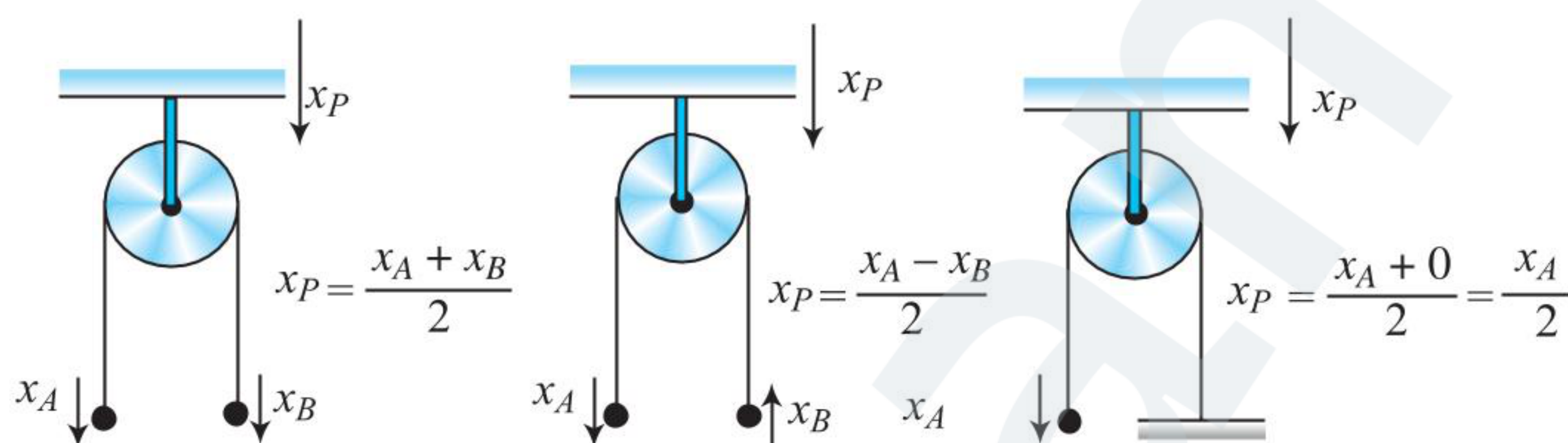
$$x_B = \frac{x_W + x_Z}{2}$$

$$2x_B = x_W + x_Z = x_A + x_C \\ = 2a_B = a_C + a_A$$



Shortcut Methods

In the cases, where pulley moves along with the blocks connected on both sides, we can say that the displacement of the pulley is the average of the displacement on both sides of the pulley.



If one end of the string is connected with the fixed end, the displacement of that end can be considered zero.

Analysis of case I using shortcut method

- As pulley M is fixed, the displacement should be zero. If the displacement of block A is x_A (down), then the displacement of other end should be x_A (up).

$$x_M = 0 = \frac{x_A + x_D}{2} \Rightarrow x_D = -x_A$$

- Displacement of block B = Displacement of pulley 2

$$x_B = \frac{x_A + 0}{2} \Rightarrow x_A = 2x_B$$

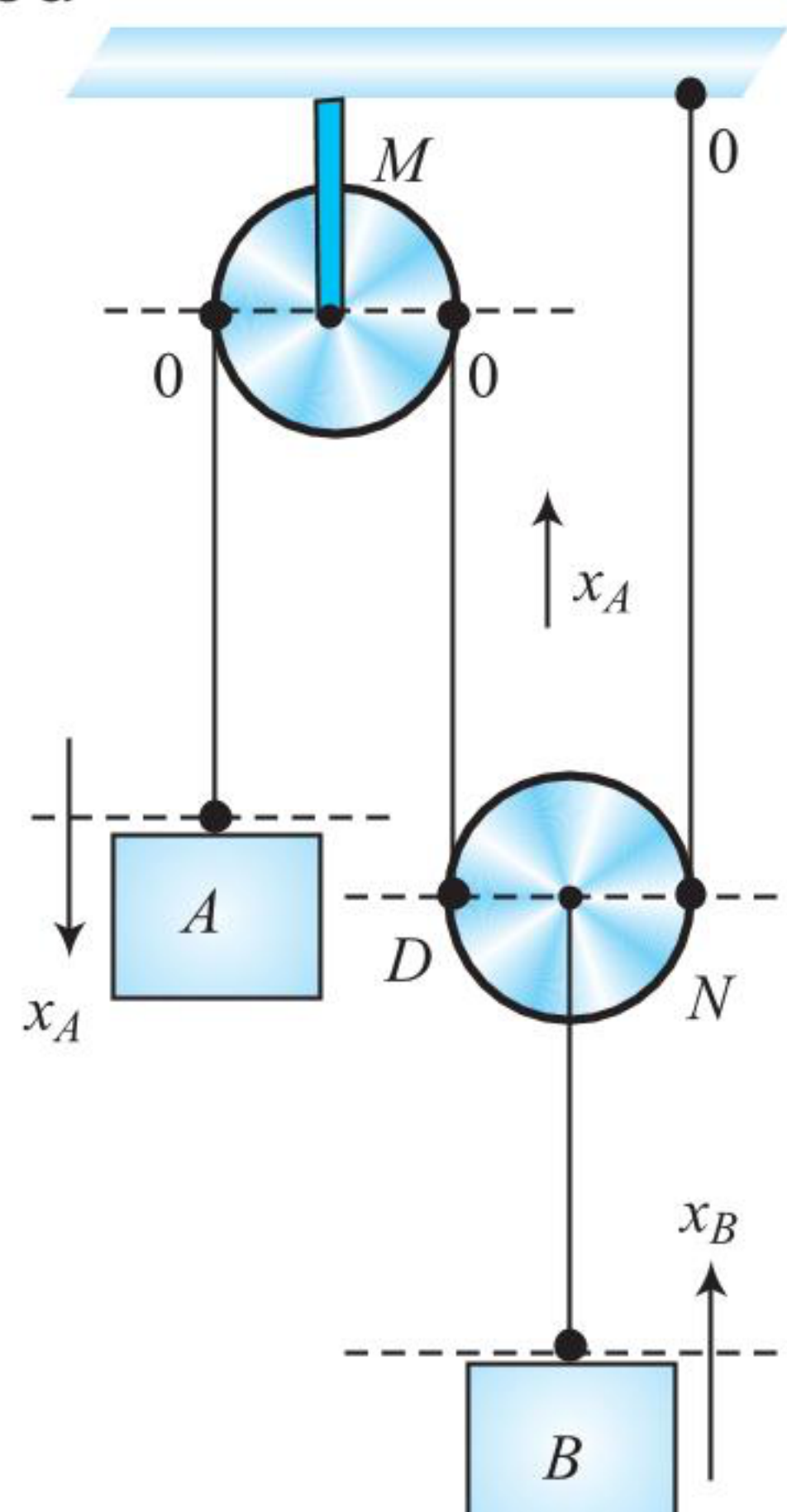
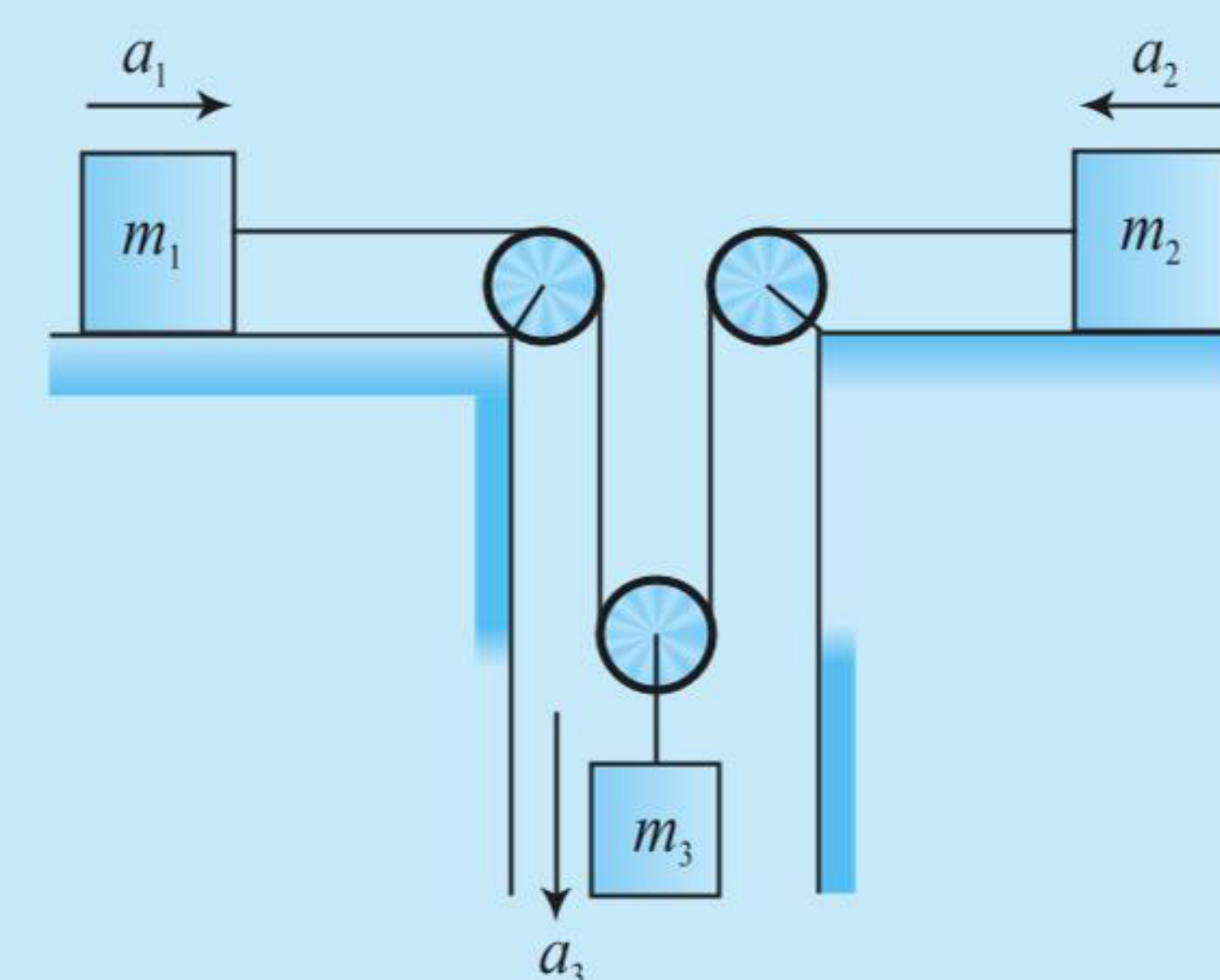
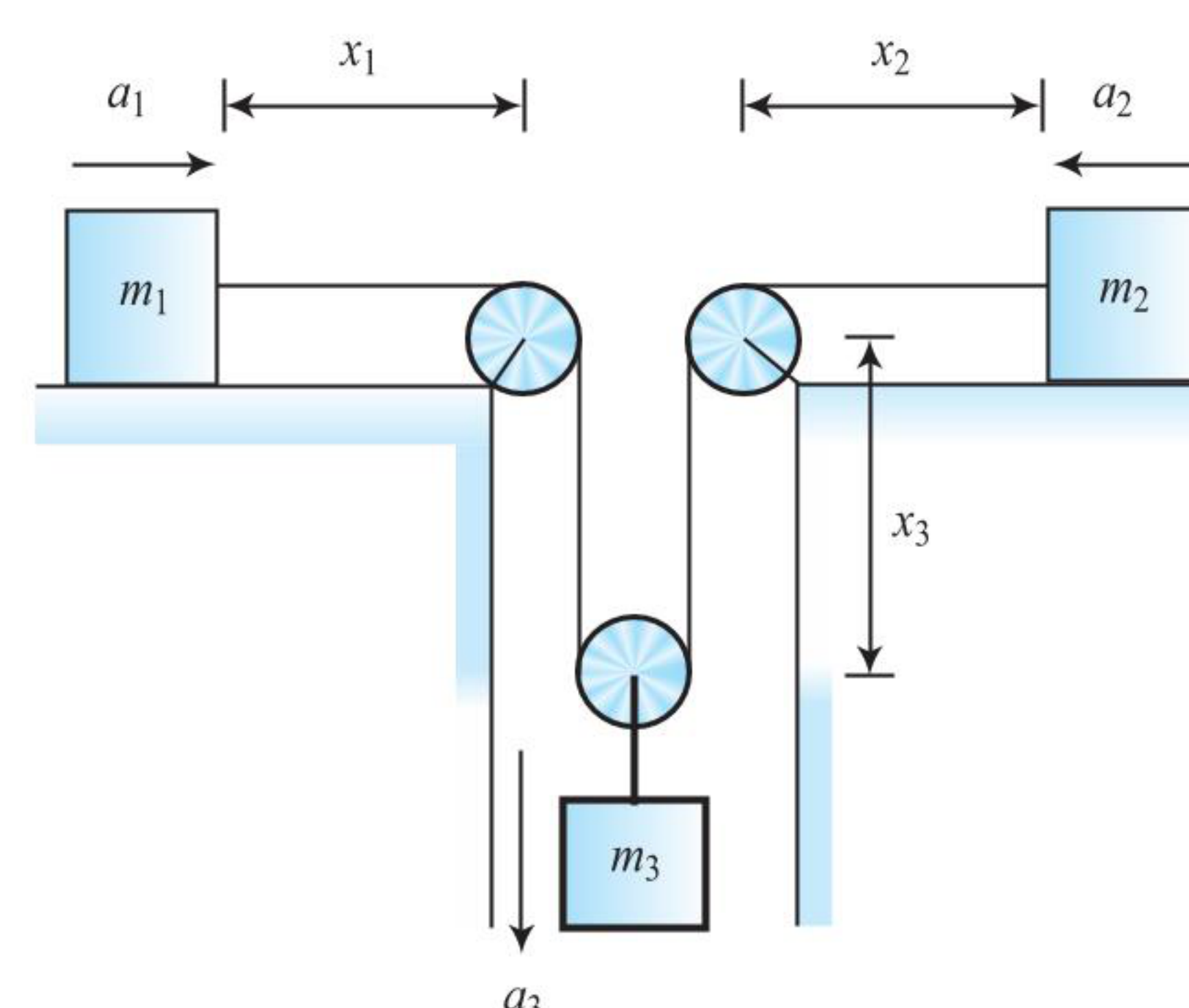


ILLUSTRATION 6.46

In the arrangement of three blocks as shown in figure, the string is inextensible. If the directions of accelerations are as shown in the figure, then determine the constraint relation.



Sol. Let us assume the respective distance of each block as shown in figure. Since the length of the string is constant, $x_1 + x_2 + 2x_3 = \text{constant}$.



On differentiating twice w.r.t. time, we get

$$\frac{d^2 x_1}{dt^2} + \frac{d^2 x_2}{dt^2} + 2 \frac{d^2 x_3}{dt^2} = 0$$

Since x_1 and x_2 are assumed to be decreasing with time,

$$\frac{d^2 x_1}{dt^2} = -a_1 \text{ and } \frac{d^2 x_2}{dt^2} = -a_2$$

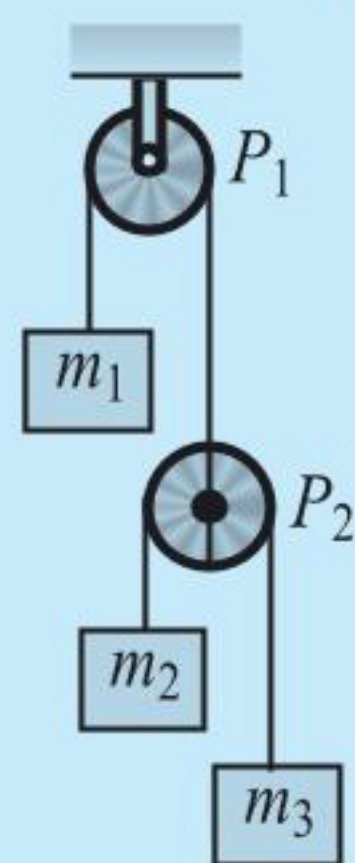
and x_3 is assumed to be increasing with time. Therefore,

$$\frac{d^2 x_3}{dt^2} = +a_3$$

Thus, $-a_1 - a_2 + 2a_3 = 0$ or $a_1 + a_2 = 2a_3$

ILLUSTRATION 6.47

A pulley-rope-mass arrangement is shown in figure. Find the acceleration of block m_1 when the masses are set free to move. Assume that the pulley and the ropes are ideal.



Sol. Constraint relations. Method 1:

For the upper string, the length of string l_1 not to change and for this string not to slacken, acceleration of m_1 w.r.t. the fixed pulley = acceleration of the movable pulley w.r.t. the fixed pulley

$$\text{or } |a_1| = |a_p|. \quad \dots(i)$$

Constraint relations for string 2

Let us assume that block m_1 is moving up with acceleration a_1 and blocks m_2 and m_3 are moving down with acceleration a_2 and a_3 , respectively w.r.t. ground.

Total length of string 2 is constant.

Therefore,

$$l_2 = (x_2 - x_{P_2}) + (x_3 - x_{P_2}) + l'_0$$

where l'_0 is the length of string on pulley 2 passing through the pulley P_2

$$l_2 = x_2 + x_3 - 2x_{P_2} + l'_0 \quad \dots(ii)$$

Differentiating Eq. (ii) w.r.t. time,

$$\frac{dl_2}{dt} = 0 \text{ (as total length of string is constant)}$$

$$\frac{dl'_0}{dt} = 0 \text{ (as length of string over pulley is constant)}$$

Differentiating Eq. (ii) twice w.r.t. time

$$2a_1 = a_3 + a_2 \quad \dots(iii)$$

Method 2: The acceleration of block m_1 and pulley 2 will be same in magnitude. Now considering pulley 2 and block 2 and 3.

Change in the length of segment (1)

$$\Delta l_1 = (+y_2) + (-y_{P_2}) = y_2 - y_{P_2}$$

Change in the length of segment (2),

$$\Delta l_2 = (y_3) + (-y_{P_2}) = y_3 - y_{P_2}$$

Total sum of change in the segment length of string should be zero.

$$\Delta l = 0 = [y_2 - y_{P_2}] + [y_3 - y_{P_2}]$$

$$\Rightarrow \Delta l = 0 = y_2 + y_3 - 2y_{P_2}$$

$$\Rightarrow 2y_{P_2} = y_3 + y_2 \Rightarrow 2a_{P_2} = a_3 + a_2$$

$$\text{or } 2a_1 = a_3 + a_2$$

This equation is same as (iii) calculated in method 2.

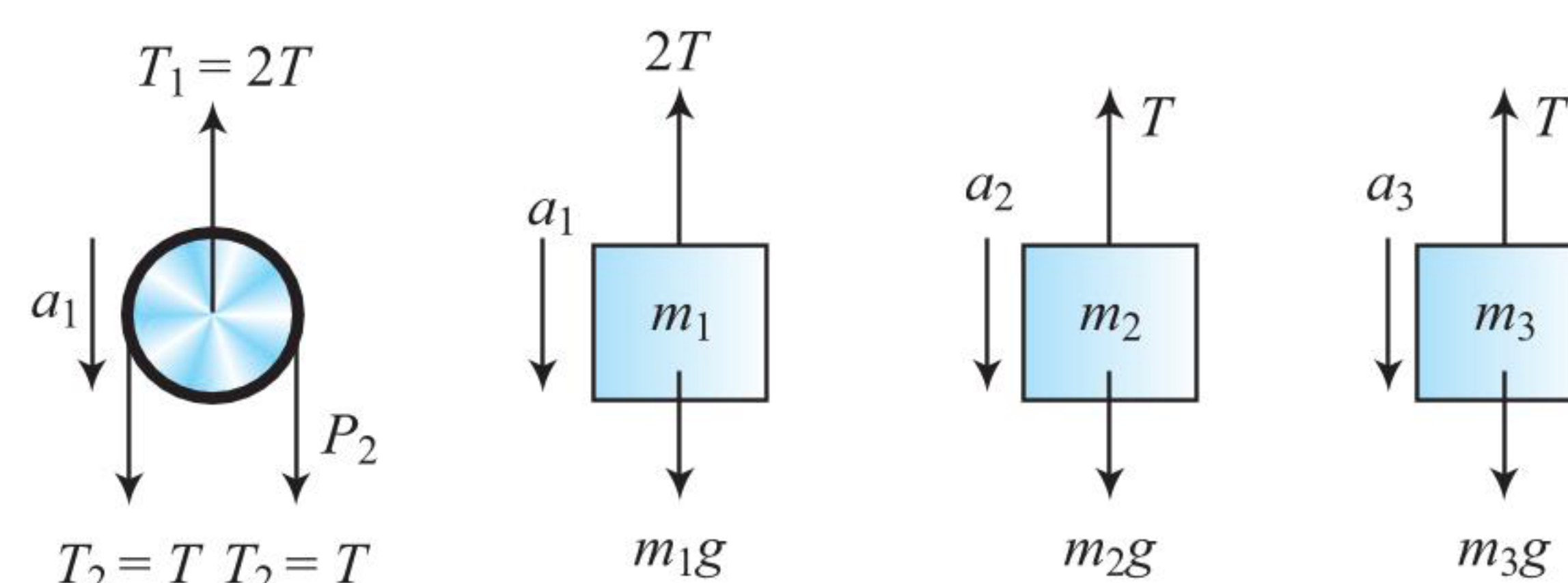
Method 3: For the rope length l_2 not to change and for this rope not to slacken acceleration of m_2 w.r.t. movable pulley = - acceleration of m_3 w.r.t. the movable pulley

$$\text{or } (a_2 - a_p) = -(a_3 - a_p)$$

$$a_2 + a_3 - 2a_p = 0 \Rightarrow 2a_0 = a_2 + a_3$$

This equation is same as Eq. (iii) as calculated in method 1 and 2.

Applying Newton's laws of motion equations,



From free-body diagram (figure),

Equations of motion

$$\text{For } m_1: 2T - m_1g = m_1a_1 \quad \dots(iv)$$

$$\text{For } m_2: m_2g - T = m_2a_2 \quad \dots(v)$$

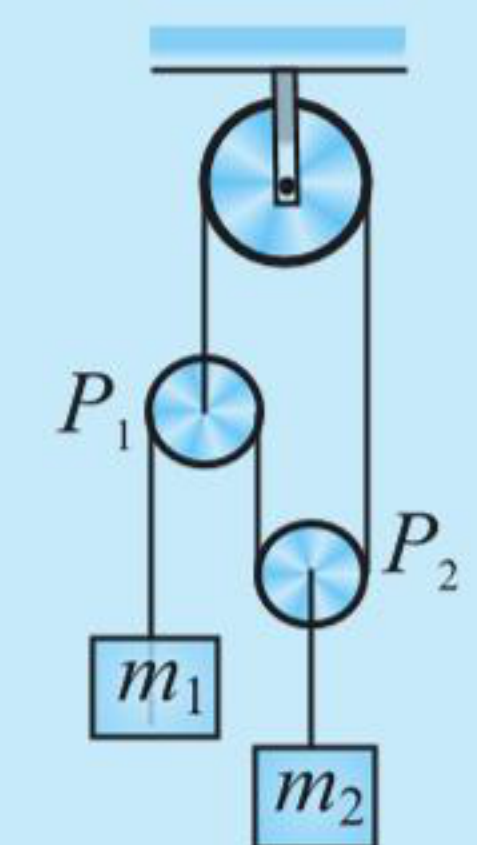
$$\text{For } m_3: m_3g - T = m_3a_3 \quad \dots(vi)$$

After solving (iii), (iv), (v), and (vi), we get

$$a_1 = \frac{4m_2m_3g - m_1(m_2 + m_3)g}{4m_2m_3 + m_1(m_2 + m_3)}$$

ILLUSTRATION 6.48

In the arrangement shown in figure, find the tensions in the rope and accelerations of the masses m_1 and m_2 and pulleys P_1 and P_2 when the system is set free to move. Assume the pulleys to be massless and strings are light and inextensible.



Sol. Applying the equations of motion for pulleys and blocks

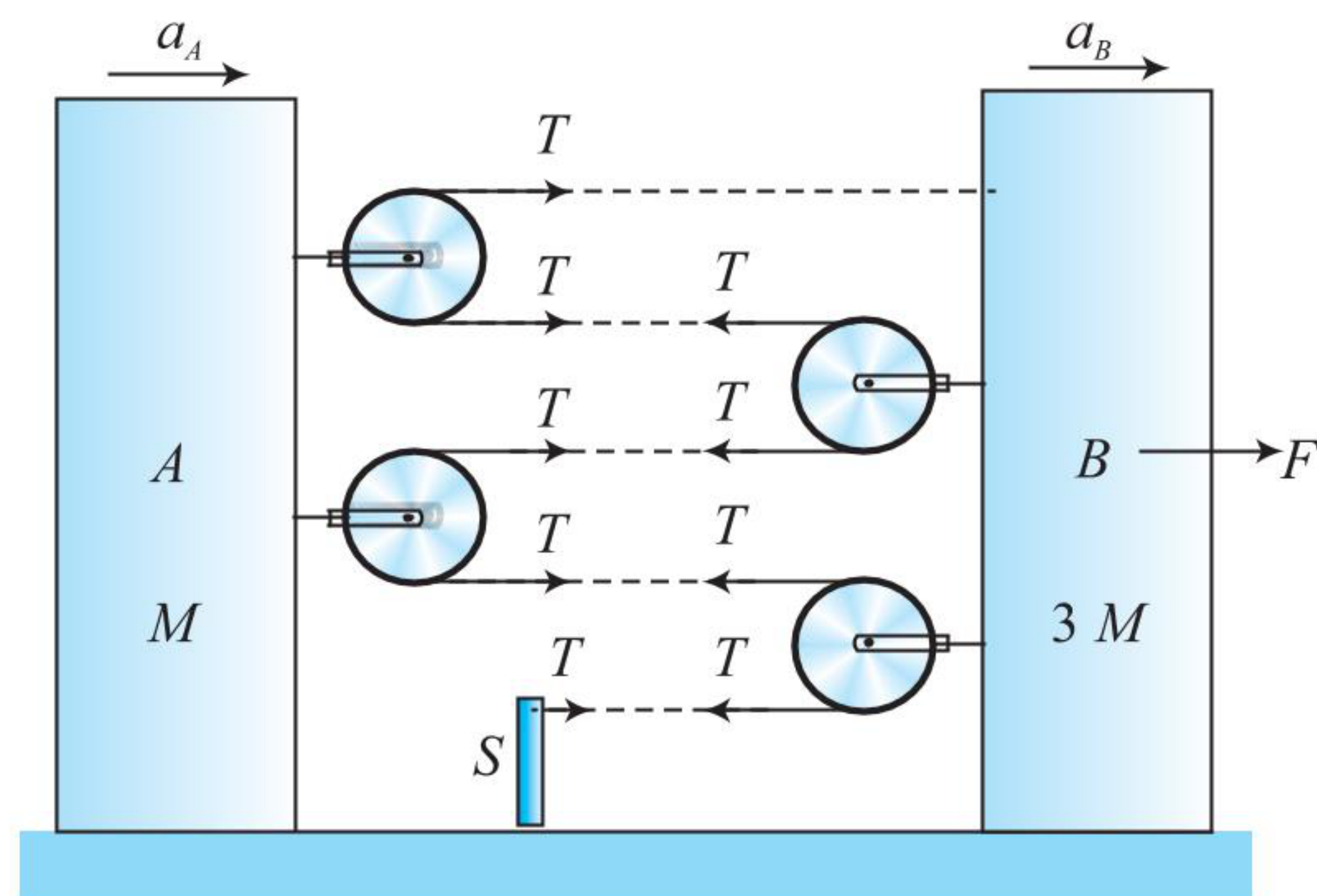
$$\text{For } m_1: T - m_1g = m_1a_1 \quad \dots(i)$$

$$\text{For } m_2: m_2g - T_2 = m_2a_2 \quad \dots(ii)$$

$$\text{For pulley } P_1: 2T - T = 0 \times a_{P_1} \quad \dots(iii)$$

$$\text{For pulley } P_2: T_2 - 2T = 0 \times a_{P_2} \quad \dots(iv)$$

Equations of motion



For A:

$$4T = m_A a_A = M a_A \quad \dots(i)$$

For B:

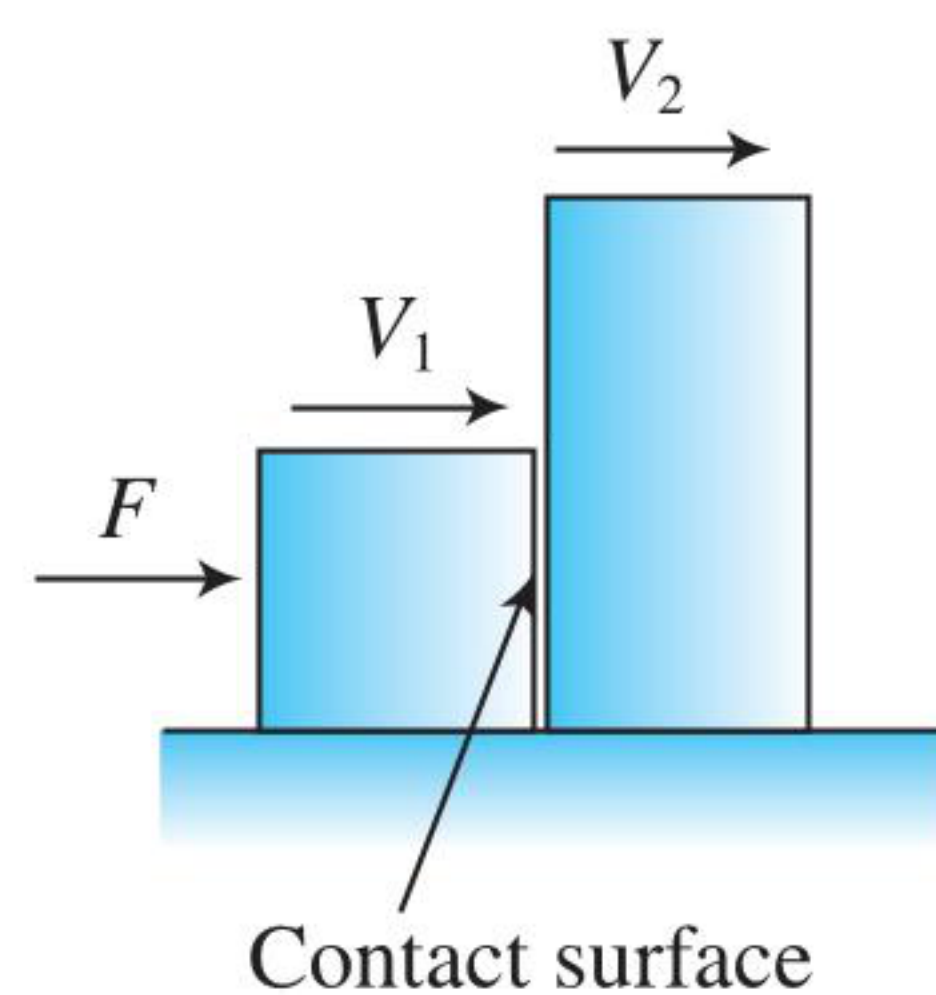
$$F - 5T = m_B a_B = 3M a_B \quad \dots(ii)$$

$$a_B = \frac{16F}{73M}$$

$$\text{and } a_A = \frac{5}{4} \times \left(\frac{16F}{73M} \right) = \frac{20F}{73M}$$

WEDGE CONSTRAINT: NORMAL CONSTRAINT

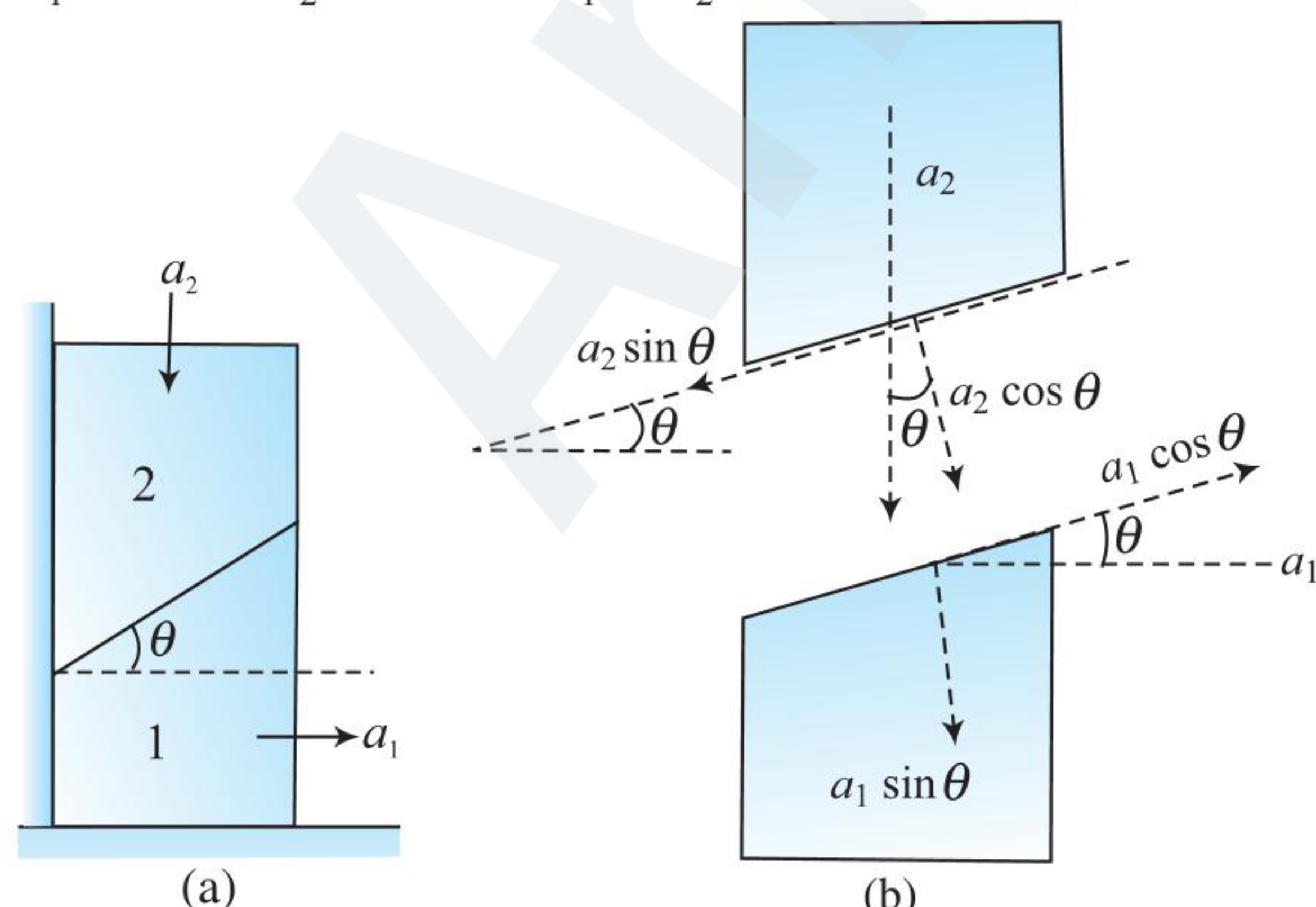
Consider two blocks moving on a surface and always remaining in contact. In order to maintain the contact component of velocity the vector perpendicular to the contact surface must be same, i.e., $\vec{v}_1 = \vec{v}_2$.



Similarly, $\vec{a}_1 = \vec{a}_2$

Now consider two wedges (1) and (2) are placed in contact. Wedge (1) is moving towards right with acceleration a_1 while wedge (2) is moving down with acceleration a_2 . If wedges (1) and (2) are to remain in contact, the component of acceleration perpendicular to the contact surface must be zero.

$$a_1 \sin \theta = a_2 \cos \theta \text{ or } a_1 = a_2 \cot \theta$$



Now finally consider a rod (1) and wedge (2) are arranged as shown in figure. The rod moves down with acceleration a_1 and wedge slides on the inclined with acceleration a_2 . As the rod

and the wedge always remain in contact with each other, then the acceleration of the rod should be equal to the component of acceleration of the wedge perpendicular to the surface of contact, i.e., $a_1 = a_2 \sin \theta$.

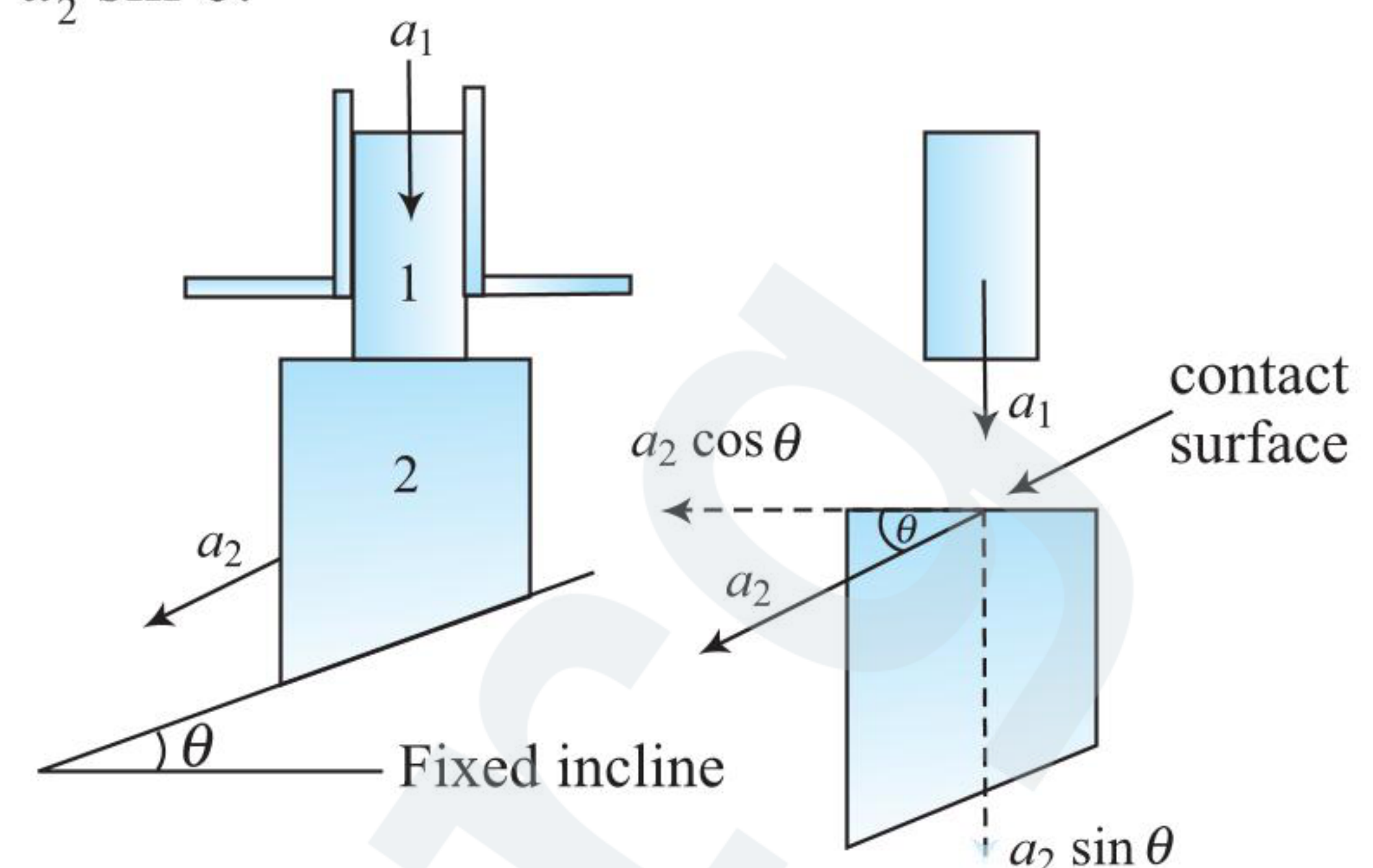
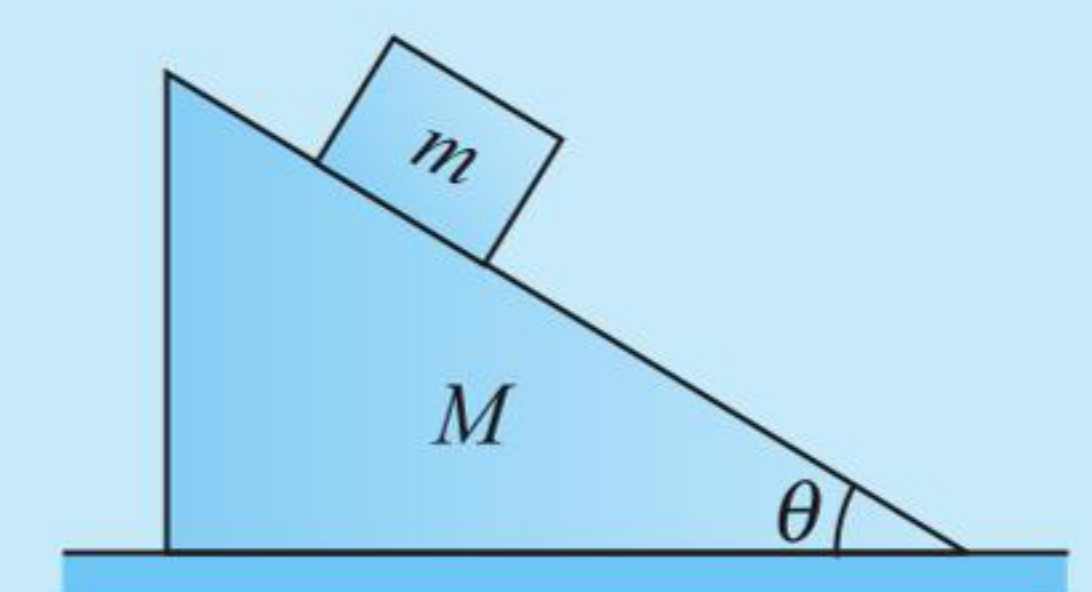


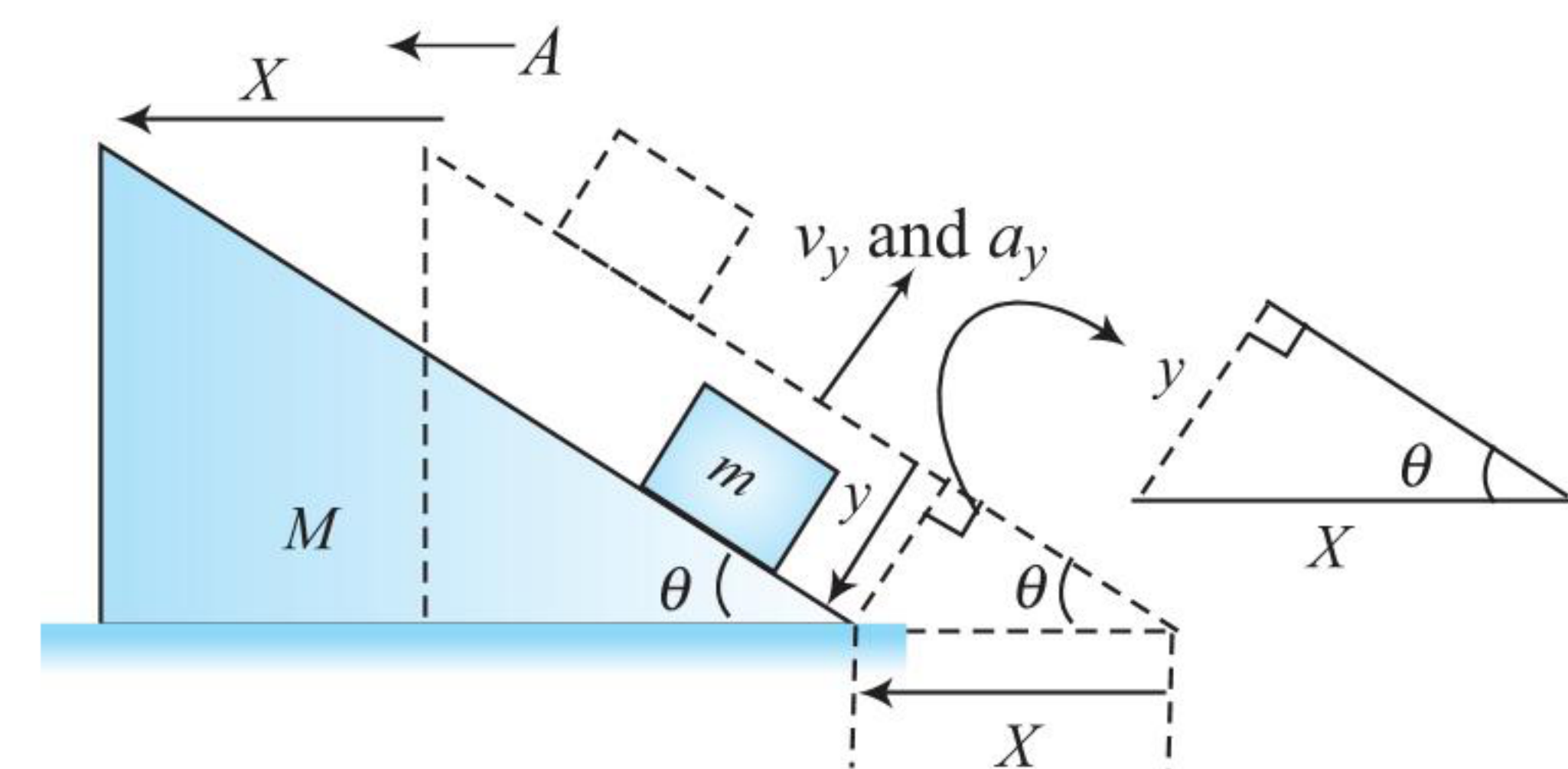
ILLUSTRATION 6.50

A block of mass m is placed on the inclined surface of a wedge as shown in figure. Calculate the acceleration of the wedge and the block when the block is released. Assume all surfaces are frictionless.



Sol. Constraint relation. Approach 1

We can observe that the wedge M can only move in horizontal direction towards left, and the block m can slide on inclined surface of M always in contact with the wedge.



- Let us define our x and y axes parallel to the incline and perpendicular to incline, respectively.
- We can observe that the displacement of m and M in $-x'$ direction will be same as the block never lose contact with the wedge.
- If the wedge moves in the horizontal direction by a distance x' , during this time, the block will move x in x' direction.
- We can relate these displacements x and X as

$$\frac{y}{X} = \sin \theta \Rightarrow y = X \sin \theta \quad \dots(i)$$

Hence, velocity relation can be written as:

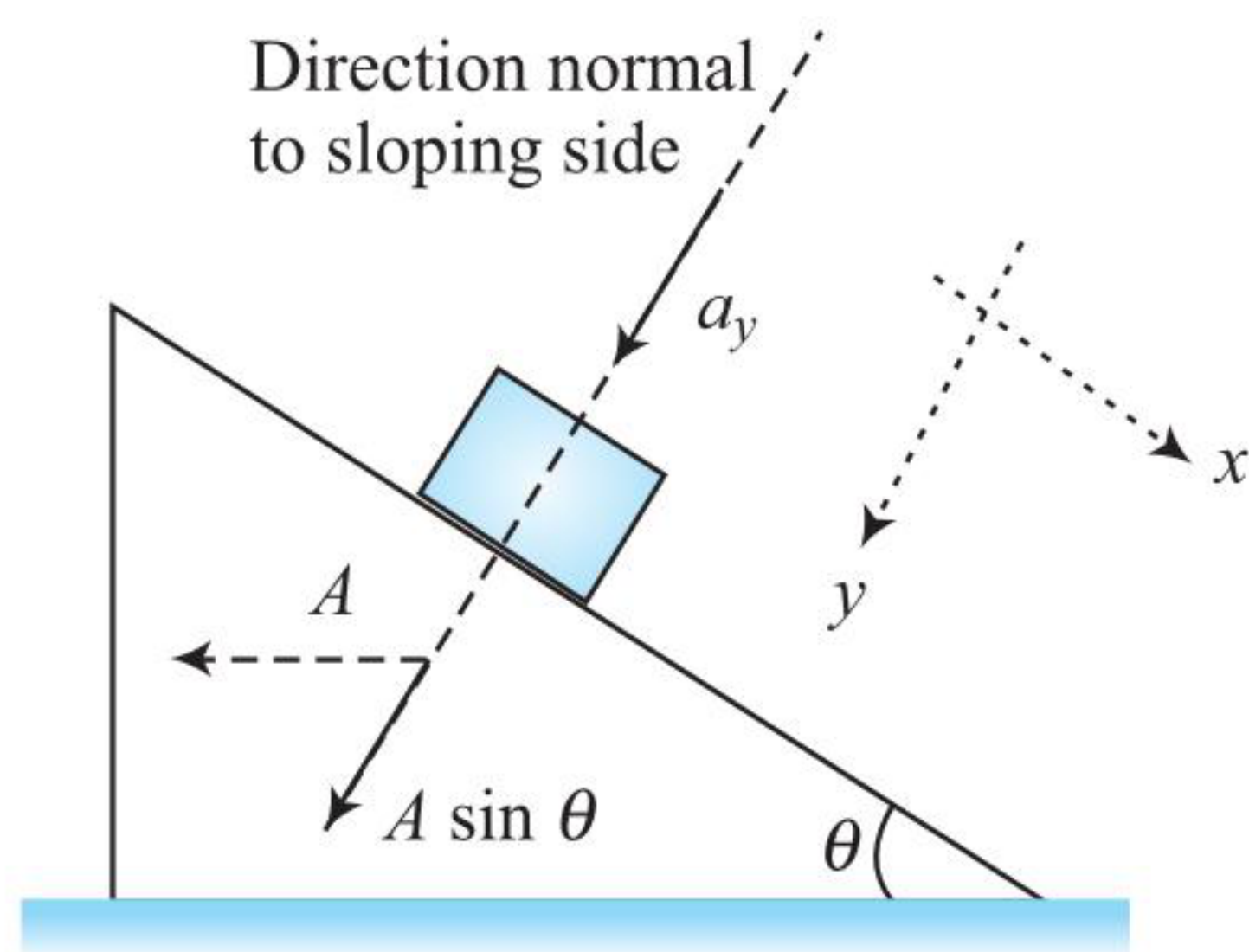
$$v_y = V \sin \theta \quad \dots(ii)$$

and acceleration relation can be written as:

$$a_y = A \sin \theta \quad \dots(iii)$$

Here v_y and a_y are the velocity and acceleration of the block, respectively, in the direction perpendicular to inclined surface.

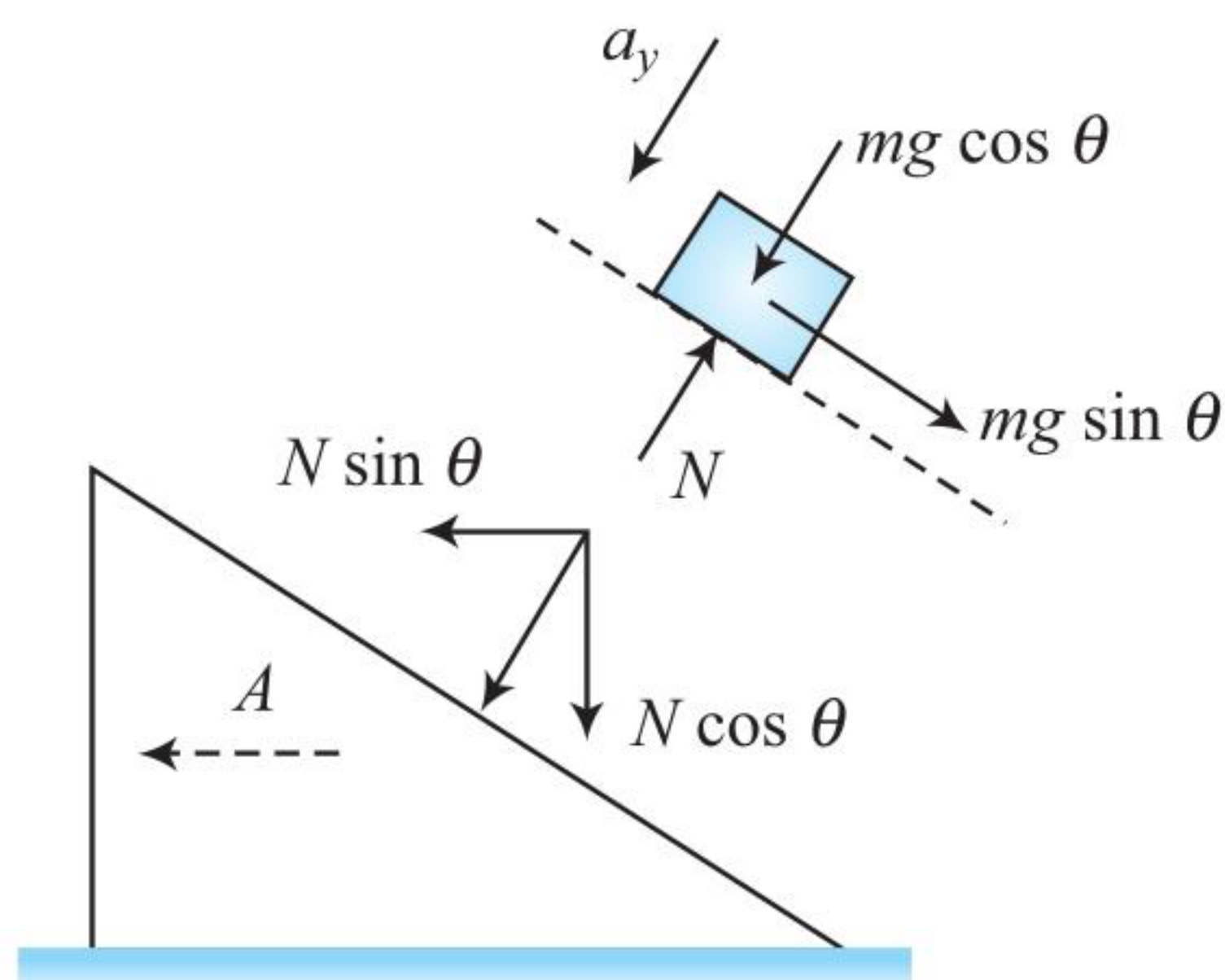
Approach 2: We consider the motion of the block parallel to incline and perpendicular to inclined surface. Let the components of acceleration of block with respect to ground along these direction are a_x and a_y , respectively.



Then we can write $a_y = A \sin \theta$.

Sol. For wedge:

$$N \sin \theta = MA \quad \dots(i)$$



For block: considering the block in the direction perpendicular to sloping surface.

$$mg \cos \theta - N = ma_y$$

But $a_y = A \sin \theta$

Hence,

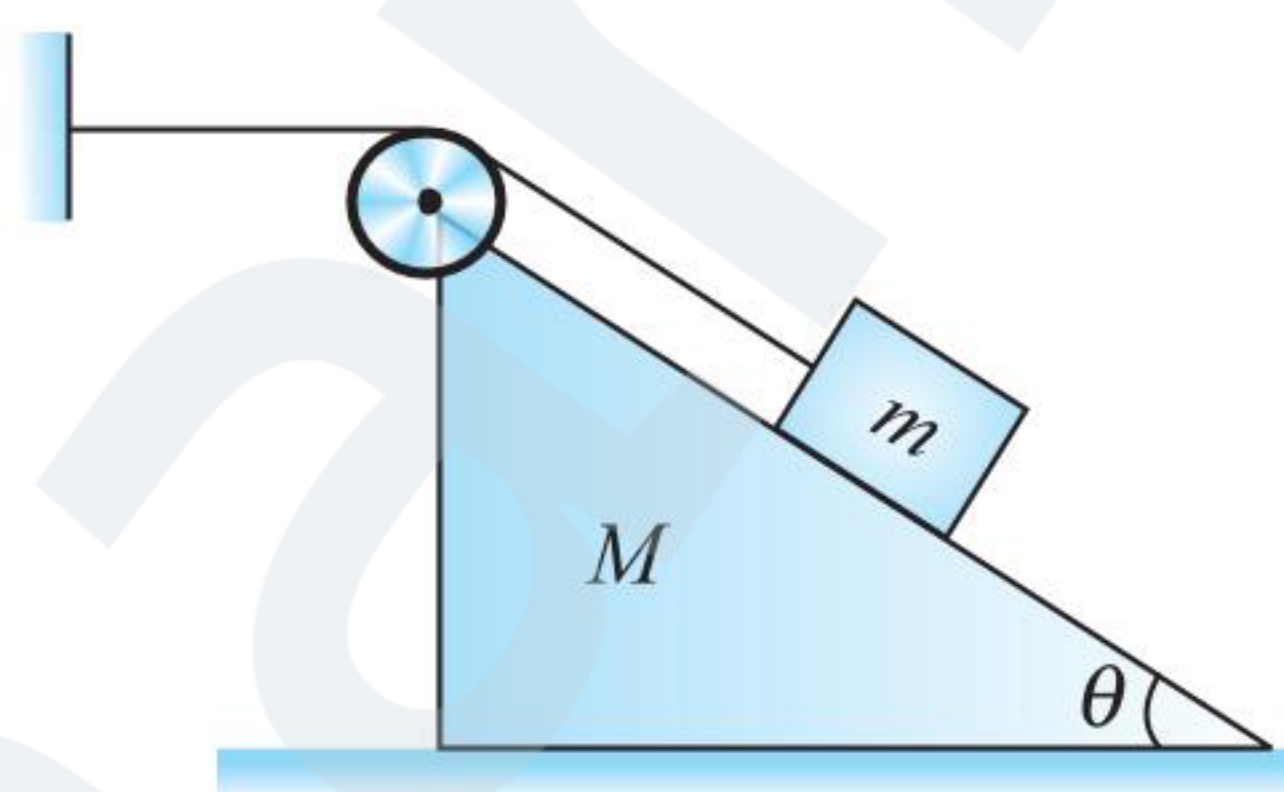
$$mg \cos \theta - N = mA \sin \theta \quad \dots(ii)$$

From (i) and (ii), we get

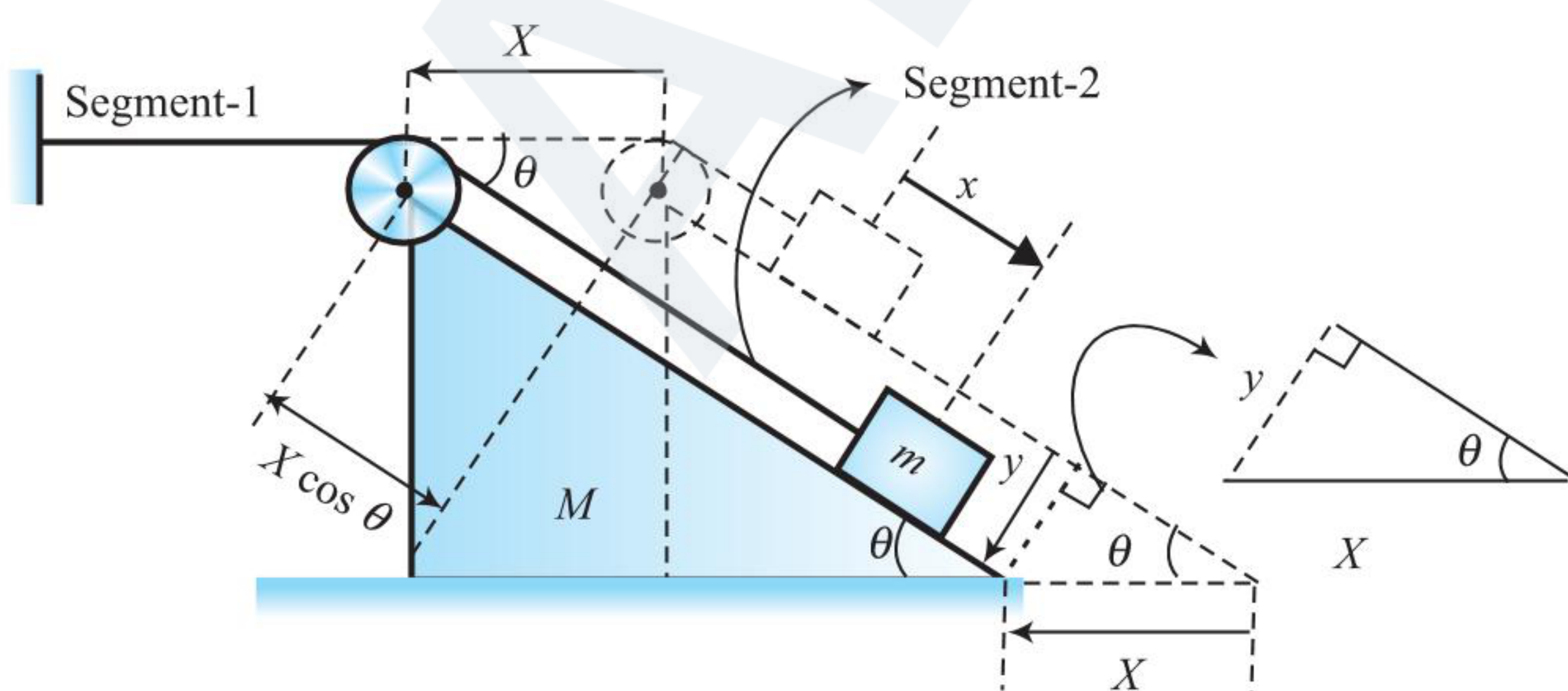
$$A = \frac{mg \sin \theta \cos \theta}{M + m \sin^2 \theta}$$

PULLEY AND WEDGE CONSTRAINT

Consider a block of mass m placed on the inclined surface of the wedge. The block is connected with a string and arranged as shown in figure. The system is released from rest.



We can divide the string in two segments, (1) and (2). let at any interval of time the wedge moves a distance X towards right and the block moves a distance x parallel to inclined direction.



From the diagrams it is clear that the length of segment 1 will decrease by X while the length of segment 2 will increase by $(x + X \cos \theta)$. As the overall length of the string is constant, then we can write

$$-X + (x + X \cos \theta) = 0$$

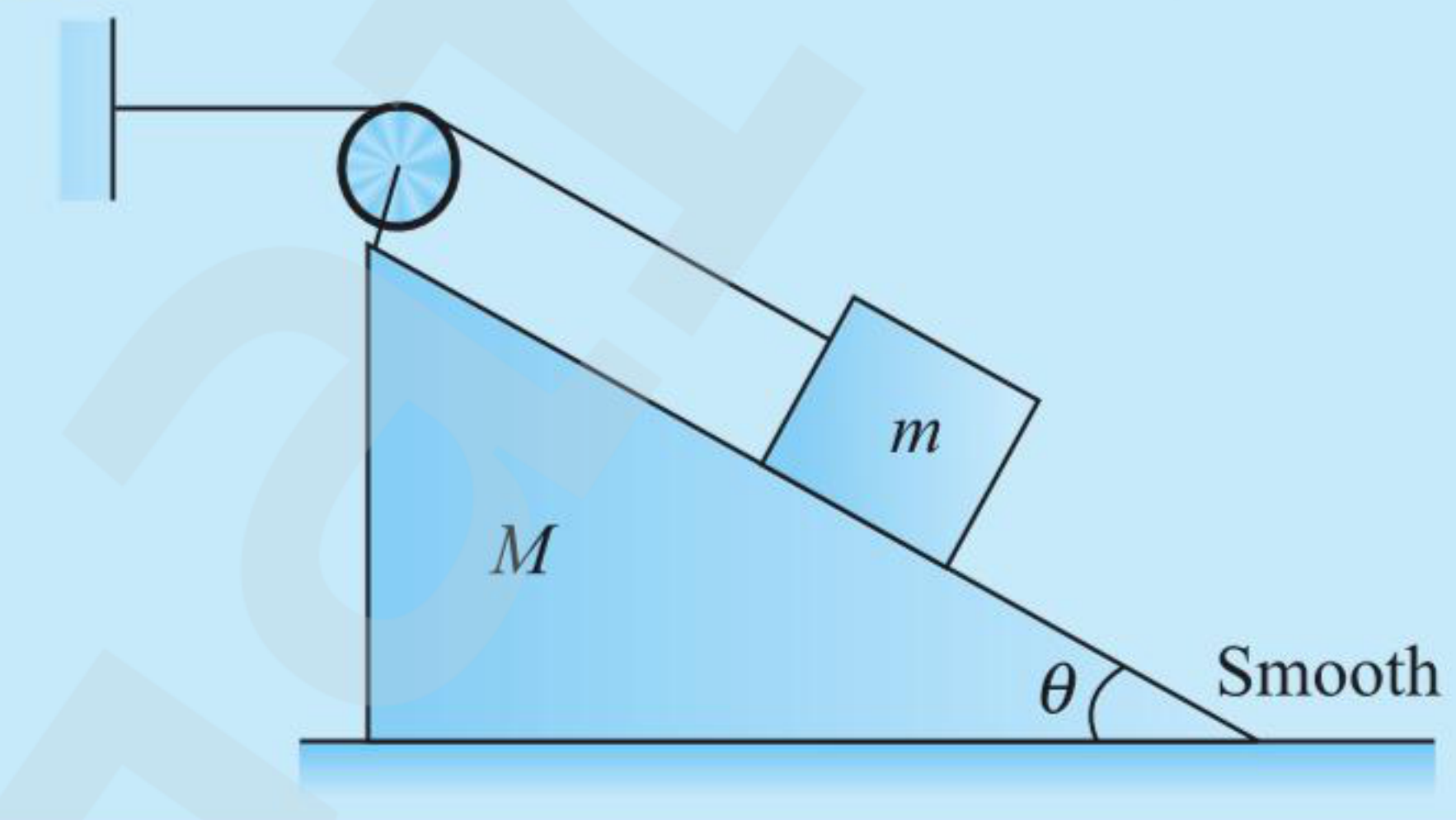
$$x = X(1 - \cos \theta) \text{ and } a_x = A(1 - \cos \theta) \quad \dots(i)$$

From wedge constant, it is clear that

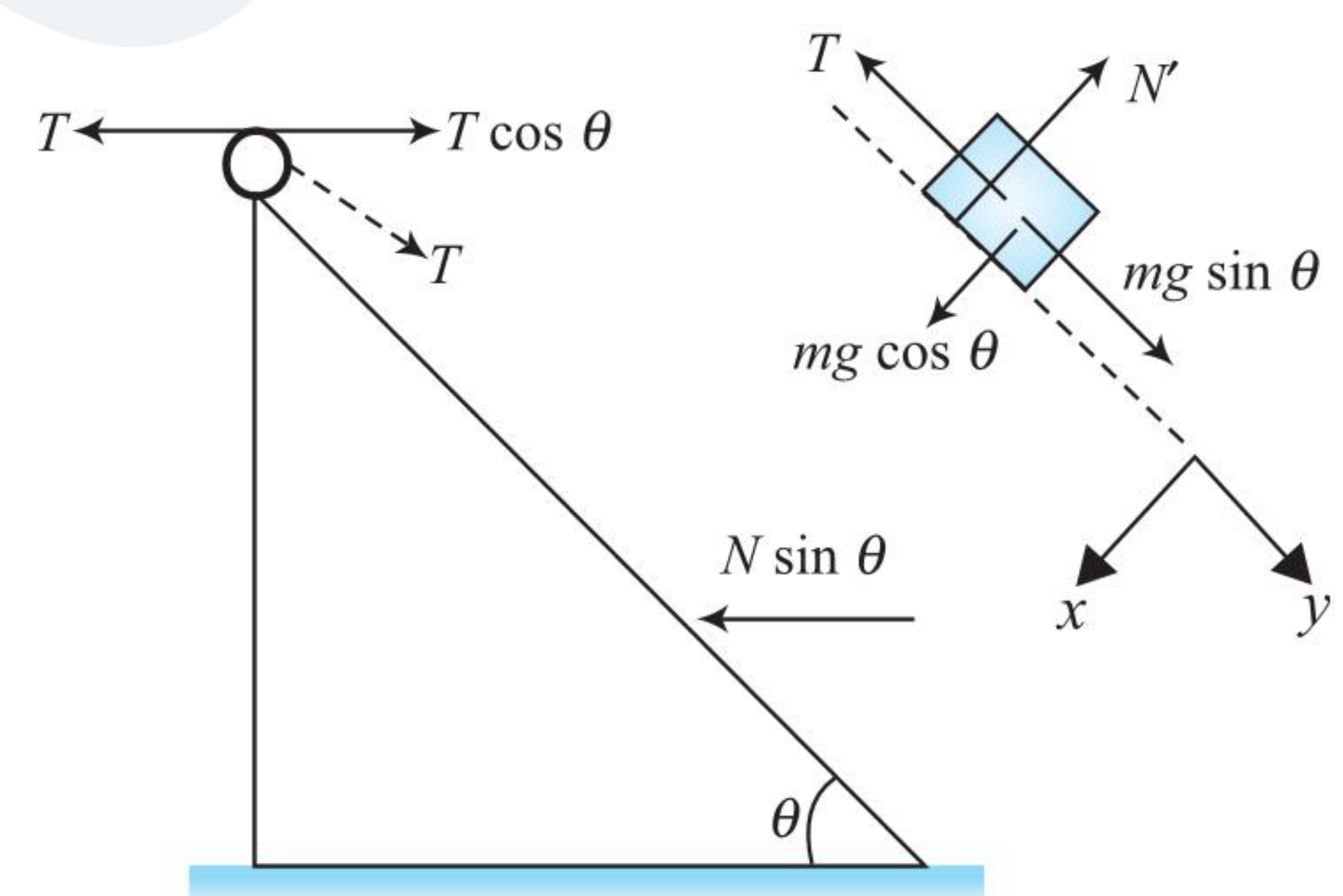
$$\frac{y}{x} = \sin \theta \Rightarrow y = X \sin \theta \text{ and } a_y = A \sin \theta$$

ILLUSTRATION 6.51

The mass of wedge, shown in figure, is M and that of the block is m . Neglecting friction at all the places and mass of the pulley, calculate the acceleration of wedge. Thread is inextensible.



Sol. Constraint relation:



It is clear from above discussion,

$$a_x = A(1 - \cos \theta)$$

and $a_y = A \sin \theta$

Equations of motion

Considering the motion of wedge in horizontal direction

For wedge:

$$T - T \cos \theta + N \sin \theta = MA \quad \dots(i)$$

Considering the motion of block parallel and perpendicular to the sloping side in horizontal direction.

For block: In the direction parallel to inclined surface,

$$mg \sin \theta - T = ma_x = MA(1 - \cos \theta)$$

or $T = mg \sin \theta - MA(1 - \cos \theta) \quad \dots(ii)$

In the direction perpendicular to inclined plane,

$$mg \cos \theta - N = ma_y = mA \sin \theta$$

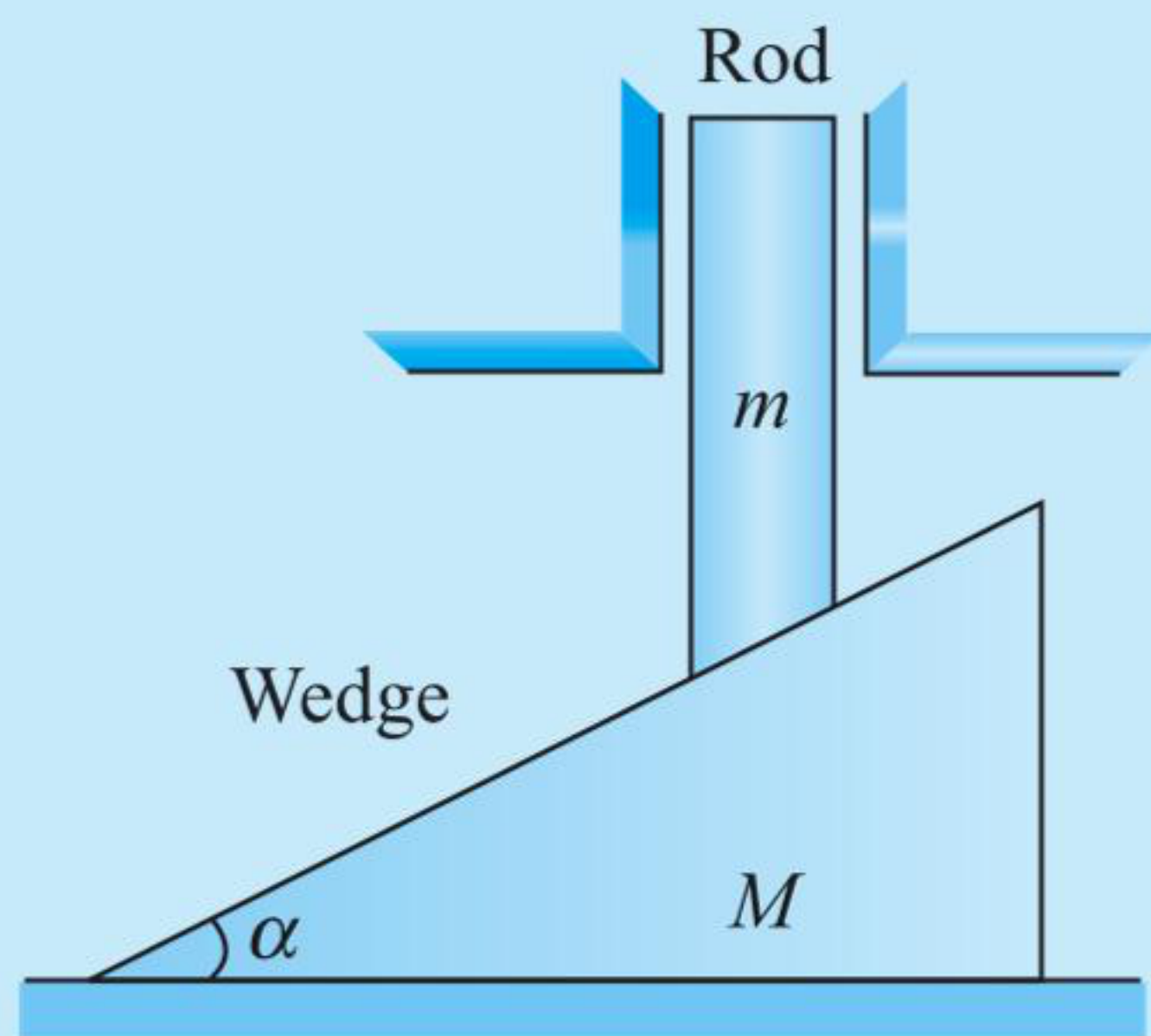
$$N = mg \cos \theta - mA \sin \theta \quad \dots(iii)$$

Substituting the value of N from (ii) and (iii) in (i), we get

$$A = \frac{mg \sin \theta}{M + 2m(1 - \cos \theta)}$$

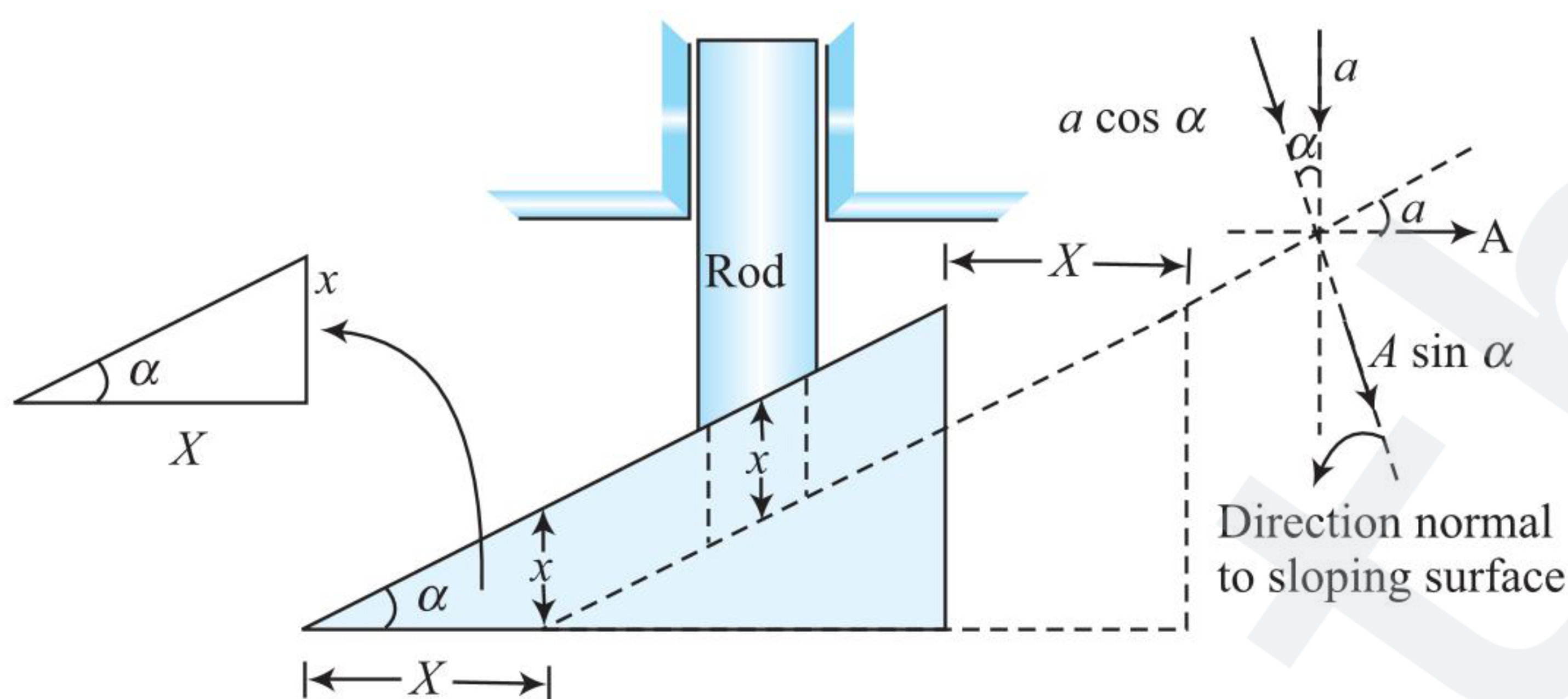
ILLUSTRATION 6.52

A rod of mass m is supported on a wedge of mass M shown in figure. Find the accelerations of rod a and wedge A in the arrangement. The friction between all contact surfaces is negligible.



Sol. The rod is constrained to move in the vertical direction (with the help of the guides) and the wedge will move along the surface in the horizontal direction. Initially, the system is held at rest.

Constraint relation: Approach 1 Let the acceleration of m w.r.t. ground be a vertically downwards and acceleration of M w.r.t. ground be A horizontally towards right.



The motion of the system is constrained by the fact that the “bottom face of the rod must always be in contact with the inclined plane.” If in time t , X is the displacement of the wedge and x is the displacement of the rod, then the constraint demands that

$$\text{From figure, we have } \frac{x}{X} = \tan \alpha$$

$$\therefore x = X \tan \alpha \quad (i)$$

Here α remains constant.

Differentiating (i) w.r.t. t twice, we get

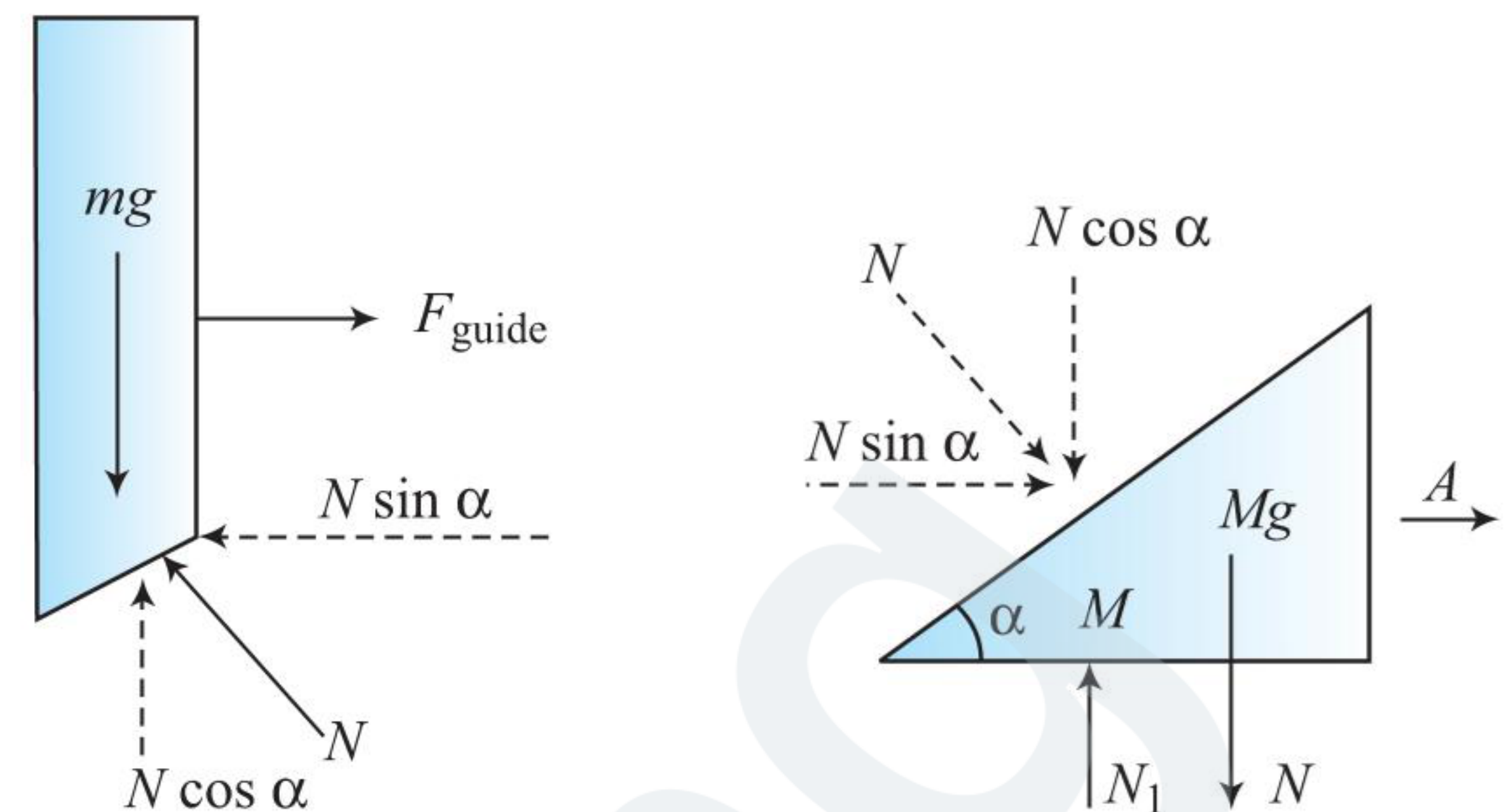
$$\left(\frac{d^2 x}{dt^2} \right) = \left(\frac{d^2 X}{dt^2} \right) \tan \alpha \quad [\tan \alpha = \text{constant}]$$

Hence, $a = A \tan \alpha$

Approach 2: The fact that the rod (or a particle on the wedge) and the wedge must not lose contact is usually called *wedge constraints*. For this, the component of the acceleration of the rod perpendicular to the wedge plane = component of acceleration of the wedge perpendicular to wedge plane.

$$a \cos \alpha = A \sin \alpha \Rightarrow a = A \tan \alpha$$

The force acting on the rod are:



- The weight mg , vertically downwards
- The normal force N , normal to the bottom surface of the rod
- The force (F_{guide}) exerted by the guide to nullify the horizontal component of N as for the rod $a_{\text{Horizontal}} = 0$.

Motion in vertical direction

$$mg - N \cos \alpha = ma \quad \dots(ii)$$

and the forces acting on the wedge are

- The weight, Mg
- N , reaction of N acting on the rod
- N_1 , normal force by the surface

The force equations are

$$N_1 - Mg - N \cos \alpha = 0 \quad (iii)$$

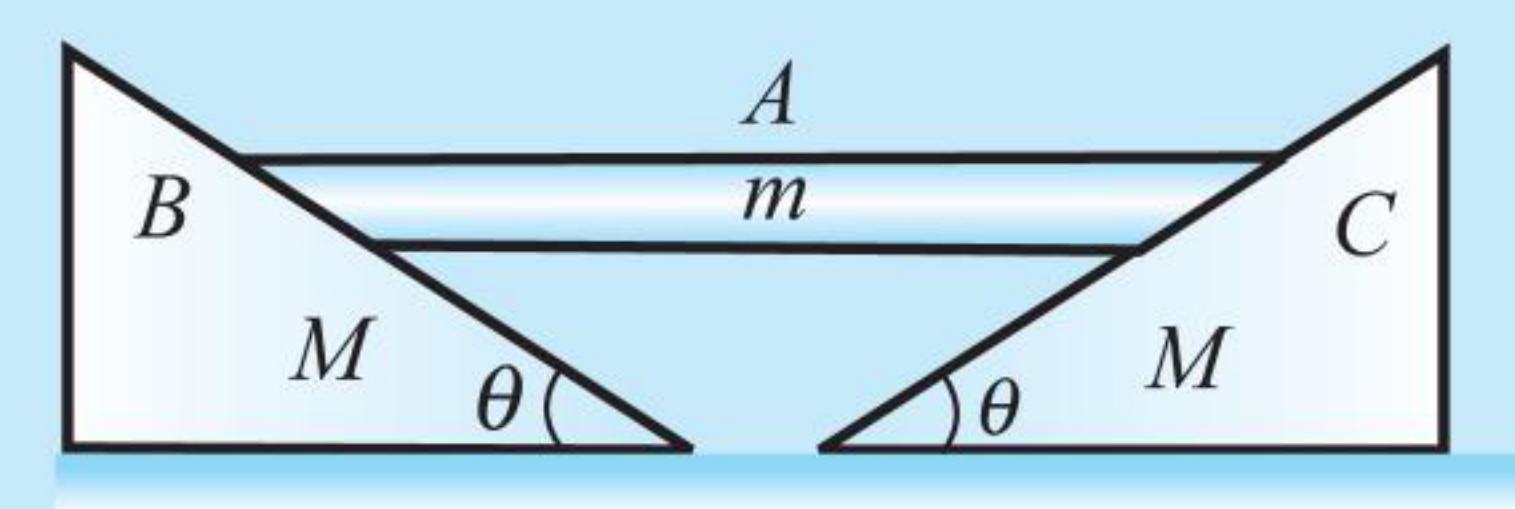
$$\text{and } N \sin \alpha = MA \quad (iv)$$

Solving the above equations for a and A , we get

$$a = \frac{mg \tan \alpha}{m \tan \alpha + M \cot \alpha} \quad \text{and} \quad A = \frac{mg}{m \tan \alpha + M \cot \alpha}$$

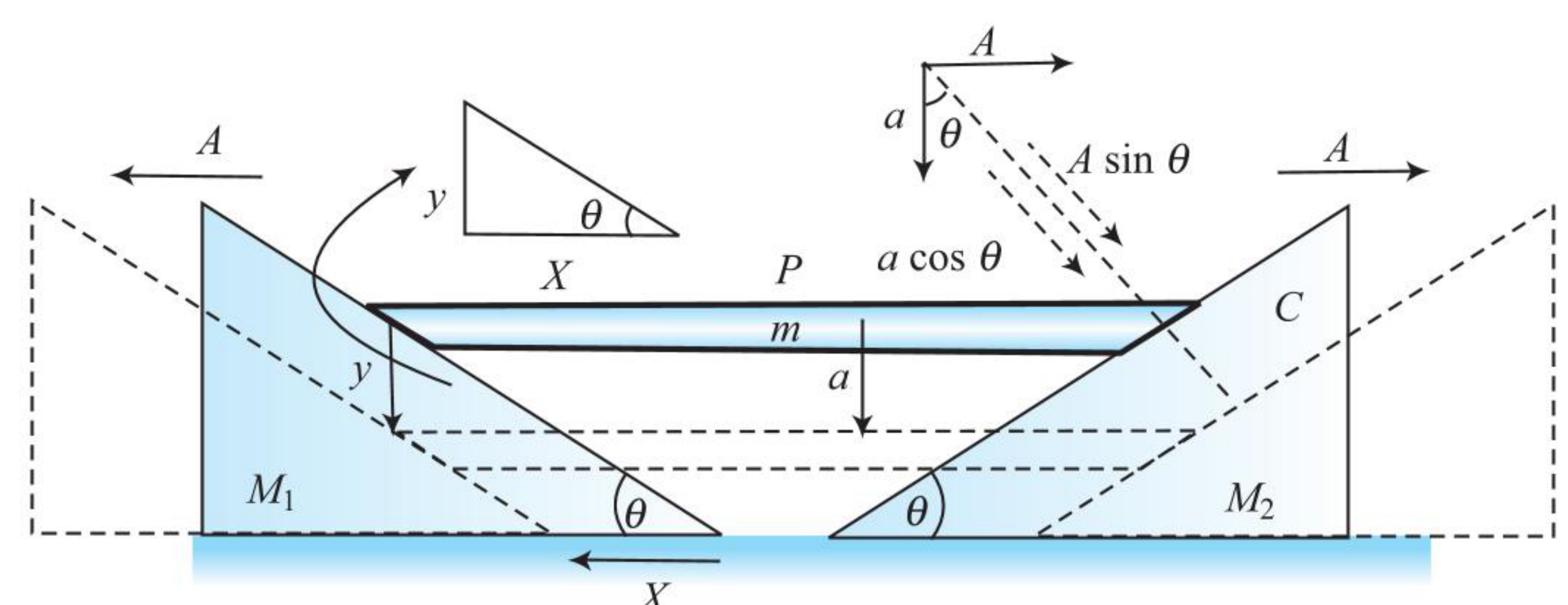
ILLUSTRATION 6.53

A plank of mass m rests symmetrically on two wedges B and C of mass M . What is the acceleration of the plank? Neglect friction between all the contact surfaces.

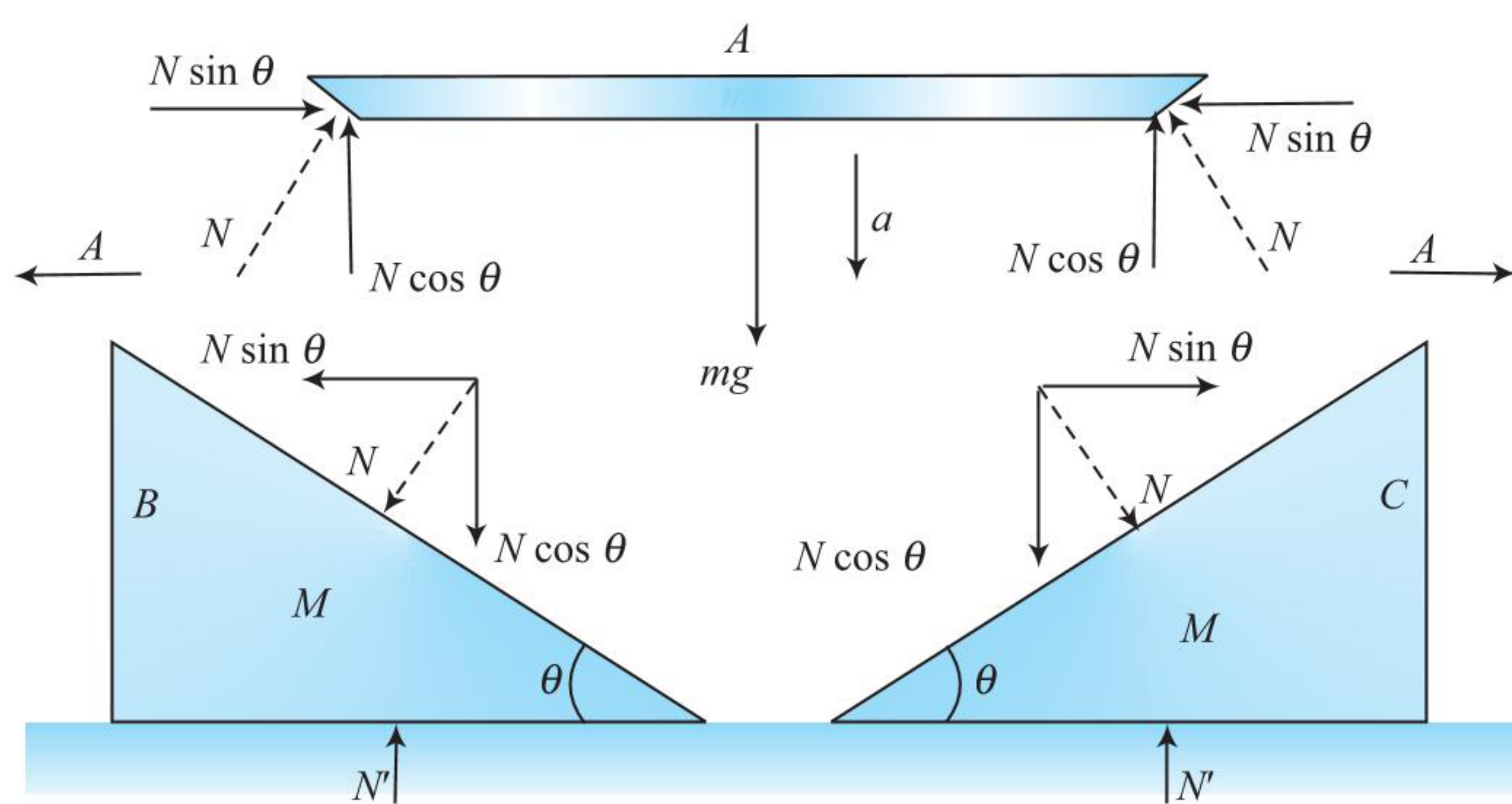


Sol. When the plank is released, let it fall through a distance y and let both the wedges move through a distance x .

$$y = x \tan \theta \quad \dots(i)$$



On differentiating this expression twice, we obtain $a = A \tan \theta$
FBD of rod and wedge



Equations of wedge:

$$\sum F_x = N \sin \theta = MA \quad \dots(ii)$$

$$\sum F_y = N' - N \cos \theta - Mg = 0 \quad \dots(iii)$$

Equations of plank:

$$\sum F_x = N \sin \theta - N \sin \theta = 0 \quad \dots(iv)$$

$$\sum F_y = mg - 2N \cos \theta = ma \quad \dots(v)$$

From (ii), $N = \frac{MA}{\sin \theta}$

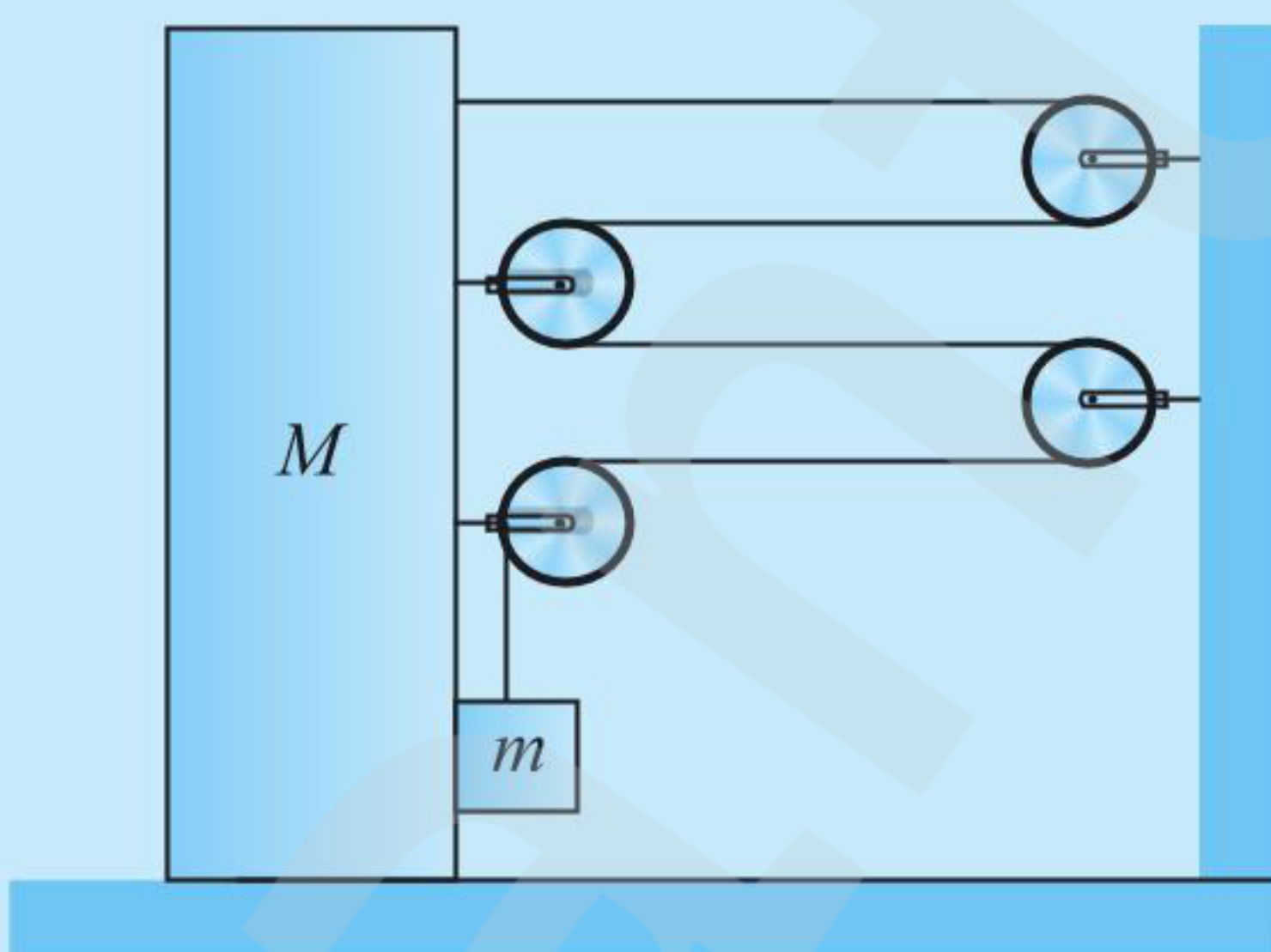
On substituting expression for N and a in (v), we obtain

$$mg - \frac{2MA \cos \theta}{\sin \theta} = \frac{MA \sin \theta}{\cos \theta}$$

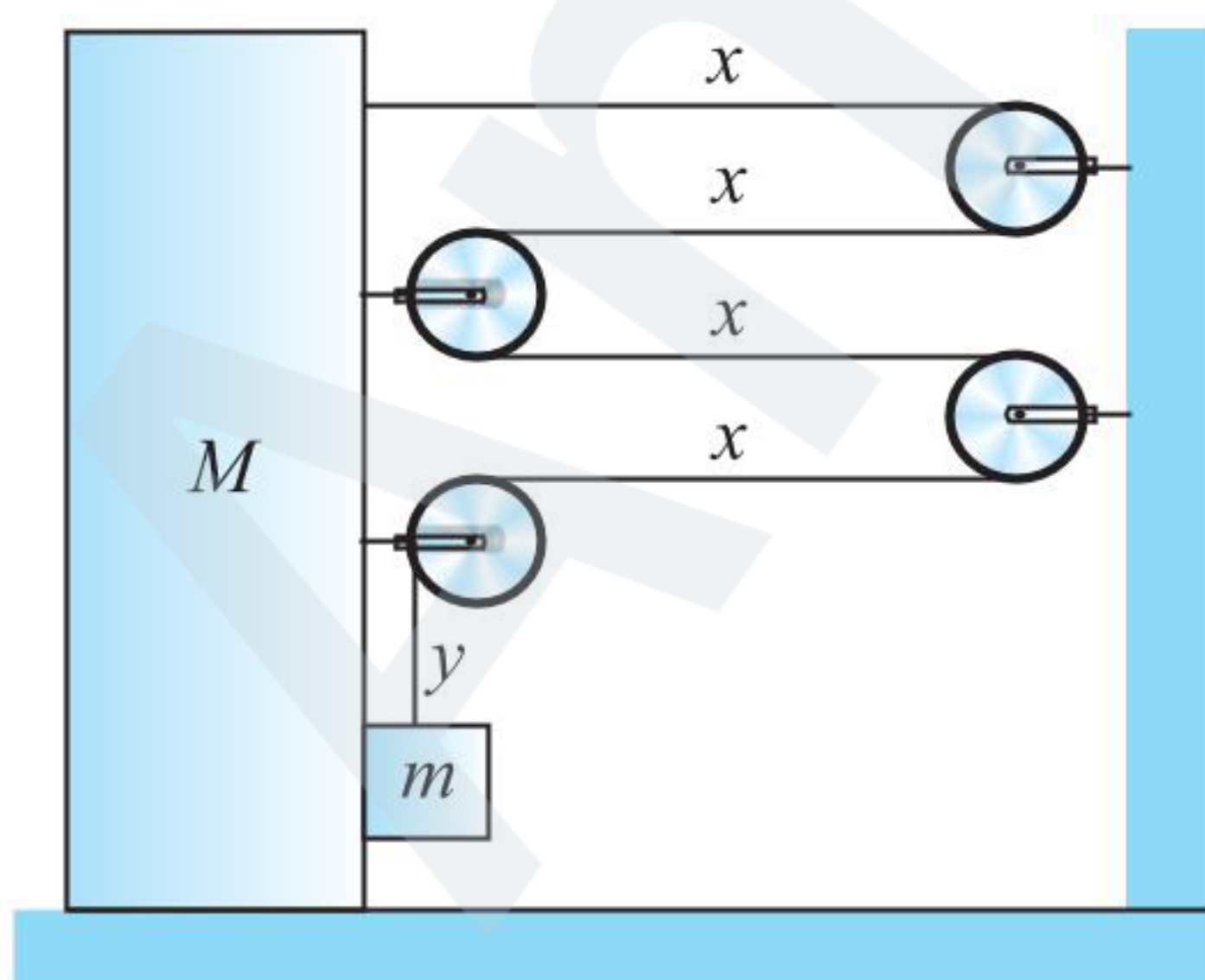
$$\Rightarrow A = \frac{mg \sin \theta \cos \theta}{m \sin^2 \theta + 2M \cos^2 \theta}$$

ILLUSTRATION 6.54

A block of mass M is connected with a particle of mass m by a light inextensible string as shown in figure. Assuming all contacting surfaces as smooth, find the acceleration of the wedge after releasing the system.



Sol. From the constraint relationship shown in figure,



$$y + 4x = l$$

$$\Rightarrow \frac{dy}{dt} + \frac{4dx}{dt} = 0$$

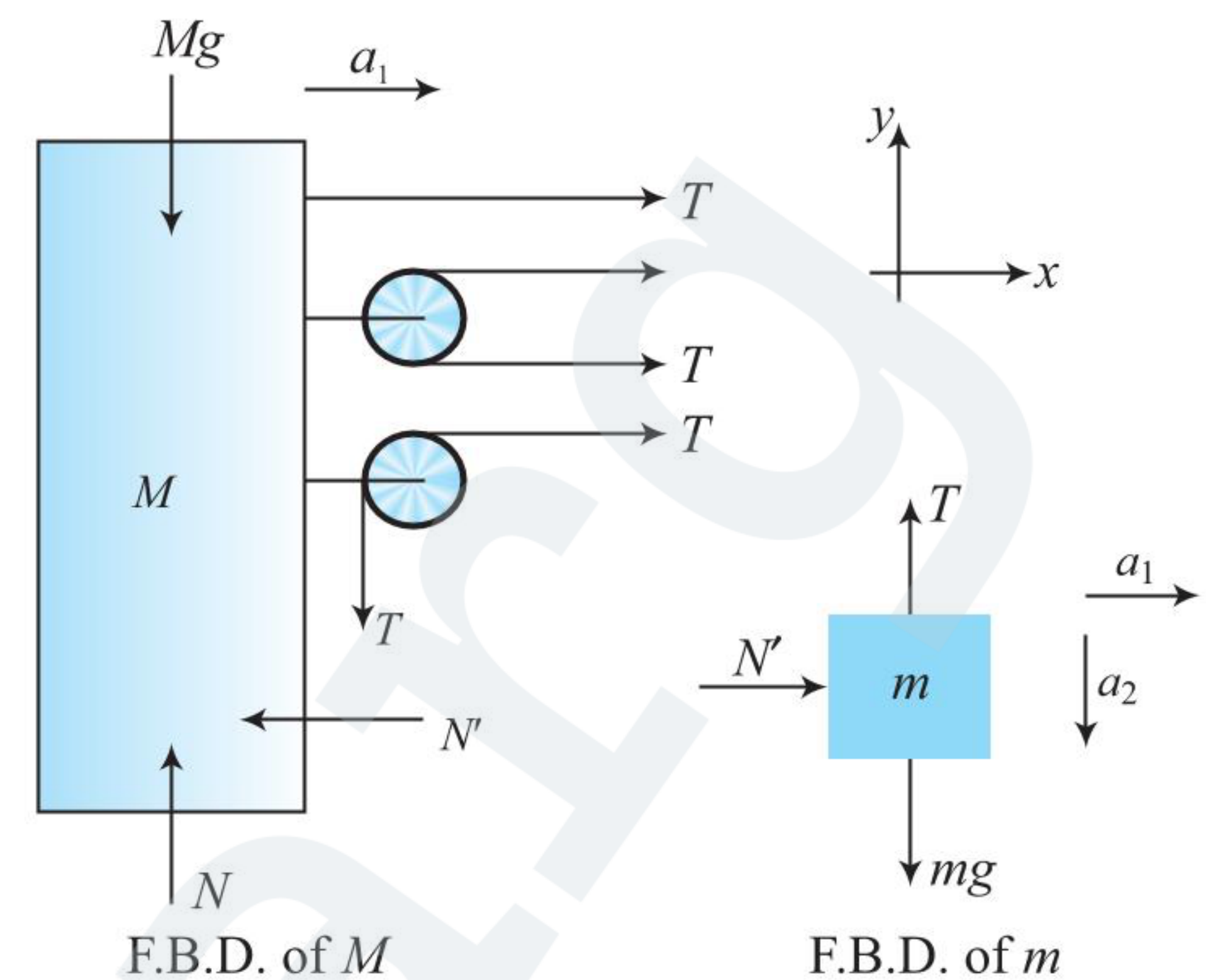
$$\Rightarrow \frac{d^2y}{dt^2} + \frac{4d^2x}{dt^2} = 0$$

$$\Rightarrow a_2 = 4a_1 \text{ (numerically)} \quad \dots(i)$$

where a_1 is the acceleration of M and a_2 is the acceleration of m , with respect to M .

Equation for M

$$\sum F_x = Ma_1 \Rightarrow 4T - N' = Ma_1 \quad \dots(ii)$$



For m , $\sum F_y = ma_2 \Rightarrow mg - T = ma_2 \quad \dots(iii)$

$$\sum F_x = ma_1 \Rightarrow N' = ma_1 \quad \dots(iv)$$

as m is always in contact with M . Hence, the acceleration of m in horizontal direction should be same as the acceleration of M .

$$(ii) + (iv) \Rightarrow a_1 = \frac{4T}{M + m} \quad \dots(v)$$

Putting $a_2 = 4a_1$ from (i) and $T = \frac{(M + m)a_1}{4}$

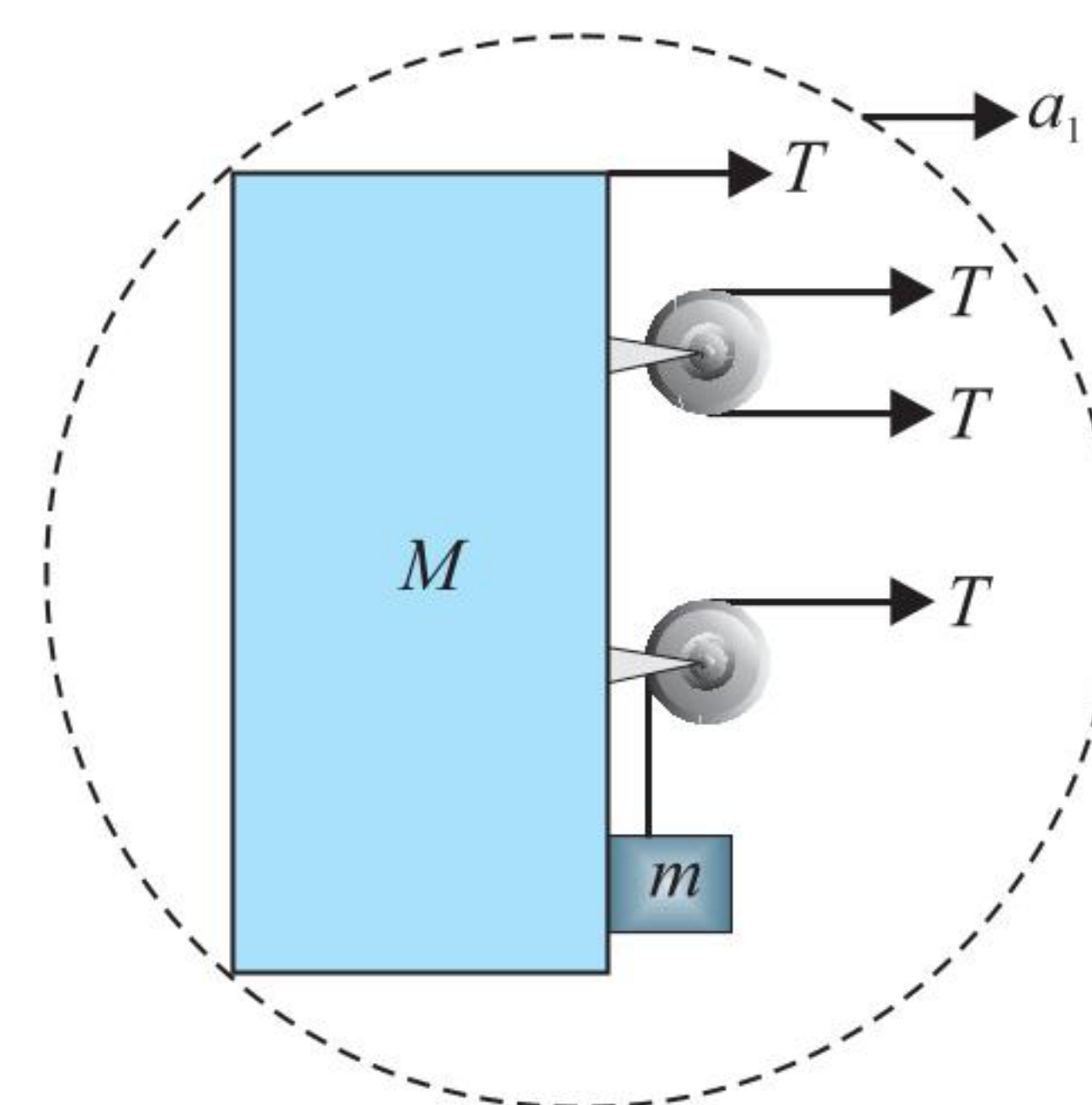
From (v) in (iii), we obtain

$$mg - \frac{(M + m)a_1}{4} = m(4a_1)$$

$$\Rightarrow a_1 = \frac{4mg}{M + 17m}$$

Approach 2:

If we take 'block (M) and particle (m)' as system, then the system has acceleration a_1 in horizontal direction. The external force on the system in horizontal direction is $4T$. Now consider the system in horizontal direction.



We can write equation of motion for system

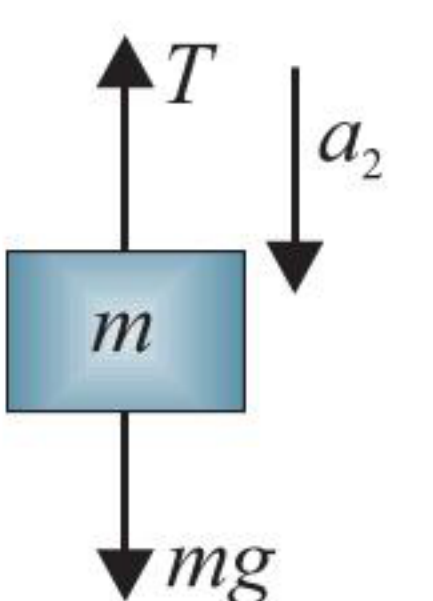
$$4T = (M + m)a_1 \quad \dots(vi)$$

Now consider the motion of the particle (m) in vertical direction.

Equation of motion of m : $mg - T = ma_2$

or $mg - T = m(4a_1) \quad \dots(vii) \quad [\text{As } a_2 = 4a_1]$

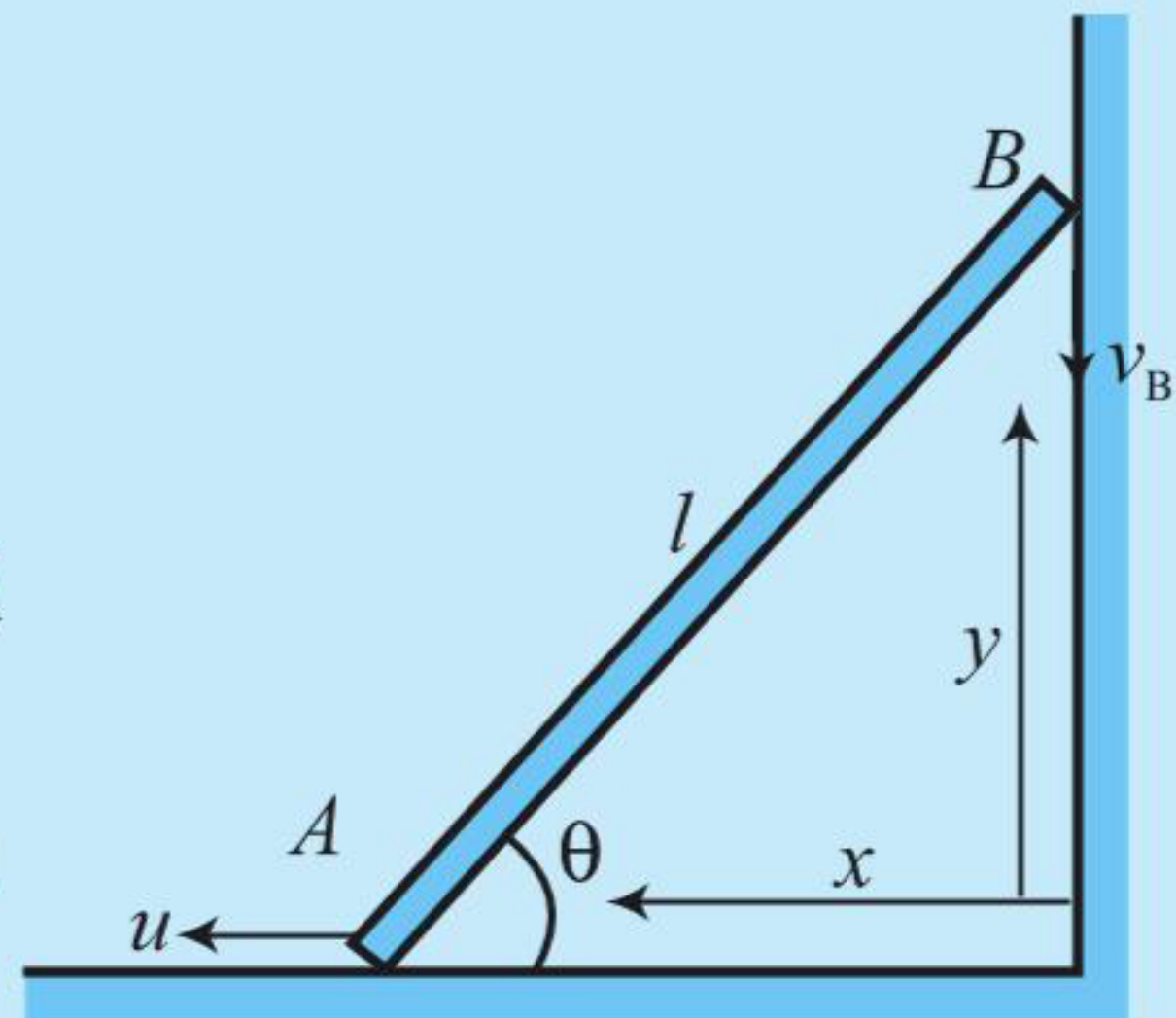
Now on solving equations (vi) and (vii), we get $a_1 = \frac{4mg}{M + 17m}$



GENERAL CONSTRAINTS

ILLUSTRATION 6.55

Figure shows a rod of length l resting on a wall and the floor. Its lower end A is pulled towards left with a constant velocity u . As a result of this, end A starts moving down along the wall. Find the velocity of the other end B downward when the rod makes an angle θ with the horizontal.



Sol. Let us first find the relation between the two displacements. Then differentiate with respect to time. Here if the distance from the corner to the point A is x and that up to B is y , the left velocity of point A can be given as $v_A = dx/dt$ and that of B can be given as $v_B = -dy/dt$ (negative sign indicates y decreasing).

If we relate x and y : $x^2 + y^2 = l^2$

Differentiating with respect to t , $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$$\Rightarrow xv_A = yv_B \Rightarrow xu = yv_B \Rightarrow v_B = u \frac{x}{y} = u \cot \theta$$

Alternatively: In cases where distance between two points is always fixed, we can say the relative velocity of one point of an object with respect to any other point of the same object in the direction of the line joining them will always remain zero, as their separation always remains constant.

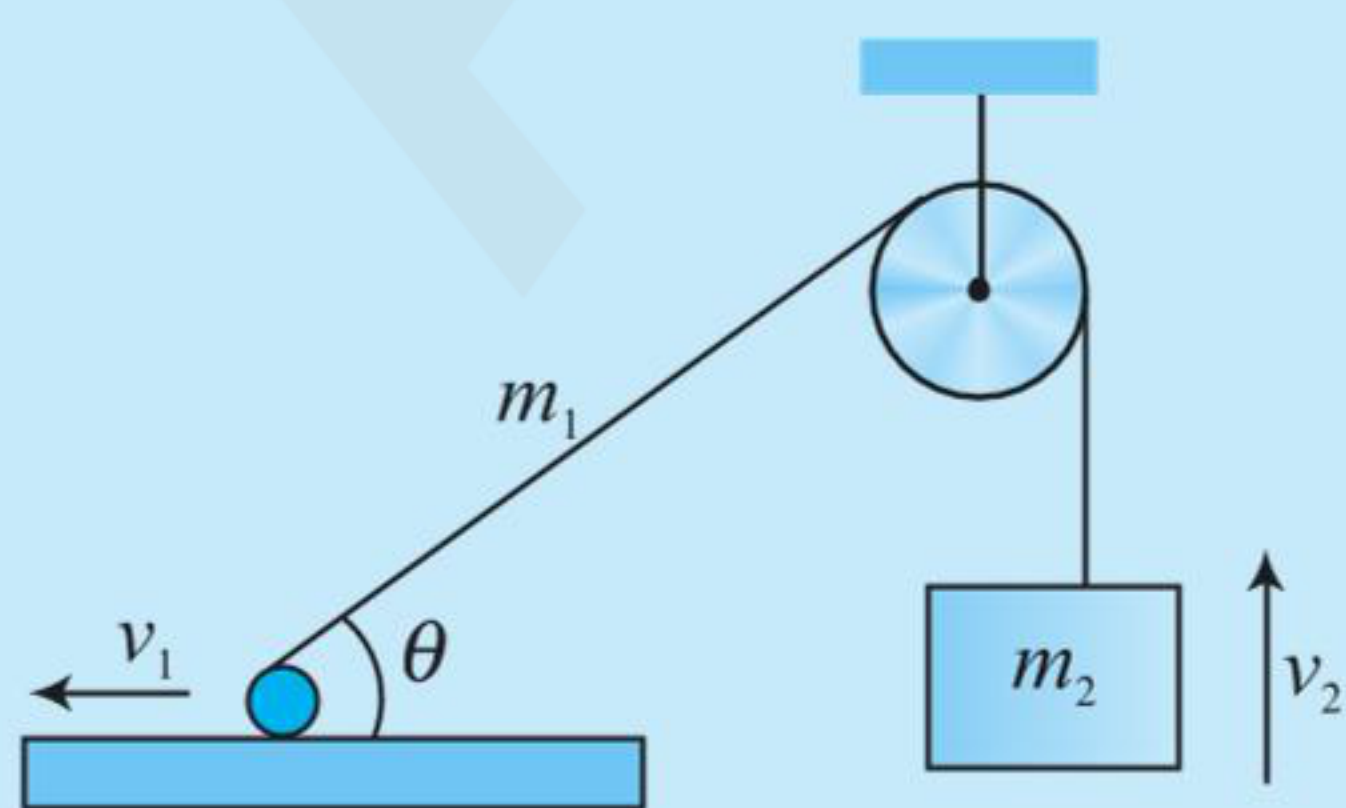
Here, in the above example, the distance between the points A and B of the rod always remains constant; thus, the two points must have the same velocity components in the direction of their line joining, i.e., along the length of the rod.

If point B is moving down with velocity v_B , its component along the length of the rod is $v_B \sin \theta$. Similarly, the velocity component of point A along the length of rod is $u \cos \theta$. Thus, we have

$$v_B \sin \theta = u \cos \theta \text{ or } v_B = u \cot \theta.$$

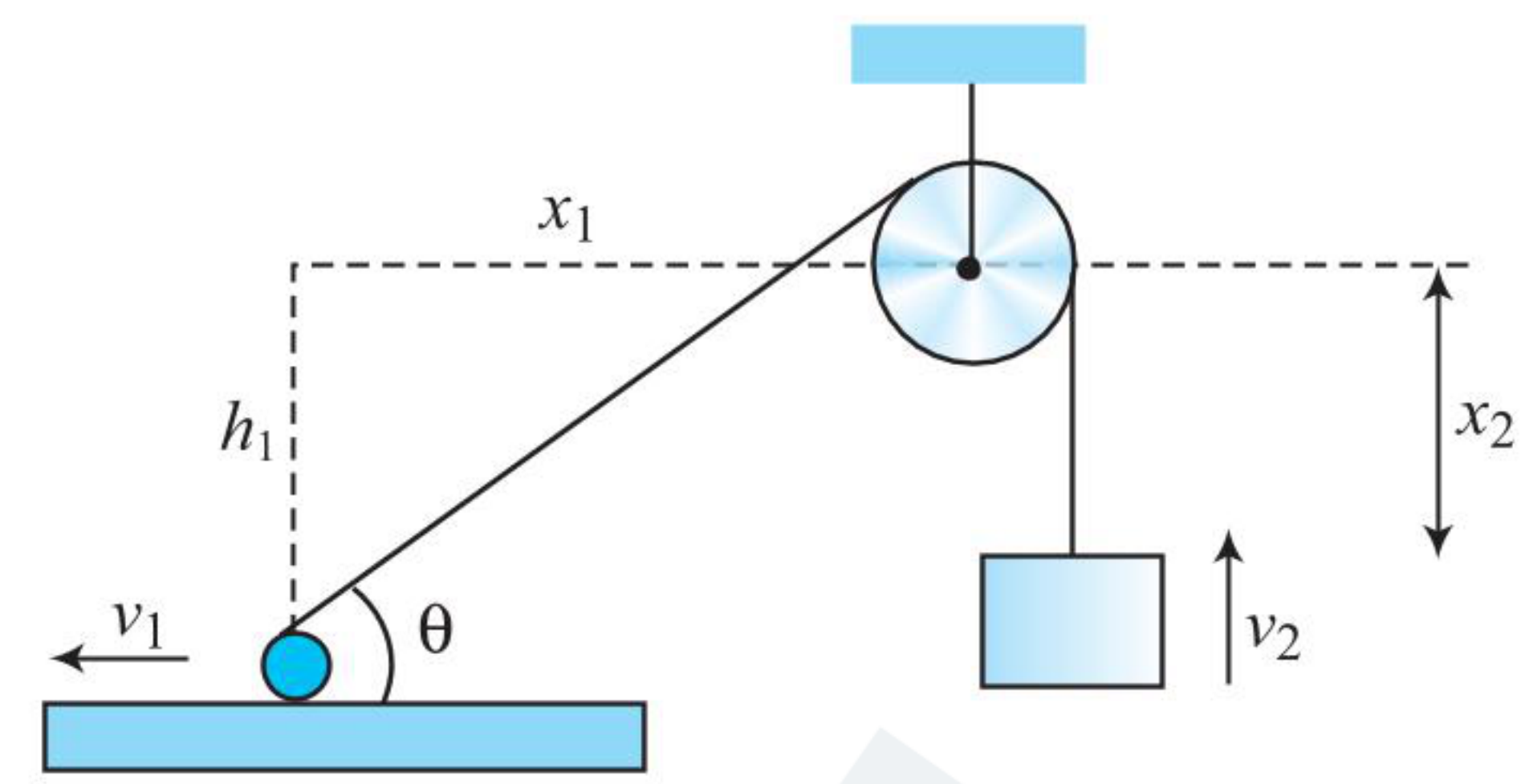
ILLUSTRATION 6.56

In figure, a ball of mass m_1 and a block of mass m_2 are joined together with an inextensible string. The ball can slide on a smooth horizontal surface. If v_1 and v_2 are the respective speeds of the ball and the block, then determine the constraint relation between the two.



Sol.

Method 1: Distances are assumed from the center of the pulley as shown in figure.



Constraint: Length of the string remains constant.

$$\sqrt{x_1^2 + h_1^2} + x_2 = \text{constant}$$

Differentiating both the sides w.r.t. time, we get

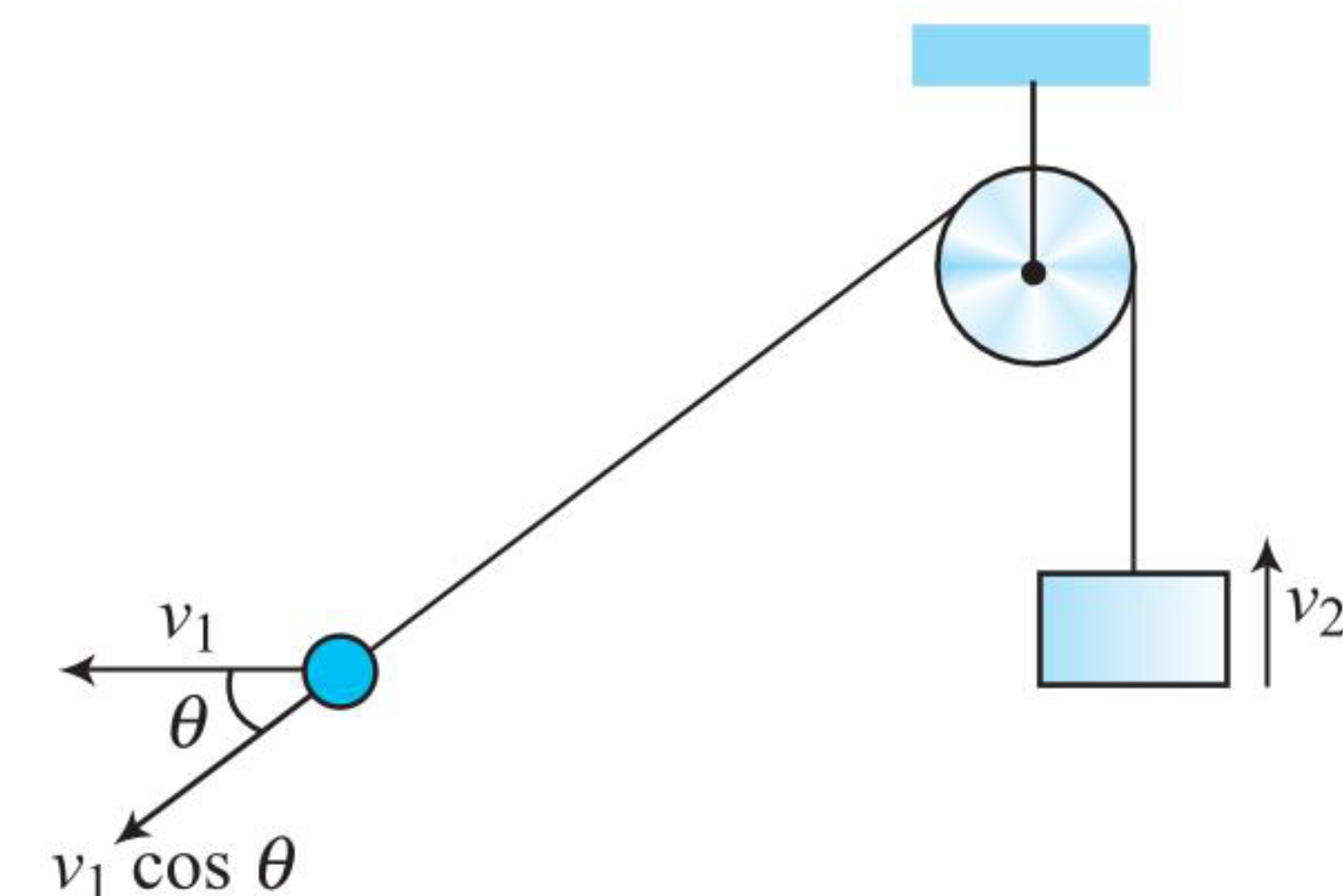
$$\frac{2x_1}{2\sqrt{x_1^2 + h_1^2}} \frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

Since the ball moves so as to increase x_1 with time and block moves so as to decrease x_2 with time,

$$\frac{dx_1}{dt} = +v_1 \quad \text{and} \quad \frac{dx_2}{dt} = -v_2$$

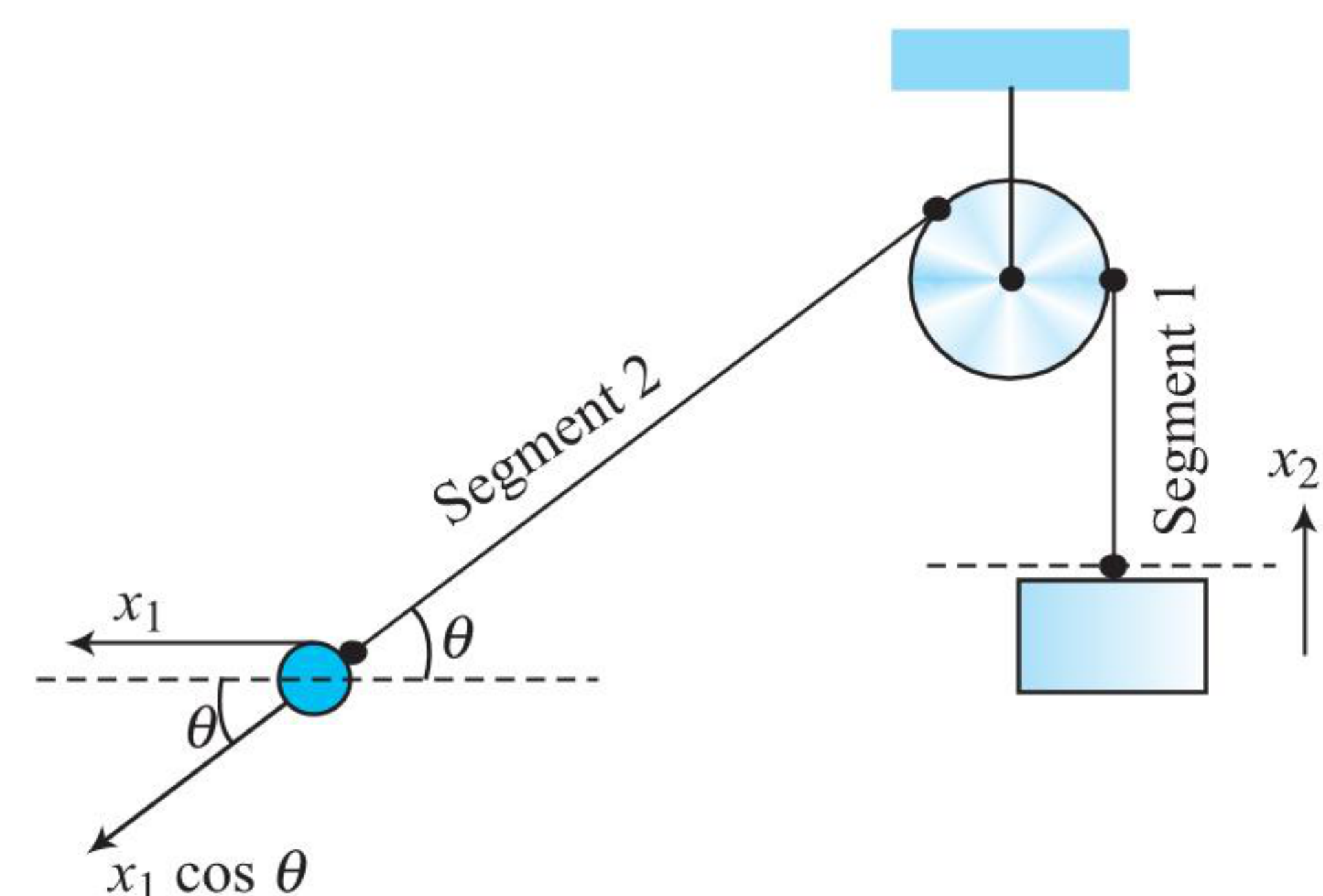
$$\text{Also } \frac{x_1}{\sqrt{x_1^2 + h_1^2}} = \cos \theta \quad \text{or} \quad v_2 = v_1 \cos \theta$$

Method 2: The problem can be solved very easily if we look at the problem from a different viewpoint and identify a different constraint, i.e., the velocity of any two points along the string is same. Obviously, from figure, we have $v_1 \cos \theta = v_2$.



Method 3: Change in length of segment I,

$$\Delta l_1 = 0 + (-x_2) = -x_2$$



Change in length of segment II,

$$\Delta l_2 = (x_1 \cos \theta) + 0 = x_1 \cos \theta$$

Total change in the length of all segments should be zero, as the length of string is constant.

$$\begin{aligned} \Delta l &= \Delta l_1 + \Delta l_2 \\ &= -x_2 + x_1 \cos \theta = 0 \end{aligned}$$

$$\Rightarrow x_2 = x_1 \cos \theta \text{ or } v_2 = v_1 \cos \theta$$

ILLUSTRATION 6.57

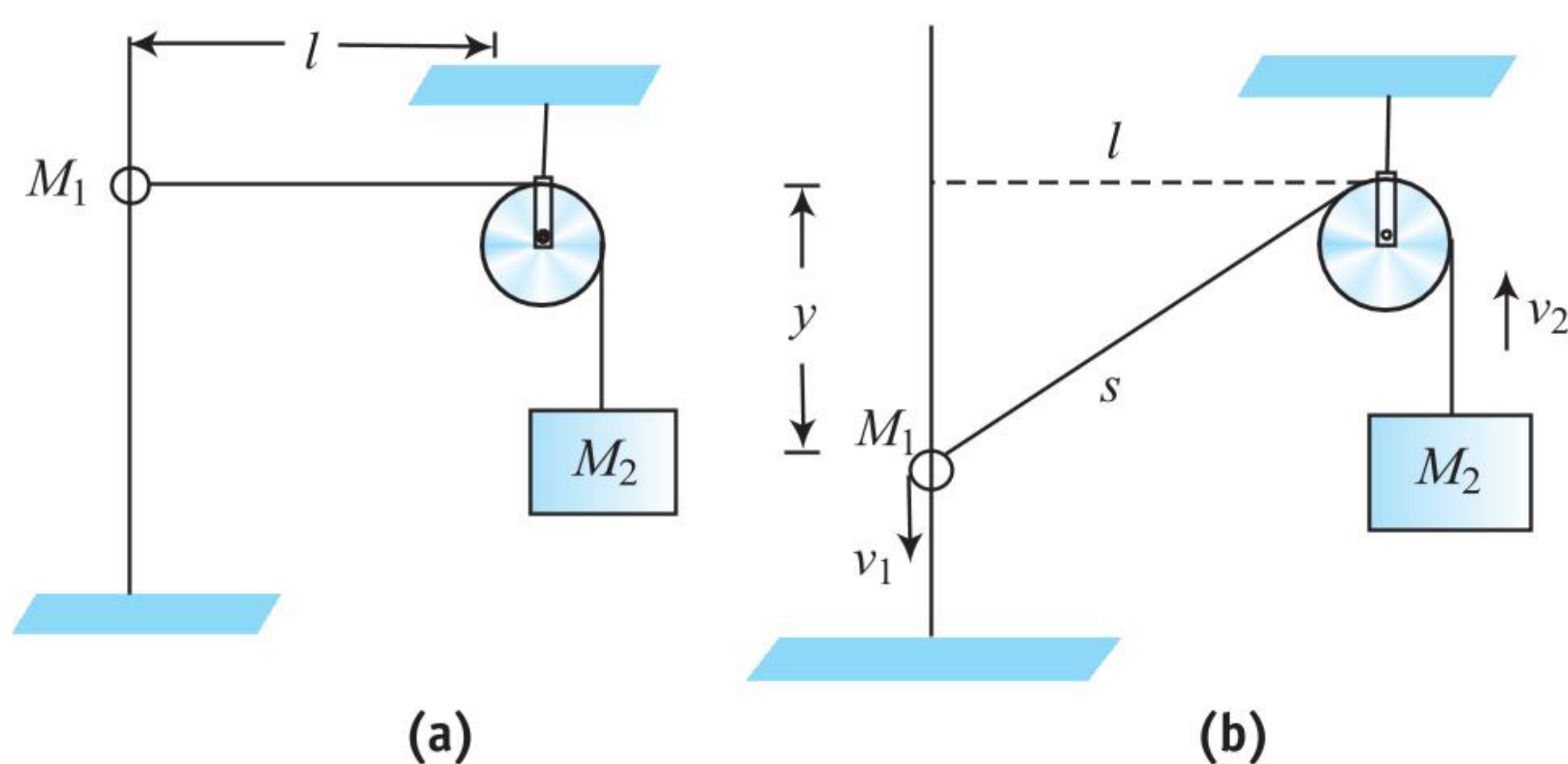
A ring M_1 and a block M_2 are held in the position shown in figure (a). Now the system is released. If $M_1 > M_2$, find V_1/V_2 when the ring m_1 slides down along the smooth fixed vertical rod by the distance h .

Sol.

Method 1: Let ring has moved down a distance y . From figure (b), we have

$$y^2 + l^2 = s^2 \quad \dots(i)$$

Here l is the length of string which is constant. Differentiating (i) w.r.t. time, we get



$$2y \frac{dy}{dt} + 0 = 2s \frac{ds}{dt} \quad \text{or} \quad \frac{dy}{dt} = \frac{s}{y} \cdot \frac{ds}{dt} \quad \dots(ii)$$

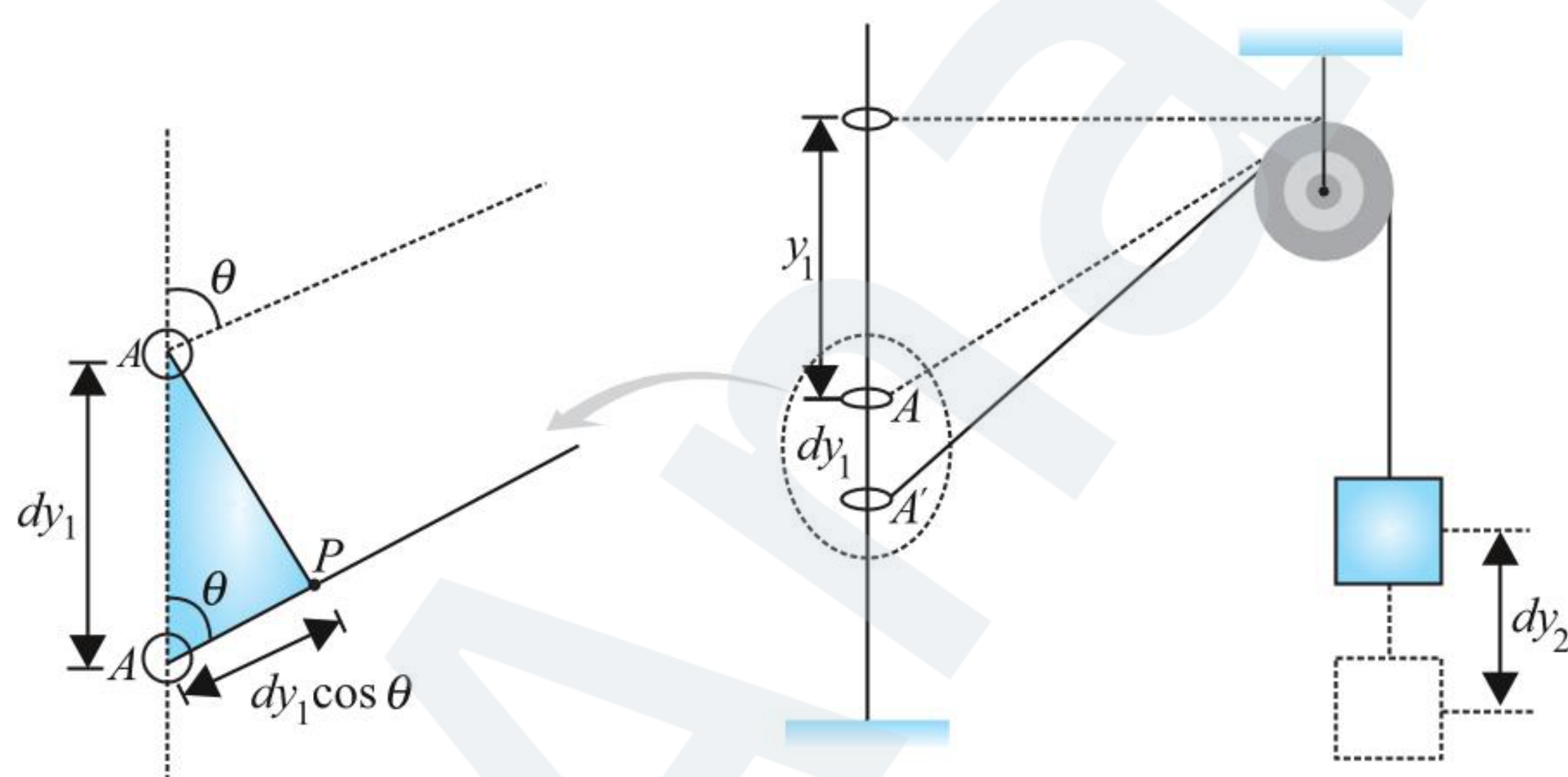
Here $\frac{dy}{dt} = v_1$ and $\frac{ds}{dt} = v_2$.

For $y = h$, $s = \sqrt{h^2 + l^2}$

Therefore, Eq. (ii) takes the form

$$v_1 = \frac{\sqrt{h^2 + l^2}}{h} \cdot v_2 \quad \text{or} \quad \frac{v_1}{v_2} = \frac{\sqrt{h^2 + l^2}}{h}$$

Method 2: Let in time t the ring falls to point A which is a distance y_1 below the ring's initial position.



At this position, the string makes an angle θ with the rod in small time interval dt , the ring moves down to dy_1 and the block rises by dy_2 . Let us draw a perpendicular AP from A to AB .

For very small displacement, $AB \approx PB$.

As the length of the string is constant

$$\therefore A'P = dy_2$$

$$\text{But } A'P = dy_1 \cos \theta$$

$$\therefore dy_1 \cos \theta = dy_2$$

Dividing both sides by dt , we get $\left(\frac{dy_1}{dt}\right) \cos \theta = \left(\frac{dy_2}{dt}\right)$.

Here $\frac{dy_1}{dt} = V_1$, the rate of change of position of ring with time

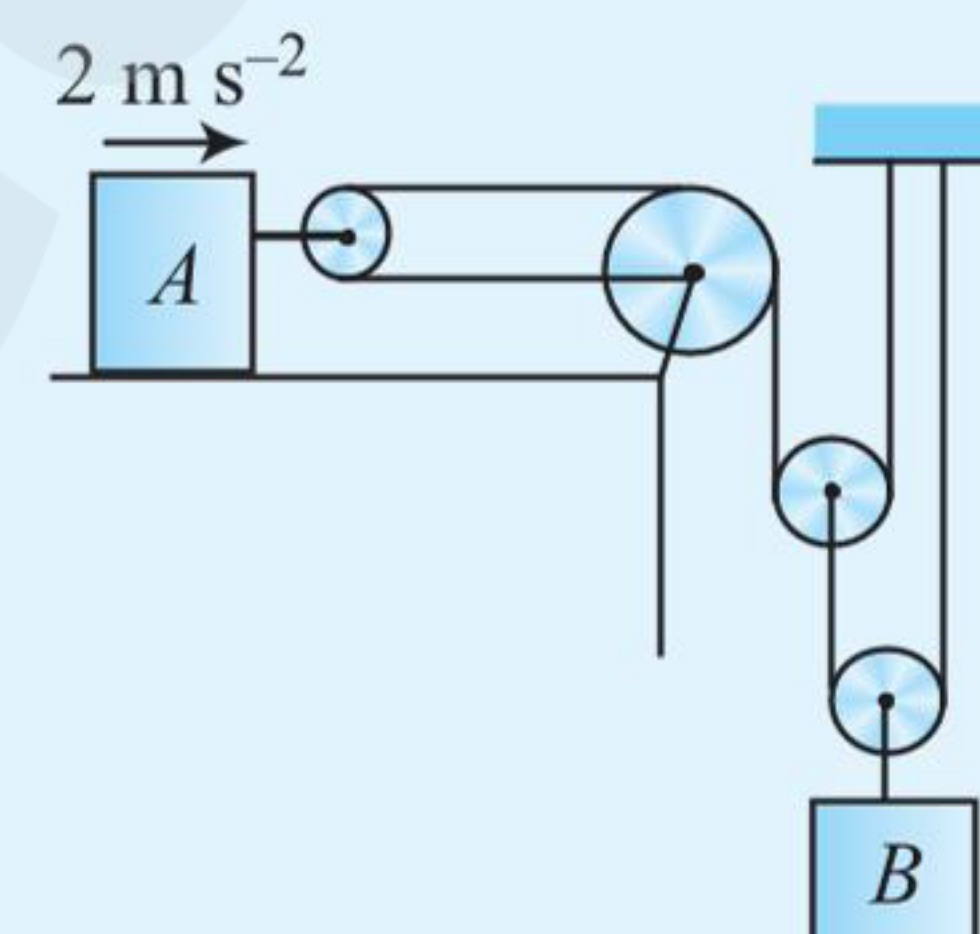
and $\frac{dy_2}{dt} = V_2$, the rate of change position of block with time.

Thus, we have $V_1 \cos \theta = V_2$

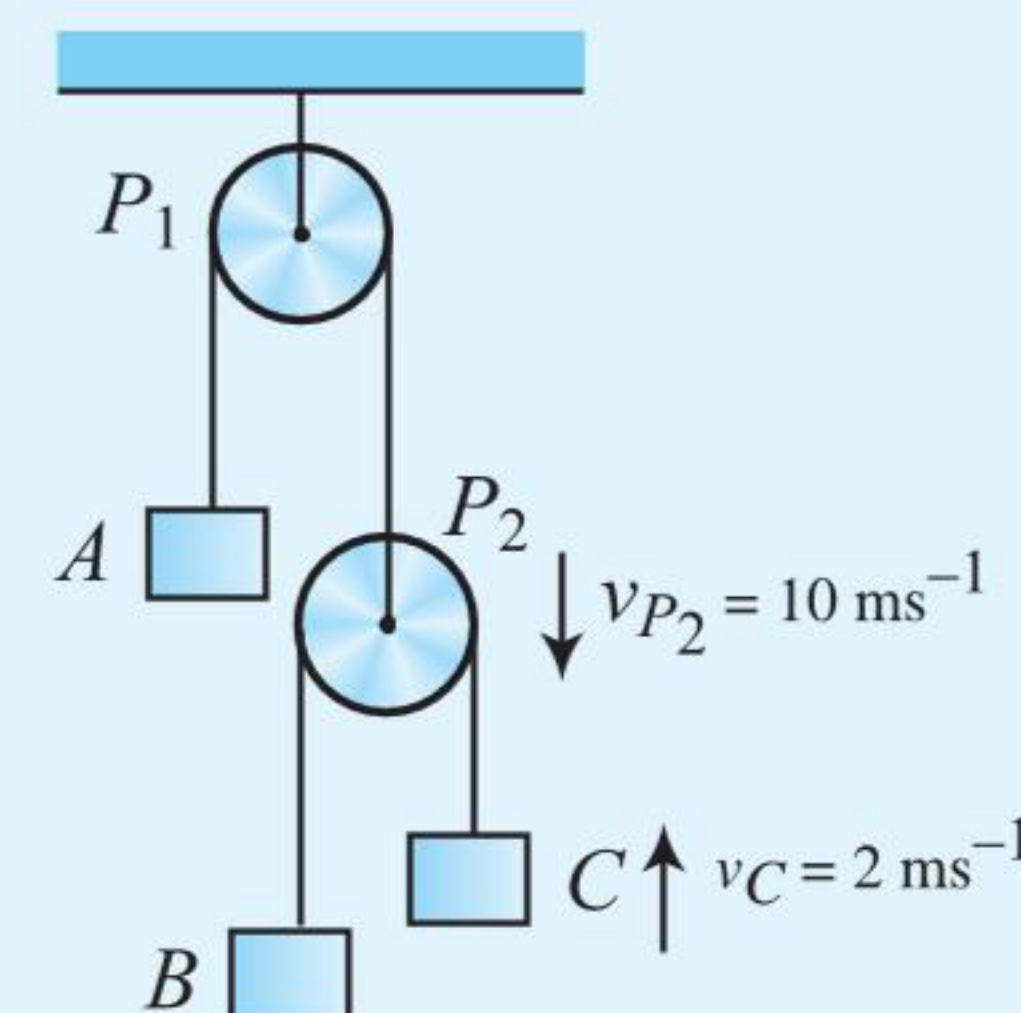
$$\text{or} \quad \frac{V_1}{V_2} = \frac{1}{\cos \theta} = \frac{1}{\frac{h}{\sqrt{h^2 + l^2}}} = \frac{\sqrt{h^2 + l^2}}{h}$$

CONCEPT APPLICATION EXERCISE 6.6

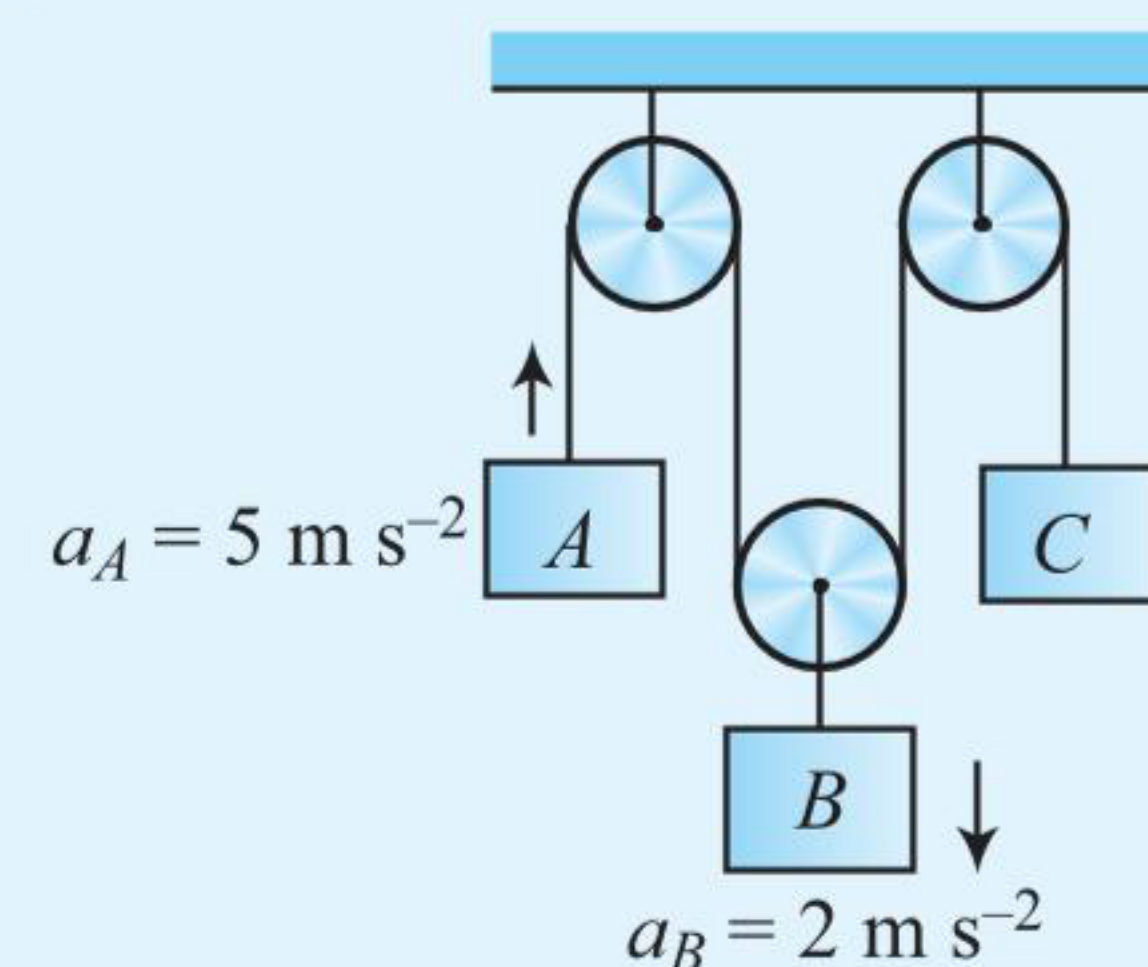
1. In the given figure, find the acceleration of B , if the acceleration of A is 2 m s^{-2} .



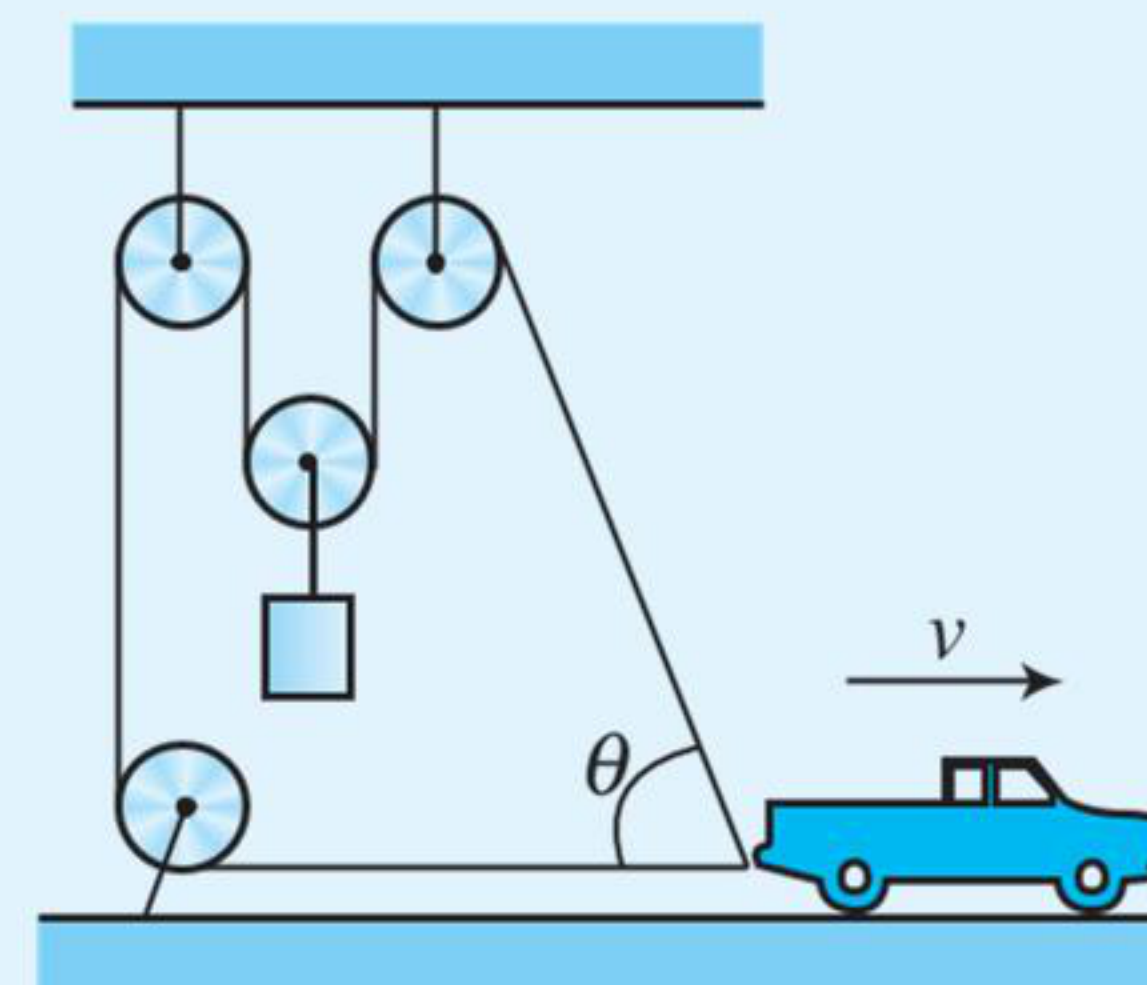
2. The three blocks shown in figure move with constant velocities. Find the velocity of blocks A and B . Given $V_{P_2} = 10 \text{ m s}^{-1} \downarrow$, $V_C = 2 \text{ m s}^{-1} \uparrow$.



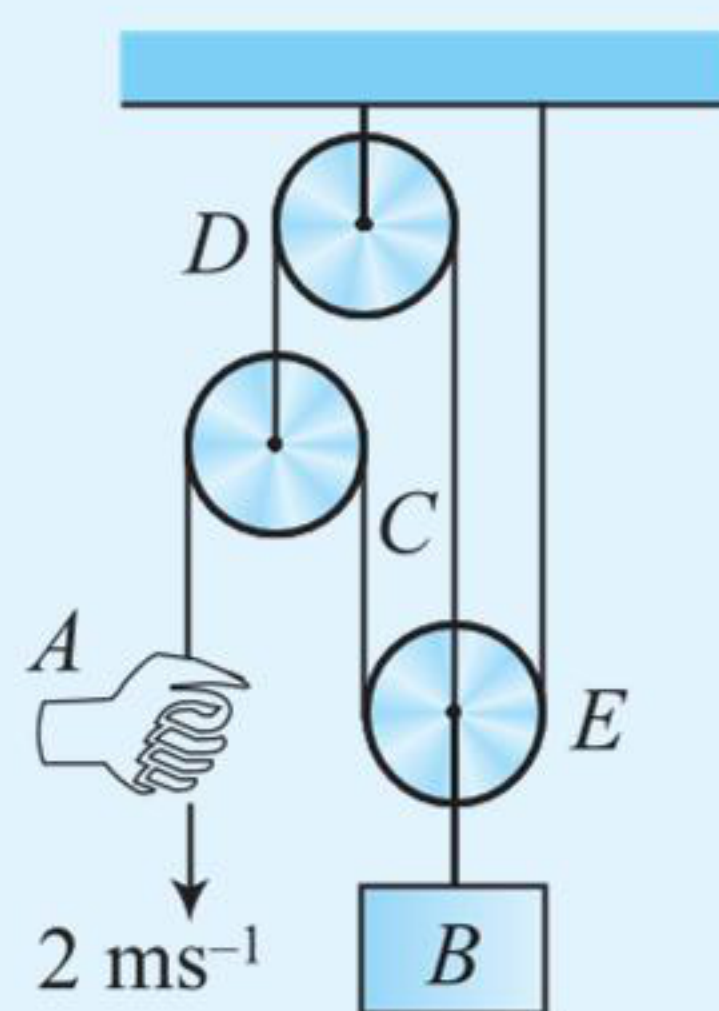
3. For the system as shown in figure, find the acceleration of C . The accelerations of A and B with respect to ground are marked.



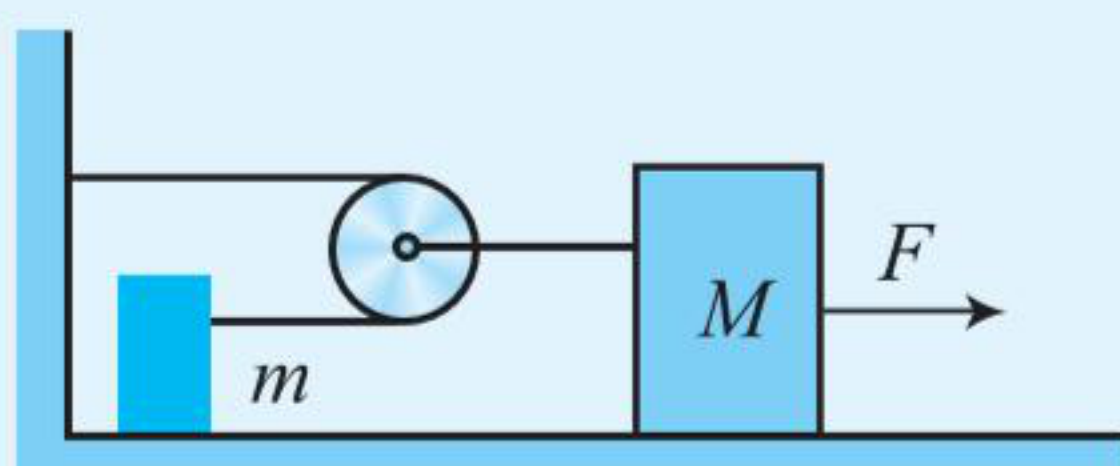
4. In the figure shown, the speed of the truck is v to the right. Find the speed with which the block is moving up at $\theta = 60^\circ$.



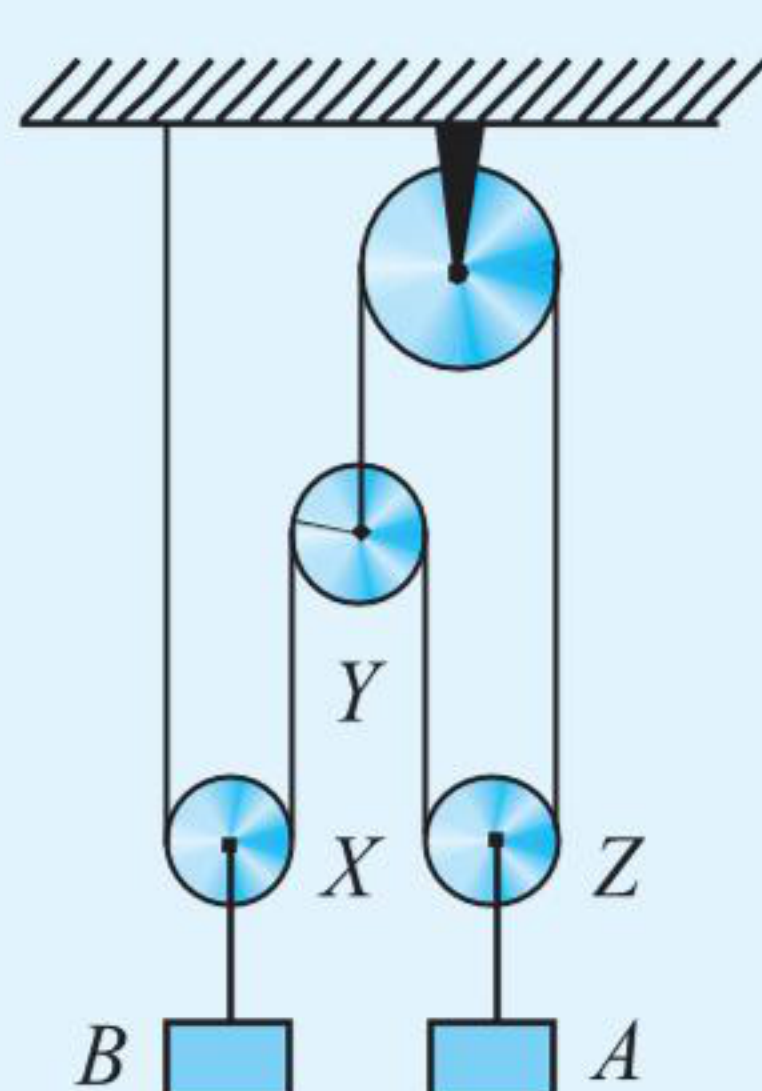
5. Determine the speed with which block B rises in figure if the end of the chord at A is pulled down with a speed of 2 ms^{-1} .



6. Find the acceleration of blocks in the given figure. The pulley and the strings are massless.

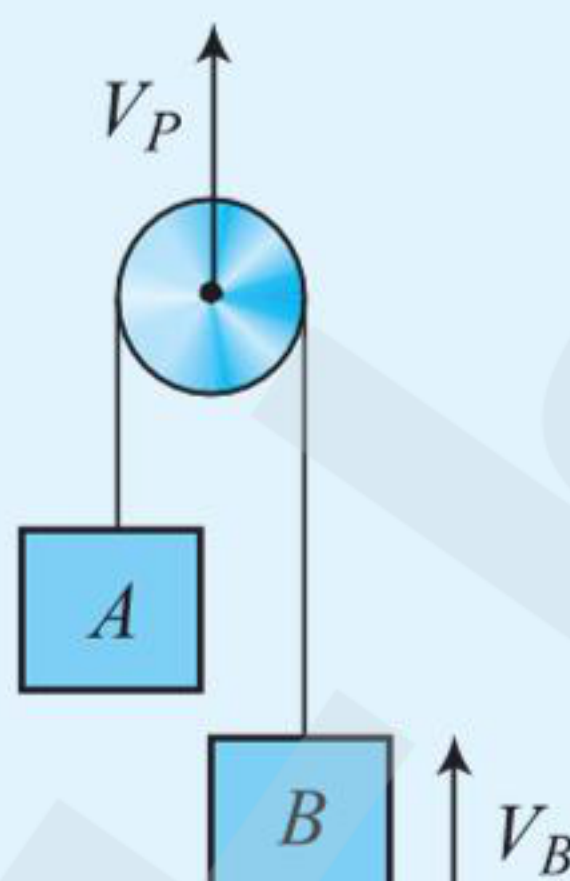


7. The given figure shows a system of four pulleys with two masses A and B . Find, at an instant



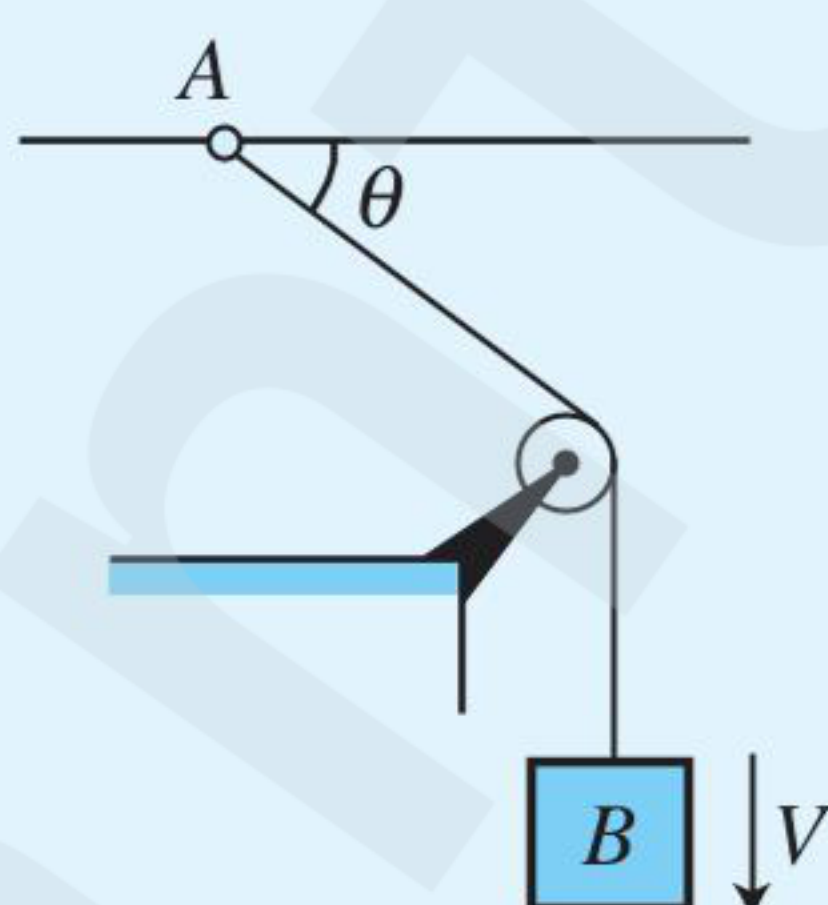
- (a) Speed of block A when block B is going up at 1 ms^{-1} and pulley Y is going up at 2 ms^{-1} .
(b) Acceleration of block A if block B is going up at 3 ms^{-2} and pulley Y is going down at 1 ms^{-2} .

8. Figure shows a pulley over which is string passes and connected to two masses A and B . Pulley moves up with a velocity V_p and mass B is also going up at a velocity V_B . Find the velocity of mass A if

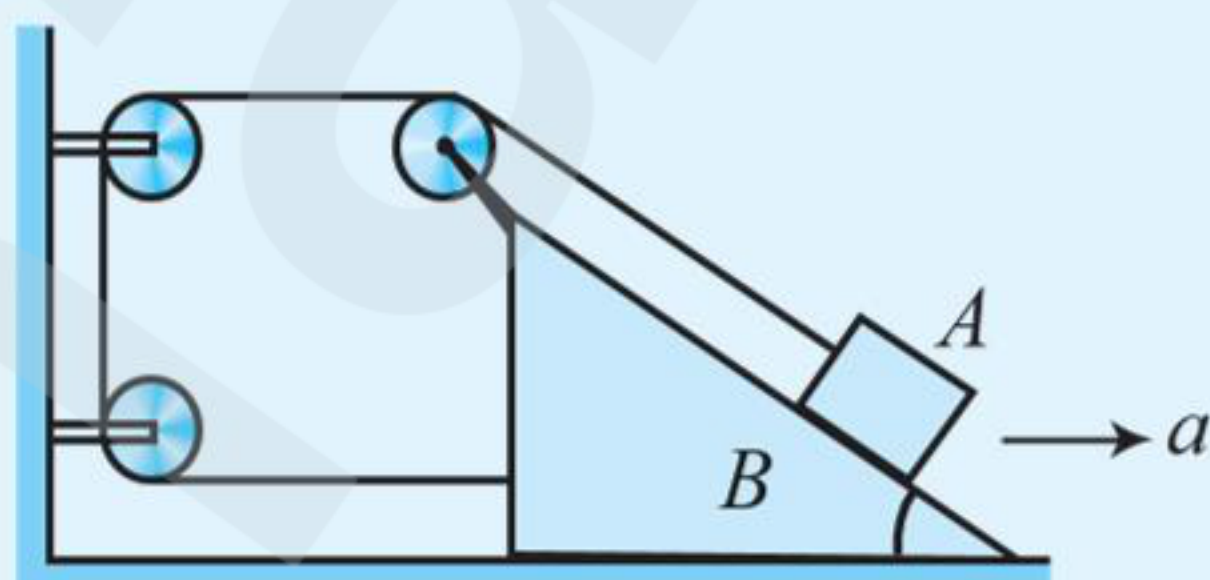


- (a) $V_p = 5 \text{ ms}^{-1}$ and $V_B = 10 \text{ ms}^{-1}$
(b) $V_p = 5 \text{ ms}^{-1}$ and $V_B = -20 \text{ ms}^{-1}$

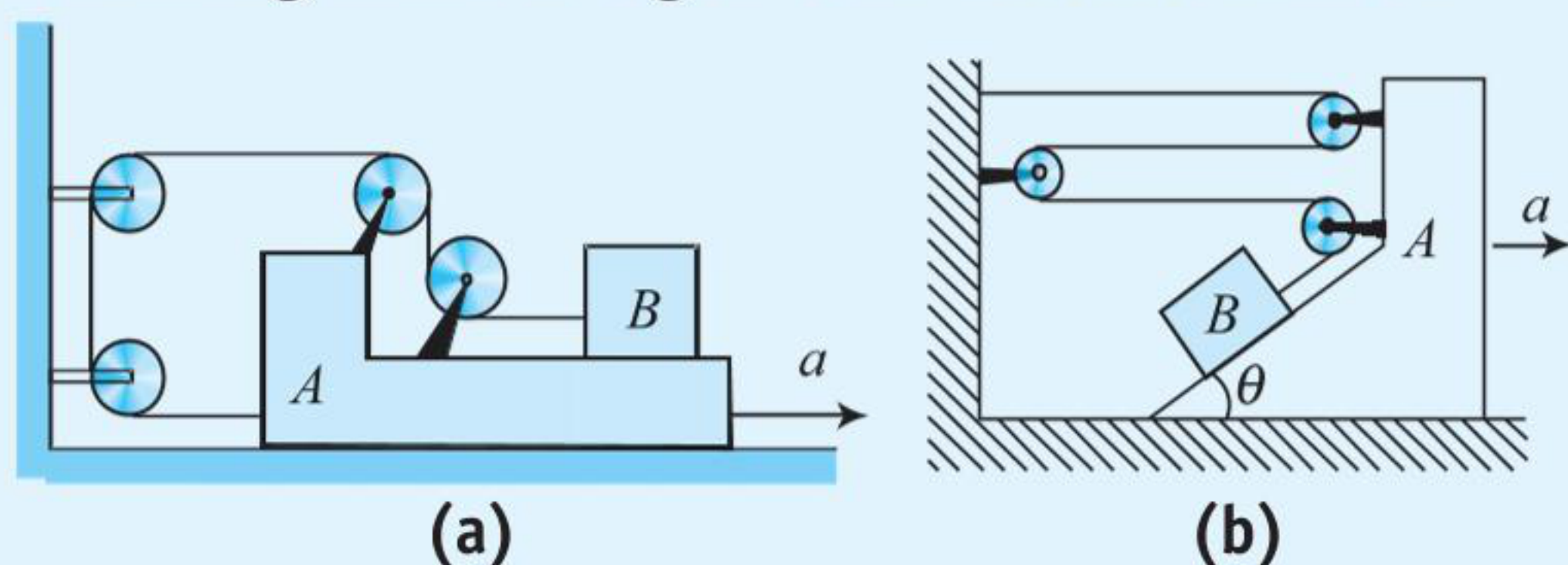
9. A ring A which can slide on a smooth wire is connected to one end of a string as shown in figure. Other end of the string is connected to a hanging mass B . Find the speed of the ring when the string makes an angle θ with the wire and mass B is going down with a velocity v .



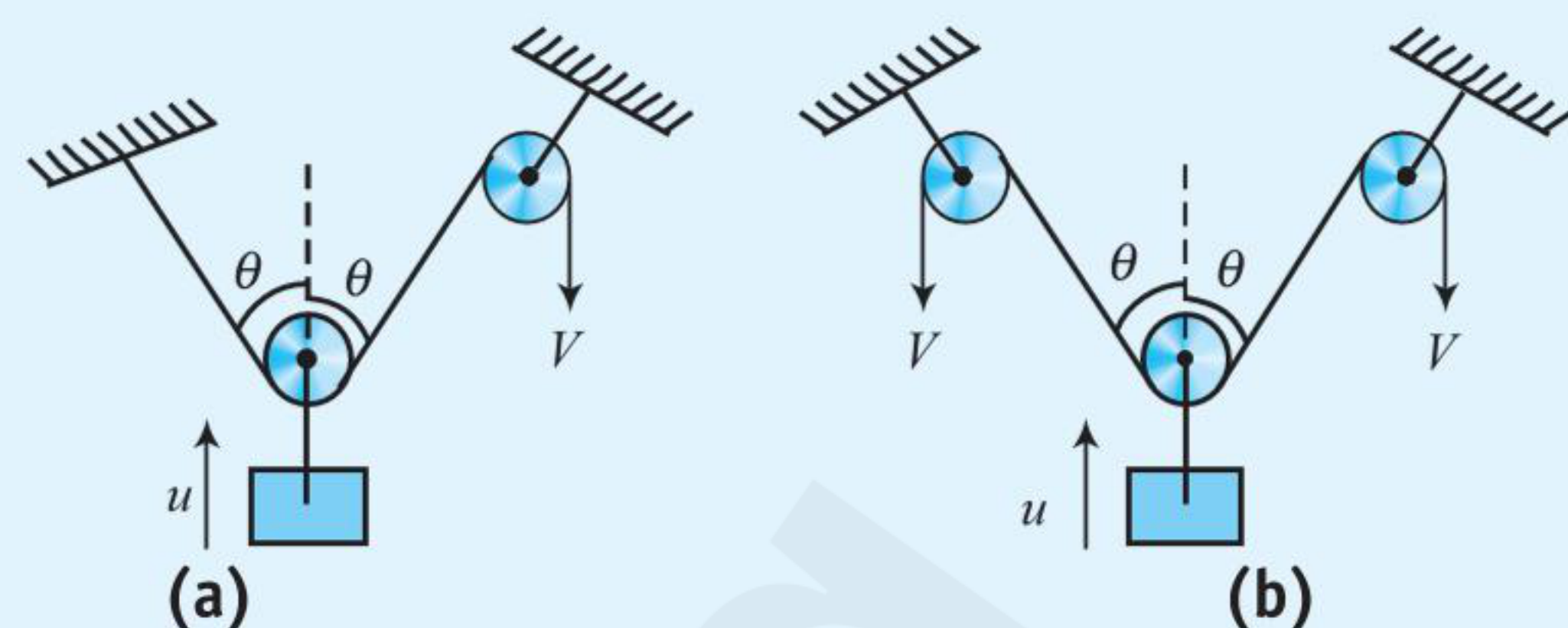
10. The given figure shows block A constrained to slide along the inclined plane of the wedge B shown. Block A is attached with a string which passes through three ideal pulleys and connected to the wedge B . If wedge is pulled toward right with an acceleration a , find



- (a) the acceleration of the block with respect to wedge.
(b) the acceleration of the block with respect to ground.
11. Find the acceleration of block B as shown in figures (a) and (b) relative to block A and relative to ground if block A is moving toward right with acceleration a .



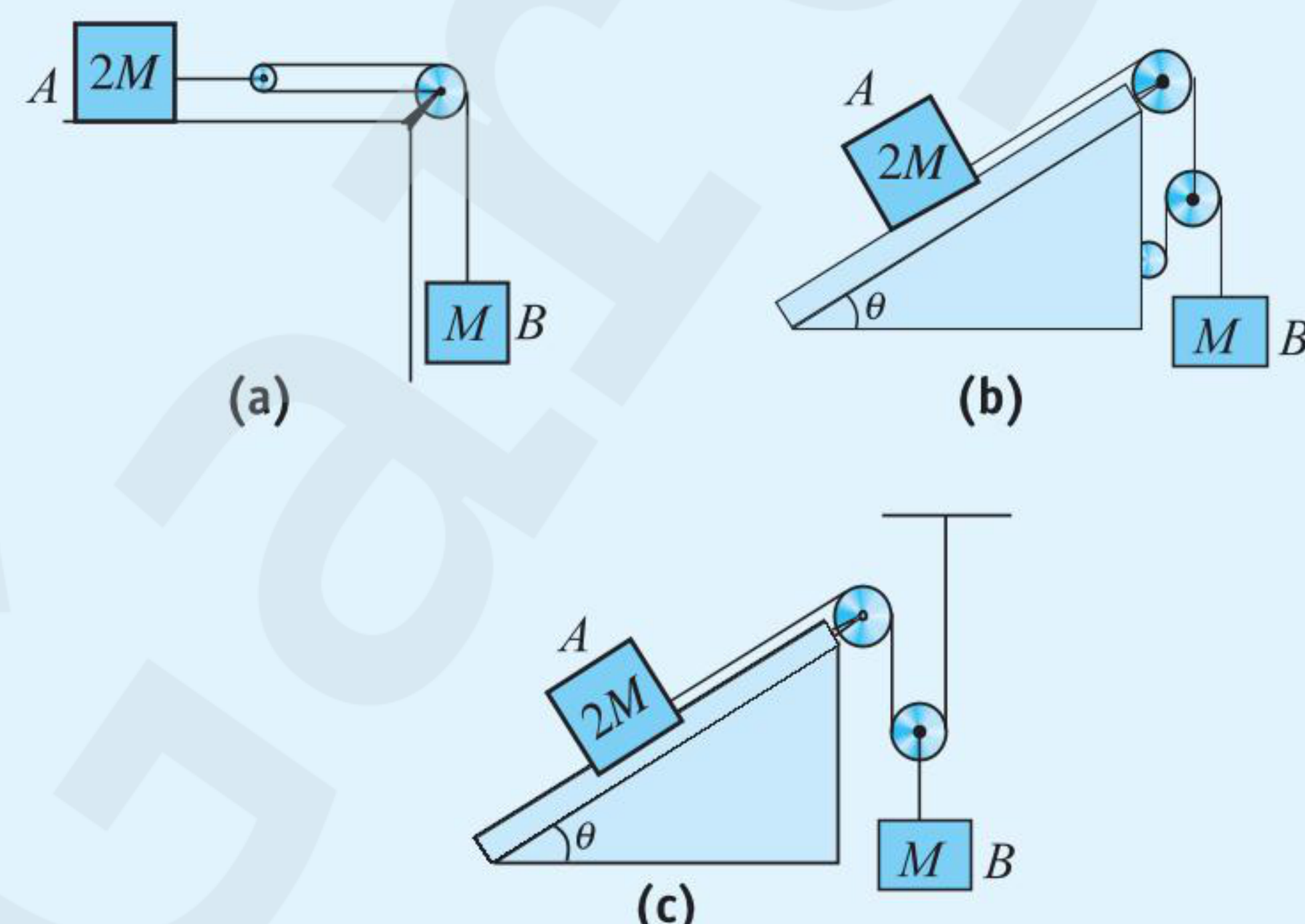
12. If the string is inextensible, determine the velocity u of each block in terms of v and θ .



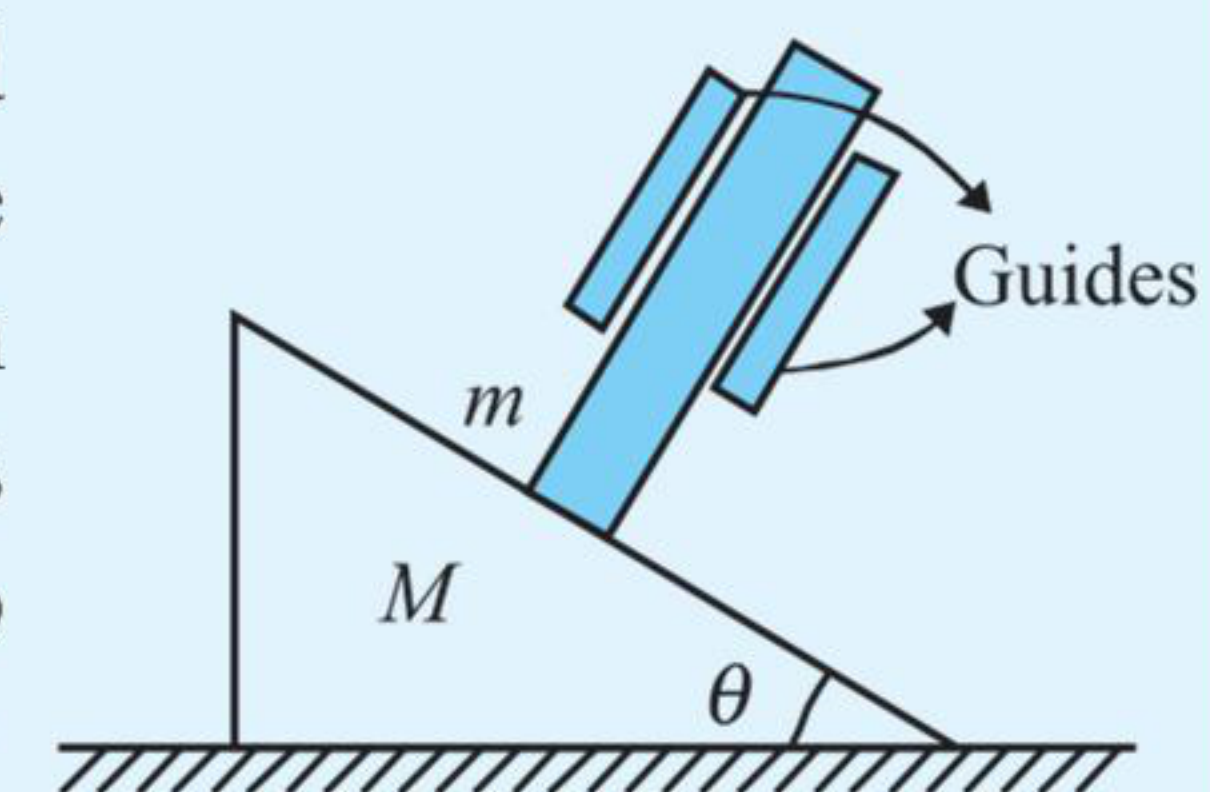
(a) Fig. (a): $u = \dots$

(b) Fig. (b): $u = \dots$

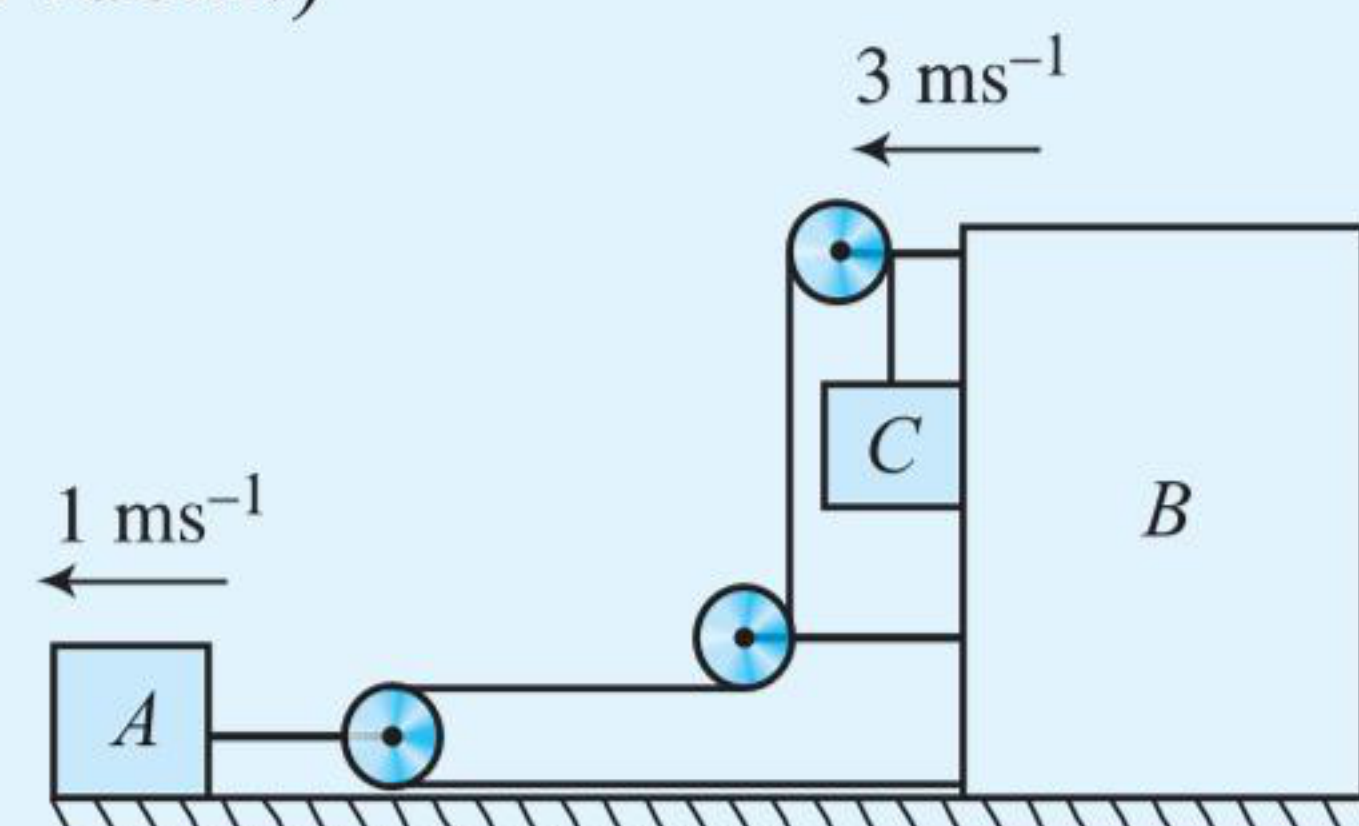
13. Calculate the accelerations of blocks A and B in cases (a), (b), and (c).



14. A rod of mass m is constrained to move along the guide and always in contact with the wedge of mass M as shown in figure. Assume no friction anywhere calculate the acceleration of the wedge and rod.



15. The velocities of A and B are shown in figure. Find the speed (in ms^{-1}) of block C . (Assume that the pulleys and string are ideal.)



ANSWERS

1. 1 ms^{-2} 2. $v_A = 10 \text{ ms}^{-1}$, $v_B = 22 \text{ ms}^{-1}$ 3. 1 ms^{-2}
4. $v_{\text{block}} = \frac{3}{4} v_{\text{truck}}$ 5. 0.5 ms^{-1}
6. $A = \frac{F}{M + 4m}$, $a = \frac{2F}{(M + 4m)}$
7. (a) 0 (b) $a_1 = 7/2 \text{ ms}^{-2}$
8. (a) $V_A = 0$ (b) $V_A = 30 \text{ ms}^{-1}$ 9. $v \sec \theta$
10. (a) $a_A = 2a$ (b) $a_1 = a\sqrt{5 - 4 \cos \theta}$
11. (a) (i) $a_{BA} = 2a$, (ii) $a_B = -a$
(b) (i) $a_{BA} = 3a$, (ii) $a_{BG} = a\sqrt{10 + 6 \cos \theta}$
12. (a) $v/(2 \cos \theta)$, $v/\cos \theta$
13. (a) $g/3$ (b) $\frac{g[\sin \theta - 1]}{3}$ (c) $\frac{g[4 \sin \theta - 1]}{9}$
14. $a_w = \frac{mg \sin \theta \cos \theta}{M + m \sin^2 \theta}$, $a_r = \frac{mg \sin^2 \theta \cos \theta}{M + m \sin^2 \theta}$
15. 5 ms^{-1}

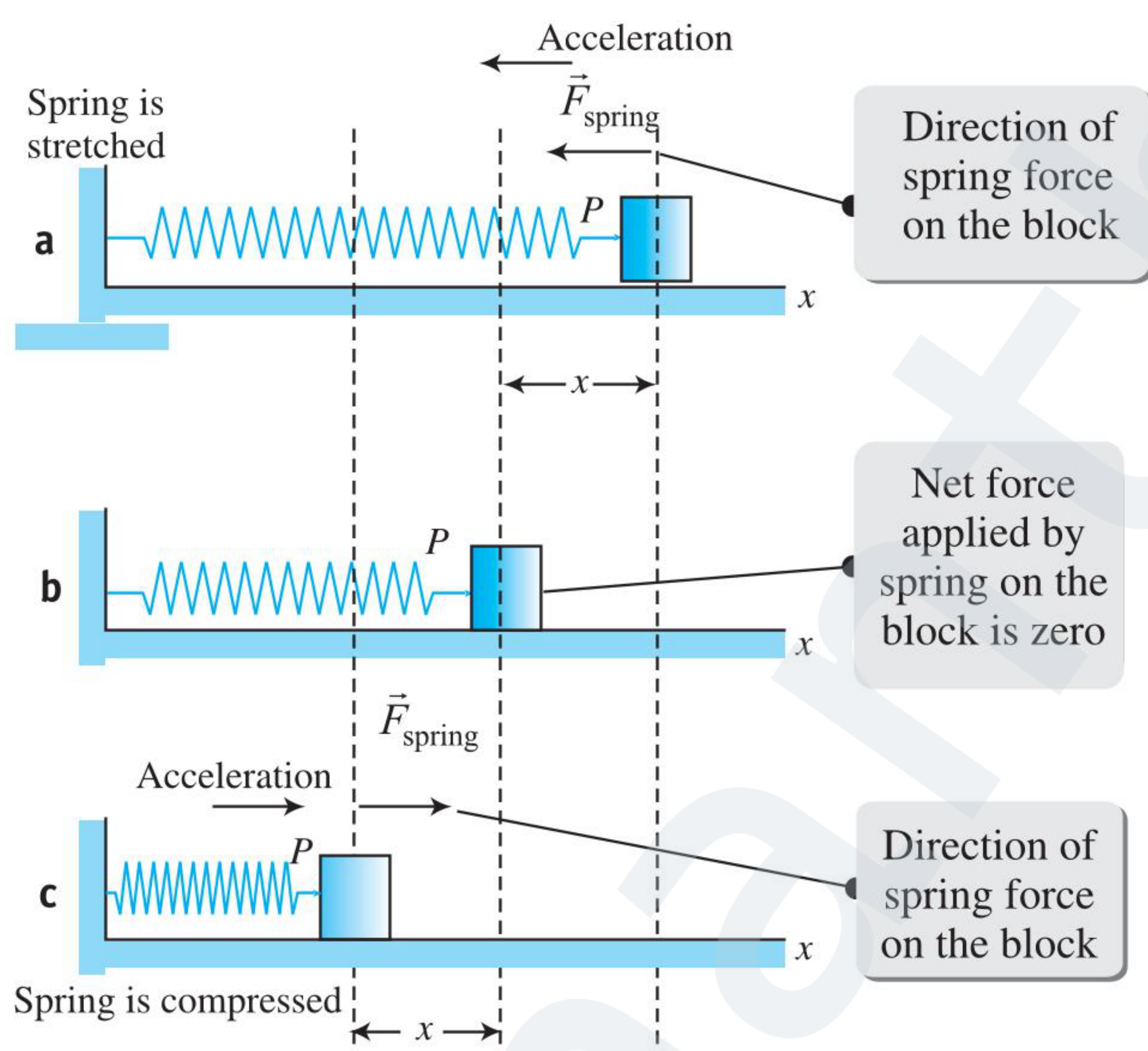
SPRING FORCE AND COMBINATIONS OF SPRINGS

In mechanics, we come across the system of bodies connected with springs. For this purpose, we need to develop the ideas how exactly a spring responds to an external force applied on it.

First of all, for the sake of simplicity, let us consider a light (massless) spring. Then fix one of its end and pull the other end slowly through a distance x (say). When we go on pulling the end P of the spring towards right, the spring pulls us towards left with a gradually increasing force F_s . Likewise, when we go on pushing the end P towards left, the spring will push us towards right with a gradually increasing force F_s .

From the above simple experiment, we can easily understand that

- the force offered by the spring, that is, “spring force” F_s points (acts) opposite to the displacement of the force end of the spring.
- the amount of springs force increase linearly with the deformation (compression or elongation) of the spring; when we plot the variation of F_s versus x , we obtain a straight line up to certain (limited) value of x , which is known as elastic limit.

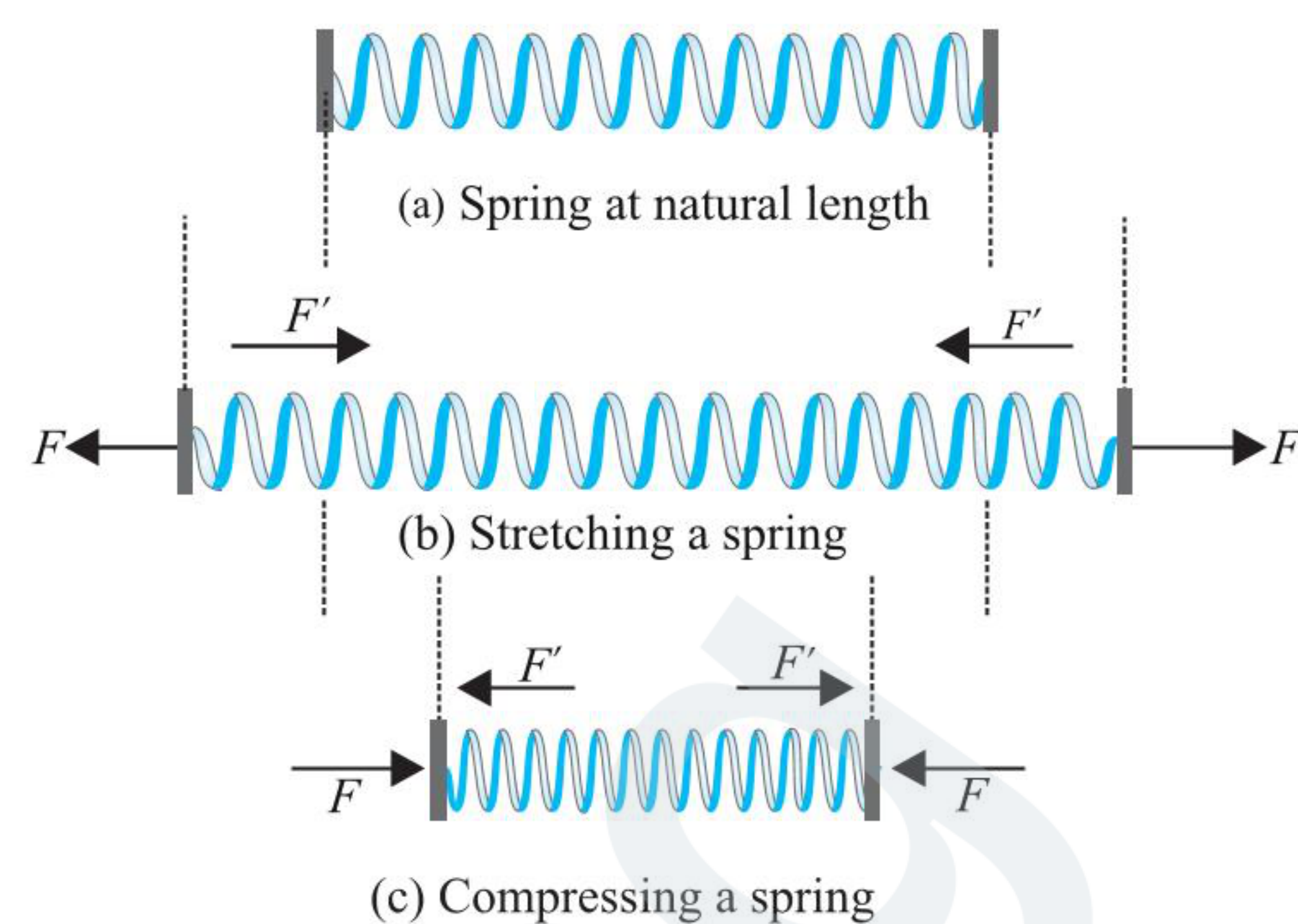


Robert Hooke experimented in seventh century who put forth all the above facts in a single vector formula in scalar form.

$$\text{Force} \propto \text{Stretch (or compression), i.e., } F = kx \quad \dots(i)$$

i.e., restoring force is linear. This force in a spring is not constant and depends on stretch (or compression) x . Greater the stretch (or compression), greater will be the force and vice-versa.

k is called the force constant of the spring and is equal to the slope of force versus–stretch curve. It has dimensions $[F/L] = [MT^{-2}]$ and units Nm^{-1} . Greater the force constant of a spring, lesser will be the stretch (or compression) for a given force and more stiffer is said to be the spring. The force constant k of a spring depends on wire (its length, radius r , and material) used to make the spring, radius of spring R , and length l of spring.



To produce extension or compression in a spring, two equal and opposite forces (F) are to be applied and in equilibrium restoring force (F') developed due to the elasticity of spring is equal to either force, i.e.,

$$F = F' \text{ and always opposite to applied force.}$$

For small stretch or compression, springs obey Hooke's law.

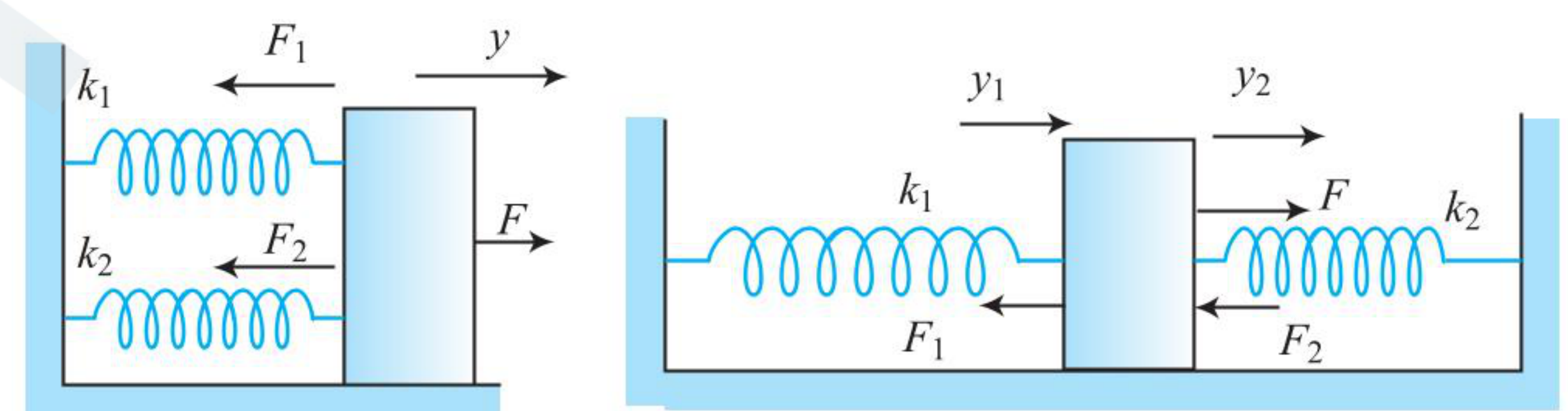
FORCE CONSTANT OF COMPOSITE SPRINGS

If a number of springs are connected to a body and we want to produce it to a single spring, following three cases of common interest are possible.

SPRINGS IN PARALLEL

This situation is shown in figure. If the force F pulls the mass m by y , the stretch in each spring will be y ,

$$\text{i.e., } y_1 = y_2 = y \quad \dots(i)$$



Now as for a spring $F = ky$ and as force constants are not equal, so $F_1 \neq F_2$, but for equilibrium,

$$F = F_1 + F_2, \text{ i.e., } ky = k_1 y_1 + k_2 y_2 \text{ [as } F = ky]$$

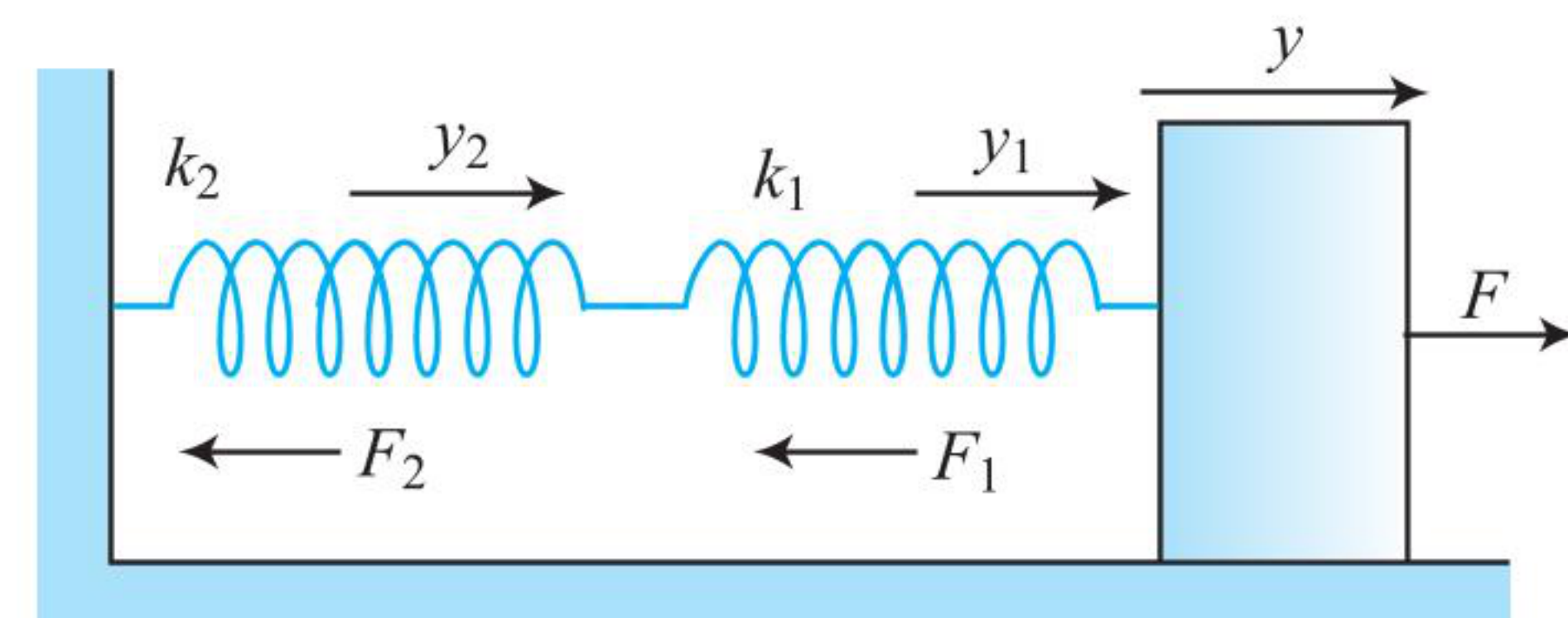
which in the light of (i) reduces to

$$k = k_1 + k_2 + \dots$$

This is like capacitors in parallel or resistance in series.

SPRINGS IN SERIES

This situation is shown in figure.



As springs are massless, force in these must be same, i.e.,

$$F_1 = F_2 = F \quad \dots(i)$$

Now as $F = ky$ and force constants are not equal, stretches will not be equal, i.e., $y \neq y_2$.

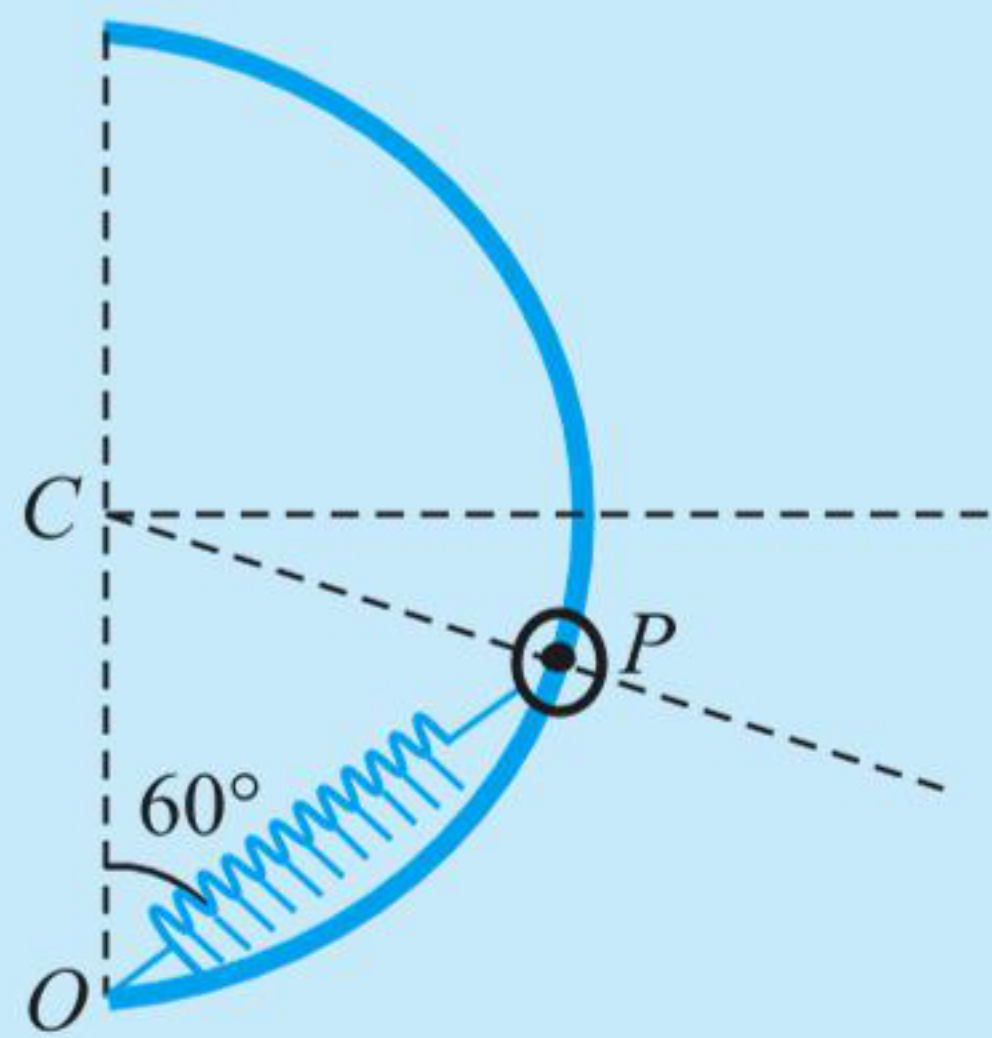
$$\text{But, } y = y_1 + y_2 \quad \text{or} \quad \frac{F}{k} = \frac{F_1}{k_1} + \frac{F_2}{k_2}$$

[as for $F = ky$, $y = (F/k)$]

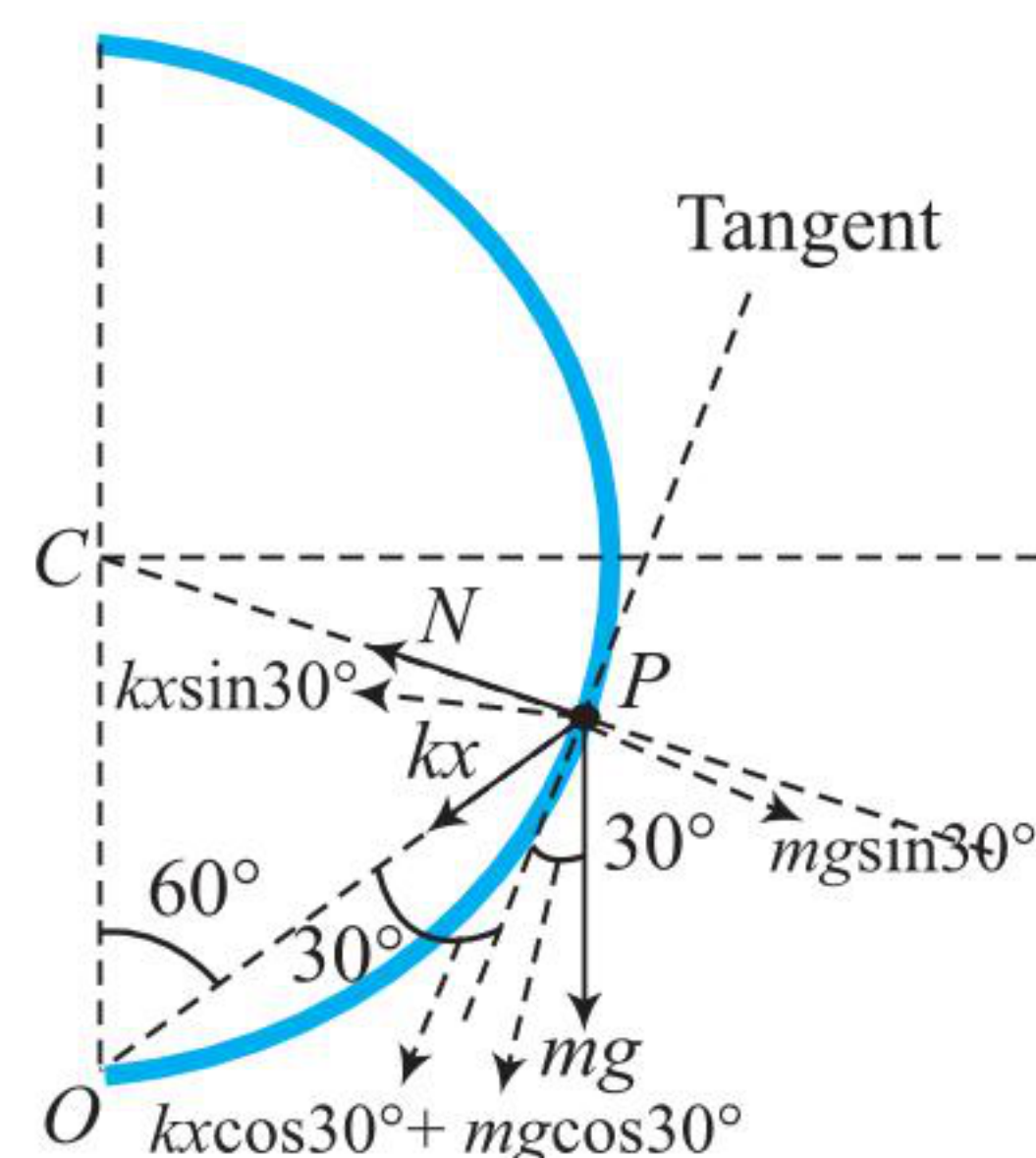
which in the light of Eq. (i) reduces to $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$

ILLUSTRATION 6.58

A smooth semicircular wire track of radius R is fixed in a vertical plane (see figure). One end of a massless spring of natural length $(3/4)R$ is attached to the lowest point O of the wire track. A small ring of mass m , which can slide on the track, is attached to the other end of the spring. The ring is held stationary of point P such that the spring makes an angle of 60° with the vertical. The spring constant $k = (mg/R)$. Consider the instant when the ring is released. Determine the tangential acceleration of the ring and the normal reaction between ring and track.


Sol.

- The free-body diagram of the ring is shown in figure. The forces acting on the ring are:



- The weight mg acting vertically downwards
- Normal force N by the wire track.
Normal force on the ring could be either radially outwards or radially inwards depending on whether the ring presses against the inner surface or outer surface of the track. To ascertain whether normal force is inwards or outwards, assume that, to begin with, it is inwards. Then from $\sum \vec{F} = m\vec{a}$, find the value of normal force. If it is positive, it is inwards and if it is negative, it is outwards.
- Force of the spring kx . In the given physical situation, the spring is extended, it will pull the ring. So the spring force kx is along the spring towards O .

- Length of the spring in the position shown = R . ($CP = CO = R$; $\angle COP = \angle OPC = 60^\circ$; $\triangle COP$ is equilateral)

$$\text{Change in length of the spring} = R - \left(\frac{3}{4}\right)R = \left(\frac{R}{4}\right)$$

$$kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$$

$$\text{Now from } F_t = ma_t,$$

$$\left(\frac{mg}{4}\right) \cos 30^\circ + mg \cos 30^\circ = ma_t \Rightarrow a_t = \frac{5\sqrt{3}}{8}g$$

Now consider in radial direction

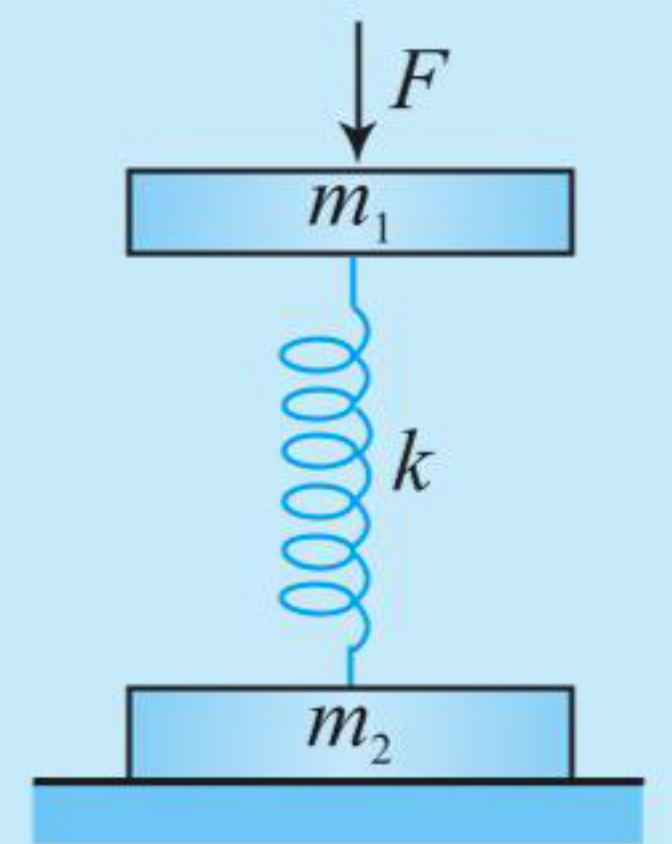
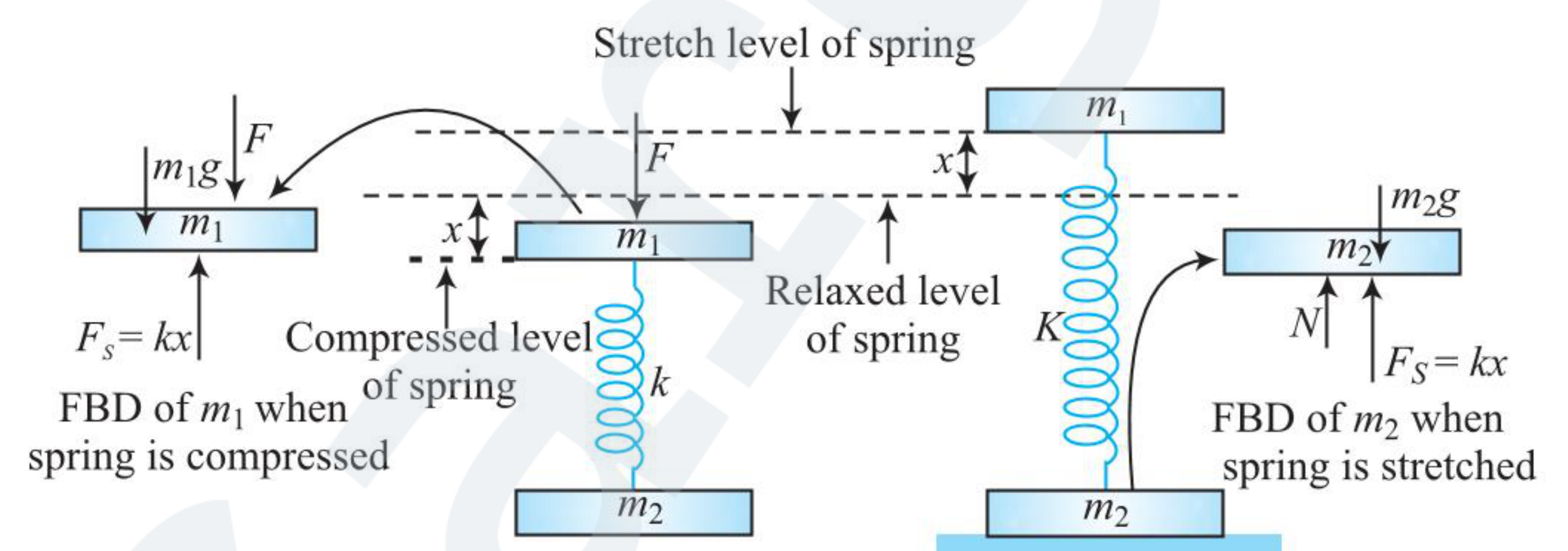
$$N + kx \sin 30^\circ = mg \sin 30^\circ$$

$$N = mg \sin 30^\circ - kx \sin 30^\circ$$

$$= mg \left(\frac{1}{2}\right) - \left(\frac{mg}{4}\right) \cdot \left(\frac{1}{2}\right) = \frac{3}{8}mg$$

ILLUSTRATION 6.59

Two disks of masses m_1 and m_2 are connected by a spring of force constant k . The lower disk of mass m_2 lies on a table and the upper disk is vertically above it (Figure). What vertical force F should be applied to the upper disk so that when the force is withdrawn, the lower disk is lifted off the table?


Sol.


The situation is shown in figure. Let the applied force F compresses the spring by x , so $F_s = kx$. After removal of force, the spring recovers its compressed length and further extends by x . At this instant the force exerted by spring is along upward direction. Thus, to lift off the lower disk, normal reaction N becomes zero.

When force is applied and spring is compressed,

$$F_s = F + m_1g \quad \dots(i)$$

When m_2 leaves contact (we have, $N = 0$) with ground and spring is stretched,

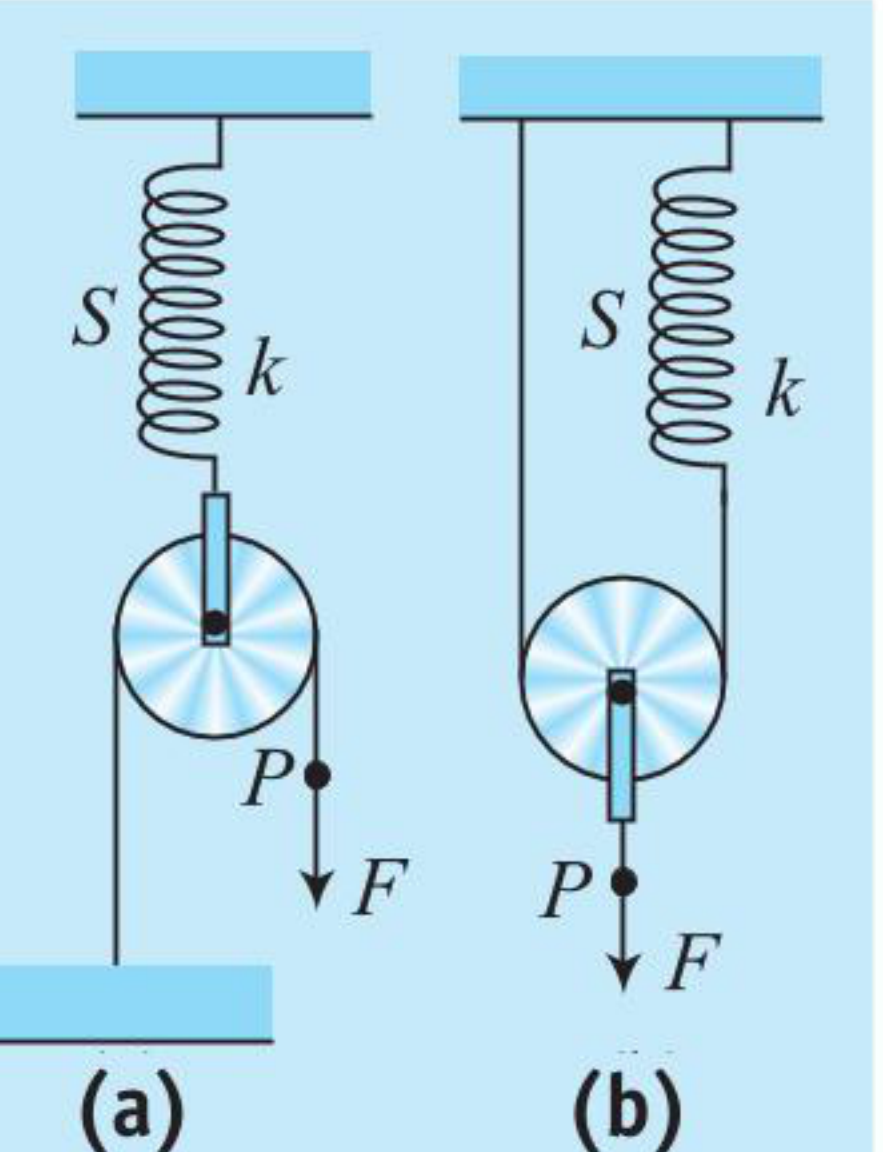
$$F_s = m_2g \quad \dots(ii)$$

From Eqs (i) and (ii), we have $F_{\min} = (m_2 - m_1)g$

So to lift off the lower disk, $F \geq (m_2 - m_1)g$

ILLUSTRATION 6.60

On applying a force F , point P is displaced vertically down by y from equilibrium position. Find force F in terms of the force constant k of the spring and displacement y , for the cases (a) and (b), as shown in figure.


Sol.

Case (a)

At point P : $F = T$

$$\dots(i)$$

And for the equilibrium of pulley,

$$2T = F_s \quad \dots(ii)$$

But as due to the shift of point P by y , the spring stretches by $(y/2)$. So

$$F_s = k(y/2) \quad \dots(iii)$$

So substituting F_s from (iii) in (ii) and then T from (ii) in (i), we get

$$F = (k/4)y \quad (A)$$

Case (b) As the tension in massless string and spring will be same,

$$T = F'_s \quad \dots(i)$$

$$\text{For pulley: } F = 2F'_s \quad \dots(ii)$$

Now if the mass M shifts by y , the spring will stretch by $2y$ (as string is inextensible)

$$F'_s = k(2y)$$

So substituting F' from Eq. (ii) in (iii),

$$F = (4k)y$$

Force of spring does not change instantaneously.

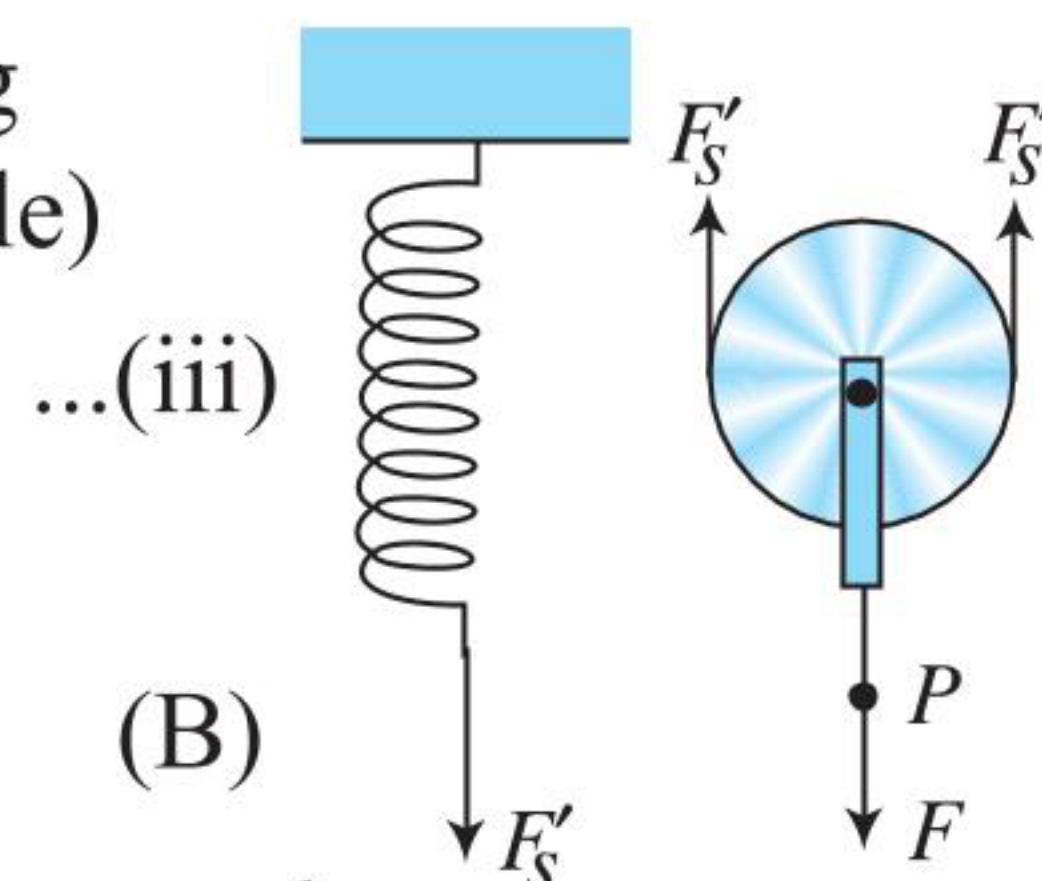
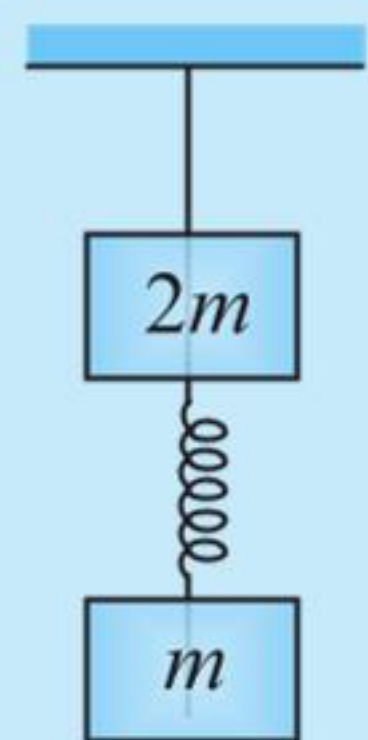


ILLUSTRATION 6.61

Two blocks are connected by a spring. The combination is suspended, at rest, from a string attached to the ceiling, as shown in figure. The string breaks suddenly.

Immediately after the string breaks, what is the initial downward acceleration of the upper block of mass $2m$?



Sol.

Step I: Discuss the problem before cutting the string:

From the force diagram of lower block,

$$kx_0 = mg$$

From the force diagram of upper block,

$$T = 2mg + kx_0$$

Step II: Discuss the problem after cutting the string,

$$2mg + kx_0 = 2ma$$

$$\text{or } 2mg + mg = 2ma$$

$$\text{or } 3mg = 2ma$$

$$\therefore a = \frac{3}{2}g$$

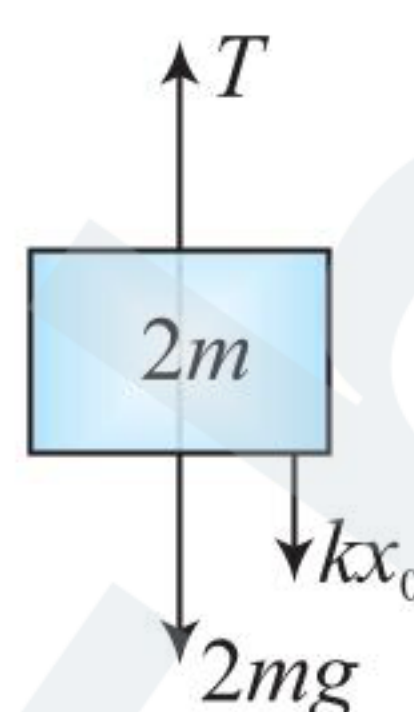
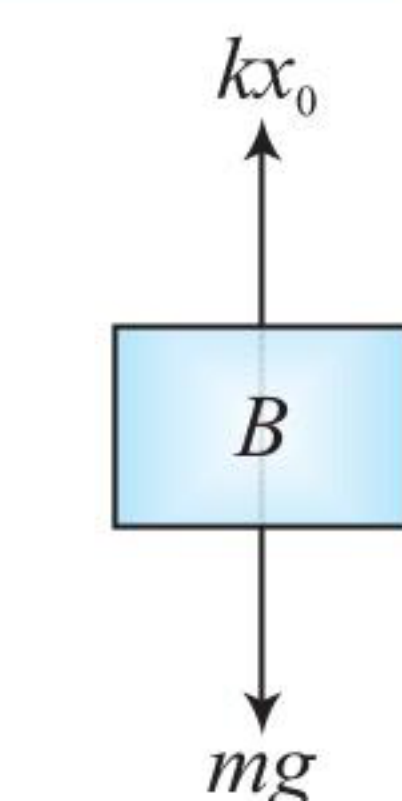
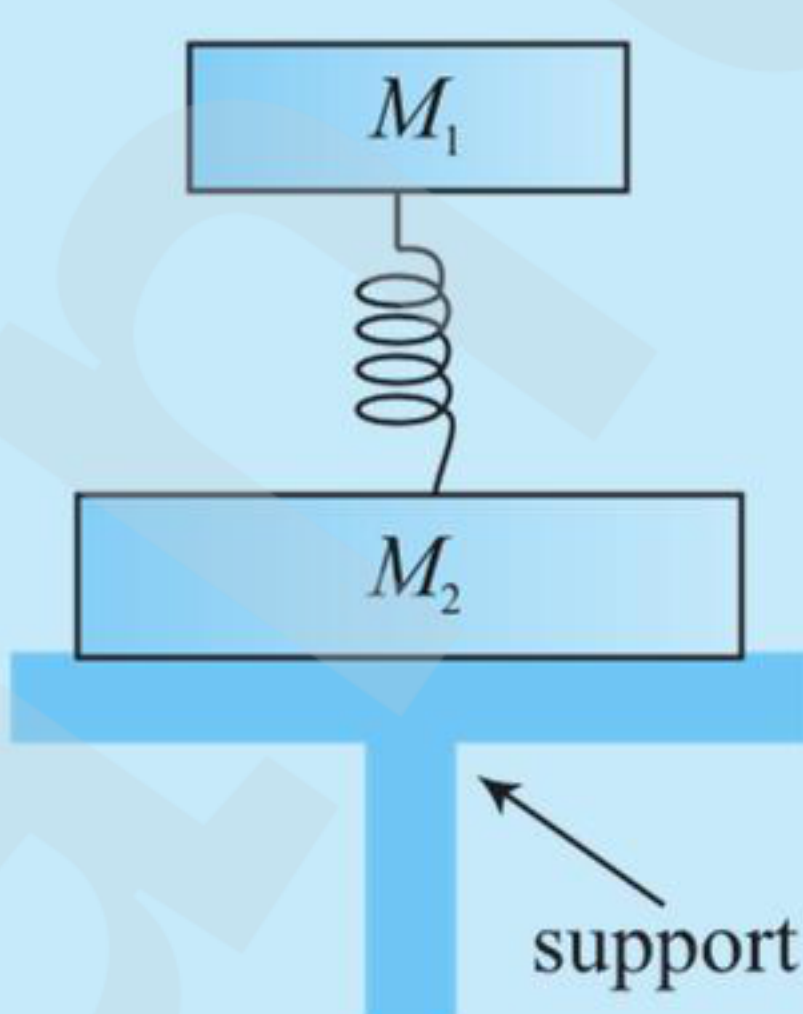
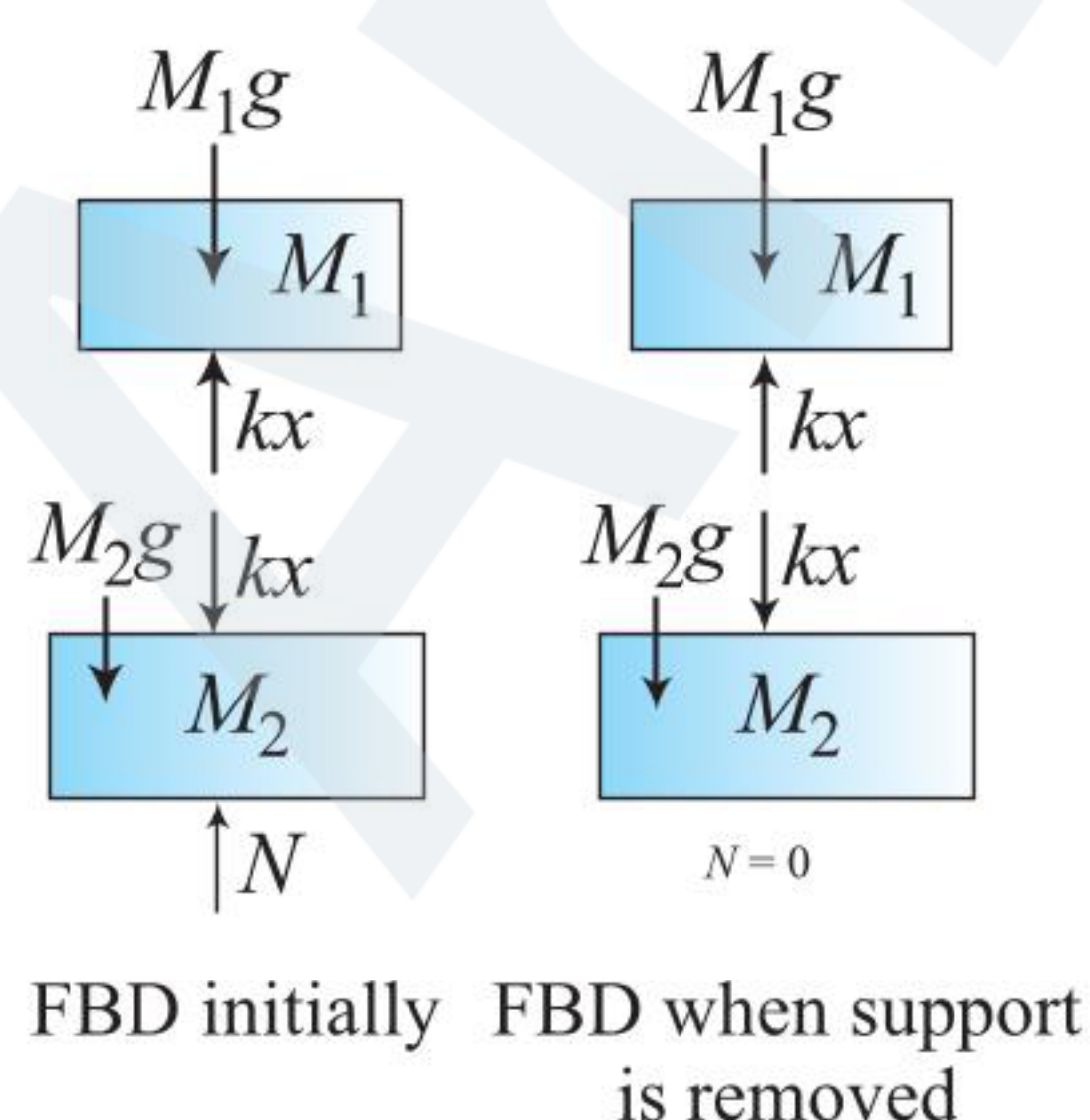


ILLUSTRATION 6.62

The system of two weights with masses M_1 and M_2 are connected with weightless spring as shown in figure. The system is resting on the support S . Find the acceleration of each of the weights just after the support S is quickly removed.



Sol. The force of spring does not change instantaneously, so find spring force at initial instant.



Initially, $M_1g = kx$

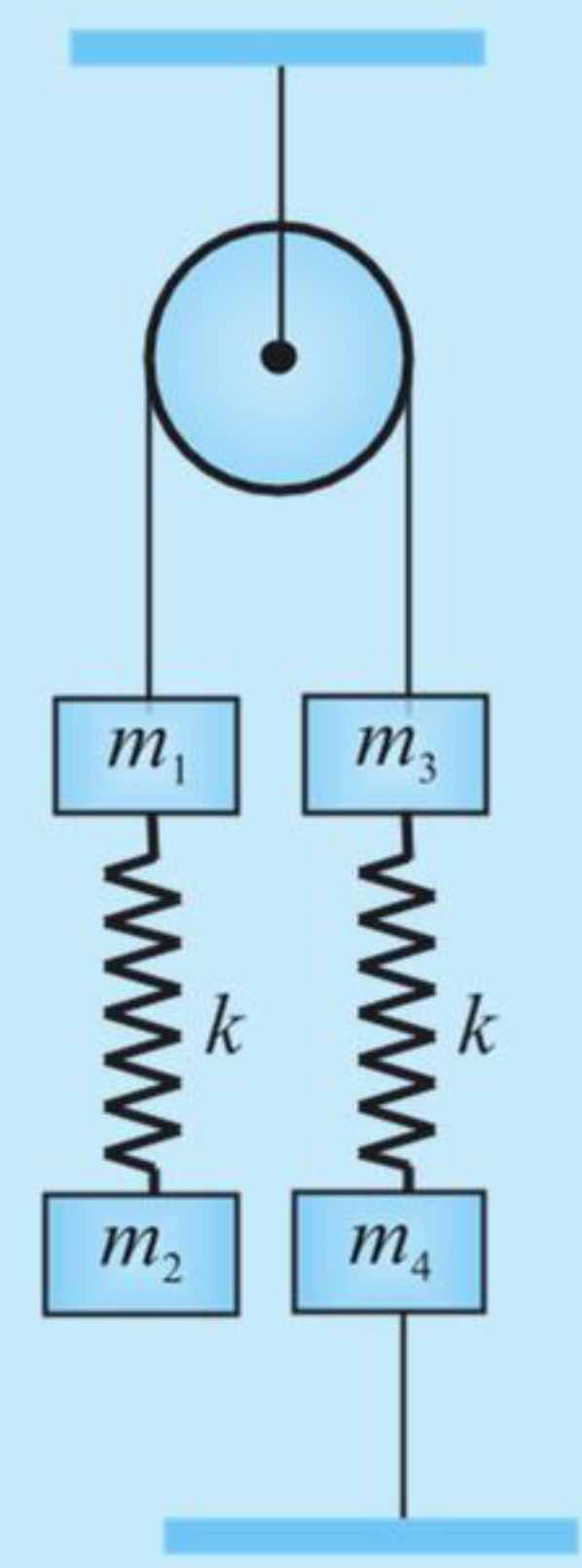
When support is removed, spring force does not change.

For M_1 : $M_1g - kx = M_1a_1$ or $a_1 = 0$

For M_2 : $M_2g + kx = M_2a_2$ or $a_2 = \frac{(M_1 + M_2)g}{M_2}$

ILLUSTRATION 6.63

Four blocks and two springs are arranged as shown in figure. The system at rest, determine the acceleration of all the loads immediately after the lower thread keeping the system in equilibrium has been cut. Assume that the threads are weightless, the mass of the pulley is negligible small, and there is no friction at the point of suspension.



Sol. For the equilibrium of system of loads,

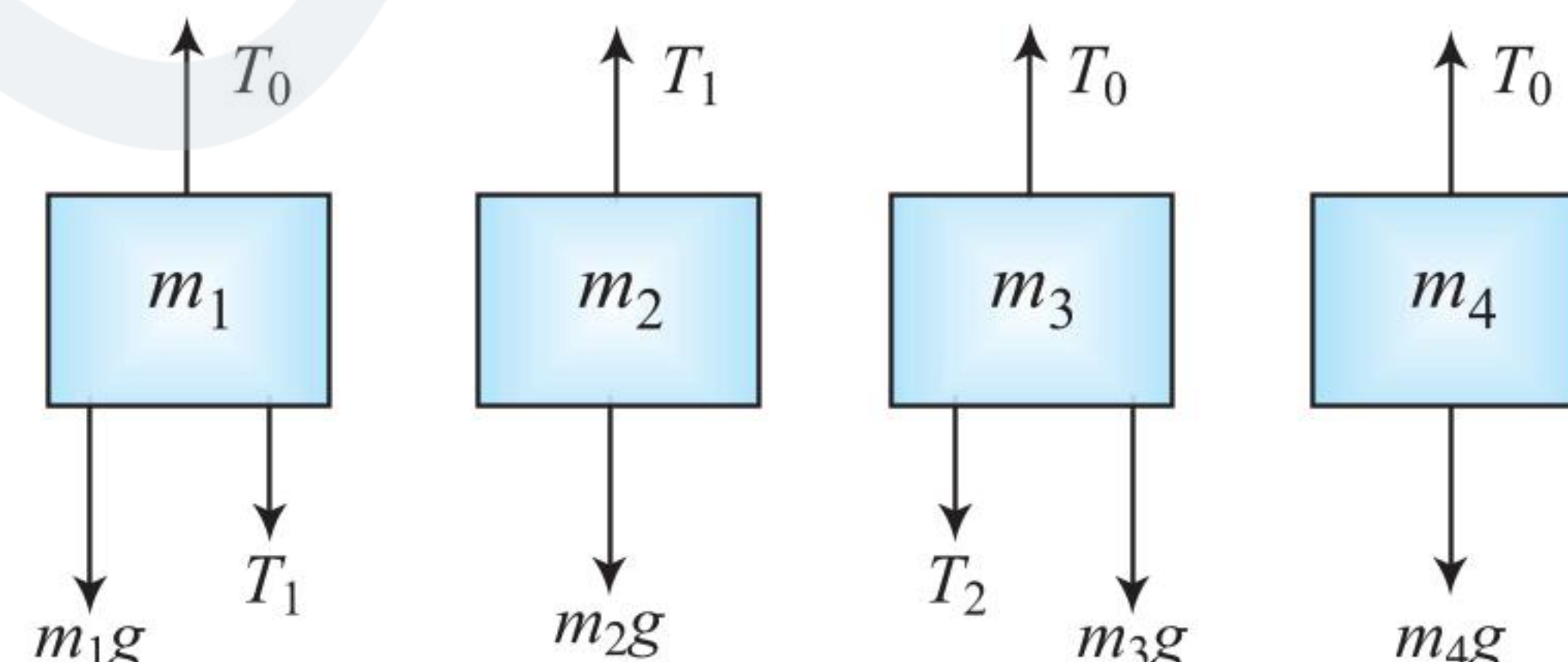
$$(m_1 + m_2)g = (m_3 + m_4)g$$

The force in the left spring, $T_1 = m_2g$

Let T_2 is the force in the right spring. For the equilibrium of load m_3 , we have

$$m_3g + T_2 = T_0 \quad \dots(i)$$

For m_1 , $m_1g + T_1 = T_0$



As $T_1 = m_2g$, therefore

$$m_1g + m_2g = T_0$$

Substituting this value in (i), we get

$$m_3g + T_2 = (m_1g + m_2g)$$

$$\text{or } T_2 = (m_1 + m_2 - m_3)g \quad \dots(ii)$$

After cutting, the lower thread, the equations of motion for the loads are

$$m_1g + T_1 - T_0 = m_1a_1 \quad \dots(iii)$$

$$m_2g - T_1 = m_2a_2 \quad \dots(iv)$$

$$T_2 + m_3g - T_0 = m_3a_3 \quad \dots(v)$$

$$\text{and } T_2 - m_4g = m_4a_4 \quad \dots(vi)$$

Solving above equations, we get

$$a_1 = a_2 = a_3 = 0 \text{ and } a_4 = \frac{(m_1 + m_2 - m_3 - m_4)g}{m_4}$$

ILLUSTRATION 6.64

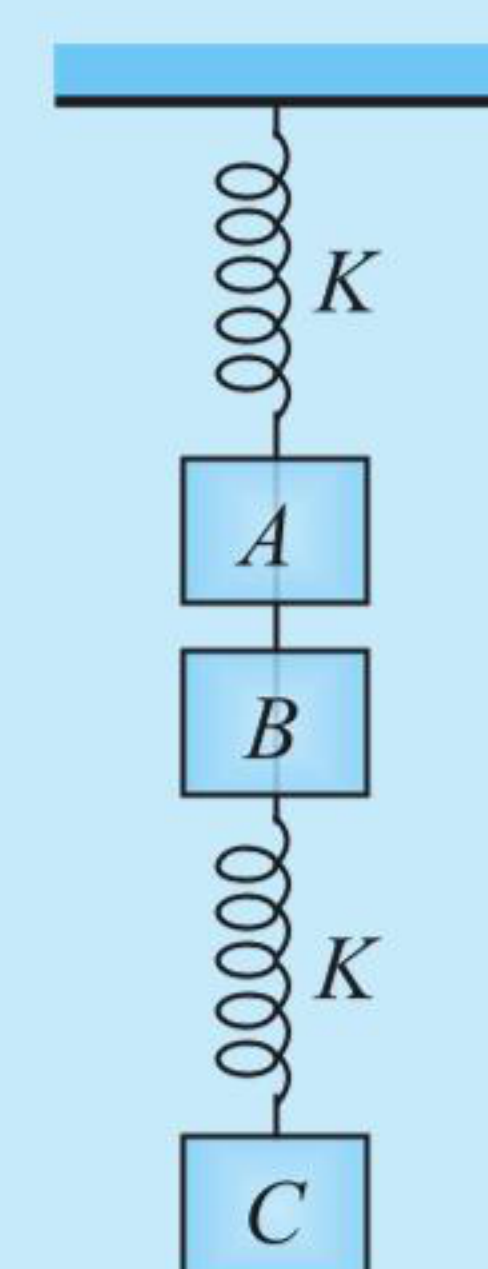
The system shown in figure is in equilibrium. Find the acceleration of the blocks A, B, and C, all of equal masses m at the instant when (assume springs to be ideal).

(a) The spring between ceiling and A is cut.

(b) The string (inextensible) between A and B is cut.

(c) The spring between B and C is cut.

Also find the tension in the string in the above three cases.



Sol. The system is in equilibrium:

From Fig. (a),

$$2mg + kx_3 = kx_1 \quad \dots(i)$$

From Fig. (b),

$$kx_3 = mg \quad \dots(ii)$$

$$\therefore 3mg = kx_1$$

- (a) When the spring between ceiling and block is cut, then the elongation of spring between B and C remains same just after cutting.

$$\therefore a_c = 0 \quad (kx_3 = mg)$$

For $(A + B)$,

$$kx_3 + 2mg = 3mg$$

$$\therefore 3mg = 2ma$$

$$\therefore a = \frac{3}{2}g = 15 \text{ m s}^{-2}$$

$$\therefore a_A = a_B = 15 \text{ m s}^{-2}$$

For tension (from FBD of B),

$$mg + kx_3 - T = ma_B$$

$$mg + mg - T = \frac{3mg}{2}$$

$$\therefore T = \frac{mg}{2}$$

- (b) When string between A and B is cut, the elongation in springs do not change just after cutting the string.

$$mg - kx_1 = ma_A$$

$$mg - 3mg = ma_A \quad \{kx_1 = 3mg\}$$

$$-2mg = ma_A$$

$$a_A = -2g$$

$$\therefore a_A = 2g \text{ (upward)}$$

For B ,

$$mg + kx_3 = ma_B$$

$$mg + mg = ma_B \quad \{\therefore kx_3 = mg\}$$

$$\therefore a_B = 2g \text{ (downward)}$$

For C ,

$$mg - kx_3 = ma_C$$

$$\text{or } mg - mg = ma_C \quad \{kx_3 = mg\}$$

$$a_C = 0$$

$$T = 0$$

- (c) When spring between B and C is cut.

For C , $mg = ma_C$

$$\therefore a_C = g \text{ (downward)}$$

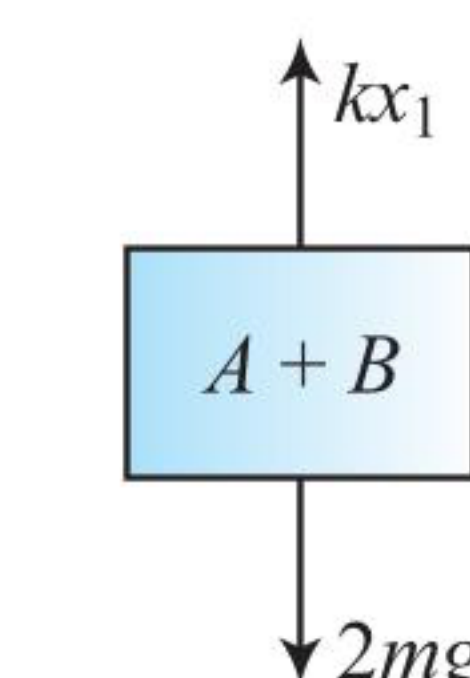
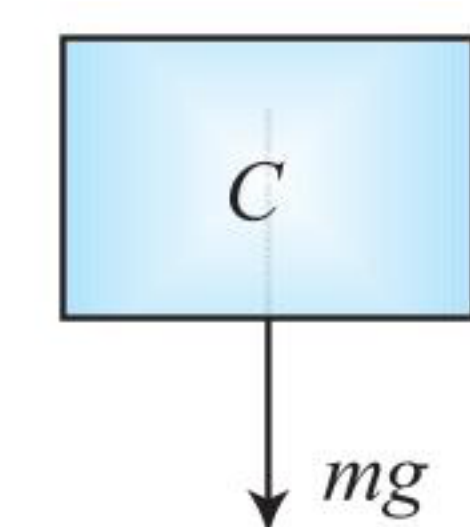
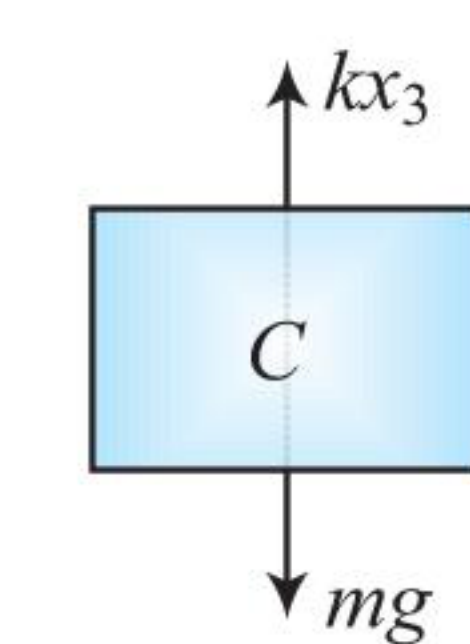
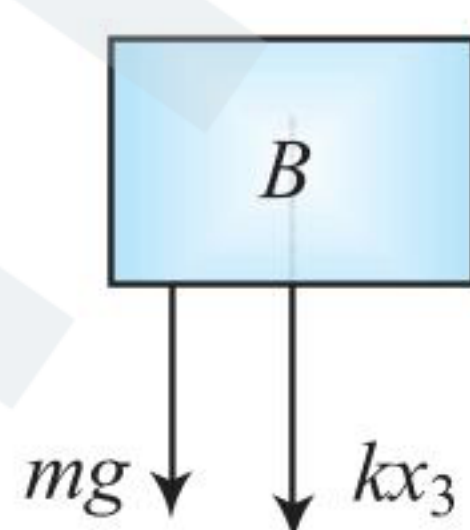
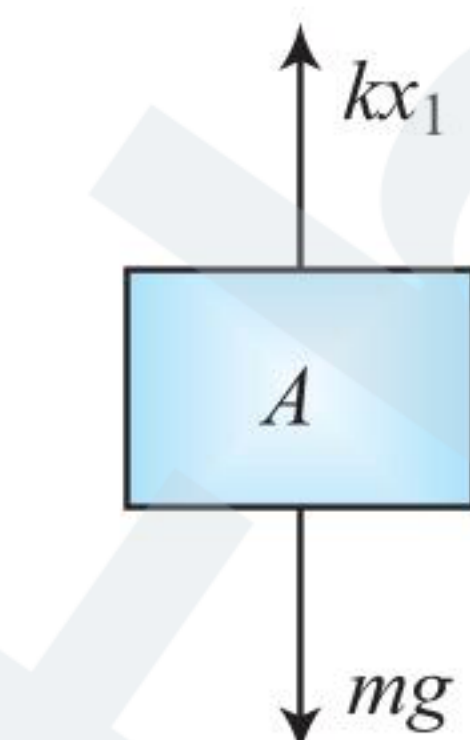
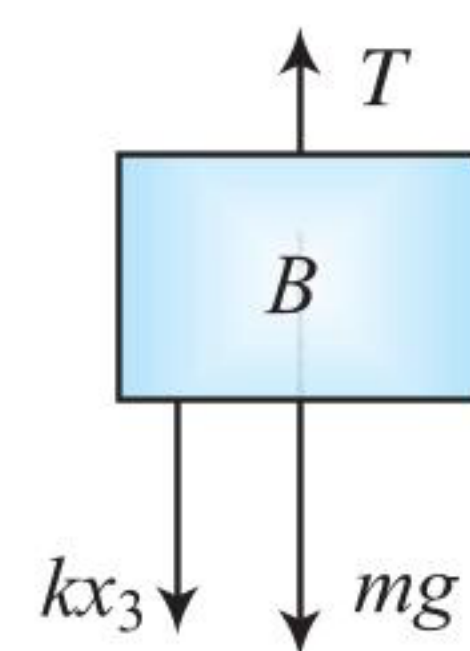
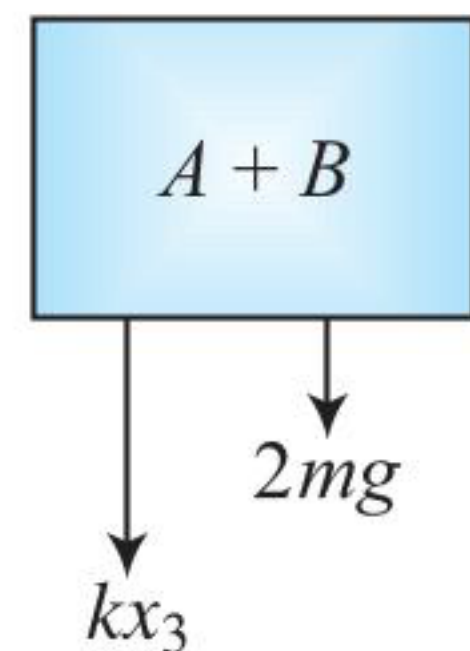
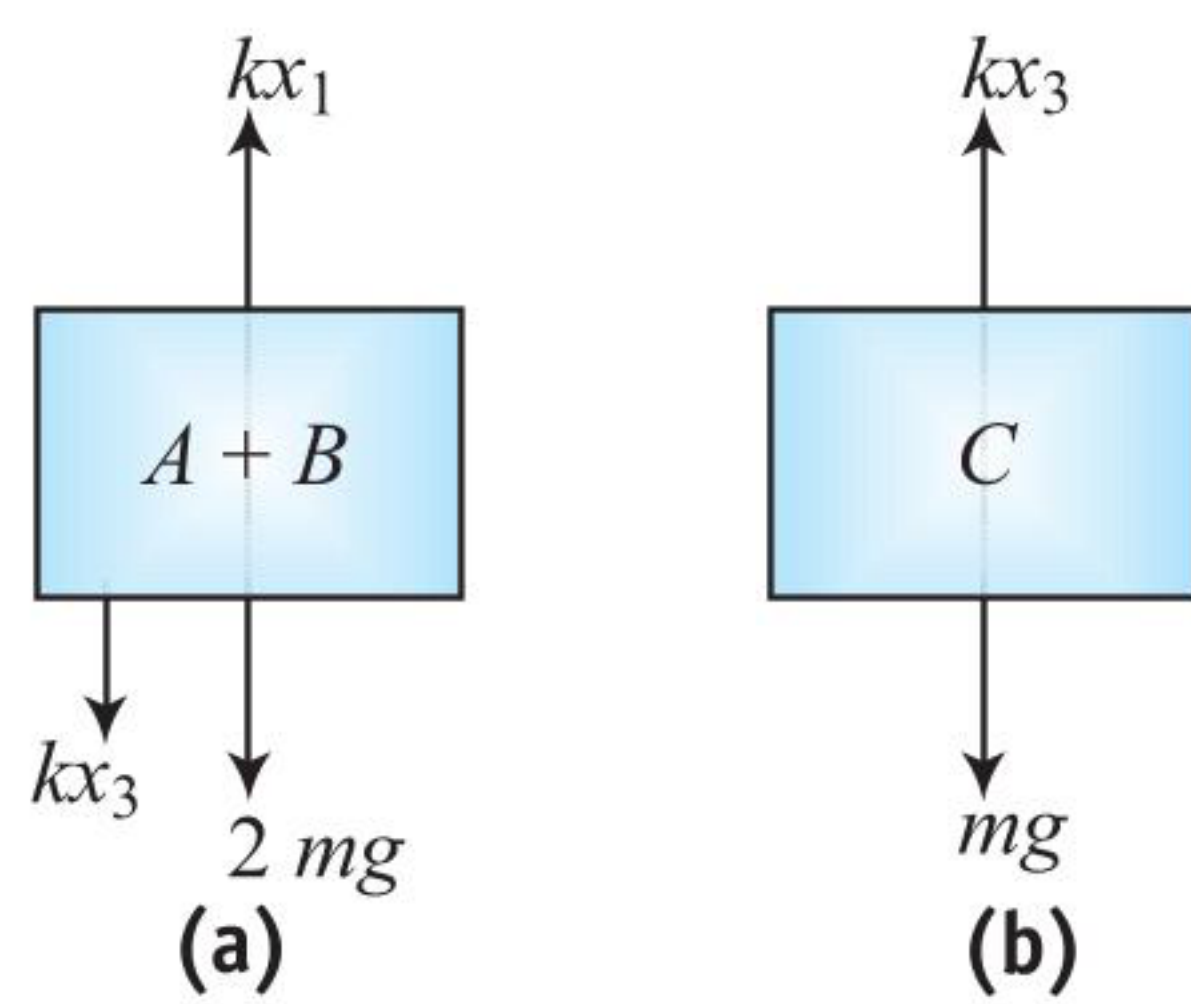
The acceleration of A and B will be equal; taking " $A + B$ " together as system

$$2mg - kx_1 = 2ma_B \quad \{\therefore a_A = a_B\}$$

$$2mg - 3mg = 2ma_B \quad \{\therefore kx_1 = 3mg\}$$

$$\therefore a_B = -\frac{g}{2}$$

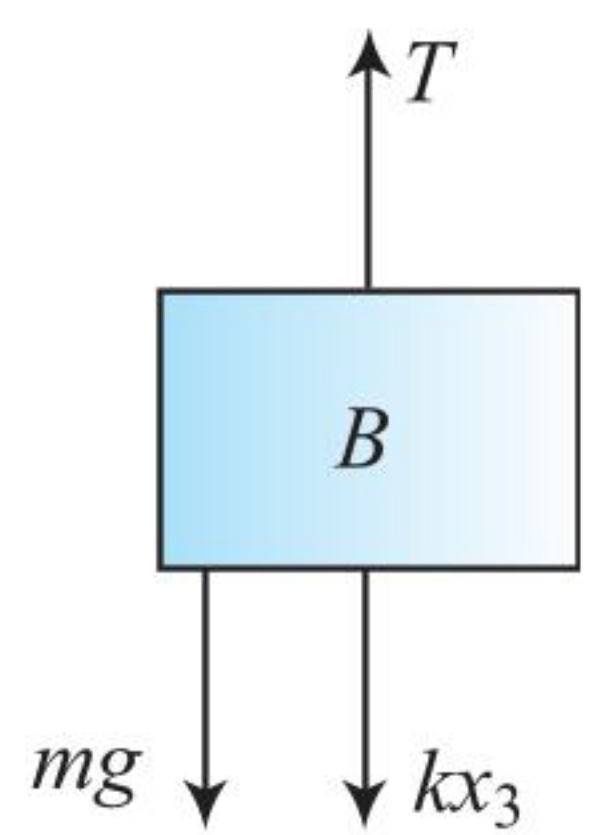
$$\therefore a_A = a_B = \frac{g}{2} \text{ (upward)}$$



For tension in string between A and B , considering the FBD of B only

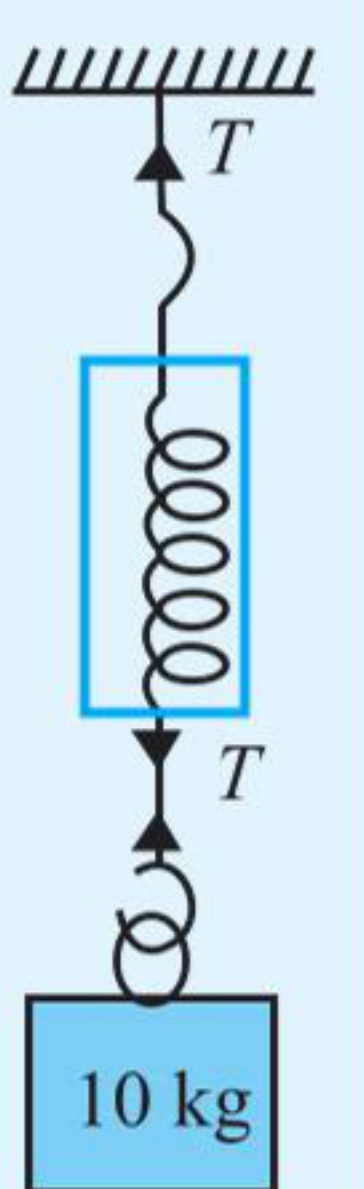
$$T - (mg) = ma_B$$

$$T = mg + \frac{mg}{2} = \frac{3mg}{2}$$

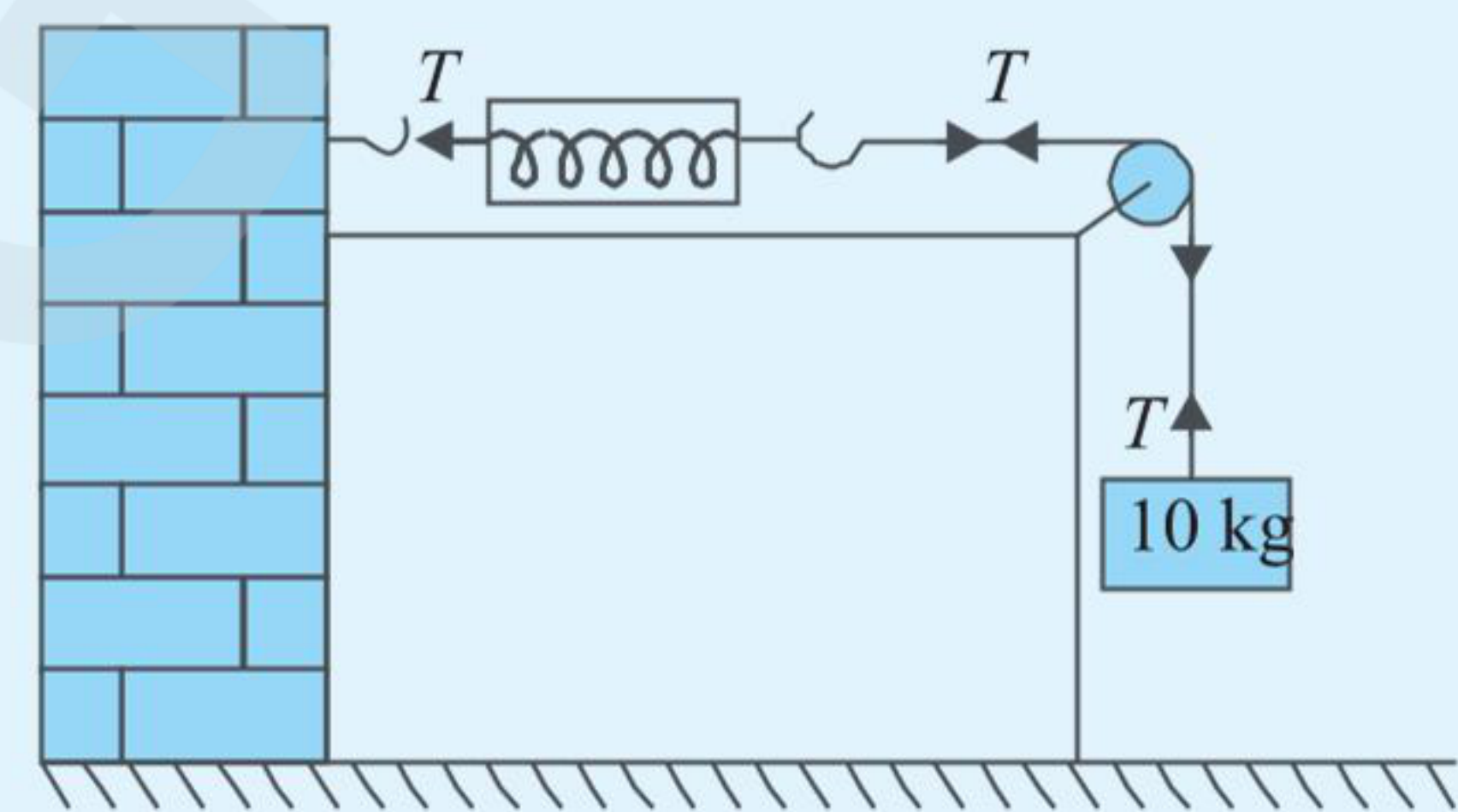


CONCEPT APPLICATION EXERCISE 6.7

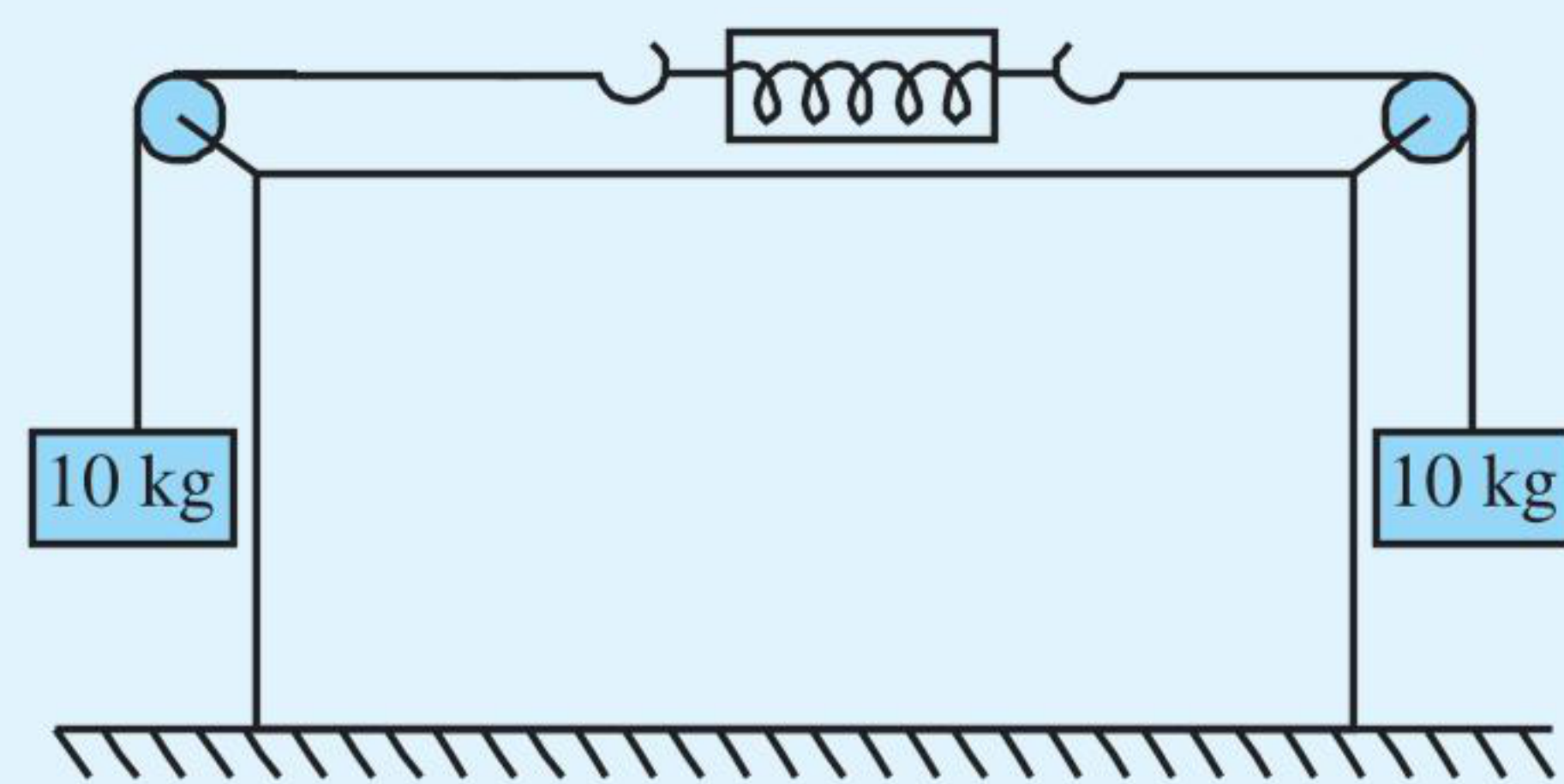
1. (a) A 10-kg block is supported by a cord that runs to a spring scale, which is supported by another cord from the ceiling as shown in figure. What is the reading on the scale?



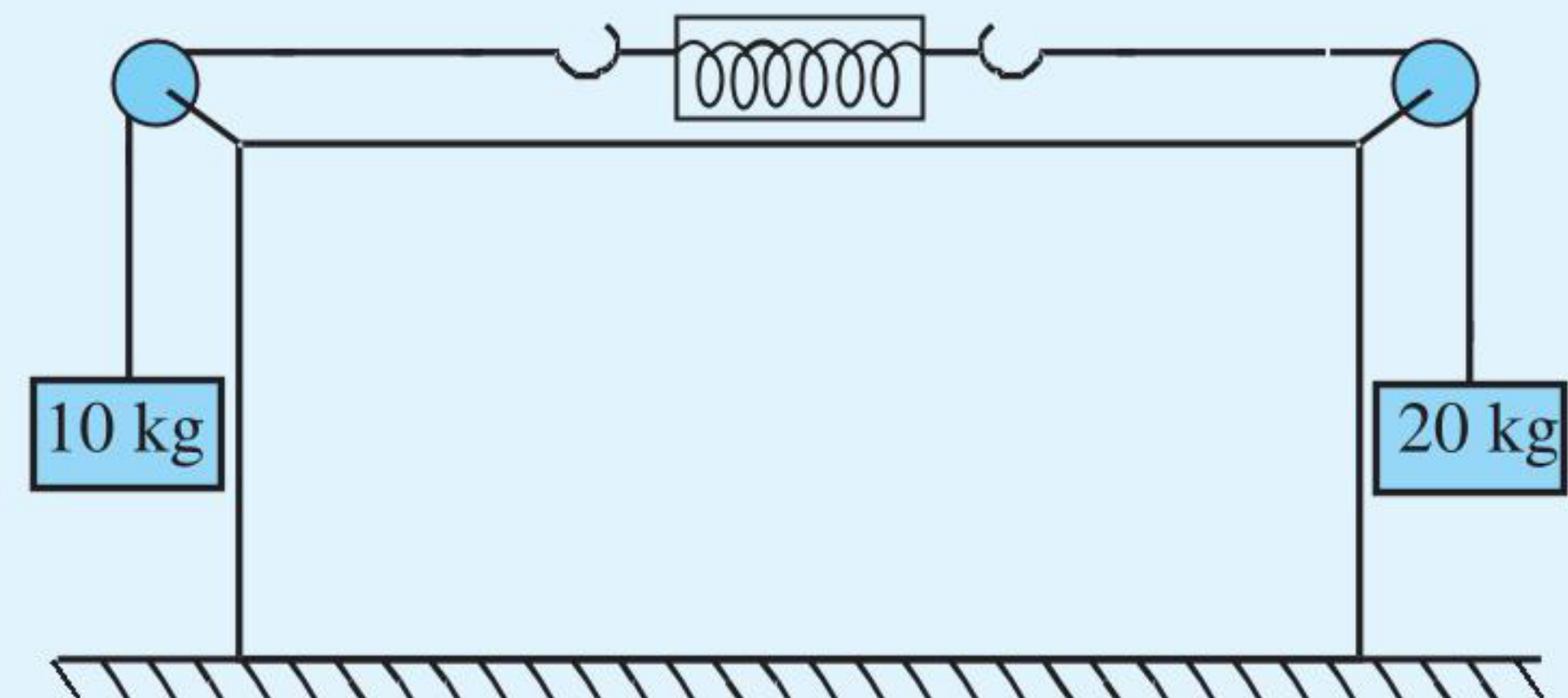
- (b) In figure, the block is supported by a cord that runs around a pulley and to a scale. The opposite end of the scale is attached by a cord to a wall. What is the reading of the scale?



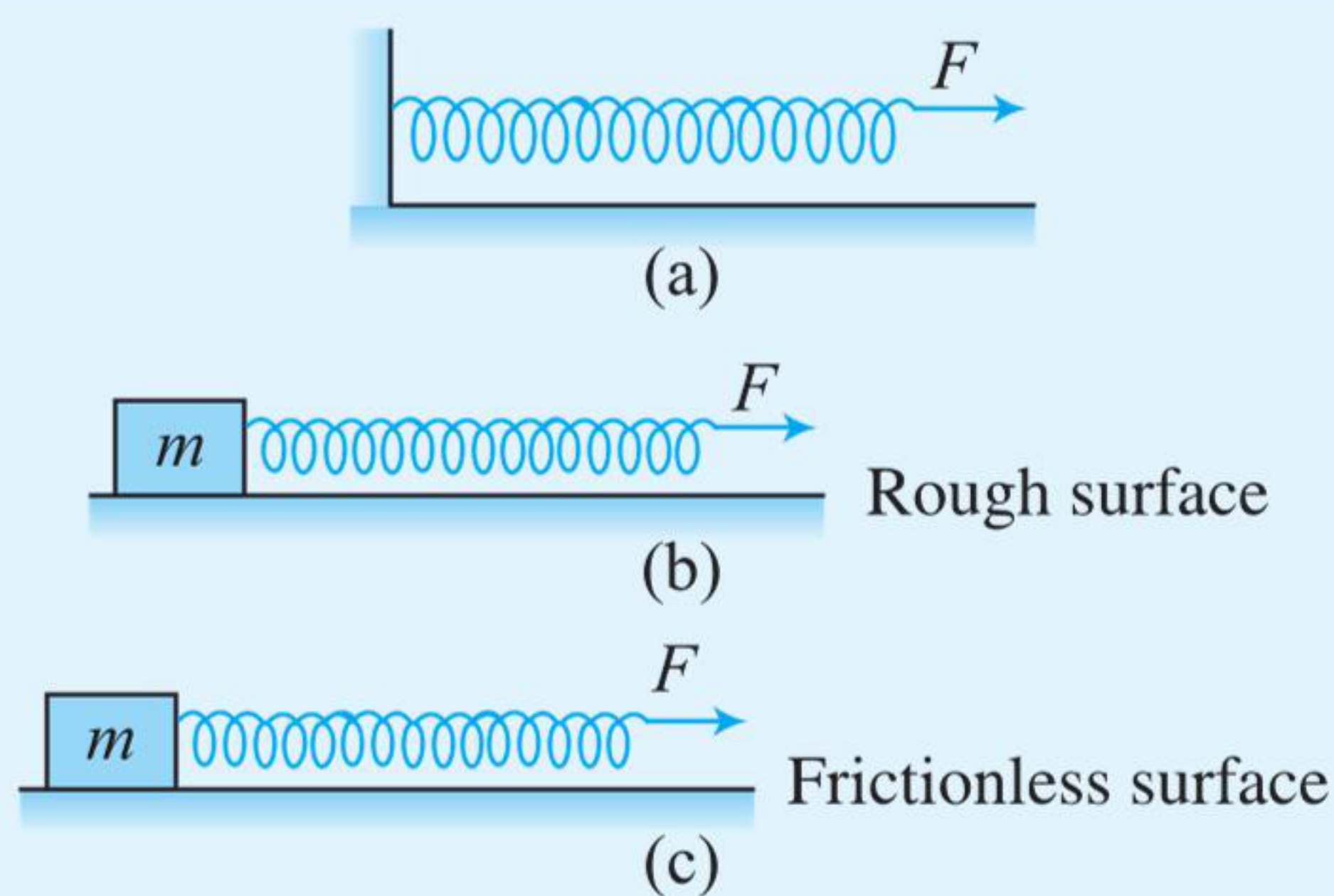
- (c) In figure, the wall has been replaced with a second 10-kg block. What is the reading on the scale now?



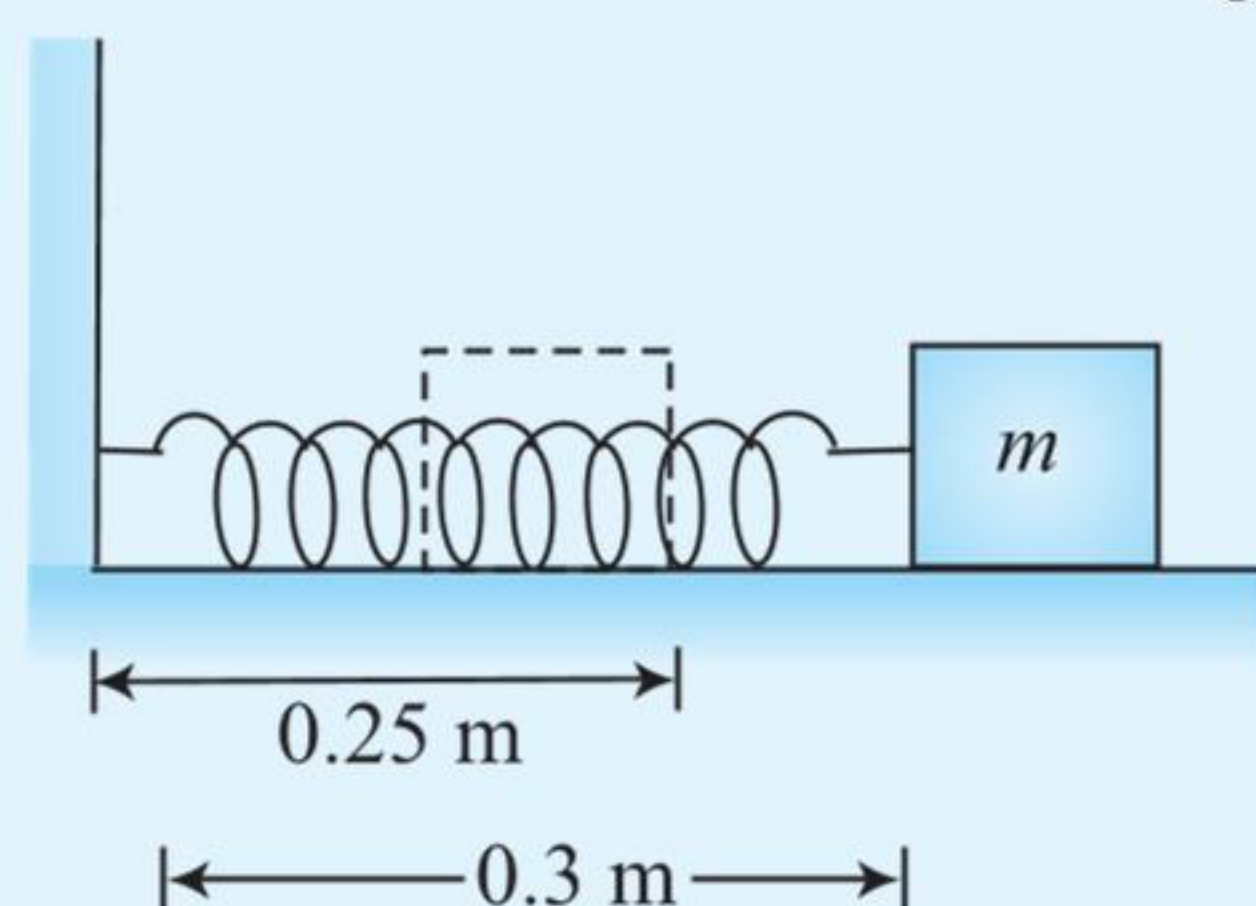
2. What is the reading of the spring balance in the following device?



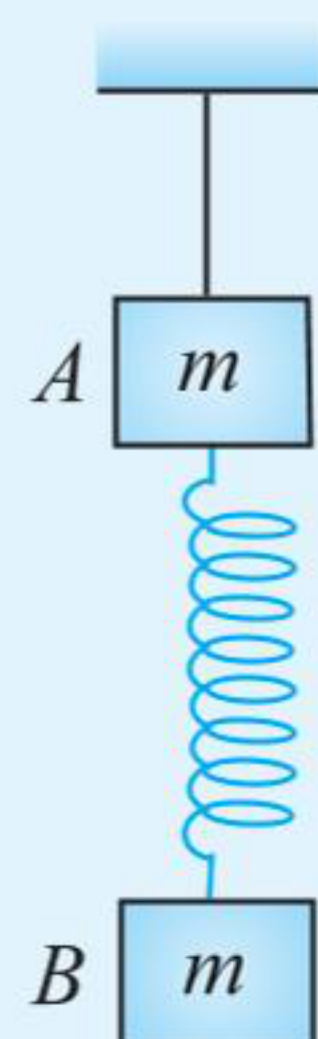
3. In the given figure, three identical massless springs are kept horizontal. The left end of the first is tied to a wall. The left end of the second spring is tied to a block of mass m placed on rough ground and the left end of the third spring is tied to a block of mass m placed on frictionless ground. The right end of each spring is pulled by a force that is increased gradually from zero to F . Extensions in these springs are x_1 , x_2 , and x_3 , respectively. Find the relationship between x_1 , x_2 , and x_3 .



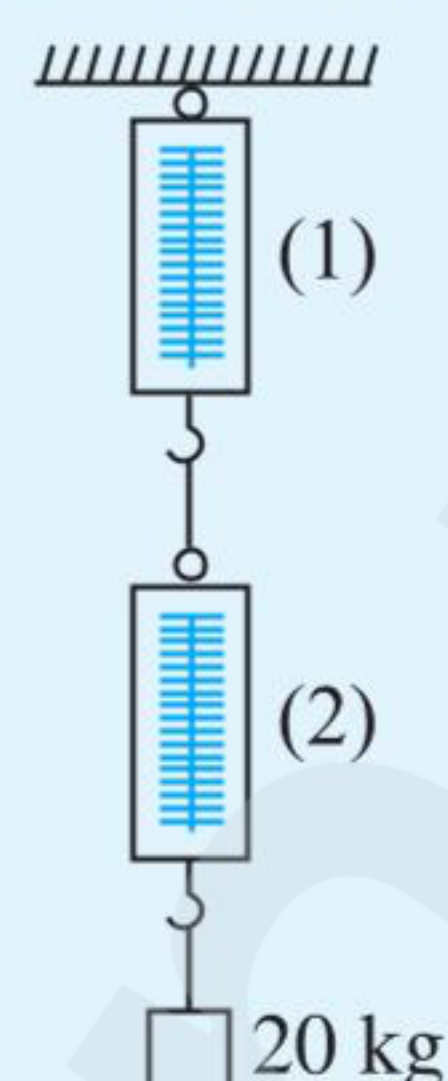
4. A smooth block of mass m is connected with a spring of stiffness $k (= 20 \text{ N m}^{-1})$ and natural length $l_0 = 0.25 \text{ m}$. If the block is pulled such that the new length of the spring becomes $l = 0.3 \text{ m}$, find the acceleration of the block at the moment when it is released from given position.



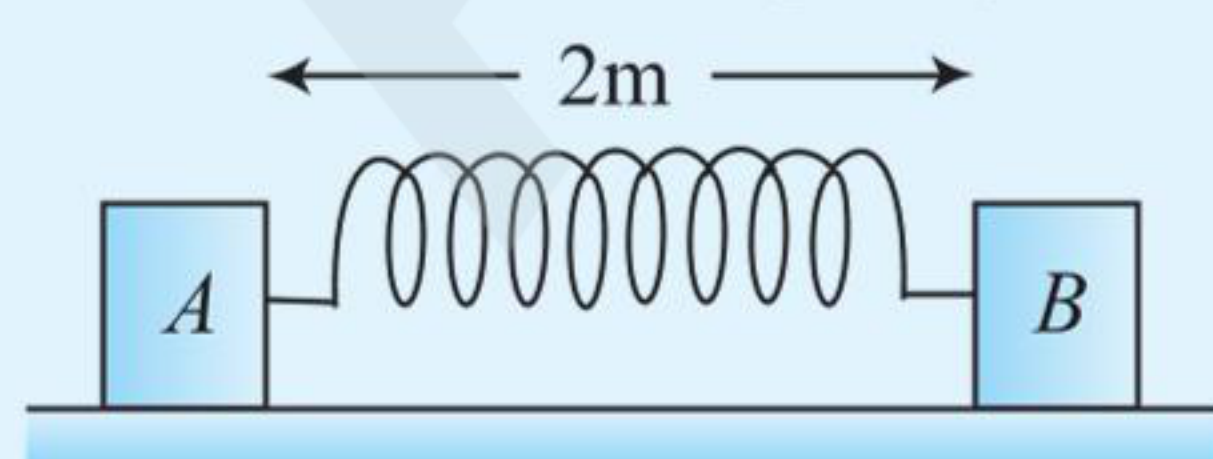
5. Two blocks A and B of same mass m attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block A and B just after the string is cut.



6. A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the:



- (a) reading of spring balance (1).
(b) reading of spring balance (2).
7. Two blocks are connected by a spring of natural length 2 m . The force constant of spring is 200 N m^{-1} .



Find the spring force in the following situations:

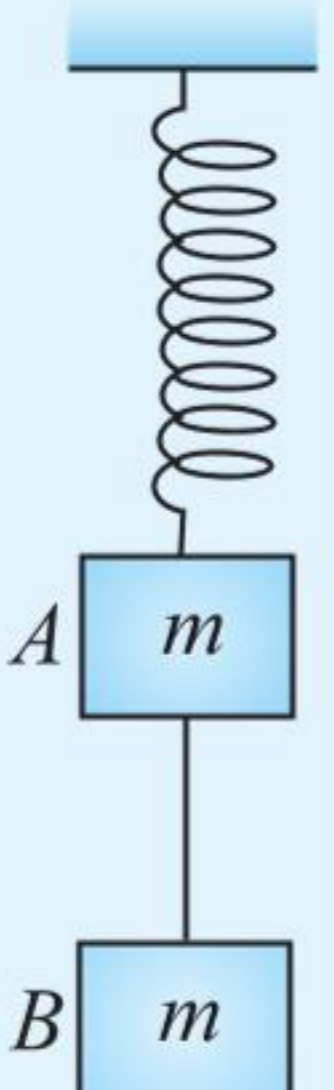
- (a) A is kept at rest and B is displaced by 1 m in right direction.
(b) B is kept at rest and A is displaced by 1 m in left direction.

- (c) A is displaced by 0.75 m in right direction and B is 0.25 m in left direction.

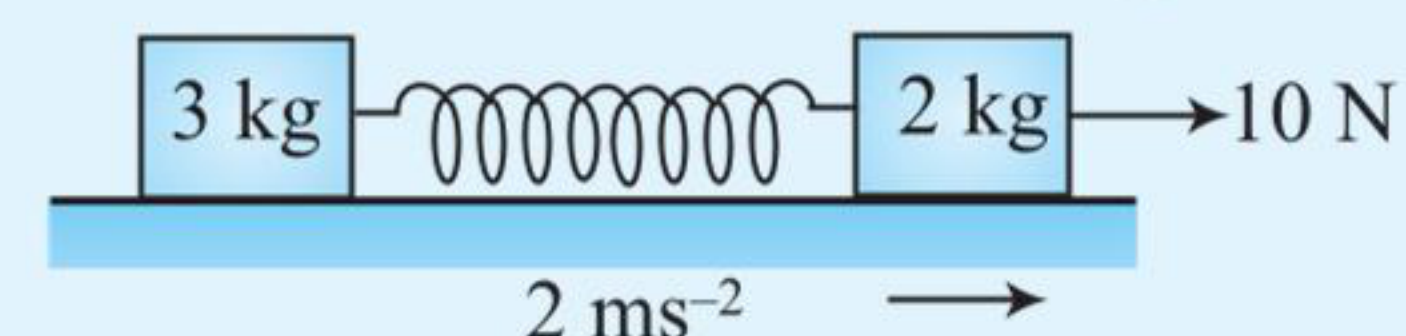
8. If the force constant of spring is 50 N m^{-1} , find mass of the block, if it rests in the given situation ($g = 10 \text{ m s}^{-2}$).



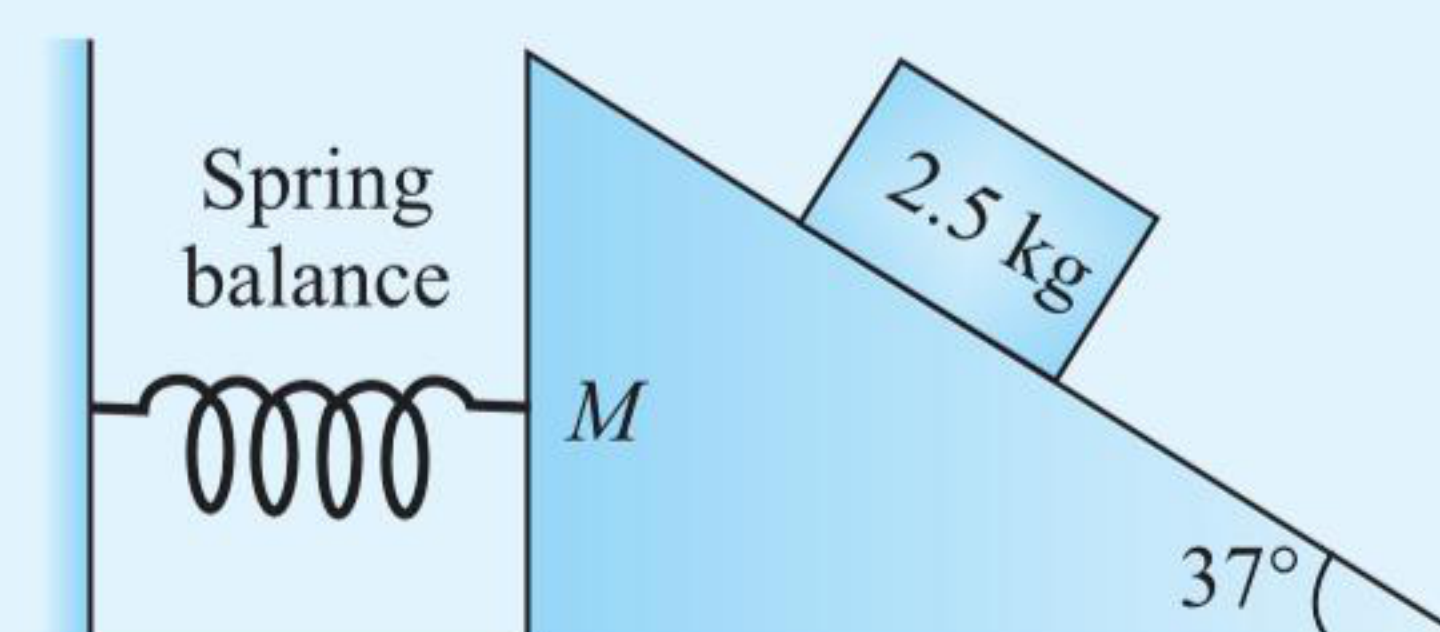
9. Two blocks A and B of same mass m attached with a light string are suspended by a spring as shown in figure. Find the acceleration of block A and B just after the string is cut.



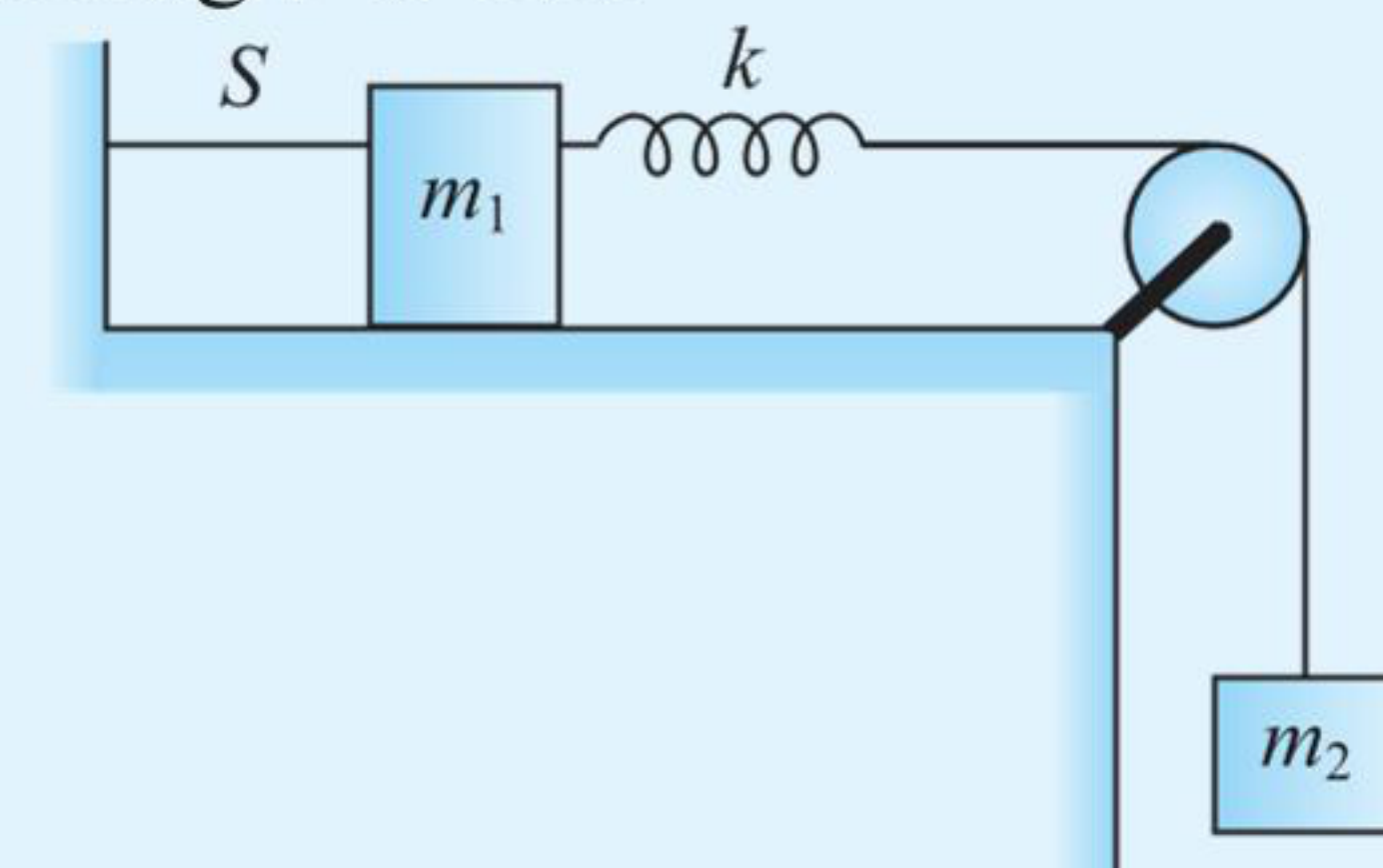
10. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 m s^{-2} as shown in figure.



11. Find the reading of spring balance as shown in figure. Assume that mass M is in equilibrium. (All surfaces are smooth.)



12. Two blocks of masses m_1 and m_2 are in equilibrium. The block m_2 hangs from a fixed smooth pulley by an inextensible string that is fitted with a light spring of stiffness k as shown in figure. Neglecting friction and mass of the string, find the acceleration of the bodies just after the string S is cut.



ANSWERS

1. In all the three cases the spring balance reads 10 kg .
2. $\frac{40}{3} \text{ kg}$ 3. Stretch of all three springs will be equal.
4. $1/m$ ($\leftarrow$) 5. $a_A = 2g$ (downwards), $a_B = 0$
6. 20 kg 7. (a) 200 N (b) 200 N (c) 200 N
8. 10 kg 9. $a_A = g$ (upwards), $a_B = g$ (downwards)
10. 2 m s^{-2} 11. 12 N 12. $a_1 = \left(\frac{m_2}{m_1}\right)g$, $a_2 = 0$

Exercises

Single Correct Answer Type

1. A wooden box is placed on a table. The normal force on the box from the table is N_1 . Now another identical box is kept on first box and the normal force on lower block due to upper block is N_2 and normal force on lower block by the table is N_3 . For this situation, mark out the correct statement(s).

- (1) $N_1 = N_2 = N_3$ (2) $N_1 < N_2 = N_3$
 (3) $N_1 = N_2 < N_3$ (4) $N_1 = N_2 > N_3$

2. Three forces are acting on a particle of mass m initially in equilibrium. If the first two forces (R_1 and R_2) are perpendicular to each other and suddenly the third force (R_3) is removed, then the acceleration of the particle is

- (1) $\frac{R_3}{m}$ (2) $\frac{R_1 + R_2}{m}$
 (3) $\frac{R_1 - R_2}{m}$ (4) $\frac{R_1}{m}$

3. Two skaters have weight in the ratio 4 : 5 and are 9 m apart, on a smooth frictionless surface. They pull on a rope stretched between them. The ratio of the distance covered by them when they meet each other will be

- (1) 5 : 4 (2) 4 : 5
 (3) 25 : 16 (4) 16 : 25

4. A plumb bob is hung from the ceiling of a train compartment. The train moves on an inclined track of inclination 30° with horizontal. The acceleration of train up the plane is $a = g/2$. The angle which the string supporting the bob makes with normal to the ceiling in equilibrium is

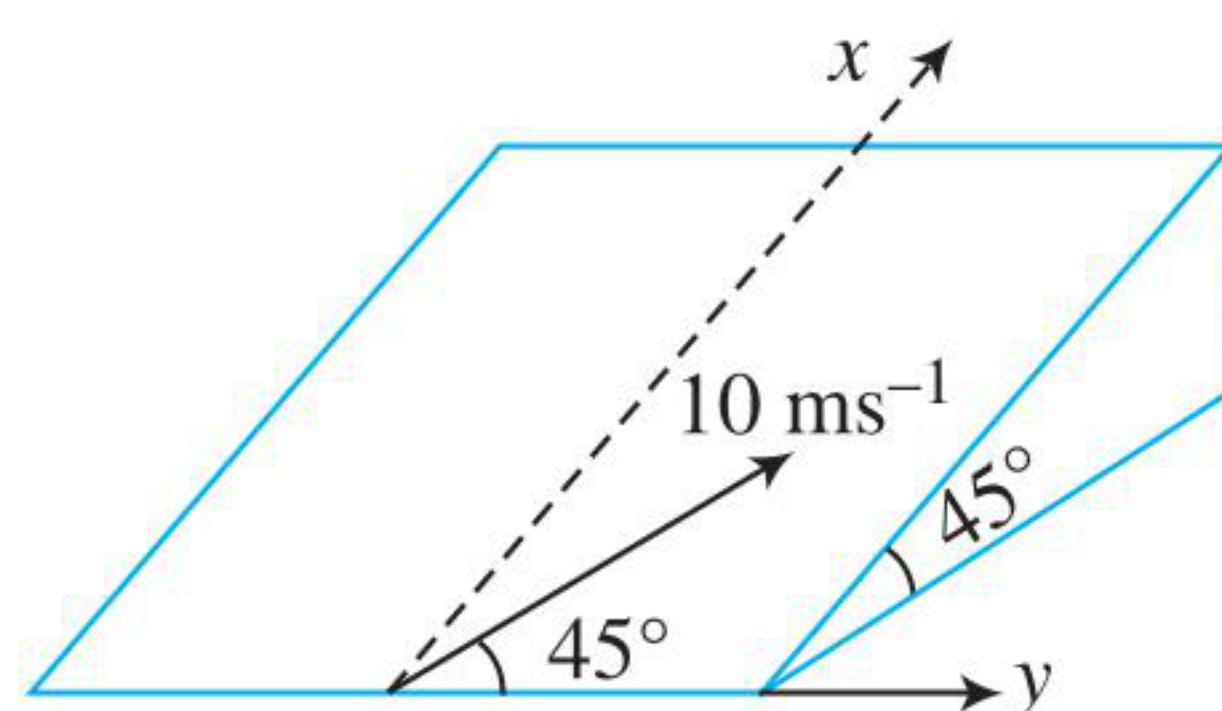
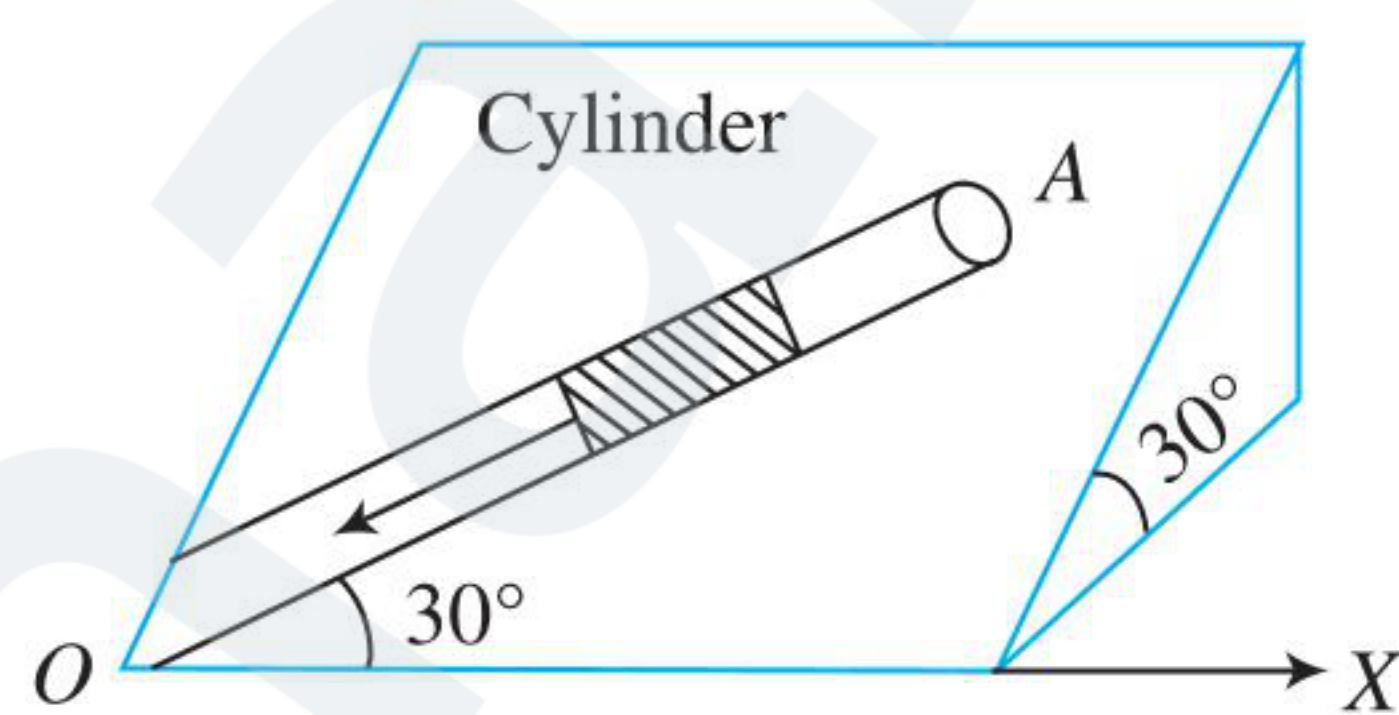
- (1) 30° (2) $\tan^{-1}(2/\sqrt{3})$
 (3) $\tan^{-1}(\sqrt{3}/2)$ (4) $\tan^{-1}(2)$

5. An inclined plane makes an angle 30° with the horizontal. A groove (OA) of length 5 m cut in the plane makes an angle 30° with OX . A short smooth cylinder is free to slide down under the influence of gravity. The time taken by the cylinder to reach from A to O is ($g = 10 \text{ ms}^{-2}$)

- (1) 4 s (2) 2 s
 (3) 3 s (4) 1 s

6. The small marble is projected with a velocity of 10 ms^{-1} in a direction 45° from the horizontal y -direction on the smooth inclined plane. Calculate the magnitude v of its velocity after 2 s.

- (1) $10\sqrt{2} \text{ ms}^{-1}$ (2) 5 ms^{-1}
 (3) 10 ms^{-1} (4) $5\sqrt{2} \text{ ms}^{-1}$



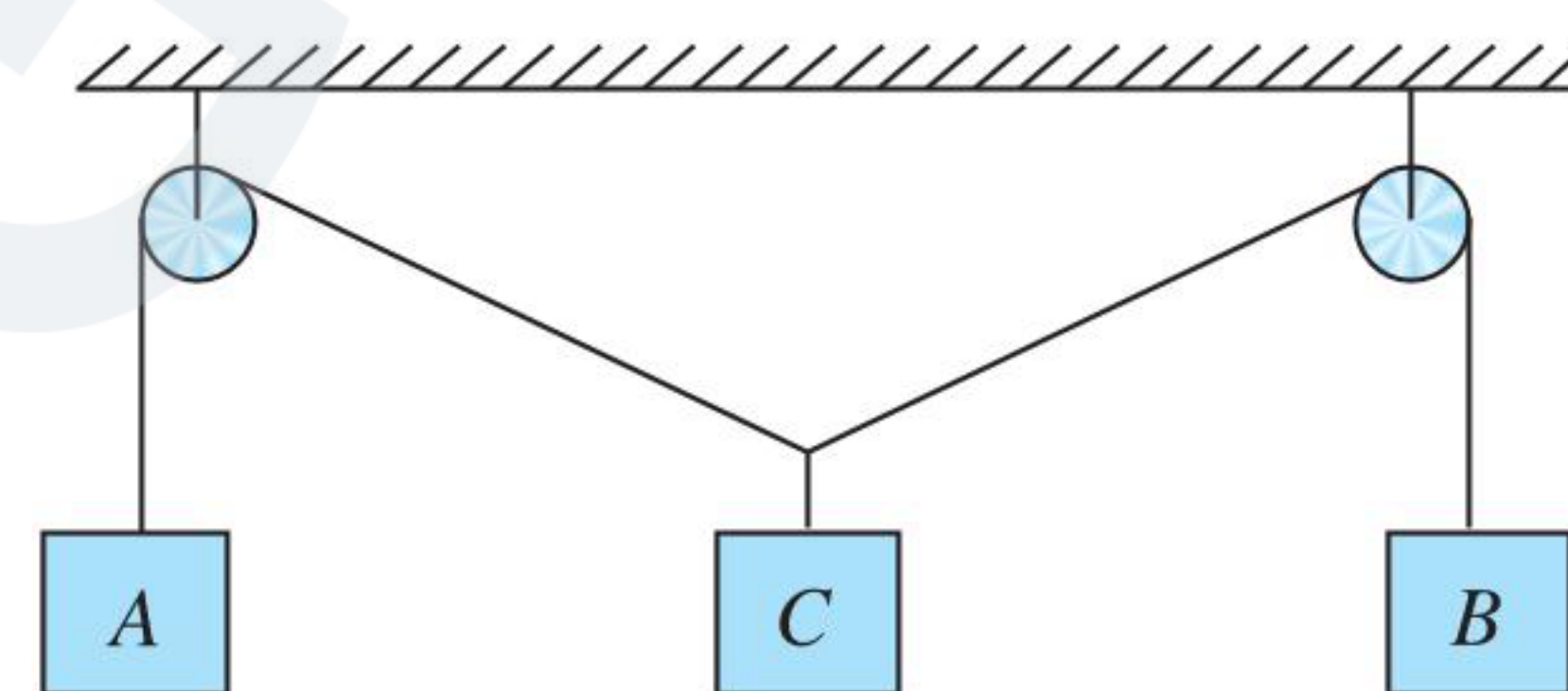
7. A lift is moving down with an acceleration a . A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift, and a man standing stationary on the ground are, respectively,

- (1) a, g (2) $(g - a), g$
 (3) a, a (4) g, g

8. A particle of mass 2 kg moves with an initial velocity of $\vec{v} = 4\hat{i} + 4\hat{j} \text{ ms}^{-1}$. A constant force of $\vec{F} = -20\hat{j} \text{ N}$ is applied on the particle. Initially, the particle was at $(0, 0)$. The x -coordinate of the particle when its y -coordinate again becomes zero is given by

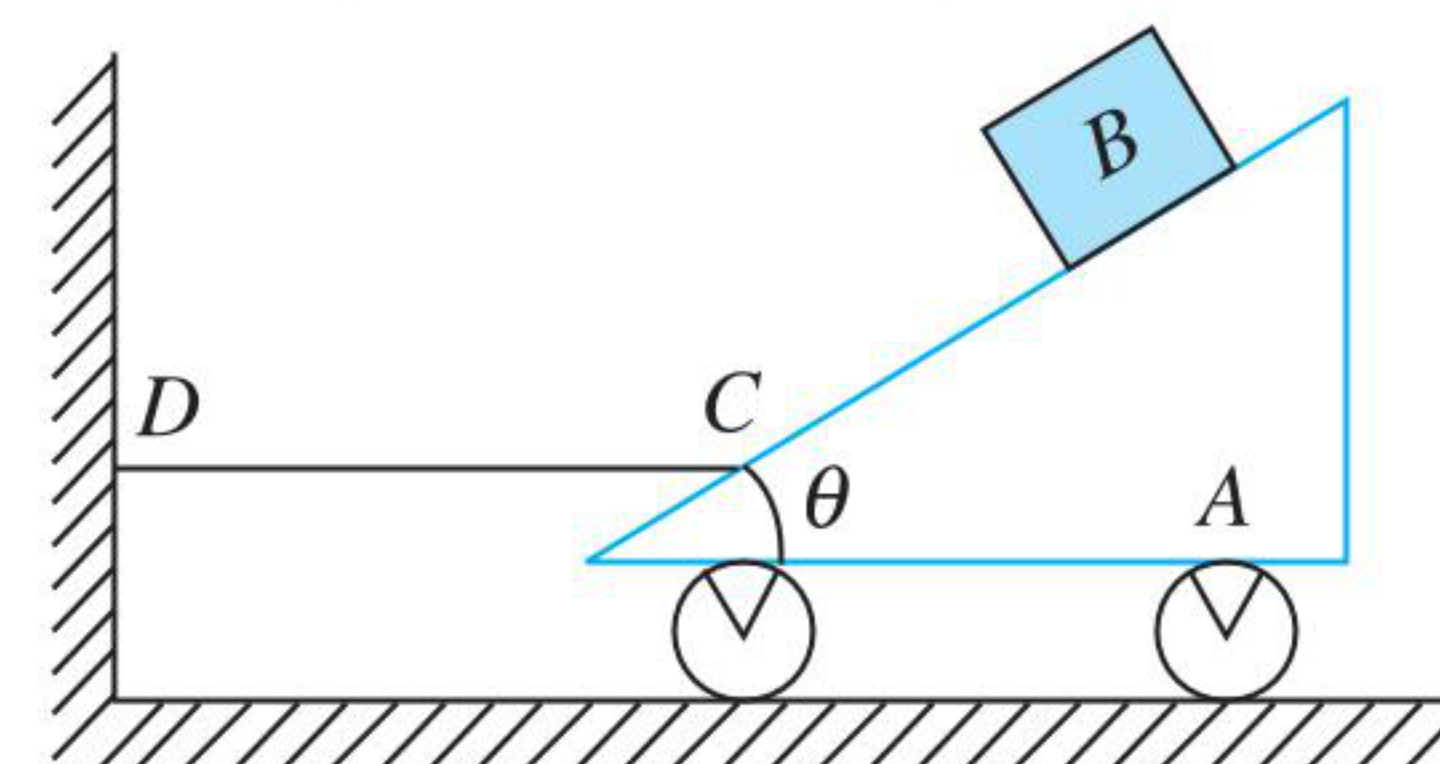
- (1) 1.2 m (2) 4.8 m
 (3) 6.0 m (4) 3.2 m

9. Three blocks A , B , and C are suspended as shown in figure. Mass of each of blocks A and B is m . If the system is in equilibrium, and mass of C is M , then



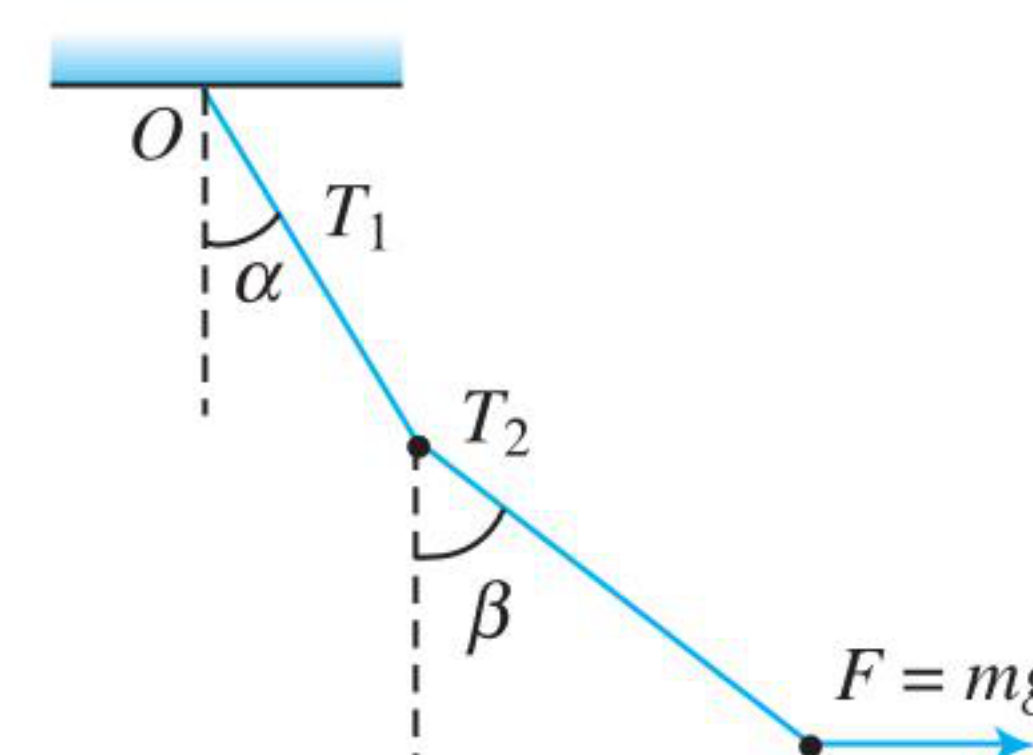
- (1) $M > 2m$ (2) $M = 2m$
 (3) $M < 2m$ (4) None of these

10. Block B has mass m and is released from rest when it is on top of wedge A , which has a mass $3m$. Determine the tension in cord CD needed to hold the wedge from moving while B is sliding down A . Neglect friction.



- (1) $2mg \cos \theta$ (2) $\frac{mg}{2} \cos \theta$
 (3) $\frac{mg}{2} \sin 2\theta$ (4) $mg \sin 2\theta$

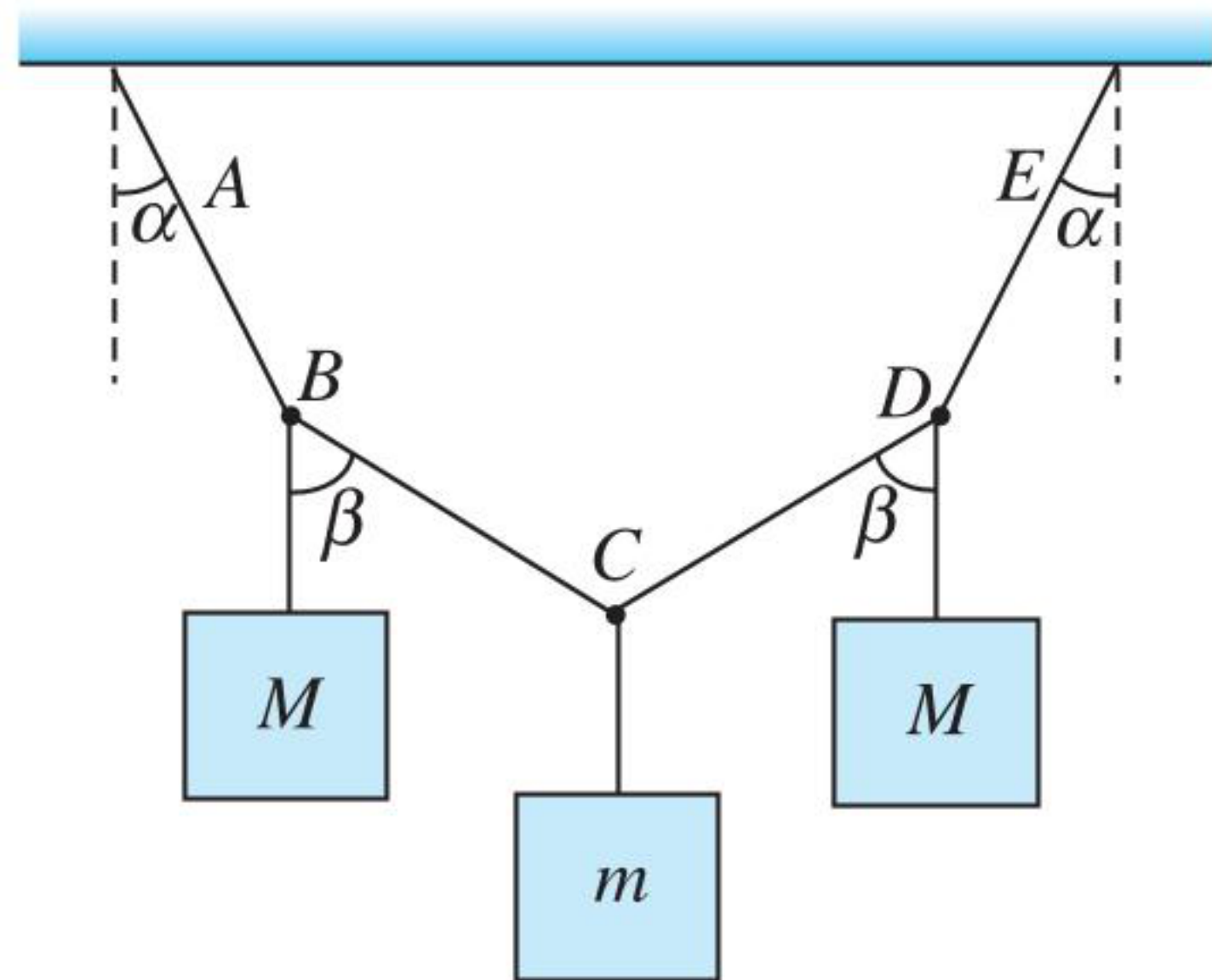
11. Two particles A and B , each of mass m , are kept stationary by applying a horizontal force $F = mg$ on particle B as shown in figure. Then



- (1) $2 \tan \beta = \tan \alpha$ (2) $2T_1 = 5T_2$
 (3) $T_1 \sqrt{2} = T_2 \sqrt{5}$ (4) None of these

12. The given figure represents a light inextensible string $ABCDE$ in which $AB = BC = CD = DE$ and to which are attached masses M , m , and M at the points B , C , and D ,

respectively. The system hangs freely in equilibrium with ends A and E of the string fixed in the same horizontal line. It is given that $\tan \alpha = 3/4$ and $\tan \beta = 12/5$. Then the tension in the string BC is

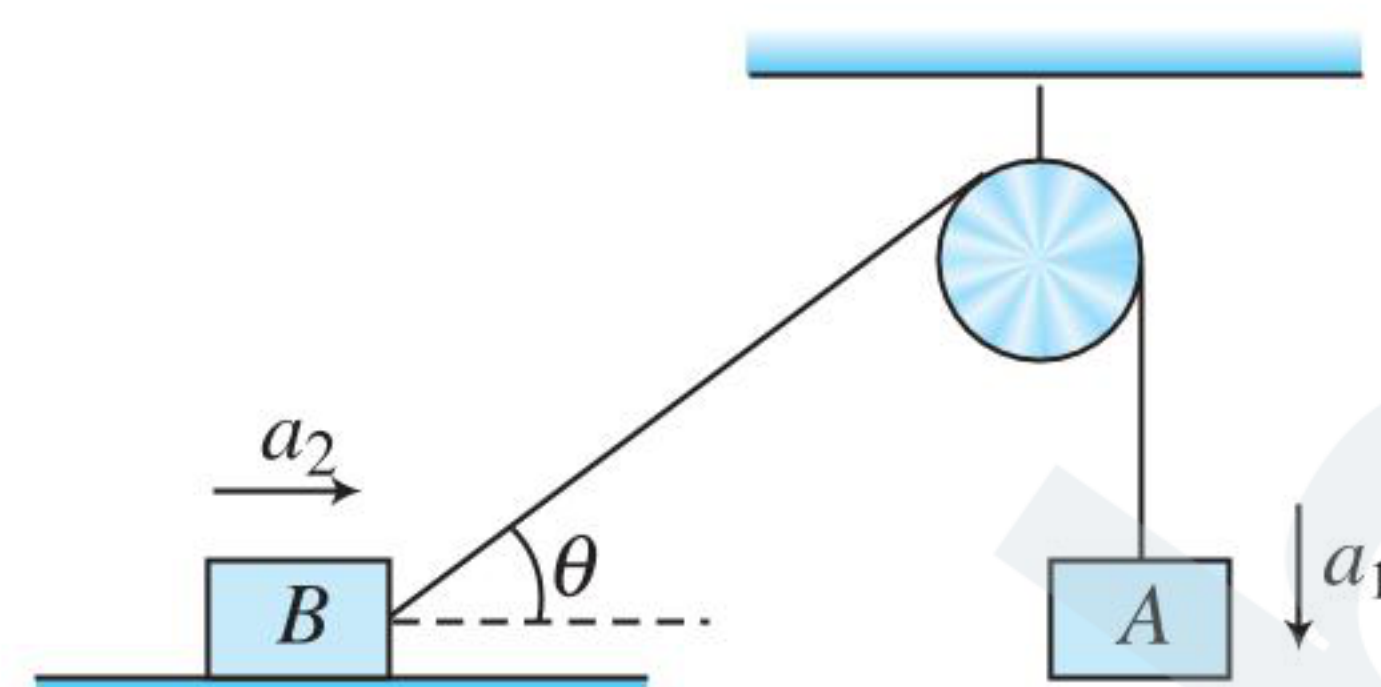


- (1) $2mg$ (2) $(13/10)mg$
(3) $(3/10)mg$ (4) $(20/11)mg$

13. A particle is moving in the x - y plane. At certain instant of time, the components of its velocity and acceleration are as follows: $v_x = 3 \text{ ms}^{-1}$, $v_y = 4 \text{ ms}^{-1}$, $a_x = 2 \text{ ms}^{-2}$ and $a_y = 1 \text{ ms}^{-2}$. The rate of change of speed at this moment is

- (1) $\sqrt{10} \text{ ms}^{-2}$ (2) 4 ms^{-2}
(3) $\sqrt{5} \text{ ms}^{-2}$ (4) 2 ms^{-2}

14. The given figure shows two blocks, each of mass m . The system is released from rest. If accelerations of blocks A and B at any instant (not initially) are a_1 and a_2 , respectively, then



- (1) $a_1 = a_2 \cos \theta$ (2) $a_2 = a_1 \cos \theta$
(3) $a_1 = a_2$ (4) None of these

15. A block is lying on the horizontal frictionless surface. One end of a uniform rope is fixed to the block which is pulled in the horizontal direction by applying a force F at the other end. If the mass of the rope is half the mass of the block, the tension in the middle of the rope will be

- (1) F (2) $2F/3$
(3) $3F/5$ (4) $5F/6$

16. A 60-kg man stands on a spring scale in a lift. At some instant, he finds that the scale reading has changed from 60 kg to 50 kg for a while and then comes back to original mark. What should be concluded?

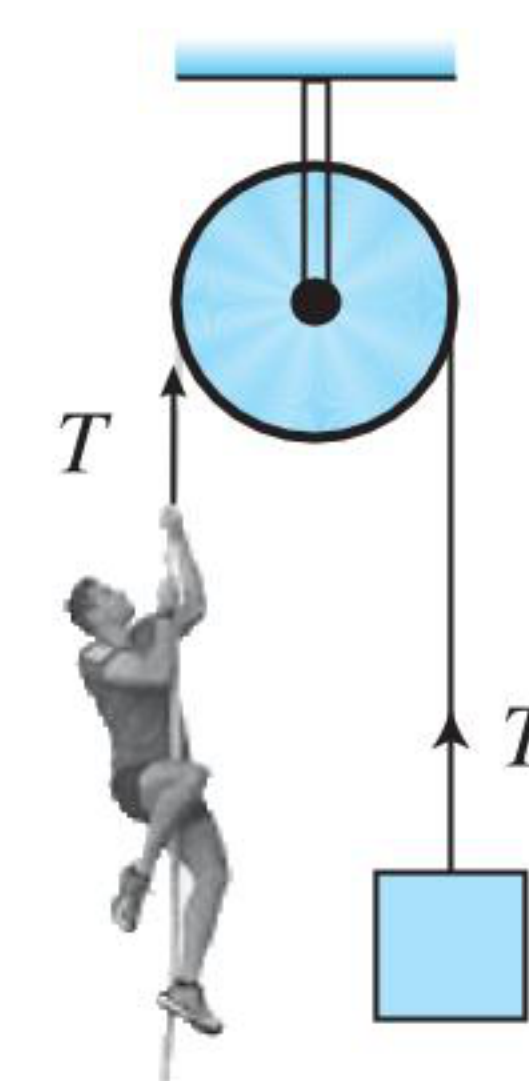
- (1) The lift was in constant motion upward.
(2) The lift was in constant motion downward.
(3) The lift while in motion downward suddenly stopped.
(4) The lift while in motion upward suddenly stopped.

17. Inside a horizontally moving box, an experimenter finds that when an object is placed on a smooth horizontal table and is released, it moves with an acceleration of 10 ms^{-2} . In this box, if 1-kg body is suspended with a light string, the tension in the string in equilibrium position. (w.r.t. experimenter) will be (take $g = 10 \text{ ms}^{-2}$)

- (1) 10 N (2) $10\sqrt{2}$ N
(3) 20 N (4) Zero

18. In order to raise a mass of 100 kg, a man of mass 60 kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with acceleration $5g/4$ relative to the rope. The tension in the rope is (take $g = 10 \text{ ms}^{-2}$)

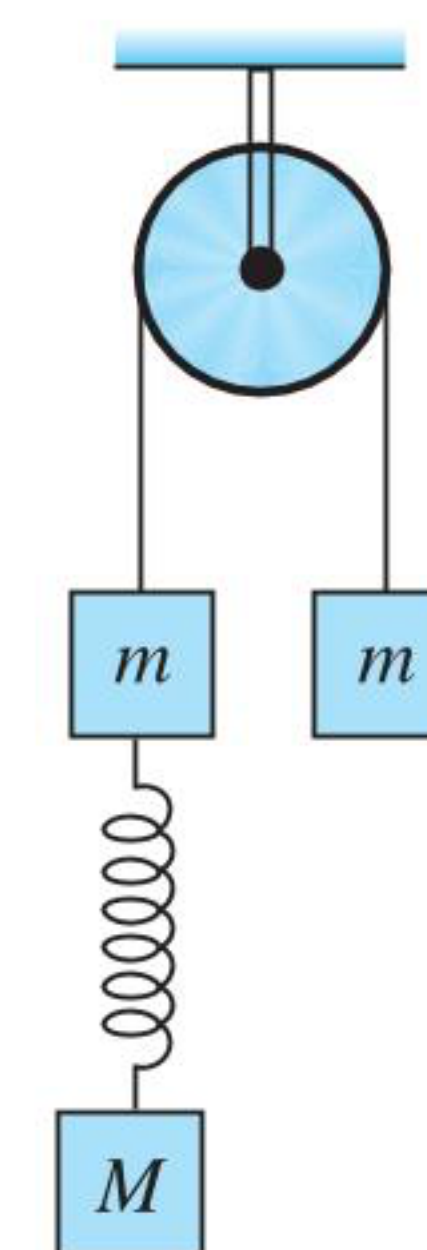
- (1) $\frac{4875}{8}$ N (2) $\frac{4875}{2}$ N
(3) $\frac{4875}{4}$ N (4) $\frac{4875}{6}$ N



19. The system shown in figure is released from rest. The spring gets elongated

- (1) If $M > m$
(2) If $M > 2m$
(3) If $M > m/2$
(4) For any value of M

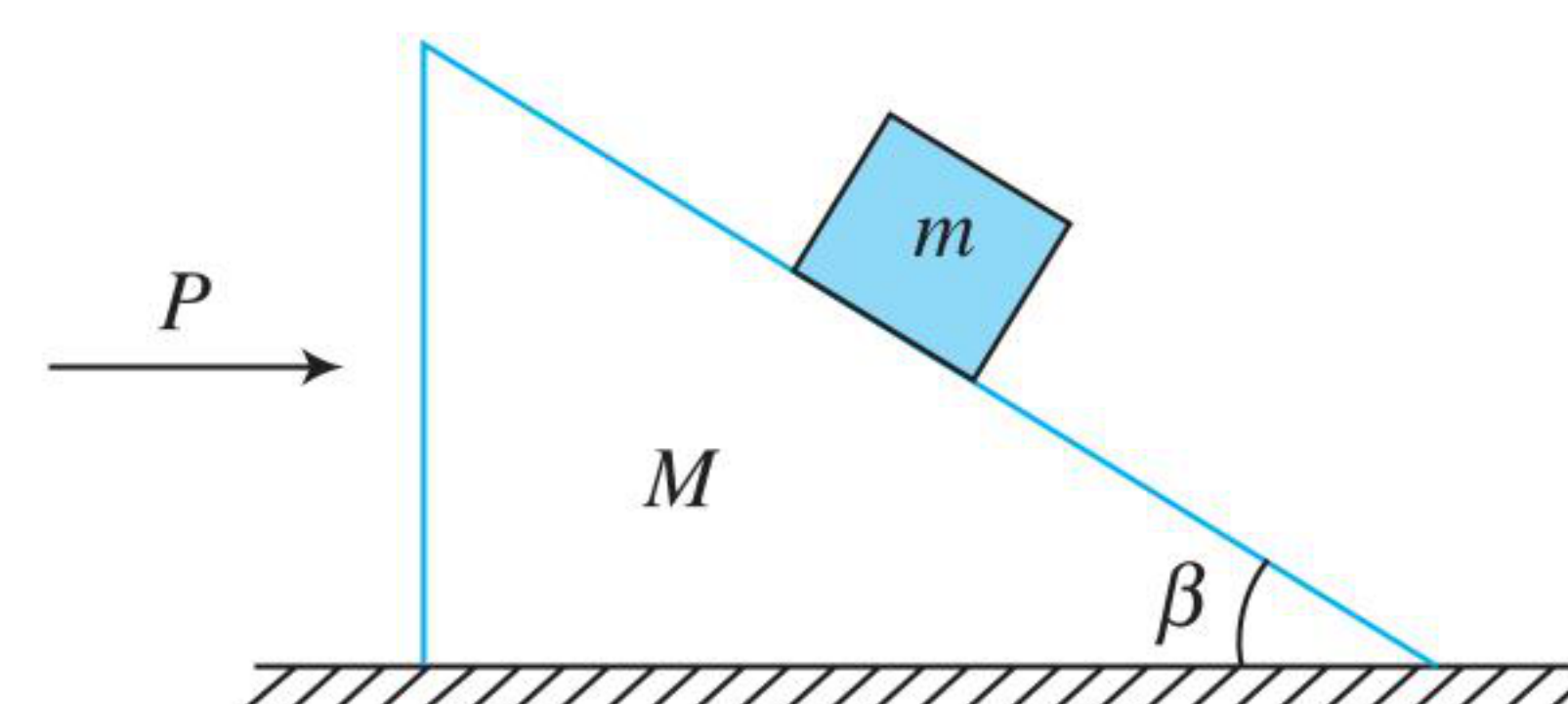
(Neglect the friction and masses of pulley, string, and spring.)



20. A balloon of mass M is descending at a constant acceleration α . When a mass m is released from the balloon, it starts rising with the same acceleration α . Assuming that its volume does not change, what is the value of m ?

- (1) $\frac{\alpha}{\alpha + g} M$ (2) $\frac{2\alpha}{\alpha + g} M$
(3) $\frac{\alpha + g}{\alpha} M$ (4) $\frac{\alpha + g}{2\alpha} M$

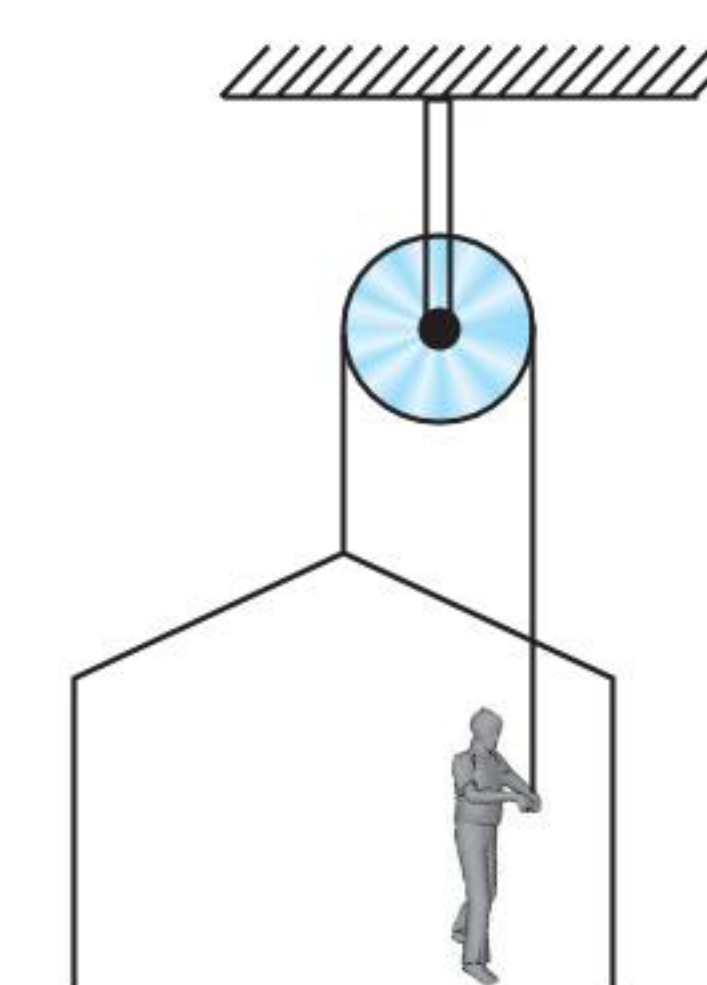
21. Two wooden blocks are moving on a smooth horizontal surface such that the mass m remains stationary with respect to the block of mass M as shown in figure. The magnitude of force P is



- (1) $(M + m)g \tan \beta$ (2) $g \tan \beta$
(3) $mg \cos \beta$ (4) $(M + m)g \csc \beta$

22. A man is raising himself as well as the crate on which he stands with an acceleration of 5 ms^{-2} by a massless rope-and-pulley arrangement. Mass of the man is 100 kg and that of the crate is 50 kg. If $g = 10 \text{ ms}^{-2}$, then the tension in the rope is

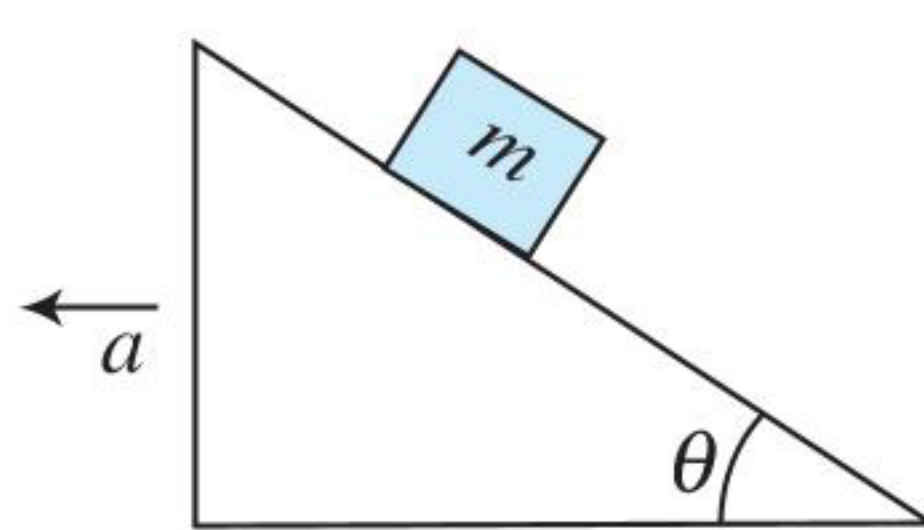
- (1) 2250 N (2) 1125 N
(3) 750 N (4) 375 N



23. In the previous problem, the contact force between the man and the crate is

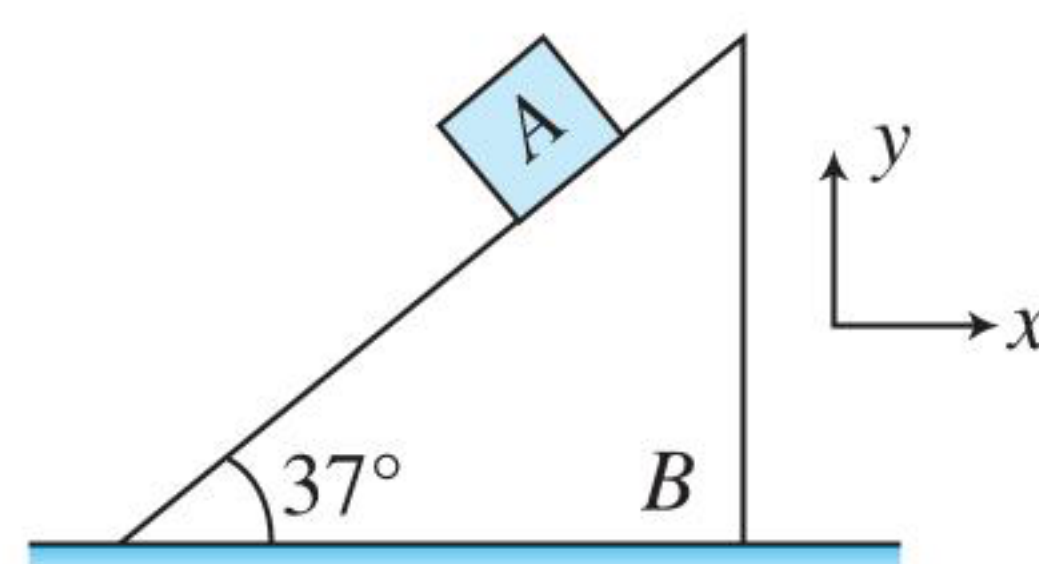
- (1) 2250 N (2) 1125 N
(3) 750 N (4) 375 N

24. A small block of mass m rests on a smooth wedge of angle θ . With what horizontal acceleration a should the wedge be pulled, as shown in figure, so that the block falls freely?



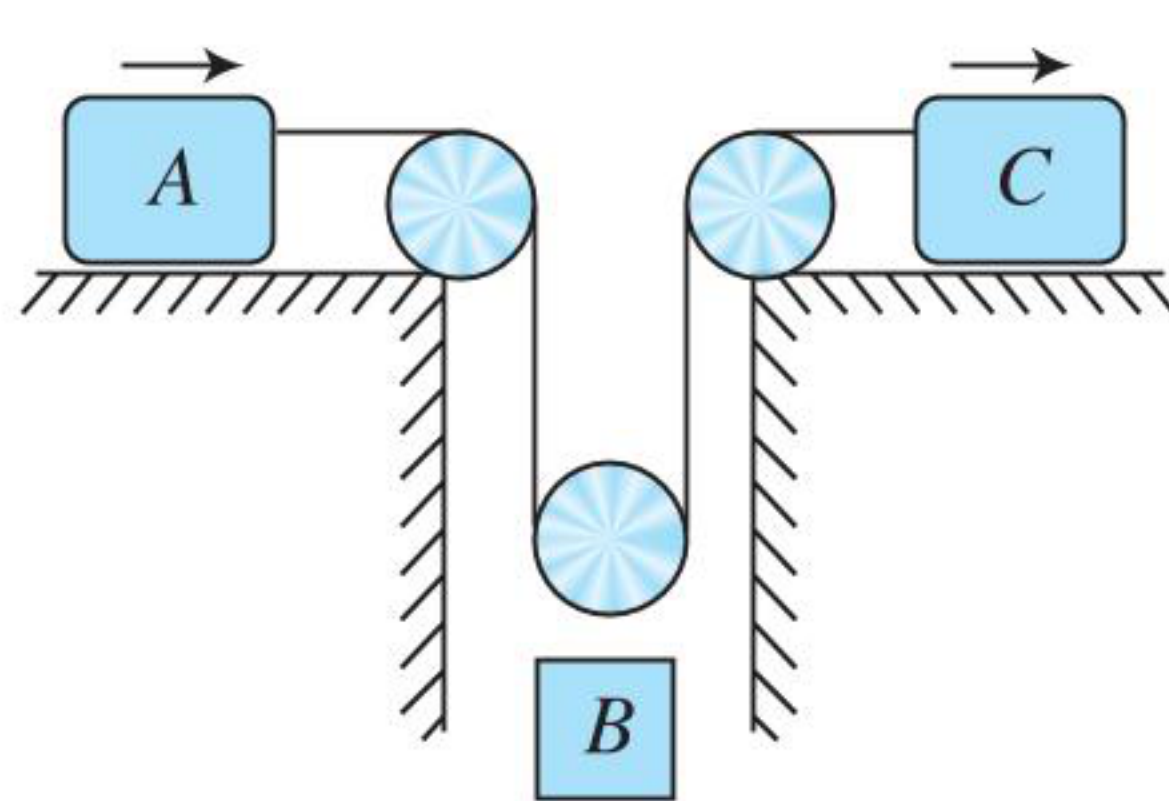
- (1) $g \cos \theta$ (2) $g \sin \theta$
(3) $g \cot \theta$ (4) $g \tan \theta$

25. In the given figure, the acceleration of A is $\vec{a}_A = 15\hat{i} + 15\hat{j}$. Then the acceleration of B is (A remains in contact with B)



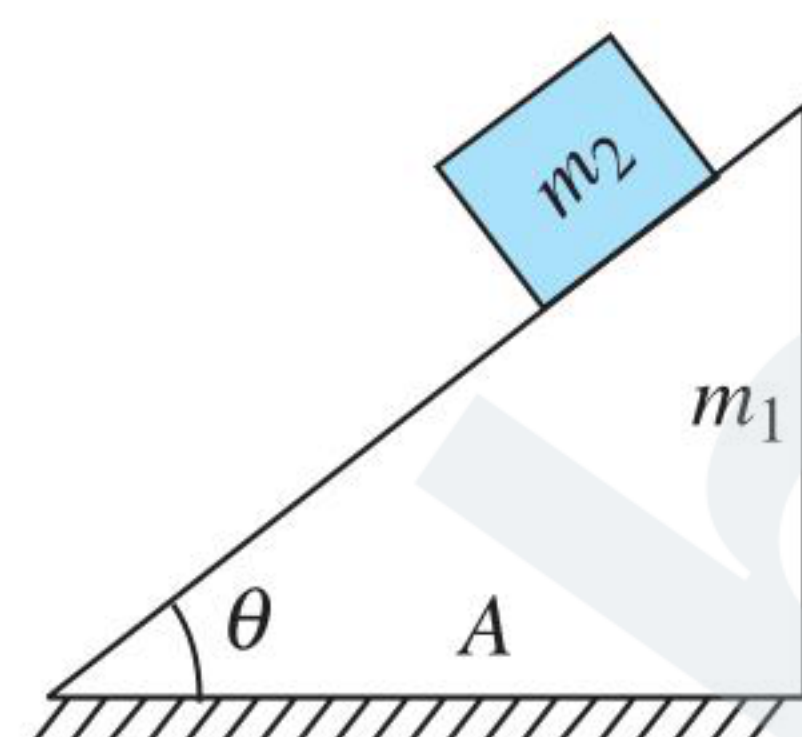
- (1) $6\hat{i}$ (2) $-15\hat{i}$
(3) $-10\hat{i}$ (4) $-5\hat{i}$

26. Blocks A and C start from rest and move to the right with acceleration $a_A = 12t \text{ m s}^{-2}$ and $a_C = 3 \text{ m s}^{-2}$. Here t is in seconds. The time when block B again comes to rest is



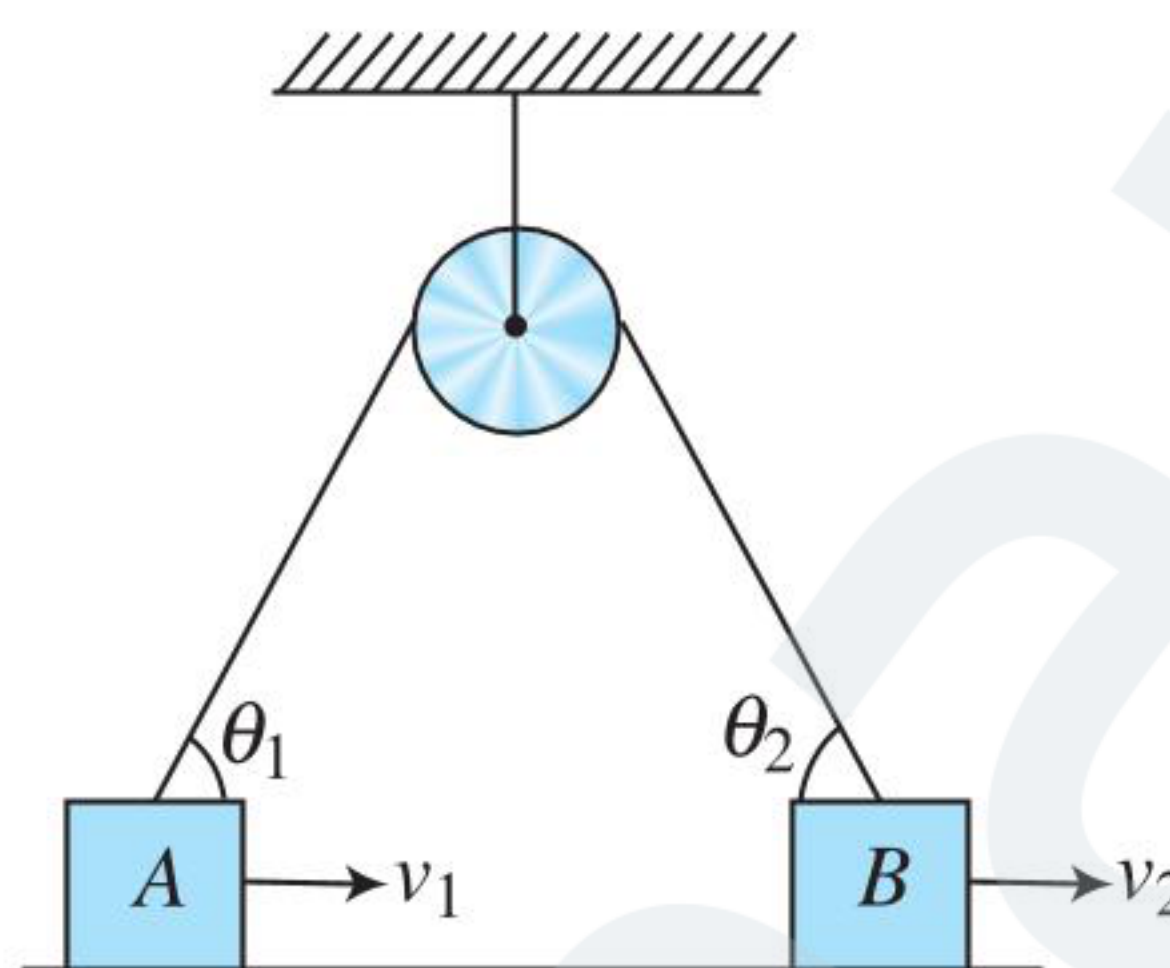
- (1) 2 s (2) 1 s
(3) 3/2 s (4) 1/2 s

27. In the given figure, the mass m_2 starts with velocity v_0 and moves with constant velocity on the surface. During motion, the normal reaction between the horizontal surface and fixed triangle block m_1 is N . Then during motion



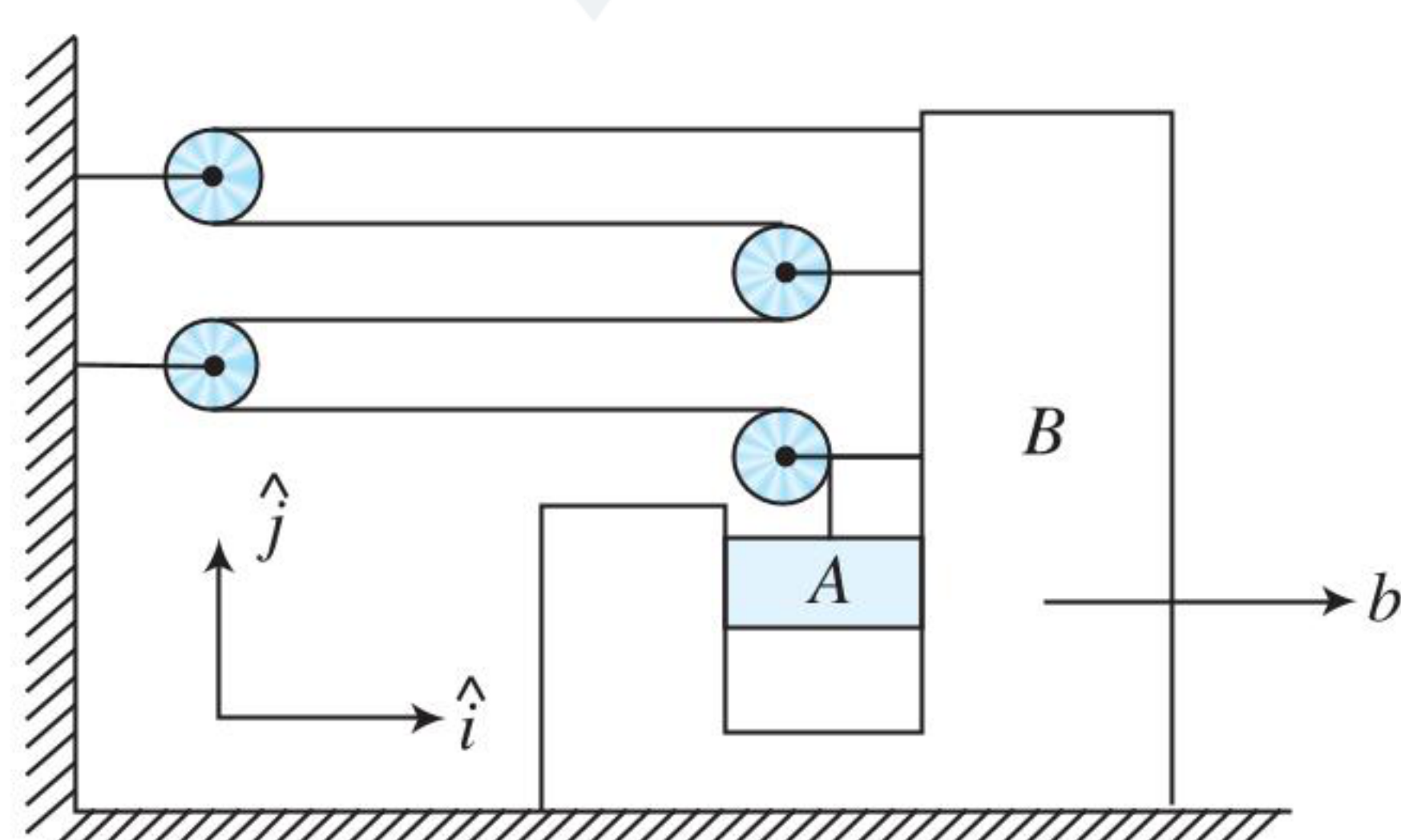
- (1) $N = (m_1 + m_2)g$ (2) $N = m_1g$
(3) $N < (m_1 + m_2)g$ (4) $N > (m_1 + m_2)g$

28. In the given figure, blocks A and B move with velocities v_1 and v_2 along horizontal direction. Find the ratio of v_1/v_2 .



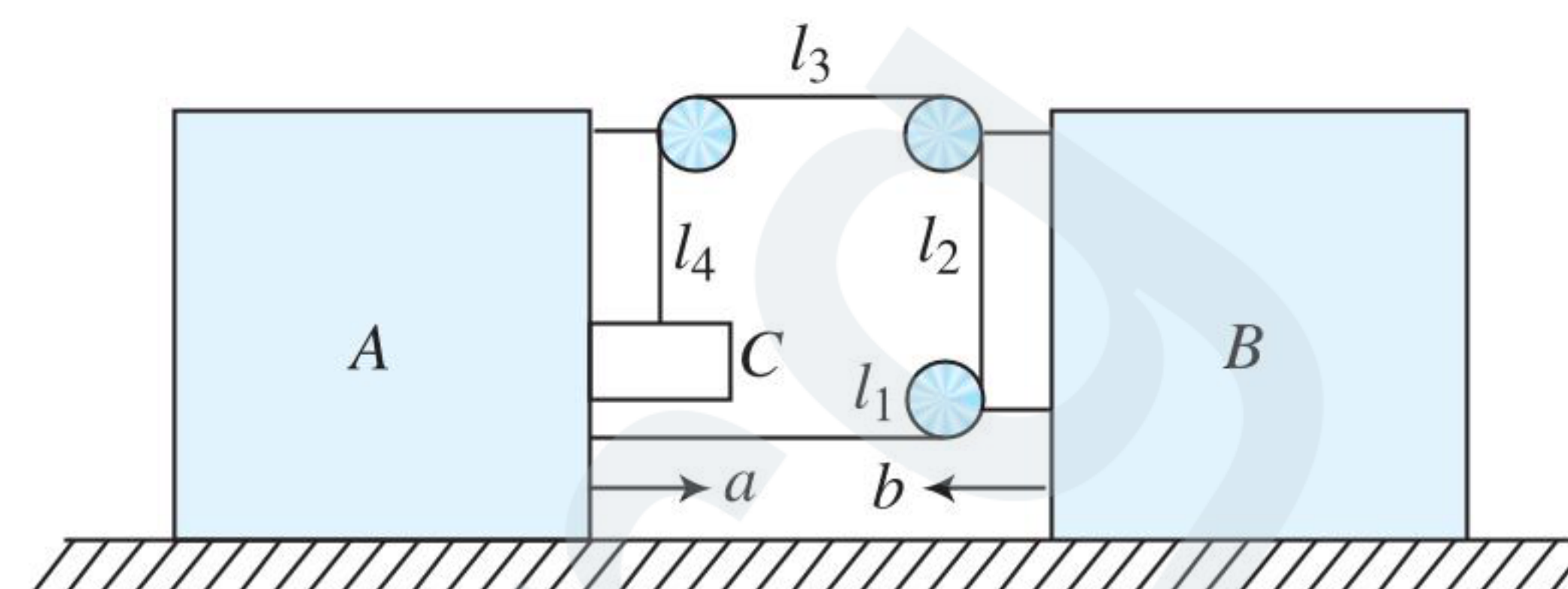
- (1) $\frac{\sin \theta_1}{\sin \theta_2}$ (2) $\frac{\sin \theta_2}{\sin \theta_1}$
(3) $\frac{\cos \theta_2}{\cos \theta_1}$ (4) $\frac{\cos \theta_1}{\cos \theta_2}$

29. If block B moves towards right with acceleration b , find the net acceleration of block A .



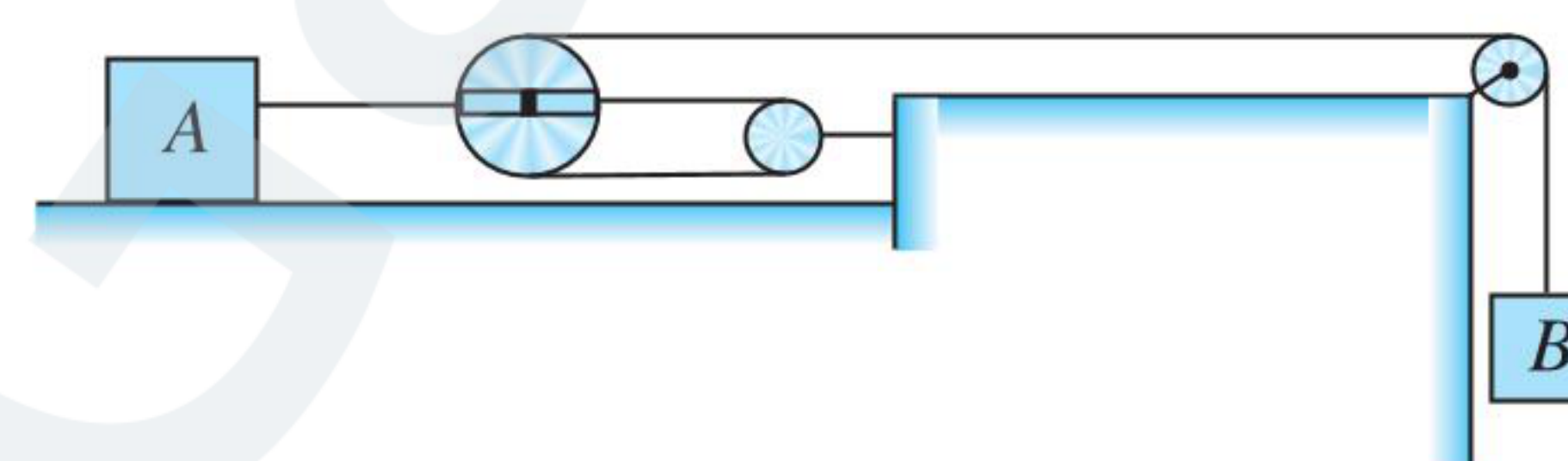
- (1) $b\hat{i} + 4b\hat{j}$ (2) $b\hat{i} + b\hat{j}$
(3) $b\hat{i} + 2b\hat{j}$ (4) None of these

30. If the blocks A and B are moving towards each other with accelerations a and b as shown in figure, then find the net acceleration of block C .



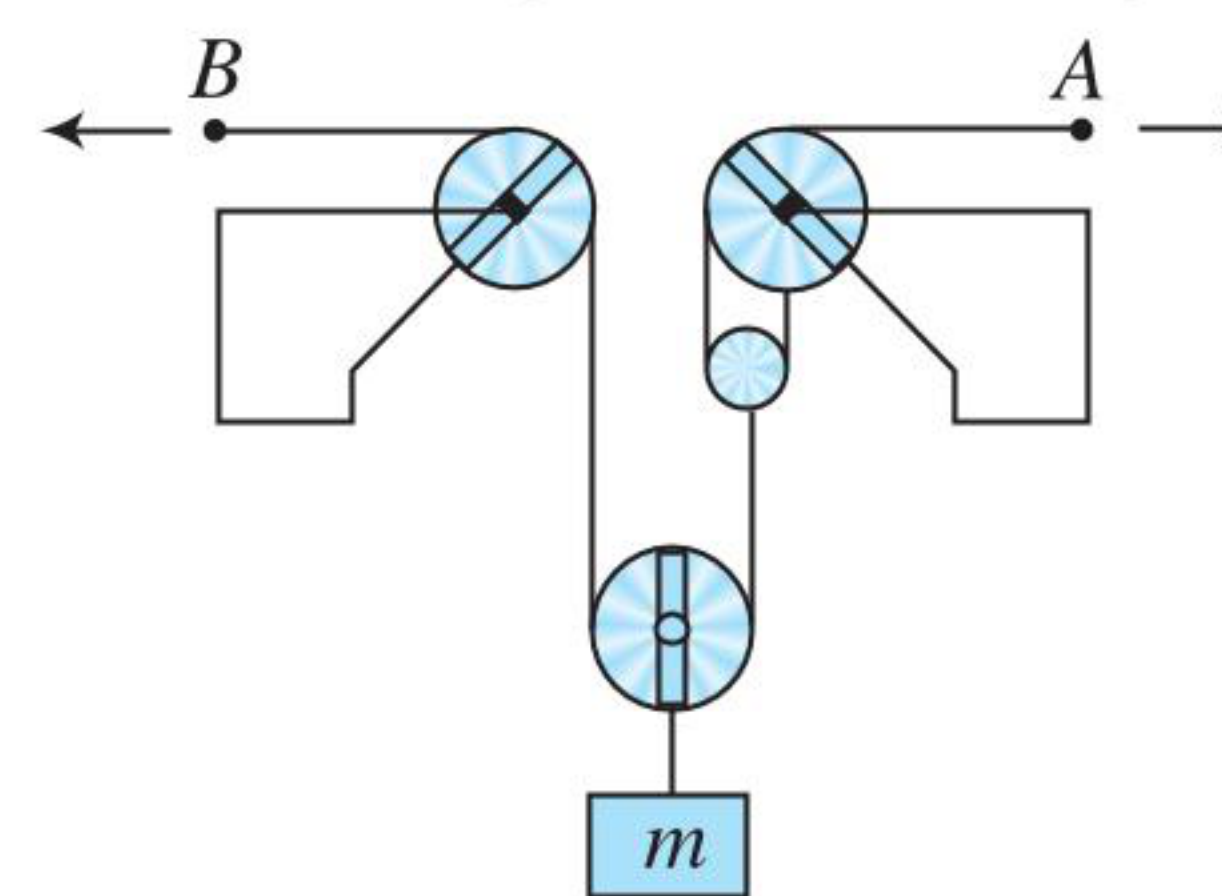
- (1) $a\hat{i} - 2(a+b)\hat{j}$ (2) $-(a+b)\hat{j}$
(3) $a\hat{i} - (a+b)\hat{j}$ (4) None of these

31. A block A has a velocity of 0.6 m s^{-1} to the right. Determine the velocity of cylinder B .



- (1) 1.2 m s^{-1} (2) 2.4 m s^{-1}
(3) 1.8 m s^{-1} (4) 3.6 m s^{-1}

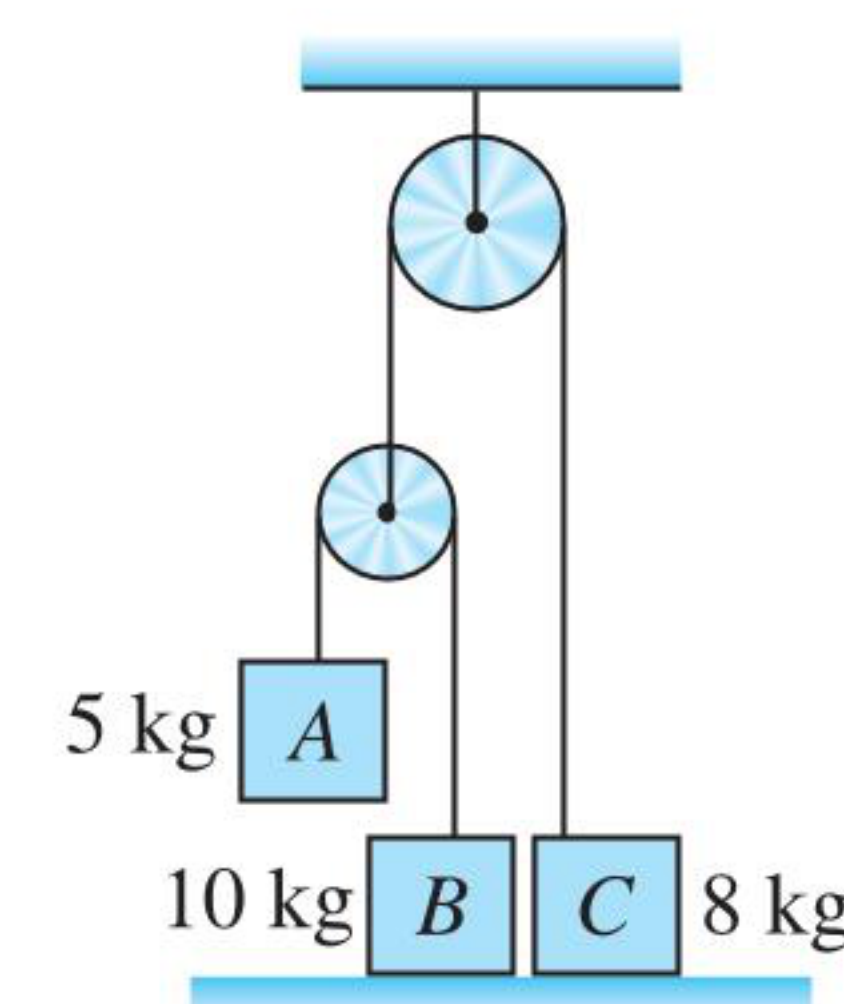
32. For the pulley system shown in figure, each of the cables at A and B is given a velocity of 2 m s^{-1} in the direction of the arrow. Determine the upward velocity v of the load m .



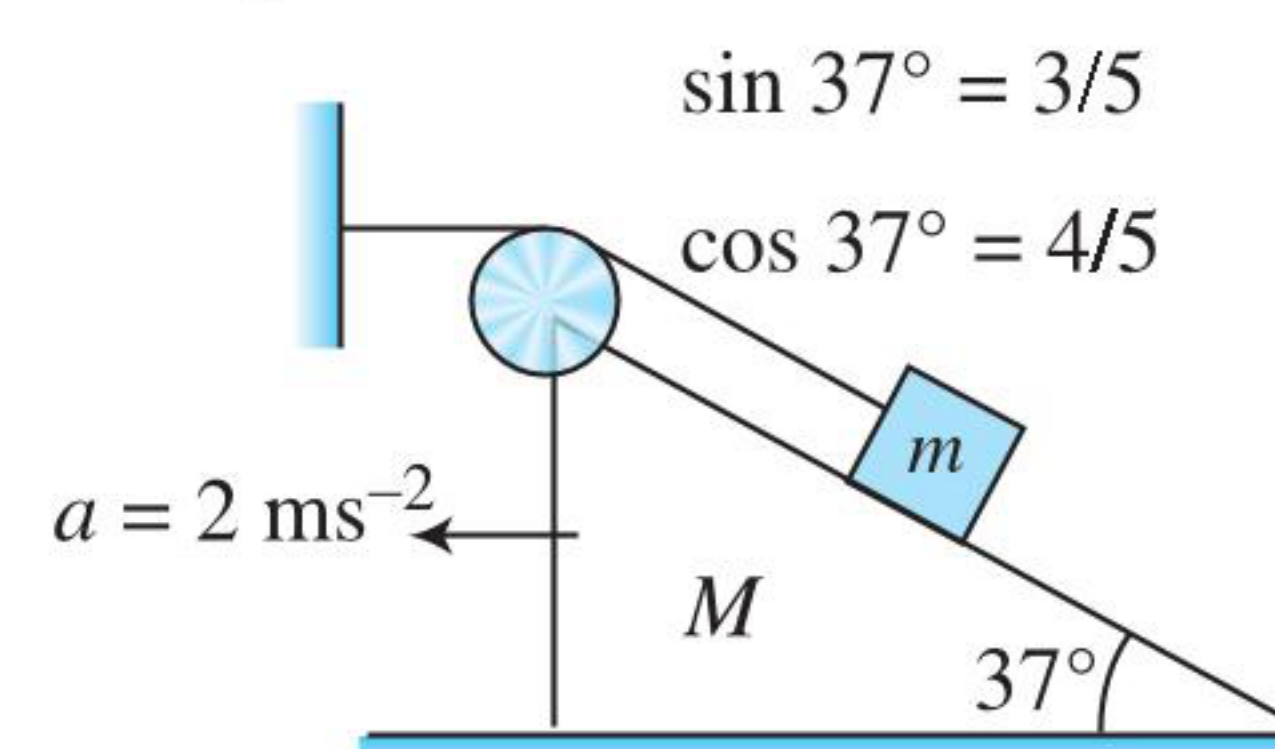
- (1) 1.5 m s^{-1} (2) 3 m s^{-1}
(3) 6 m s^{-1} (4) 4.5 m s^{-1}

33. In the following arrangement, the system is initially at rest. The 5-kg block is now released. Assuming the pulleys and string to be massless and smooth, the acceleration of block C will be

- (1) Zero (2) 2.5 m s^{-2}
(3) $10/7 \text{ m s}^{-2}$ (4) $5/7 \text{ m s}^{-2}$

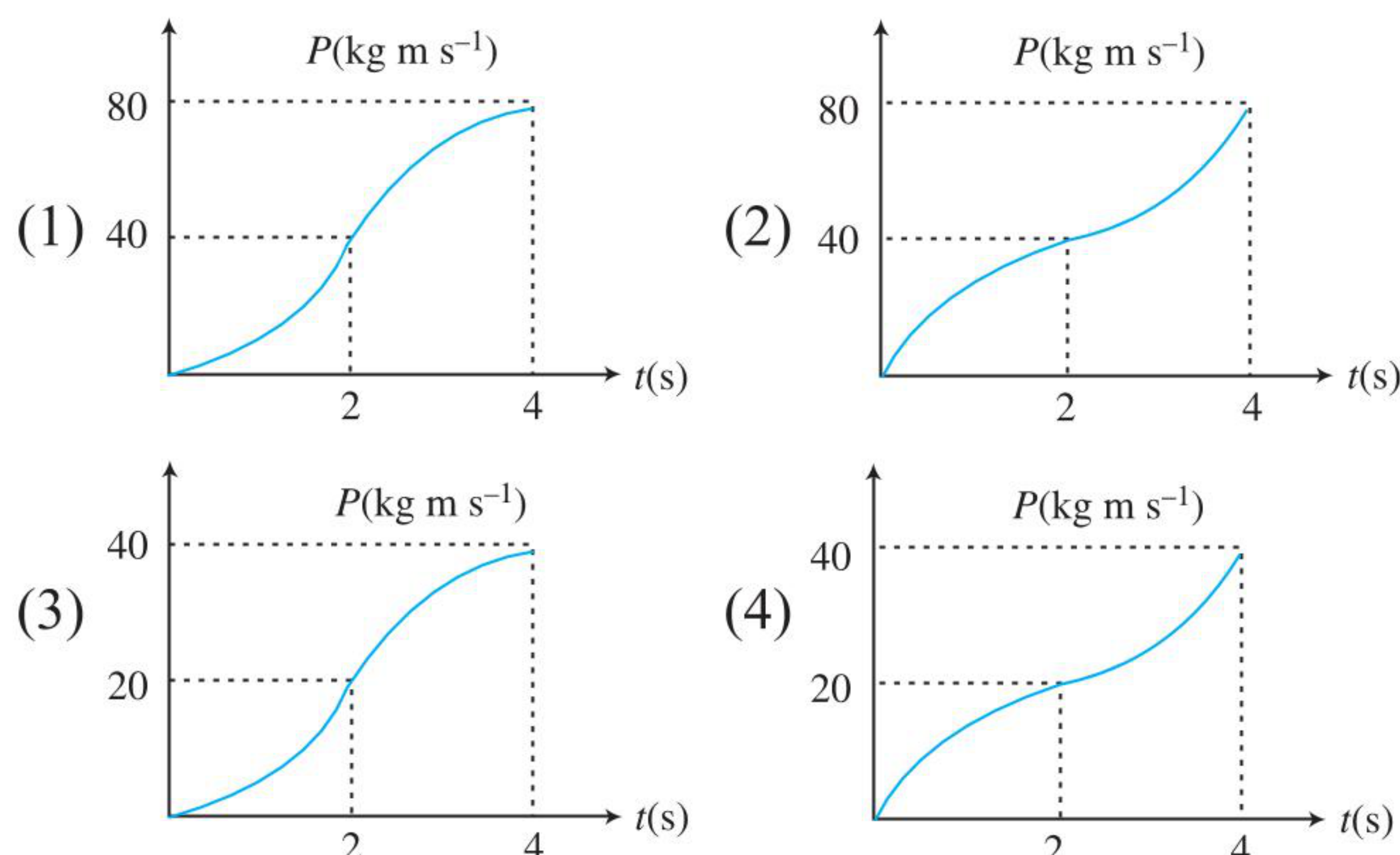
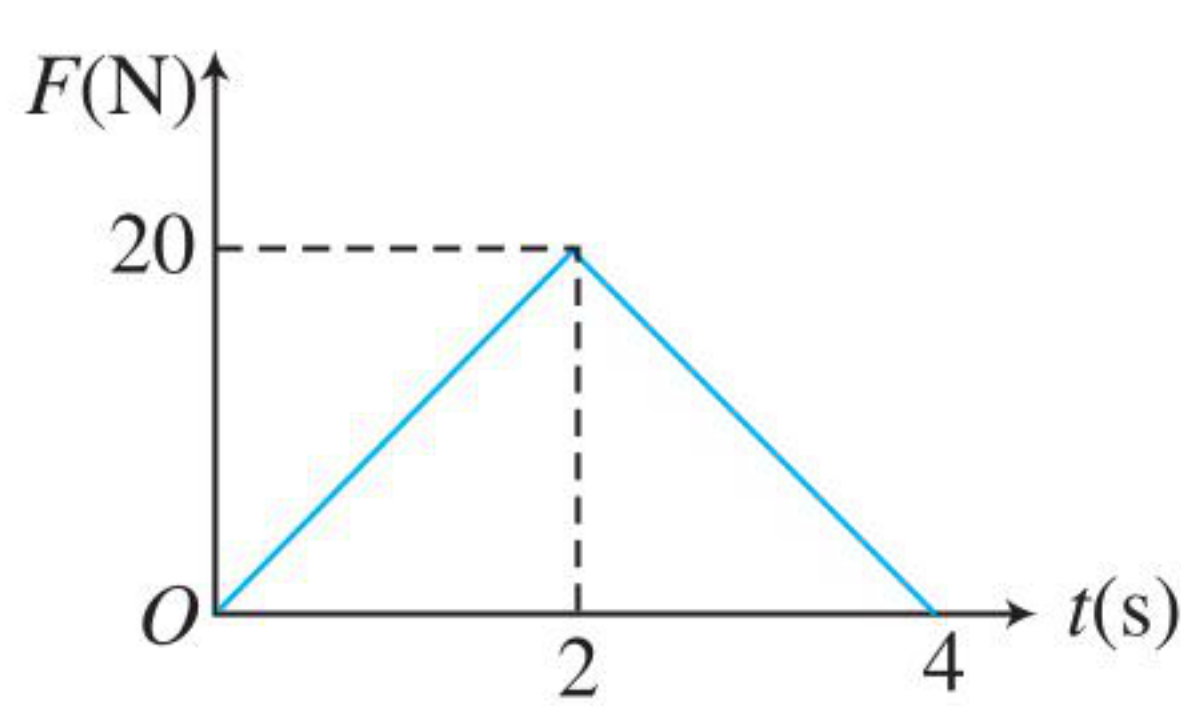


34. As shown in figure, if acceleration of M with respect to ground is 2 m s^{-2} , then

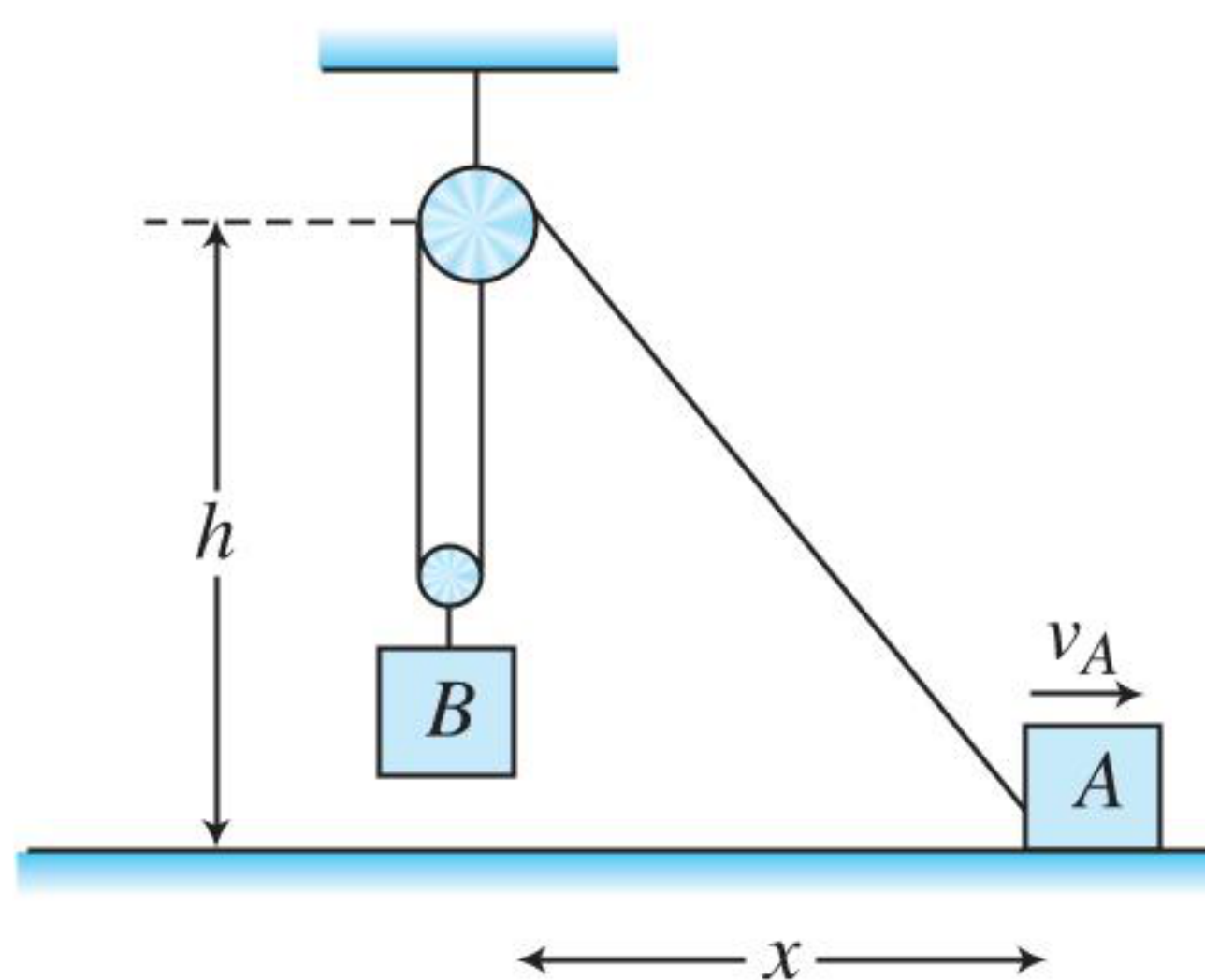


- (1) Acceleration of m with respect to M is 5 m s^{-2} .
(2) Acceleration of m with respect to ground is 5 m s^{-2} .
(3) Acceleration of m with respect to M is 2 m s^{-2} .
(4) Acceleration of m with respect to ground is 10 m s^{-2} .

35. Figure shows the variation of force acting on a body with time. Assuming the body to start from rest, the variation of its momentum with time is best represented by which plot?

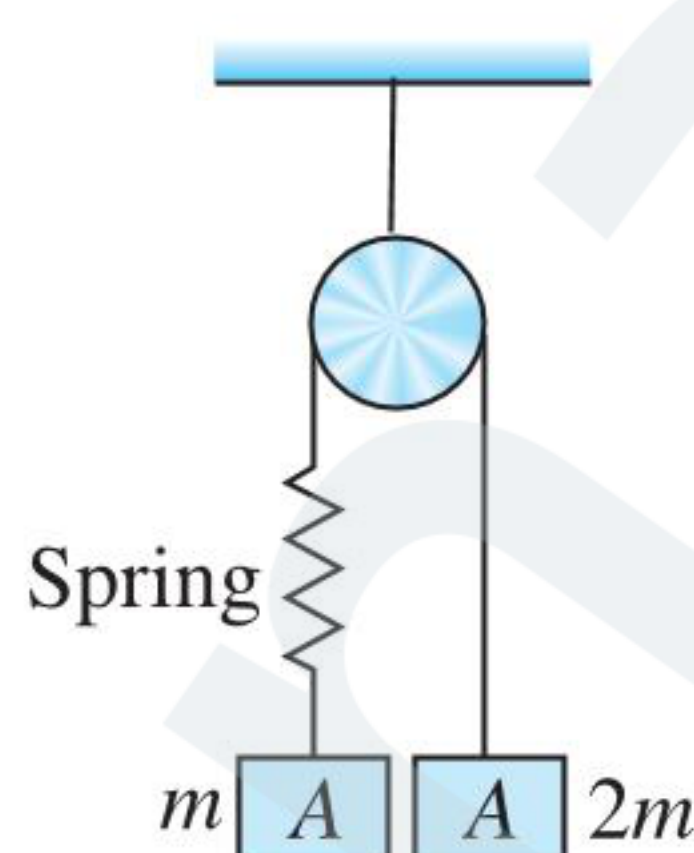


36. If block A is moving horizontally with velocity v_A , then find the velocity of block B at the instant as shown in Figure.



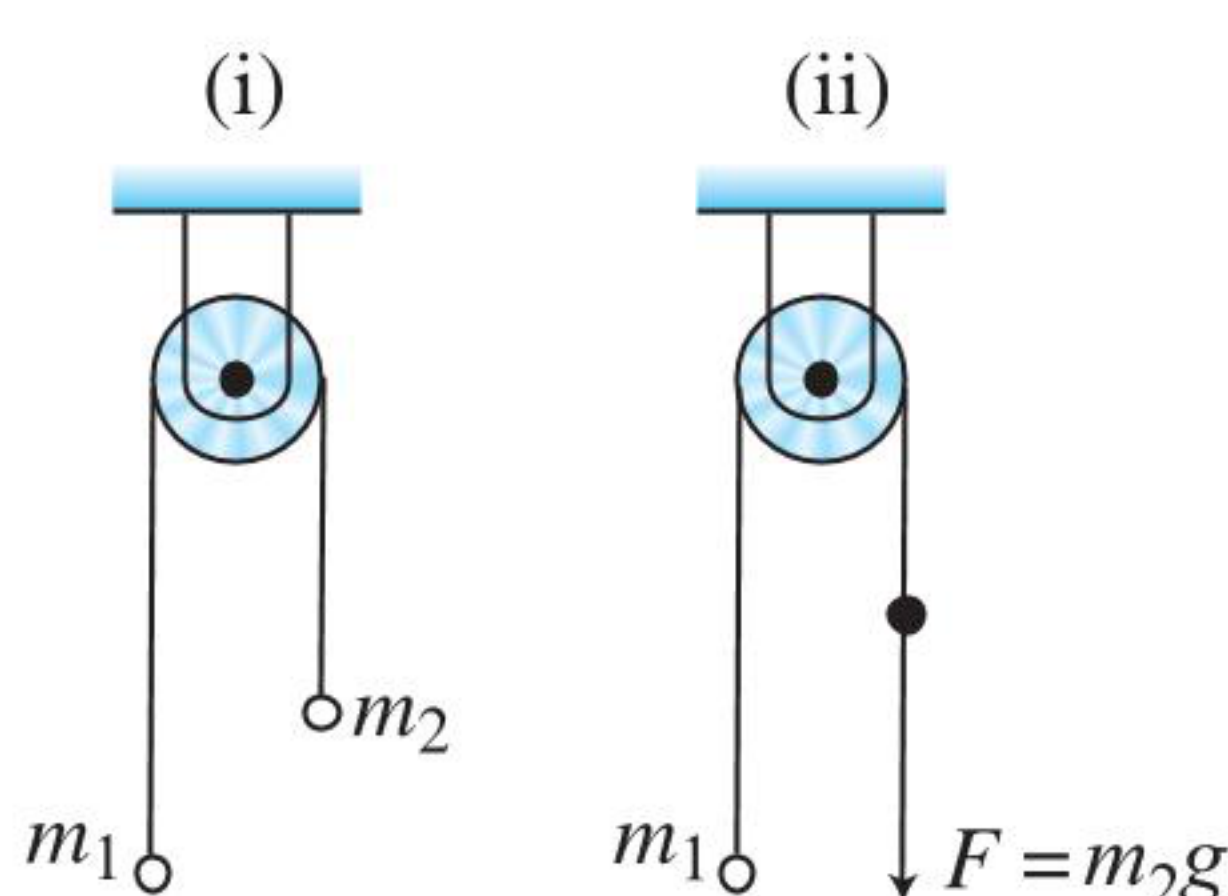
(1) $\frac{h v_A}{2\sqrt{x^2 + h^2}}$ (2) $\frac{x v_A}{\sqrt{x^2 + h^2}}$
 (3) $\frac{x v_A}{2\sqrt{x^2 + h^2}}$ (4) $\frac{h v_A}{\sqrt{x^2 + h^2}}$

37. Two blocks A and B of masses m and $2m$, respectively, are held at rest such that the spring is in natural length. Find out the accelerations of both the blocks just after release.



(1) $g \downarrow, g \downarrow$ (2) $\frac{g}{3} \downarrow, \frac{g}{3} \uparrow$
 (3) $0, 0$ (4) $g \downarrow, c$

38. In two pulley-particle systems (i) and (ii), the acceleration and force imparted by the string on the pulley and tension in the strings are (a_1, a_2) , (N_1, N_2) , and (T_1, T_2) , respectively. Ignoring friction in all contacting surfaces, study the following statements:

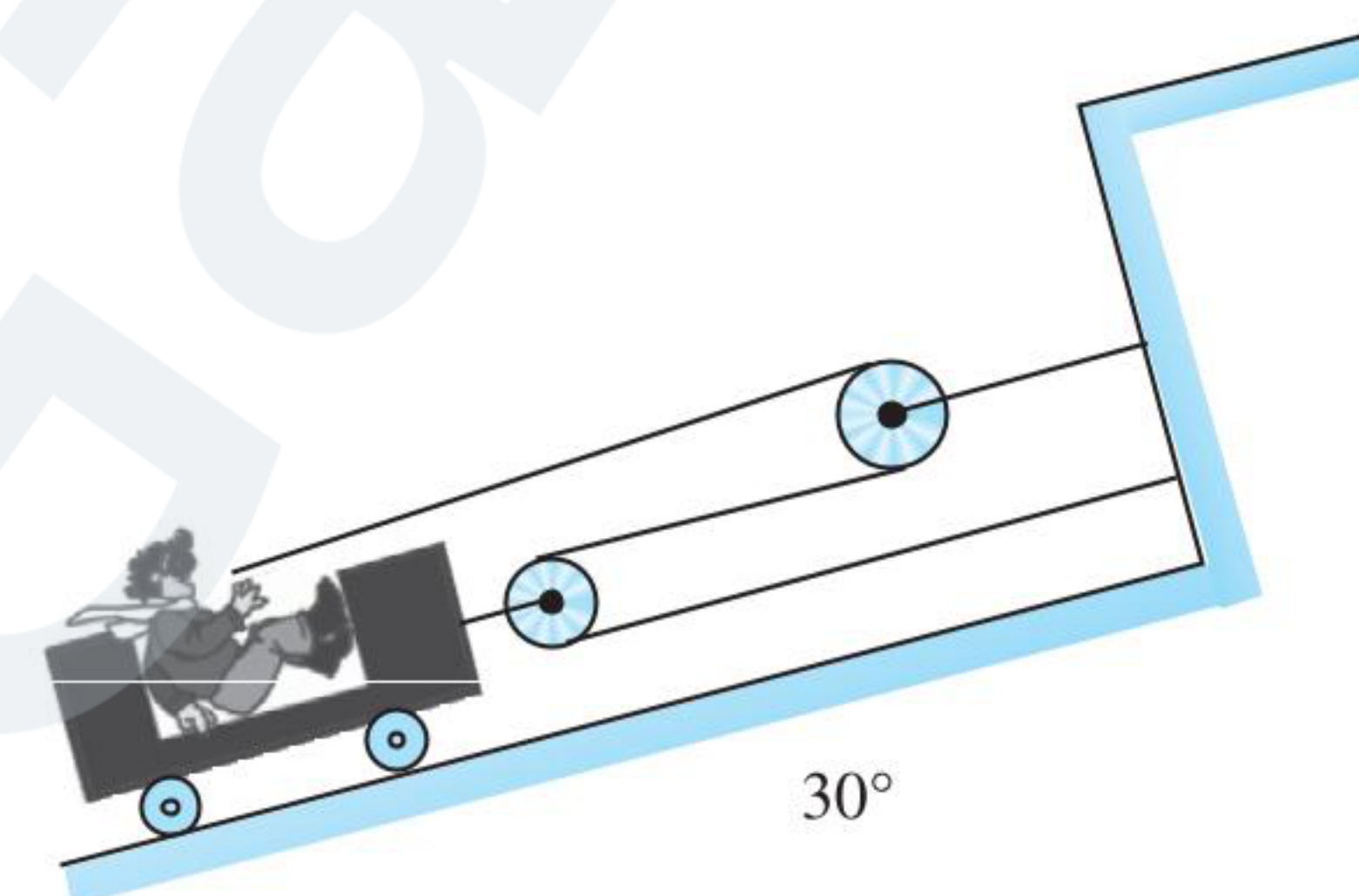


(1) $\frac{a_1}{a_2} = 1$ (2) $\frac{T_1}{T_2} < 1$
 (3) $\frac{N_1}{N_2} > 1$ (4) $\frac{a_1}{a_2} < 1$

Now mark the correct answer:

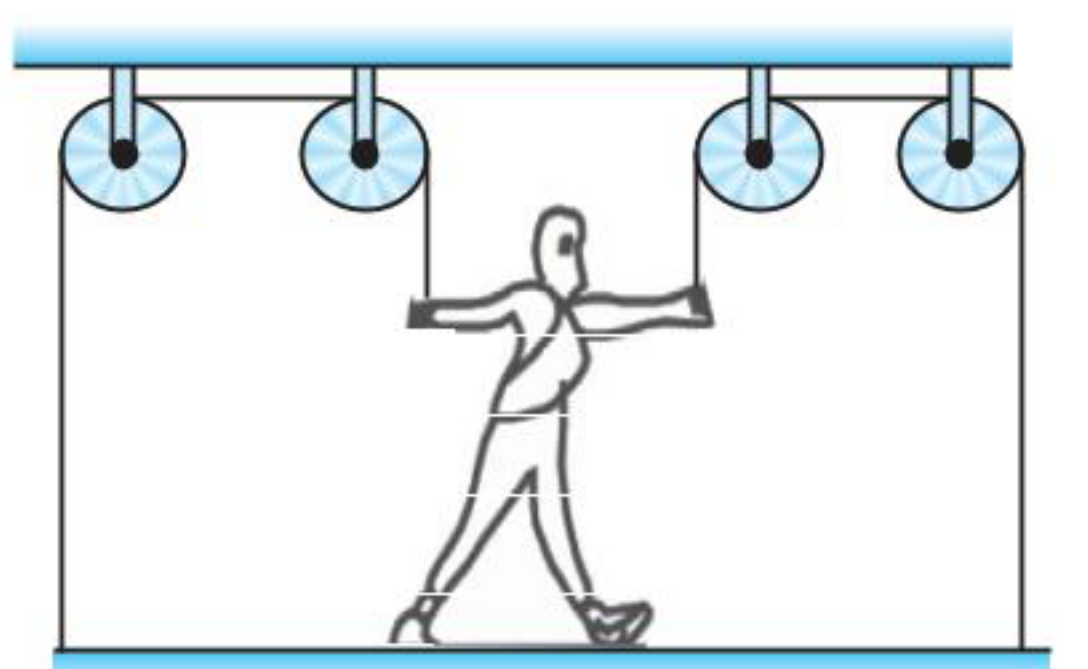
- (1) Relations (ii) and (iii) always follow.
 (2) Relations (ii) and (iv) always follow.
 (3) Only relation (i) always follows.
 (4) Only relation (iv) always follows.

39. A man pulls himself up the 30° incline by the method shown in figure. If the combined mass of the man and cart is 100 kg , determine the acceleration of the cart if the man exerts a pull of 250 N on the rope. Neglect all friction and the mass of the rope, pulleys, and wheels.



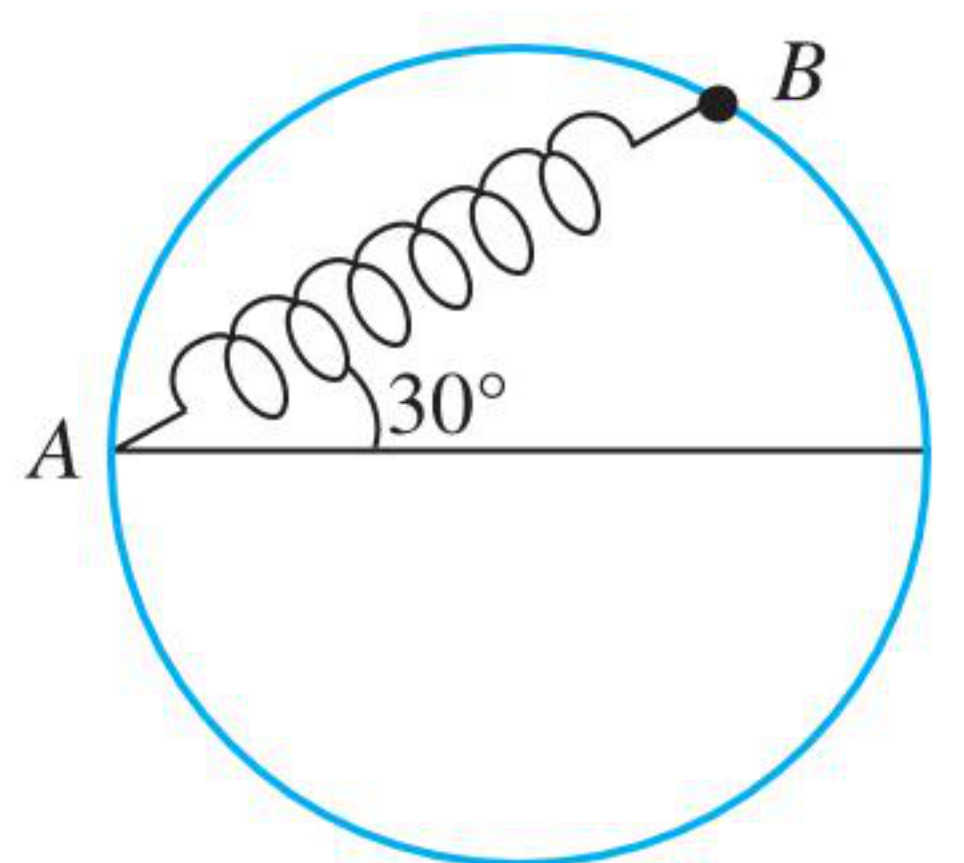
(1) 4.5 m s^{-2} (2) 2.5 m s^{-2}
 (3) 3.5 m s^{-2} (4) 1.5 m s^{-2}

40. A painter of mass M stands on a platform of mass m and pulls himself up by two ropes which hang over pulley as shown in figure. He pulls each rope with force F and moves upward with a uniform acceleration a . Find a , neglecting the fact that no one could do this for long time.



(1) $\frac{4F + (2M + m)g}{M + 2m}$ (2) $\frac{4F + (M + m)g}{M + 2m}$
 (3) $\frac{4F - (M + m)g}{M + m}$ (4) $\frac{4F - (M + m)g}{2M + m}$

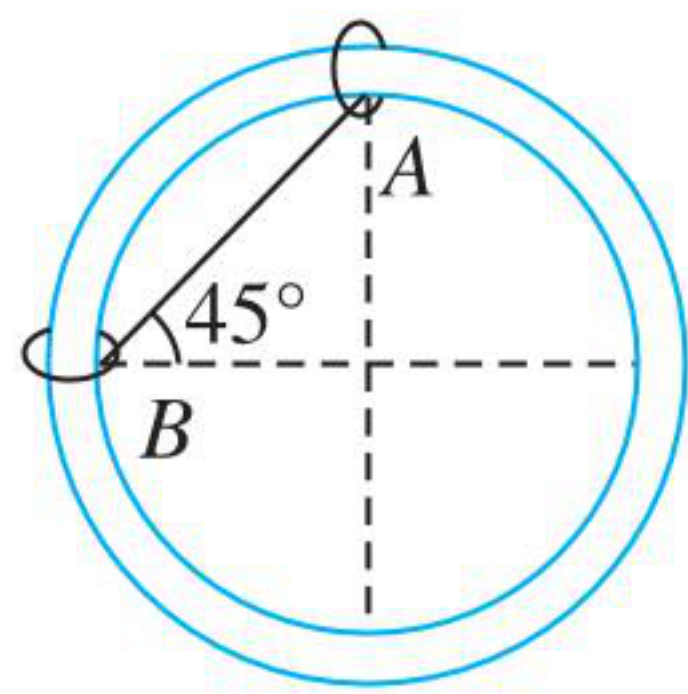
41. A bead of mass m is attached to one end of a spring of natural length R and spring constant $K = \frac{(\sqrt{3} + 1)mg}{R}$. The other end of the spring is fixed at a point A on a smooth vertical ring of radius R as shown in figure. The normal reaction at B just after it is released to move is



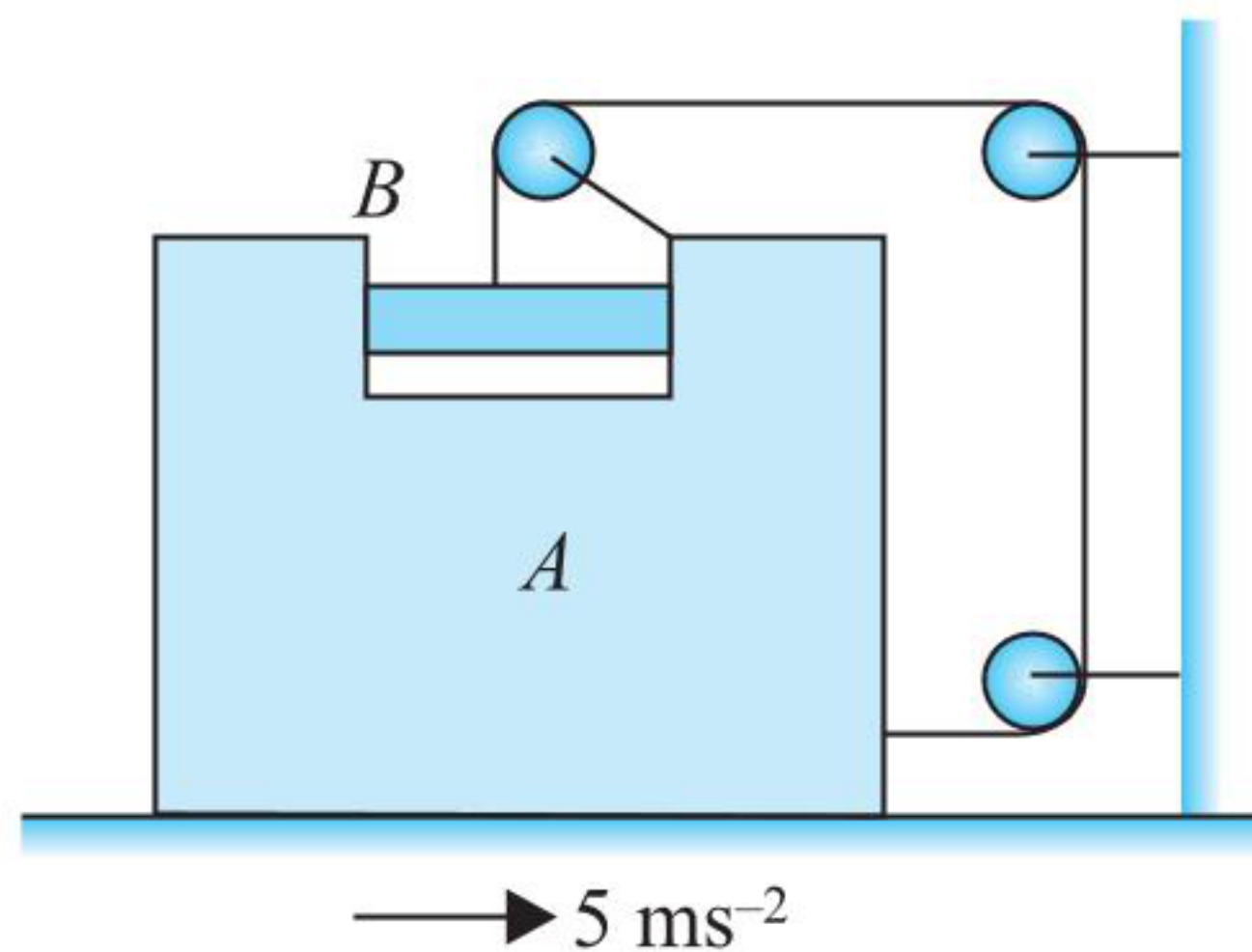
(1) $mg/2$ (2) $\sqrt{3} mg$
 (3) $3\sqrt{3} mg$ (4) $\frac{3\sqrt{3} mg}{2}$

42. Two objects A and B , each of mass m , are connected by a light inextensible string. They are restricted to move on a frictionless ring of radius R in a vertical plane (as shown in figure). The objects are released from rest at the position shown. Then the tension in the cord just after release is

- (1) Zero
 (2) mg
 (3) $\sqrt{2} mg$
 (4) $mg/\sqrt{2}$

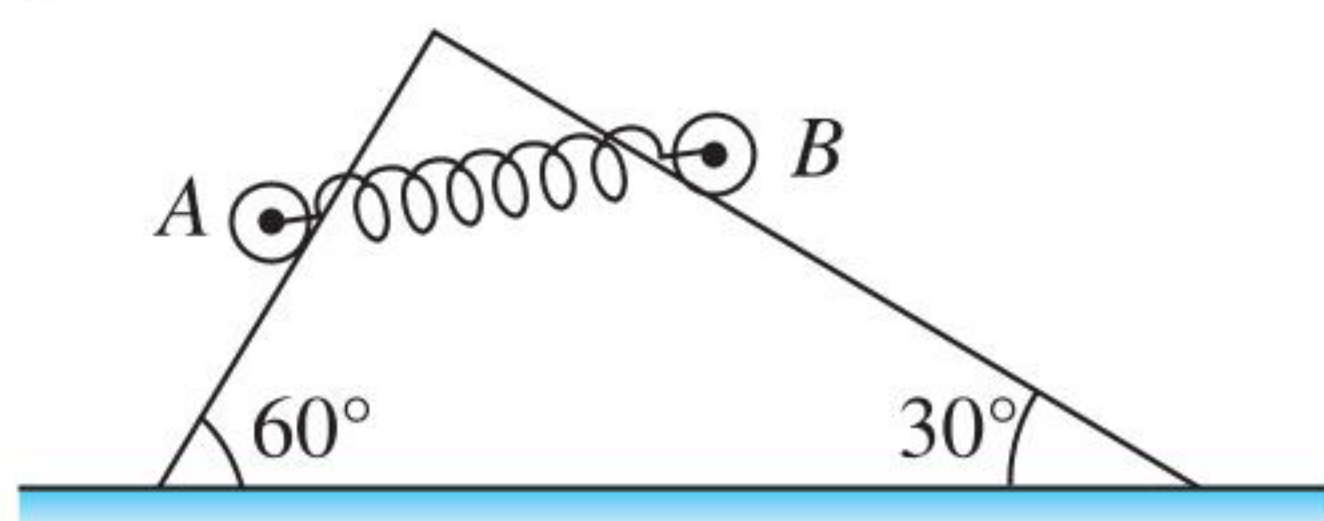


43. If block A is moving with an acceleration of 5 ms^{-2} , the acceleration of B w.r.t ground is



- (1) 5 ms^{-2}
 (2) $5\sqrt{2} \text{ ms}^{-2}$
 (3) $5\sqrt{5} \text{ ms}^{-2}$
 (4) 10 ms^{-2}

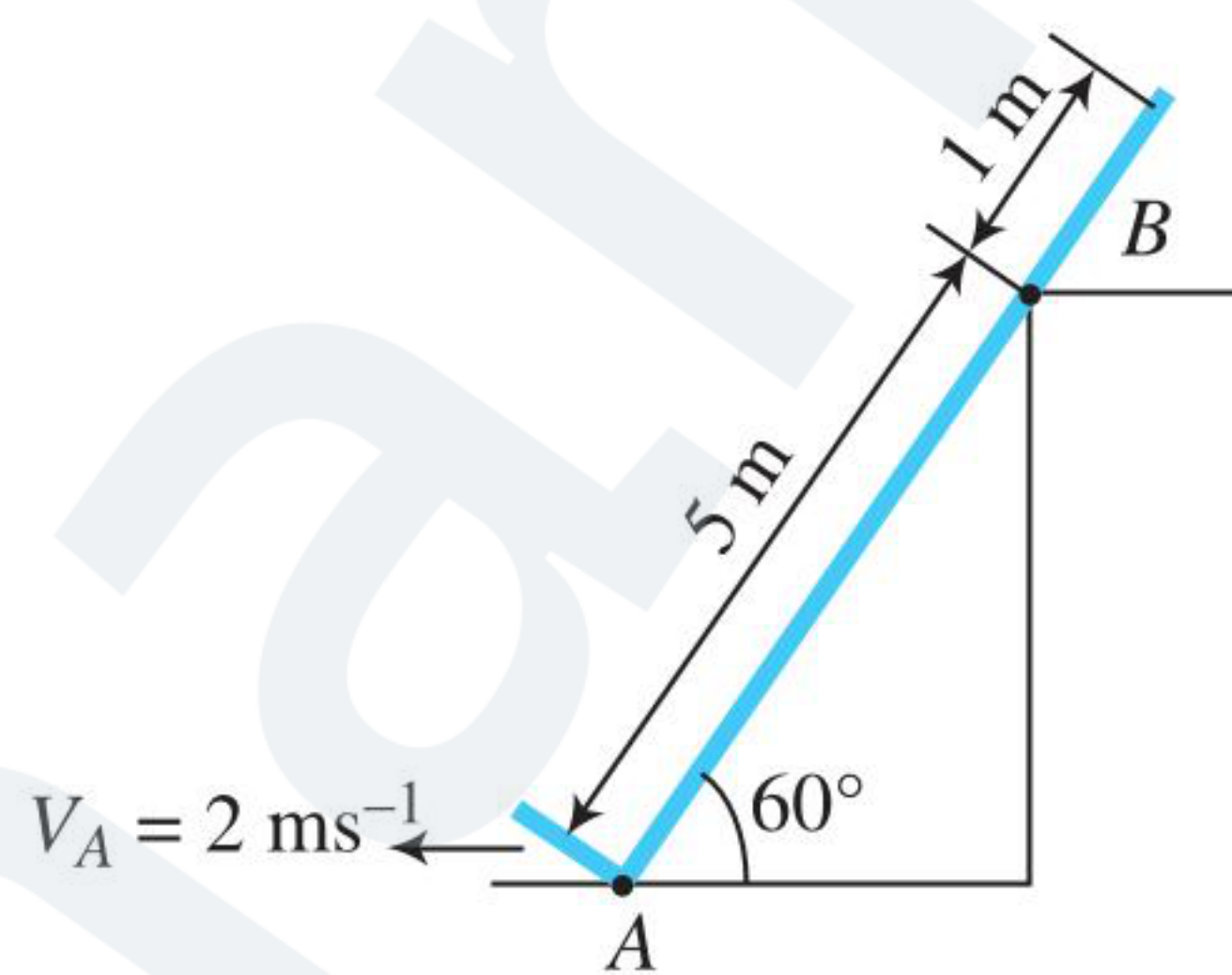
44. Two uniform solid cylinders A and B each of mass 1 kg are connected by a spring of constant 200 Nm^{-1} at their axes and are placed on a fixed wedge as shown in the figure. There is no friction between cylinders and wedge. The angle made by the line AB with the horizontal, in equilibrium, is



- (1) 0°
 (2) 15°
 (3) 30°
 (4) None of these

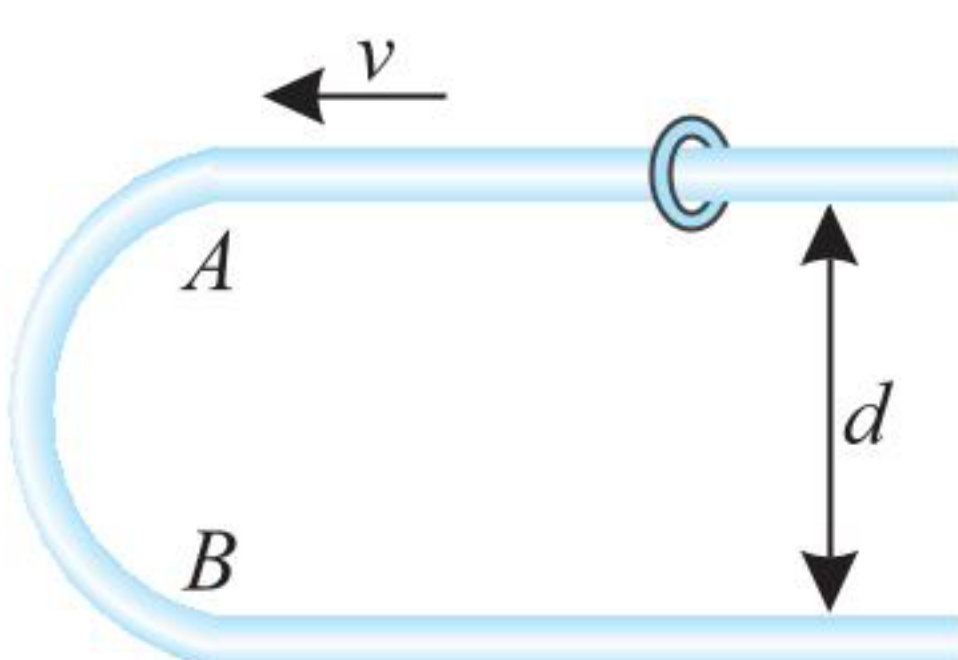
45. The velocity of point A on the rod is 2 ms^{-1} (leftwards) at the instant shown in figure. The velocity of the point B on the rod at this instant is

- (1) $\frac{2}{\sqrt{3}} \text{ ms}^{-1}$
 (2) 1 ms^{-1}
 (3) $\frac{1}{2\sqrt{3}} \text{ ms}^{-1}$
 (4) $\frac{\sqrt{3}}{2} \text{ ms}^{-1}$



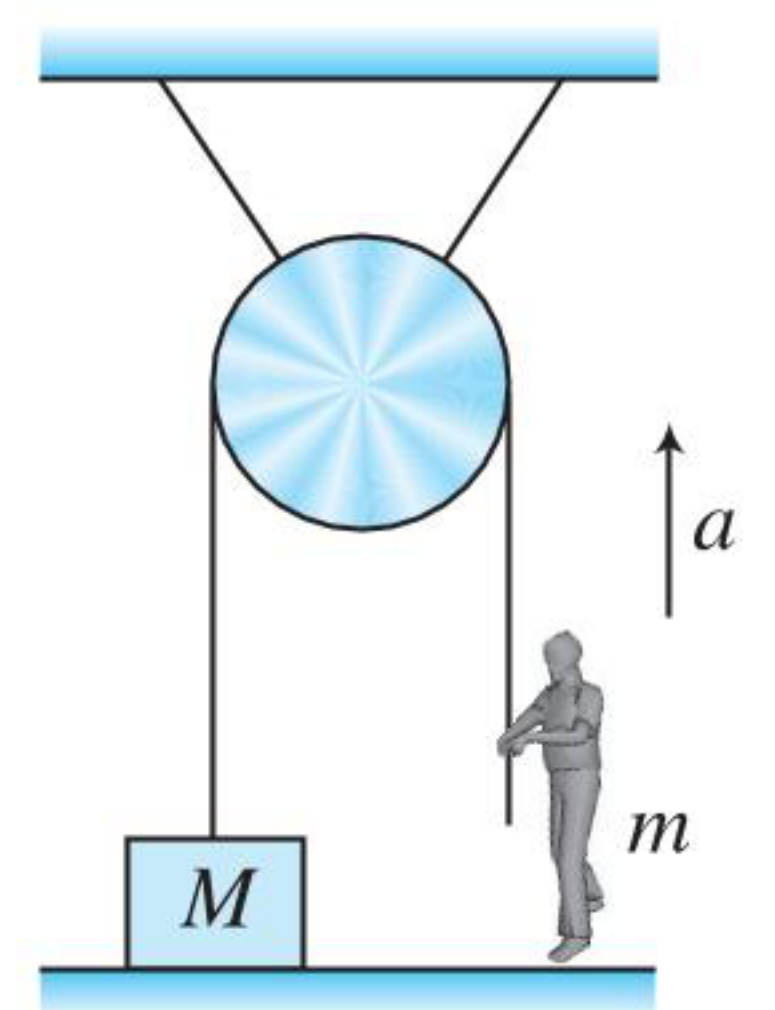
46. A fixed U-shaped smooth wire has a semi-circular bending between A and B as shown in figure. A bead of mass m moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B . The average force exerted by the bead on the part AB of the wire is

- (1) 0
 (2) $\frac{4mv^2}{\pi d}$
 (3) $\frac{2mv^2}{\pi d}$
 (4) None of these

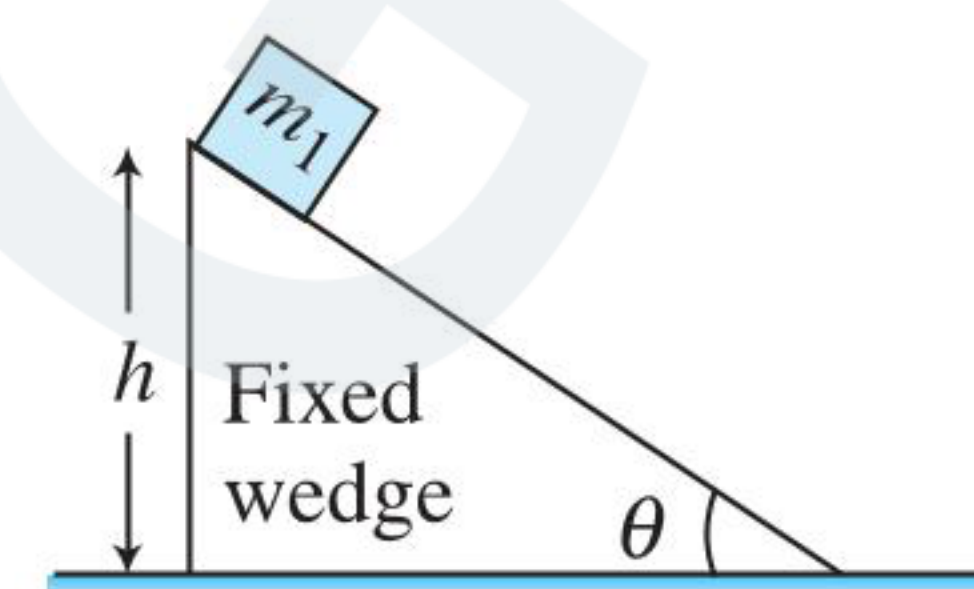


47. In figure, the block of mass M is at rest on the floor. The acceleration with which a boy of mass m should climb along the rope of negligible mass so as to lift the block from the floor is

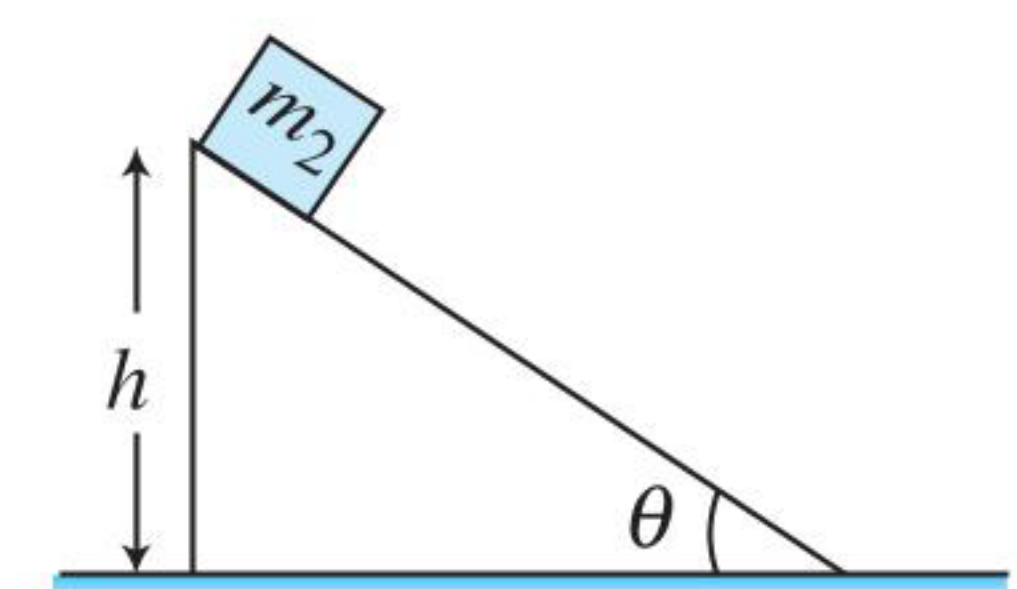
- (1) $\left(\frac{m}{M} - 1\right)g$
 (2) $\left(\frac{M}{m} - 1\right)g$
 (3) $\frac{M}{m}g$
 (4) $> \frac{M}{m}g$



48. A block of mass m_1 lies on the top of fixed wedge as shown in Fig. (a) and another block of mass m_2 lies on top of wedge which is free to move as shown in Fig. (b). At time $t = 0$, both the blocks are released from rest from a vertical height h above the respective horizontal surface on which the wedge is placed as shown. There is no friction between block and wedge in both the figures. Let T_1 and T_2 be the time taken by the blocks, respectively, to just reach the horizontal surface. Then



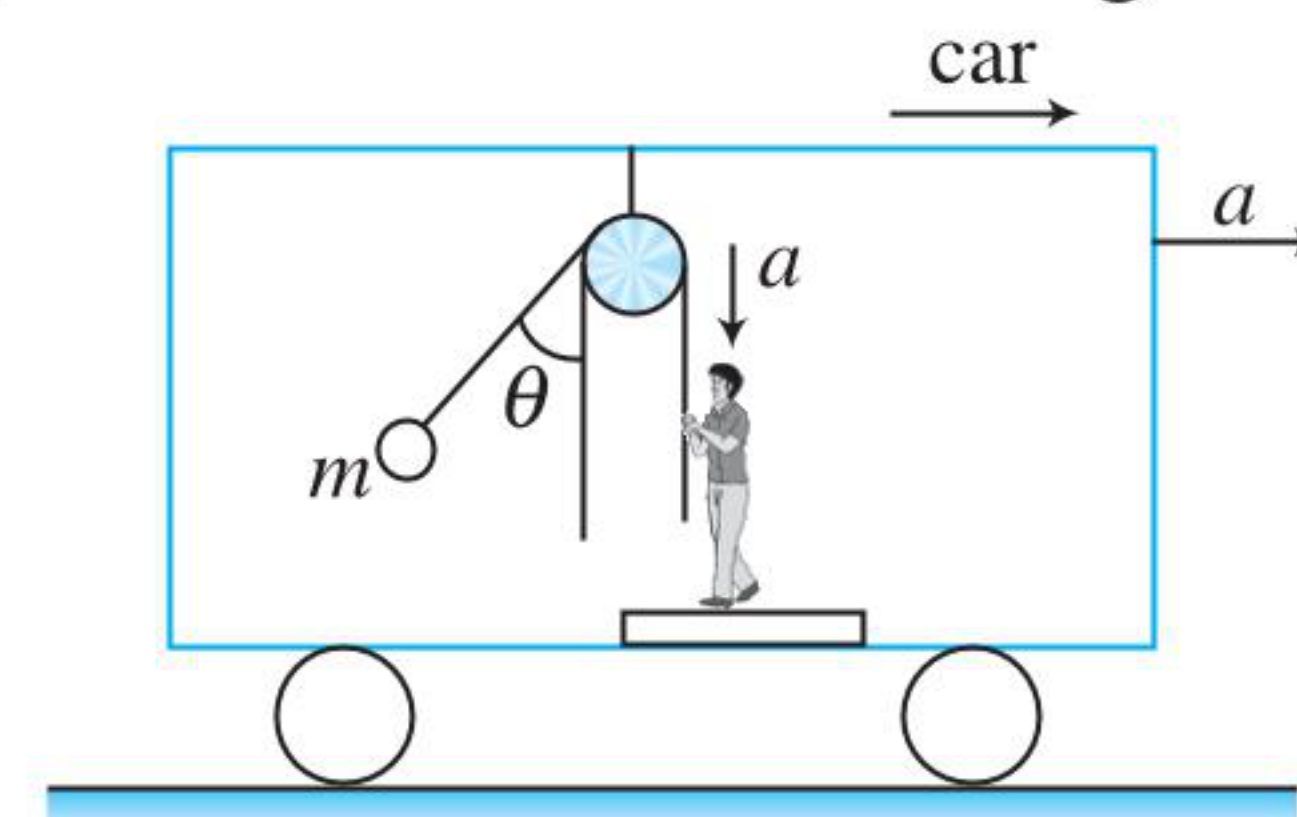
(a) Horizontal surface



(b) Smooth horizontal surface

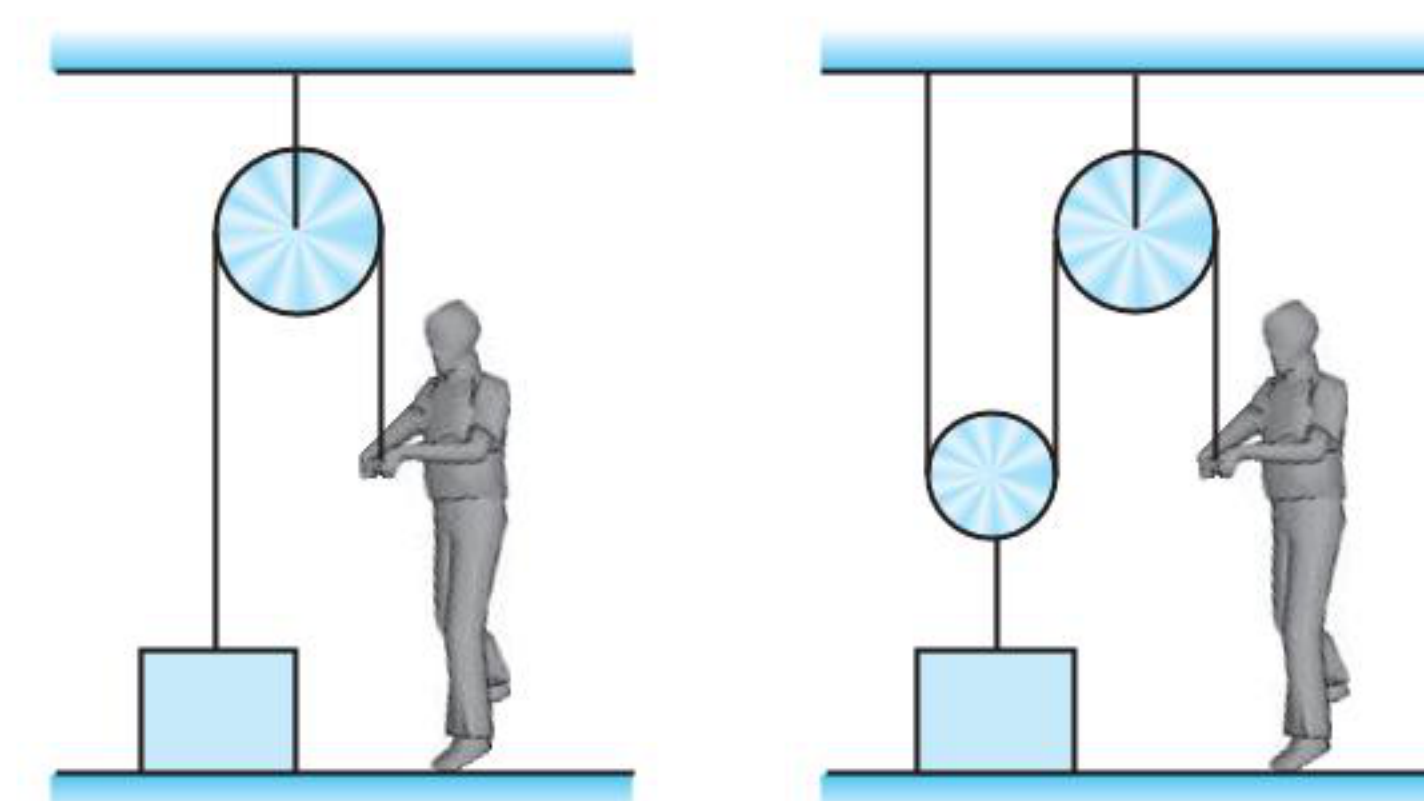
- (1) $T_1 > T_2$
 (2) $T_1 < T_2$
 (3) $T_1 = T_2$
 (4) Data insufficient

49. A bob is hanging over a pulley inside a car through a string. The second end of the string is in the hands of a person standing in the car. The car is moving with constant acceleration a directed horizontally as shown in Figure. The other end of the string is pulled with constant acceleration a vertically. The tension in the string is equal to



- (1) $m\sqrt{g^2 + a^2}$
 (2) $m\sqrt{g^2 + a^2} - ma$
 (3) $m\sqrt{g^2 + a^2} + ma$
 (4) $m(g + a)$

50. In figure, a person wants to rise a block lying on the ground to a height h . In both the cases, if the time required is same, then in which case he has to exert more force? Assume pulleys and strings light.

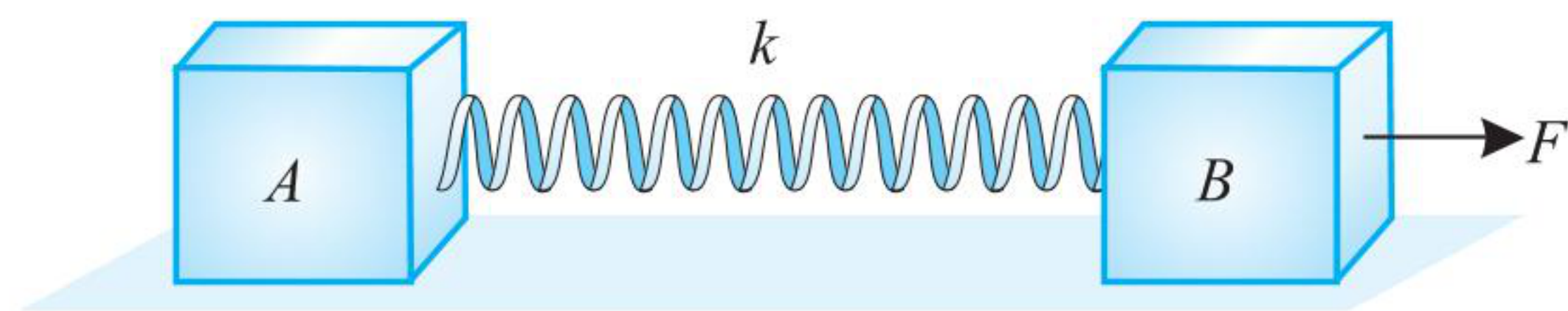


(i)

(ii)

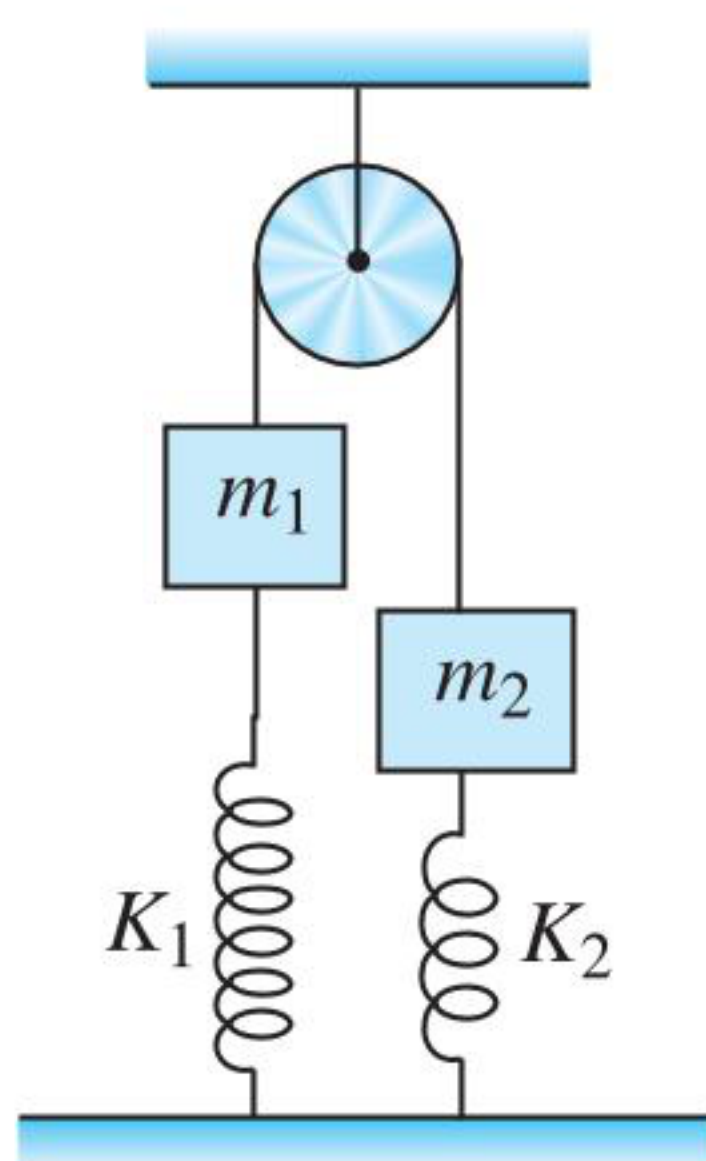
- (1) (i)
 (2) (ii)
 (3) Same in both
 (4) Cannot be determined

51. Two identical particles A and B , each of mass m , are interconnected by a spring of stiffness k . If particle B experiences a force F and the elongation of the spring is x , the acceleration of particle B relative to particle A is equal to



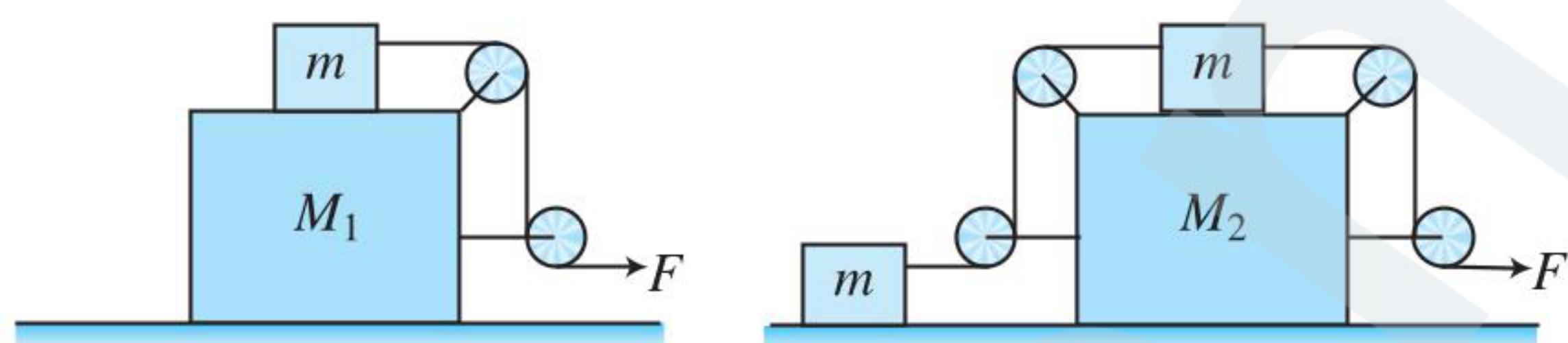
- (1) $\frac{F}{2m}$ (2) $\frac{F - kx}{m}$
 (3) $\frac{F - 2kx}{m}$ (4) $\frac{kx}{m}$

52. The system shown in figure is in equilibrium. Masses m_1 and m_2 are 2 kg and 8 kg, respectively. Spring constants k_1 and k_2 are 50 Nm^{-1} and 70 Nm^{-1} , respectively. If the compression in second spring is 0.5 m. What is the compression in first spring? (Both springs have natural length initially.)



- (1) 1.3 m (2) -0.5 m
 (3) 0.5 m (4) 0.9 m

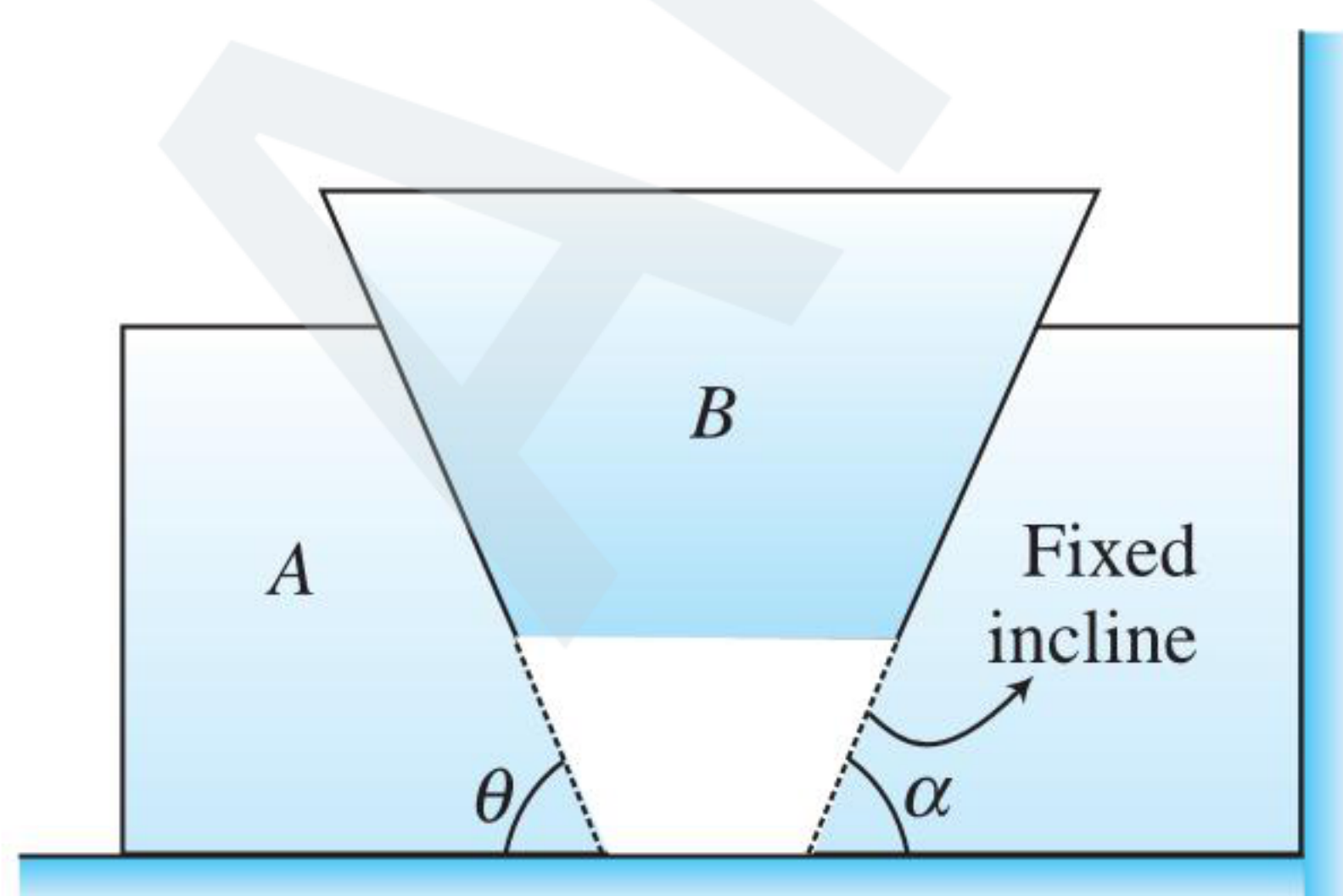
53. In the situation shown in figure, all the strings are light and inextensible and pulleys are light. There is no friction at any surface and all blocks are of cuboidal shape. A horizontal force of magnitude F is applied to right most free end of string in both cases shown in the figure. At the instant shown, the tension in all strings are non-zero. Let the magnitudes of acceleration of large blocks (of mass M) in Fig.(a) and Fig. (b) be a_1 and a_2 , respectively. Then,



Smooth horizontal surface (a) Smooth horizontal surface (b)

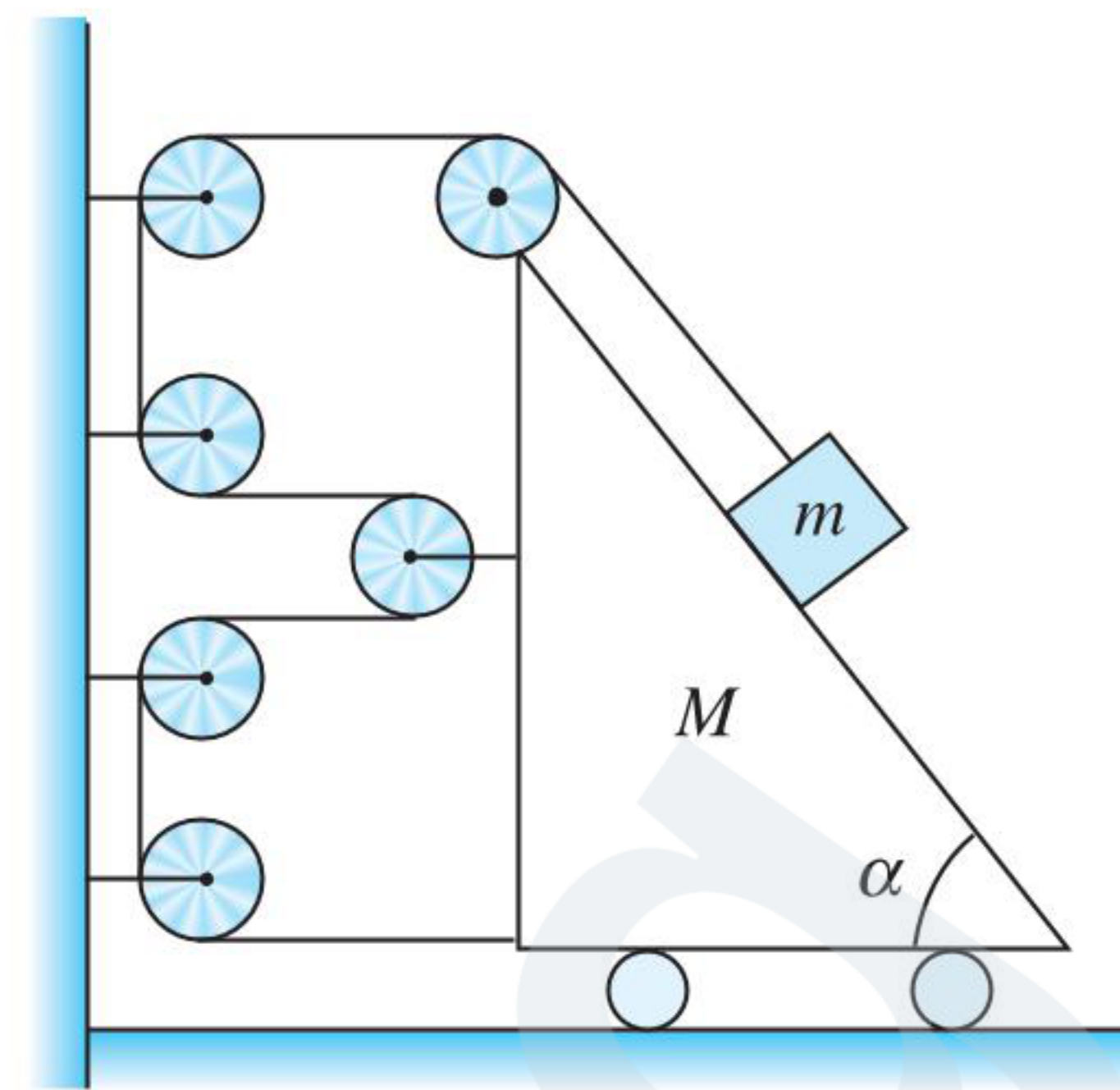
- (1) $a_1 = a_2 \neq 0$ (2) $a_1 = a_2 = 0$
 (3) $a_1 > a_2$ (4) $a_1 < a_2$

54. In the arrangement shown in figure, if the acceleration of B is $\vec{a}$, then find the acceleration of A .



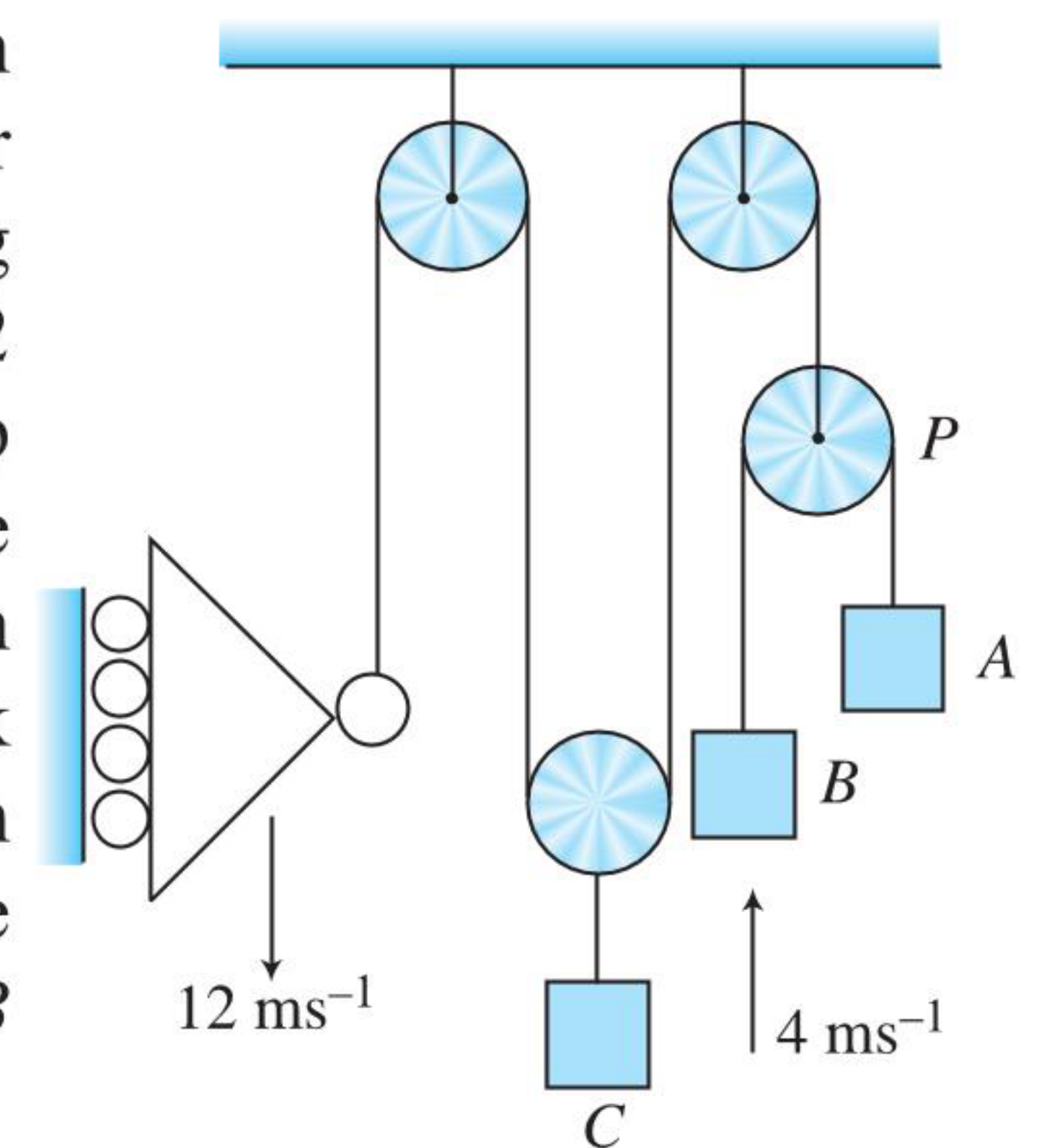
- (1) $a \sin \alpha$ (2) $a \cot \theta$
 (3) $a \tan \theta$ (4) $a(\sin \alpha \cot \theta + \cos \alpha)$

55. If the acceleration of wedge in the shown arrangement is $a \text{ ms}^{-2}$ towards left, then at this instant, acceleration of the block (magnitude only) would be



- (1) $4a \text{ ms}^{-2}$ (2) $a\sqrt{17 - 8 \cos \alpha} \text{ ms}^{-2}$
 (3) $(\sqrt{17})a \text{ ms}^{-2}$ (4) $\sqrt{17} \cos \frac{\alpha}{2} \times a \text{ ms}^{-2}$

56. In the arrangement shown in figure at a particular instant, the roller is coming down with a speed of 12 ms^{-1} and C is moving up with 4 ms^{-1} . At the same instant, it is also known that w.r.t. pulley P , block A is moving down with speed 3 ms^{-1} . Determine the motion of block B (velocity) w.r.t. ground.

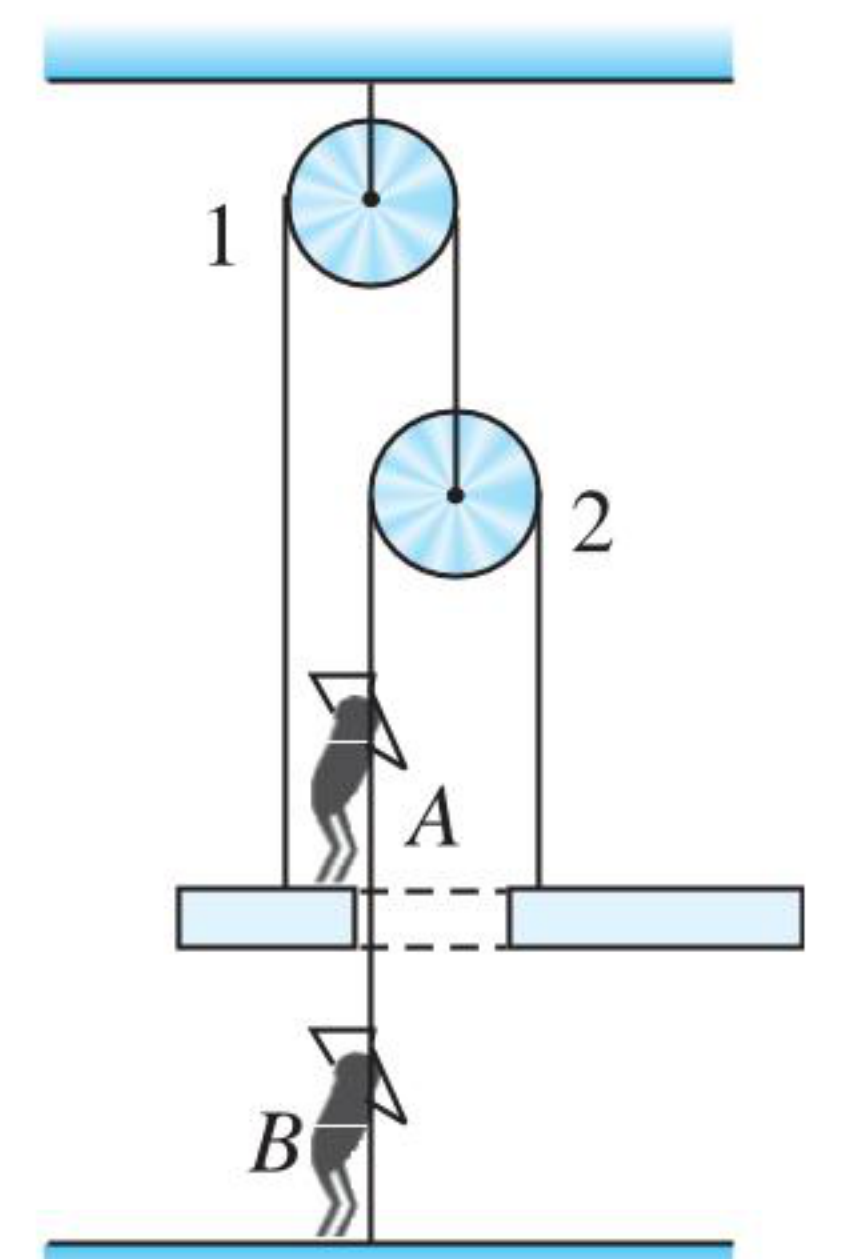


- (1) 4 ms^{-1} in downward direction
 (2) 3 ms^{-1} in upward direction
 (3) 7 ms^{-1} in downward direction
 (4) 7 ms^{-1} in upward direction

57. A particle of mass 2 kg moves with an initial velocity of $(4\hat{i} + 2\hat{j}) \text{ ms}^{-1}$ on the x - y plane. A force $\vec{F} = (2\hat{i} - 8\hat{j}) \text{ N}$ acts on the particle. The initial position of the particle is (2 m, 3 m). Then for $y = 3 \text{ m}$,

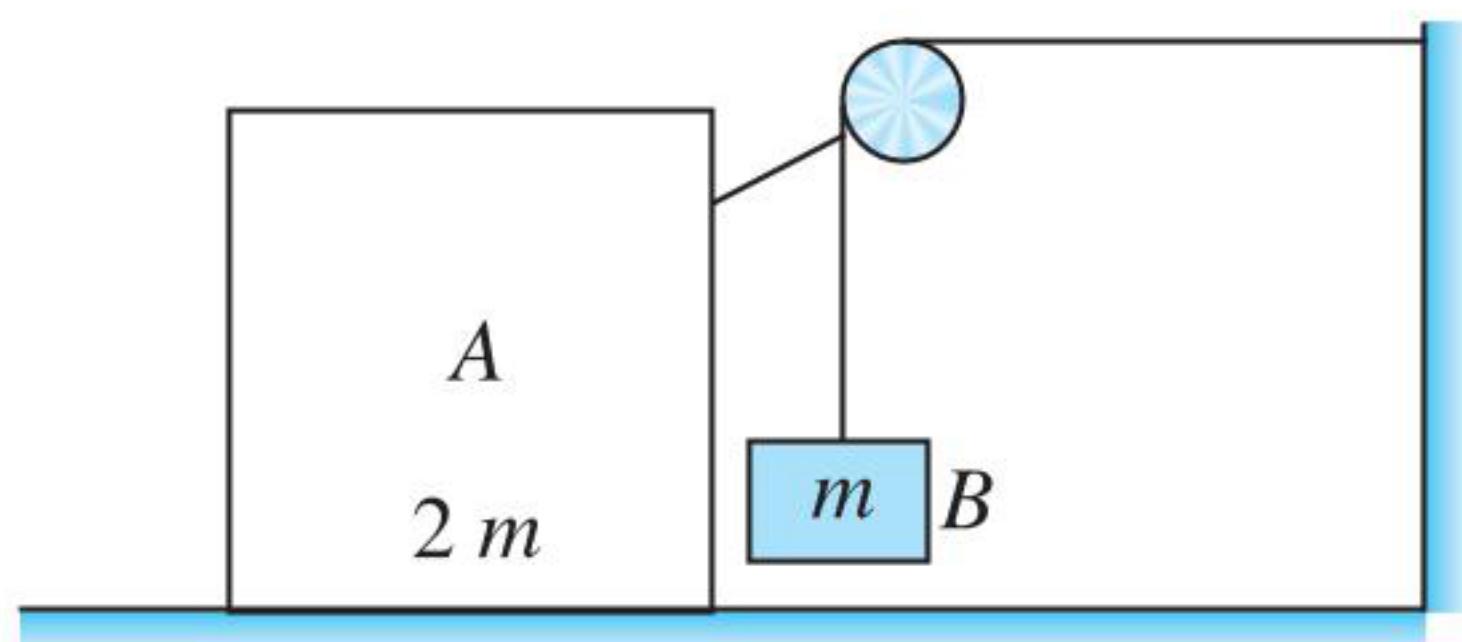
- (1) Possible value of x is only $x = 2 \text{ m}$
 (2) Possible value of x is not only $x = 2 \text{ m}$, but there exists some other value of x also
 (3) Time taken is 2 s
 (4) All of the above

58. In figure, man A is standing on a movable plank while man B is standing on a stationary platform. Both are pulling the string down such that the plank moves slowly up. As a result of this the string slips through the hands of the men. Find the ratio of length of the string that slips through the hands of A and B .



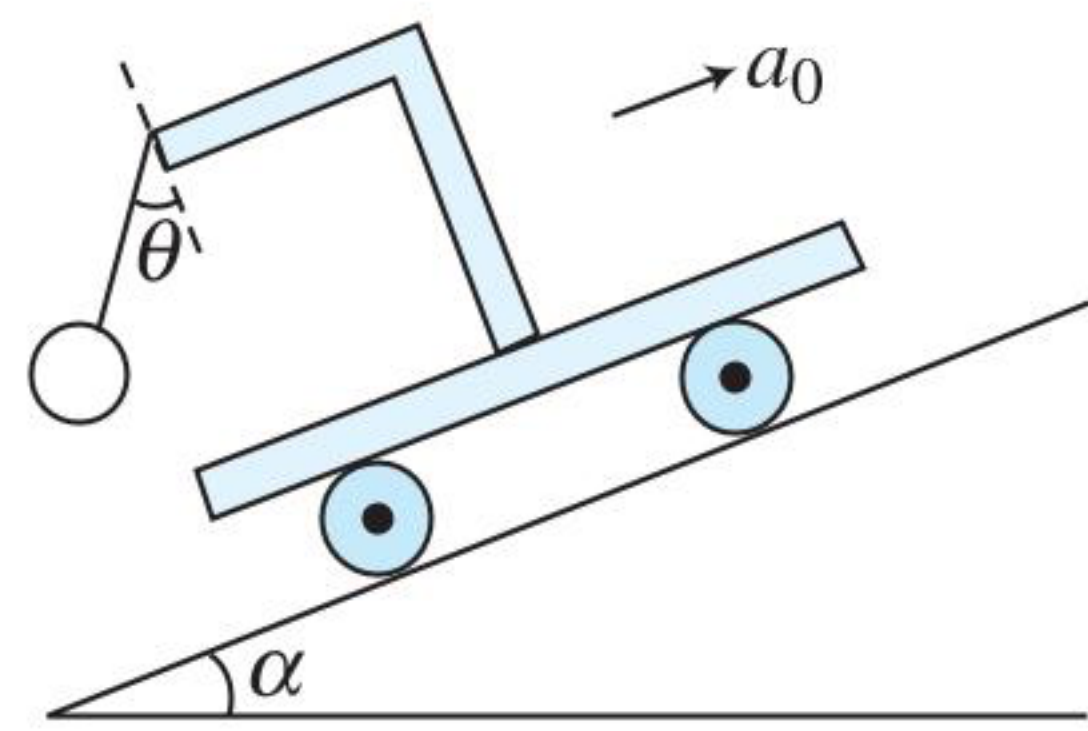
- (1) $3/2$ (2) $3/4$
 (3) $4/3$ (4) $2/3$

59. In the system shown, all the surfaces are frictionless while pulley and string are massless. The mass of block A is $2m$ and that of block B is m . The acceleration of block B immediately after system is released from rest is



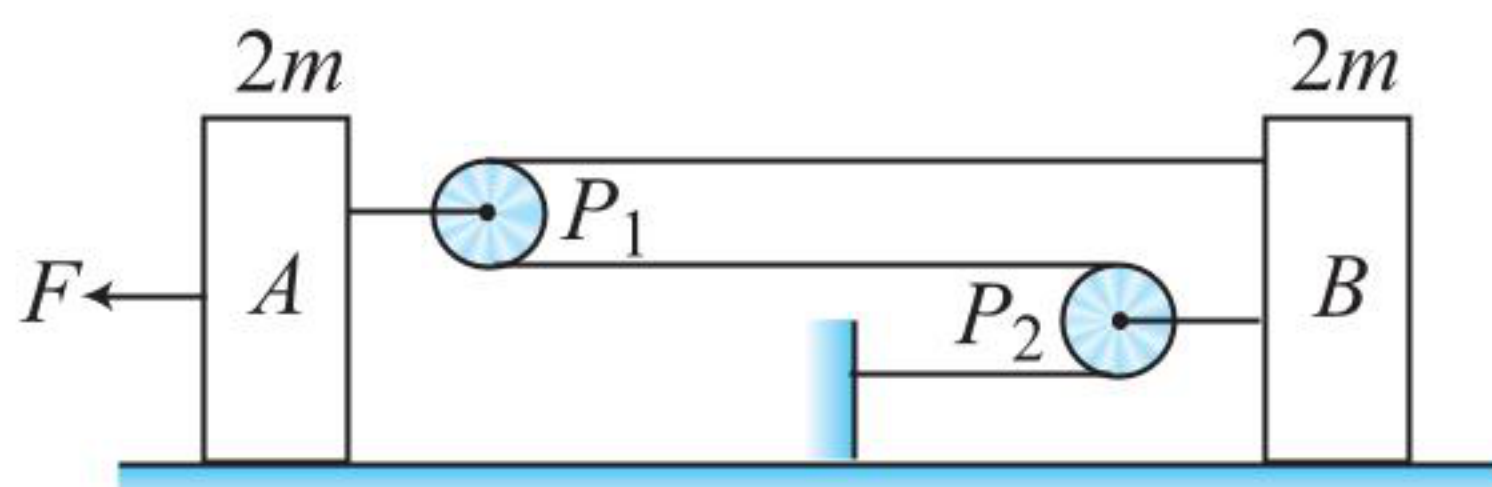
- (1) $g/2$ (2) g
 (3) $g/3$ (4) none of these

60. A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is



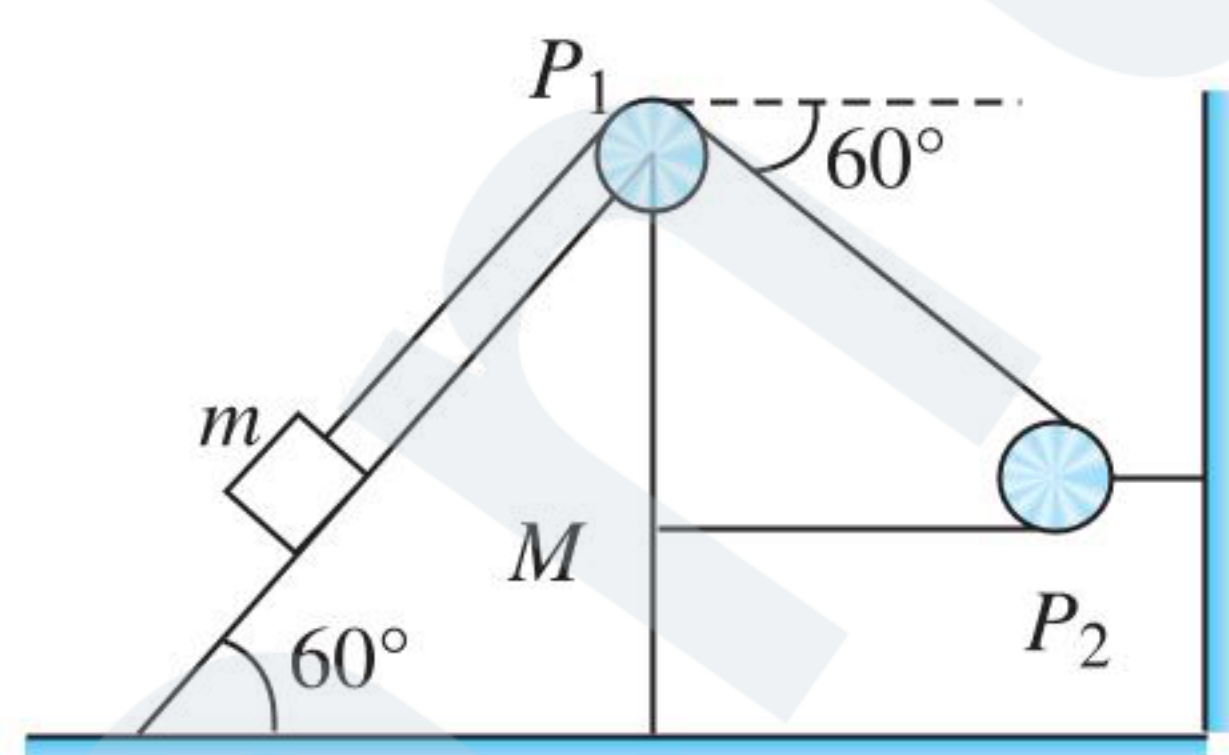
- (1) $\theta = \tan^{-1} \alpha$ (2) $\theta = \tan^{-1} \left(\frac{a_0}{g} \right)$
 (3) $\theta = \tan^{-1} \left(\frac{g}{a_0} \right)$ (4) $\theta = \tan^{-1} \left(\frac{a_0 + g \sin \alpha}{g \cos \alpha} \right)$

61. The acceleration of the block B in figure, assuming the surfaces and the pulleys P_1 and P_2 are all smooth, is



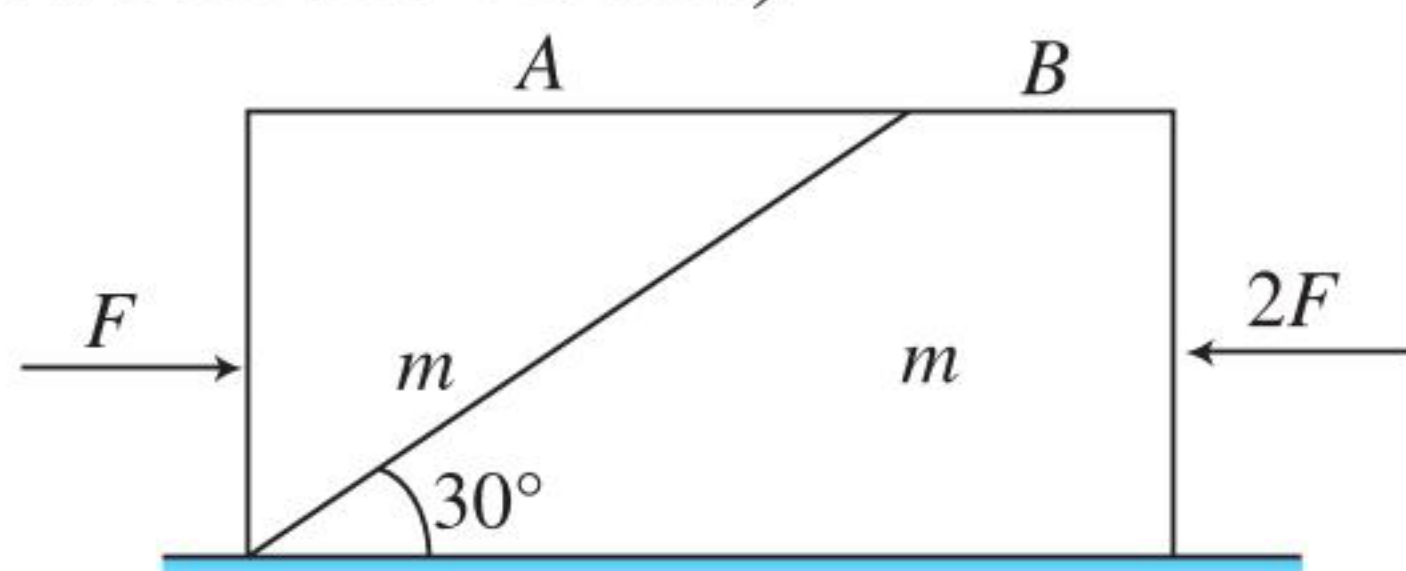
- (1) $\frac{F}{4m}$ (2) $\frac{3F}{13m}$
 (3) $\frac{F}{2m}$ (4) $\frac{3F}{17m}$

62. In the arrangement shown in figure the block of mass $m = 2$ kg lies on the wedge of mass $M = 8$ kg. The initial acceleration of the wedge, if the surfaces are smooth, is



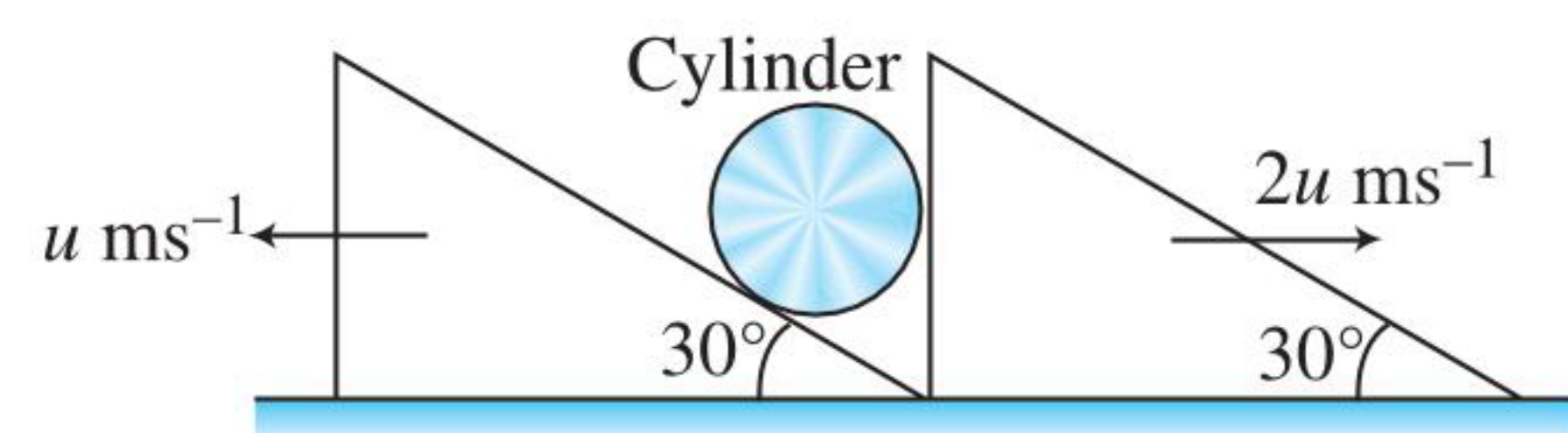
- (1) $\frac{\sqrt{3}g}{23} \text{ m s}^{-2}$ (2) $\frac{3\sqrt{3}g}{23} \text{ m s}^{-2}$
 (3) $\frac{3g}{23} \text{ m s}^{-2}$ (4) $\frac{g}{23} \text{ m s}^{-2}$

63. Two blocks A and B each of mass m are placed on a smooth horizontal surface. Two horizontal forces F and $2F$ are applied on blocks A and B , respectively, as shown in figure. Block A does not slide on block B . Then the normal reaction acting between the two blocks is (assume no friction between the blocks)



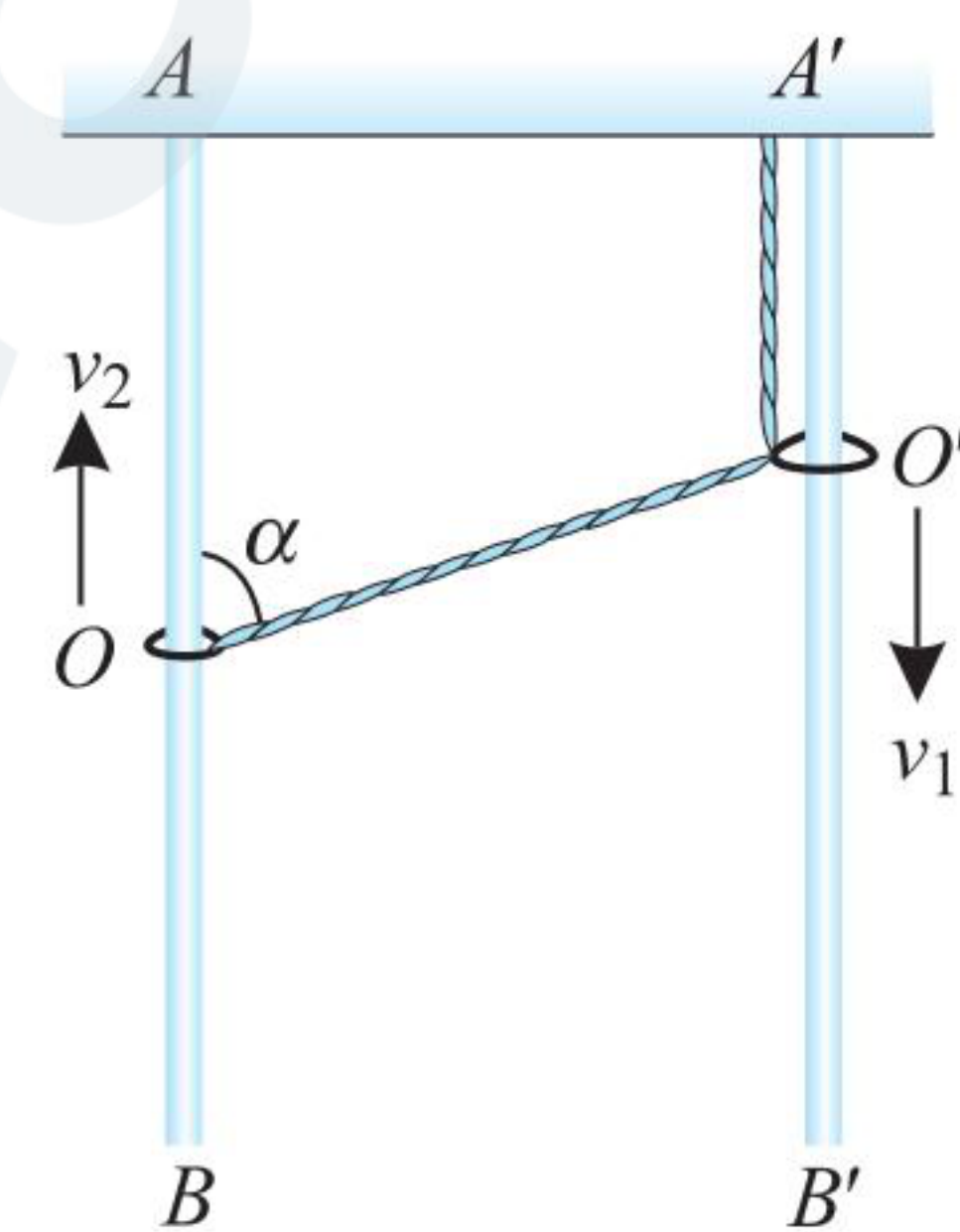
- (1) F (2) $F/2$
 (3) $\frac{F}{\sqrt{3}}$ (4) $3F$

64. A system is shown in Figure. Assume that the cylinder remains in contact with the two wedges. Then the velocity of cylinder is



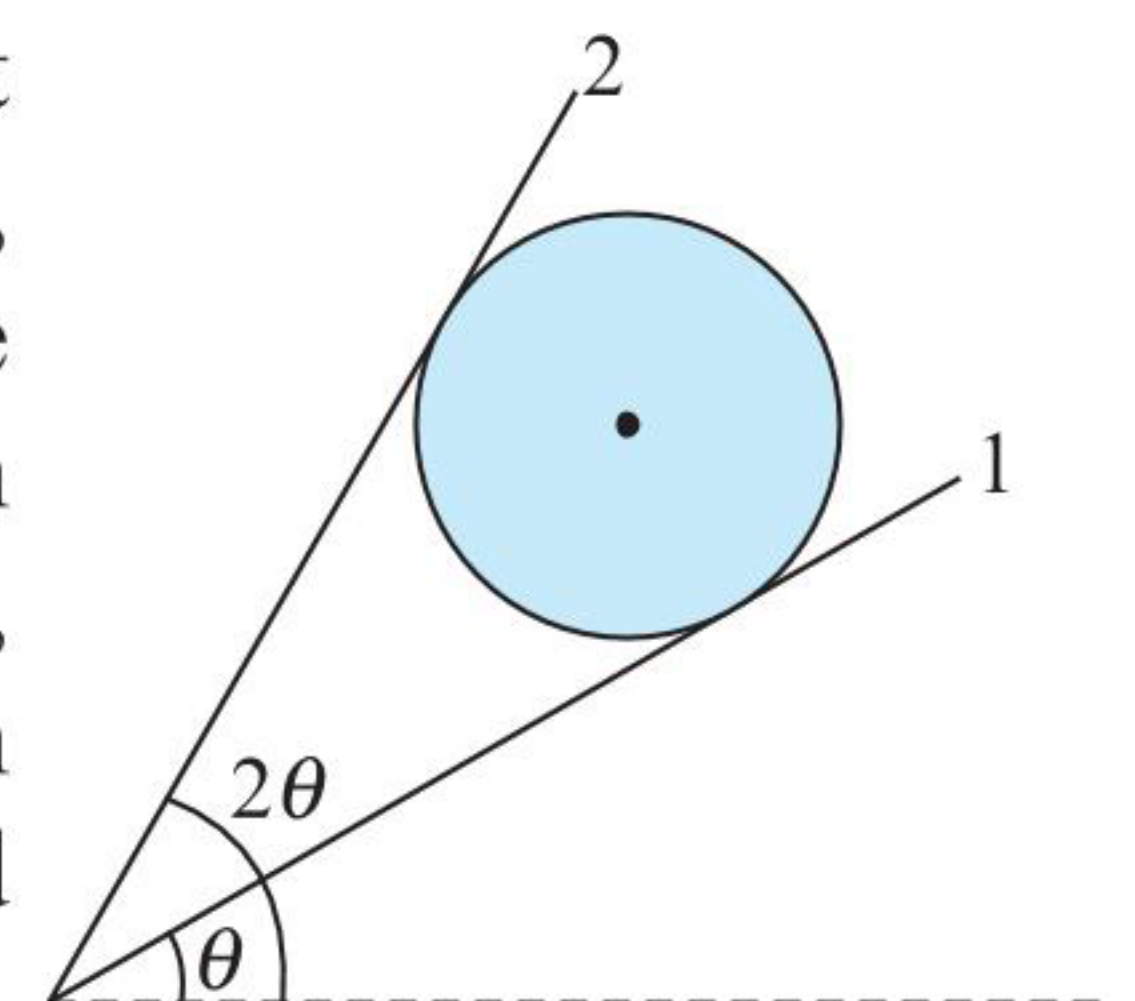
- (1) $\sqrt{19 - 4\sqrt{3}} \text{ m s}^{-1}$ (2) $\frac{\sqrt{13}u}{2} \text{ m s}^{-1}$
 (3) $\sqrt{3}u \text{ m s}^{-1}$ (4) $\sqrt{7} \text{ m s}^{-1}$

65. Two small rings O and O' are put on two vertical stationary rods AB and $A'B'$, respectively. One end of an inextensible thread is tied at point A' . The thread passes through ring O' and its other end is tied to ring O . Assuming that ring O' moves downwards at a constant velocity v_1 , then velocity v_2 of the ring O , when $\angle AOO' = \alpha$, is



- (1) $v_1 \left[\frac{2 \sin^2 \alpha/2}{\cos \alpha} \right]$ (2) $v_1 \left[\frac{2 \cos^2 \alpha/2}{\sin \alpha} \right]$
 (3) $v_1 \left[\frac{3 \cos^2 \alpha/2}{\sin \alpha} \right]$ (4) None of these

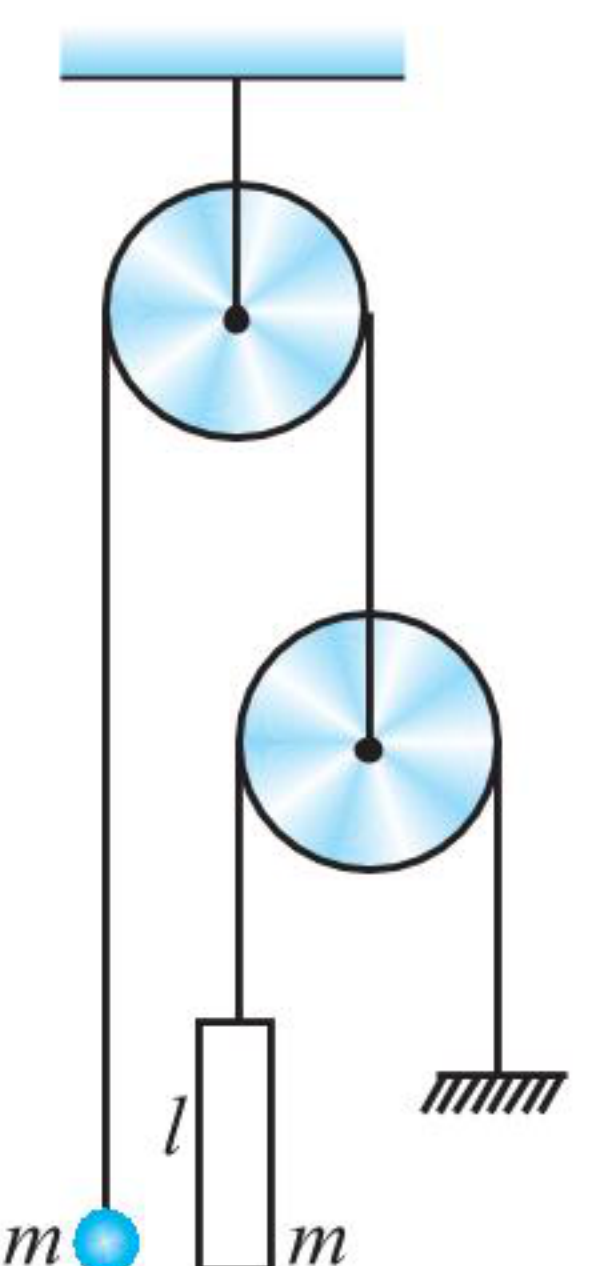
66. A sphere of mass m is kept between two inclined walls, as shown in the figure. If the coefficient of friction between each wall and the sphere is zero, then the ratio of normal reaction (N_1/N_2) offered by the walls 1 and 2 on the sphere will be



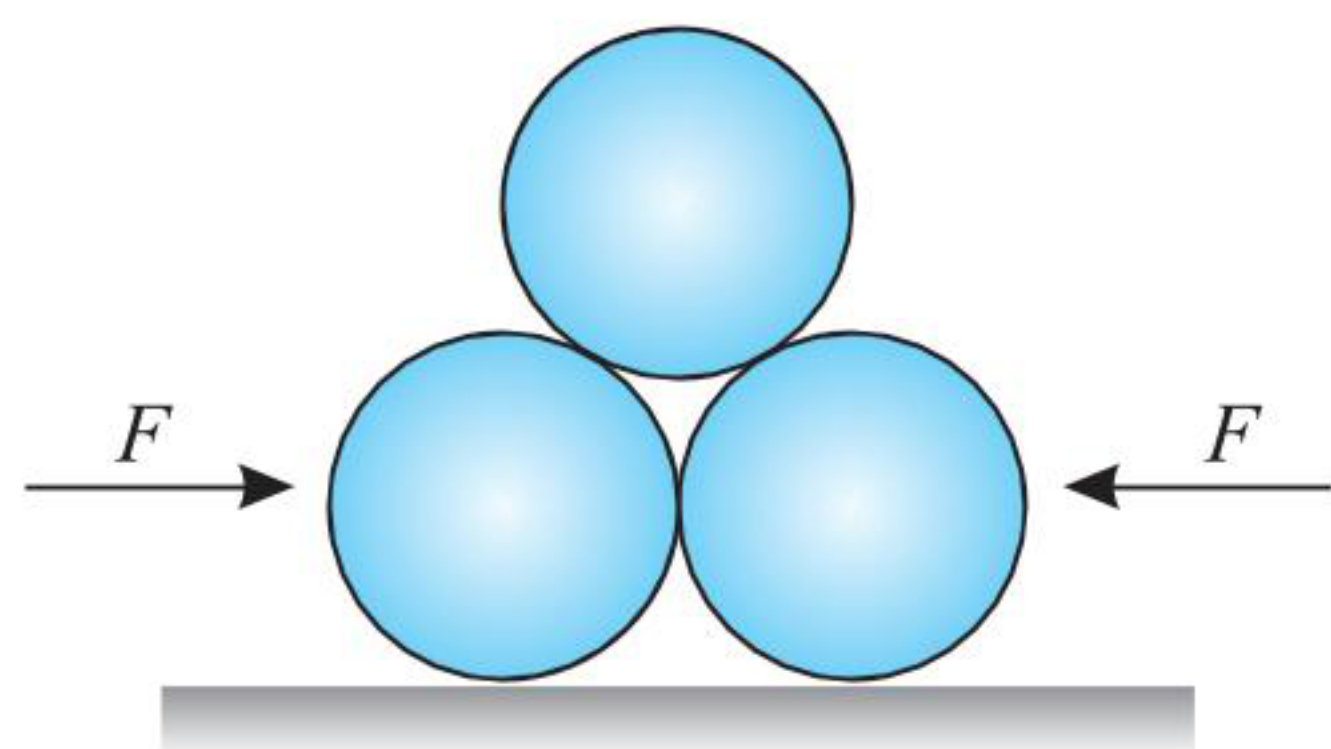
- (1) $\tan \theta$ (2) $\tan 2\theta$
 (3) $2 \cos \theta$ (4) $\cos 2\theta$

67. In the figure shown, all pulleys are massless and frictionless. The time taken by the ball to reach the upper end of the rod is:

- (1) $\sqrt{\frac{10l}{3g}}$ (2) $\sqrt{\frac{5l}{3g}}$
 (3) $\sqrt{\frac{3l}{4g}}$ (4) $\sqrt{\frac{3l}{10g}}$

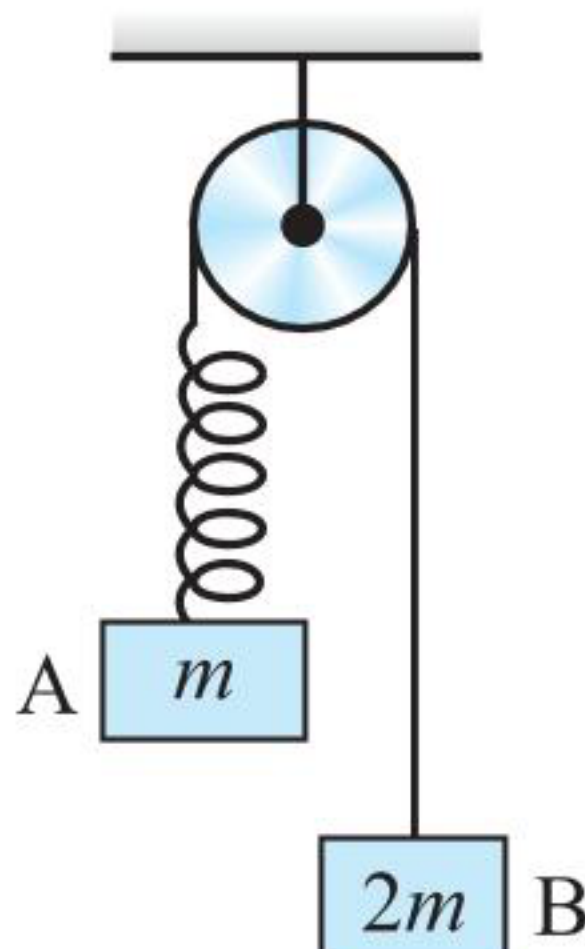


68. Two smooth cylindrical bars weighing W N each lie next to each other in contact. A similar third bar is placed over the two bars as shown in figure. Neglecting friction, the minimum horizontal force on each lower bar necessary to keep them together is



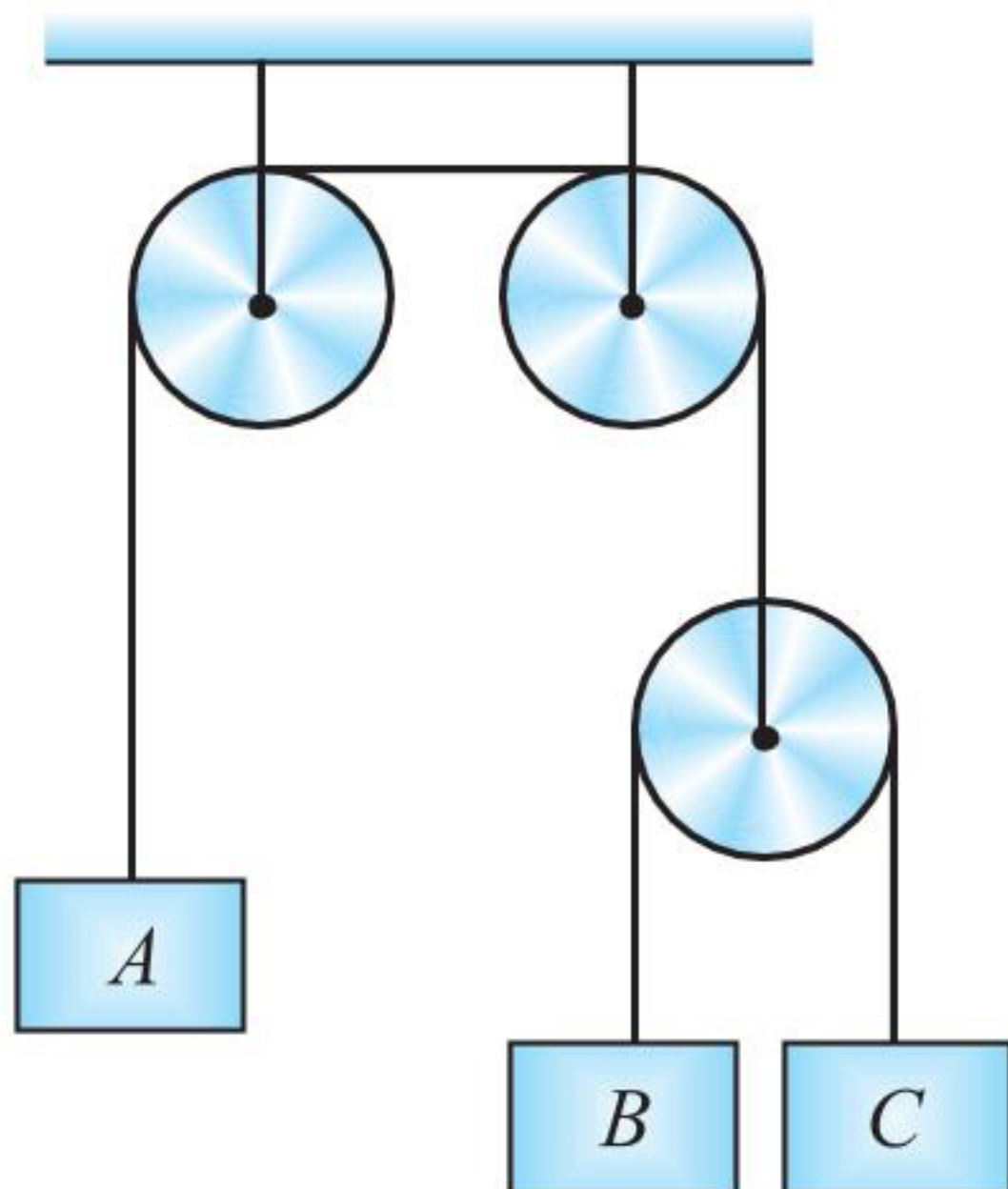
- (1) $\frac{W}{2}$ (2) EW
 (3) $\frac{W}{\sqrt{3}}$ (4) $\frac{W}{2\sqrt{3}}$

69. In the figure a block A of mass m is attached at one end of a light spring and the other end of the spring is connected to another block B of mass $2m$ through a light string. A is held and B has obtained equilibrium. Now A is released. The acceleration of A just after that instant is a . The same thing is repeated for B . In that case the acceleration of B is b . Then the value of a/b is



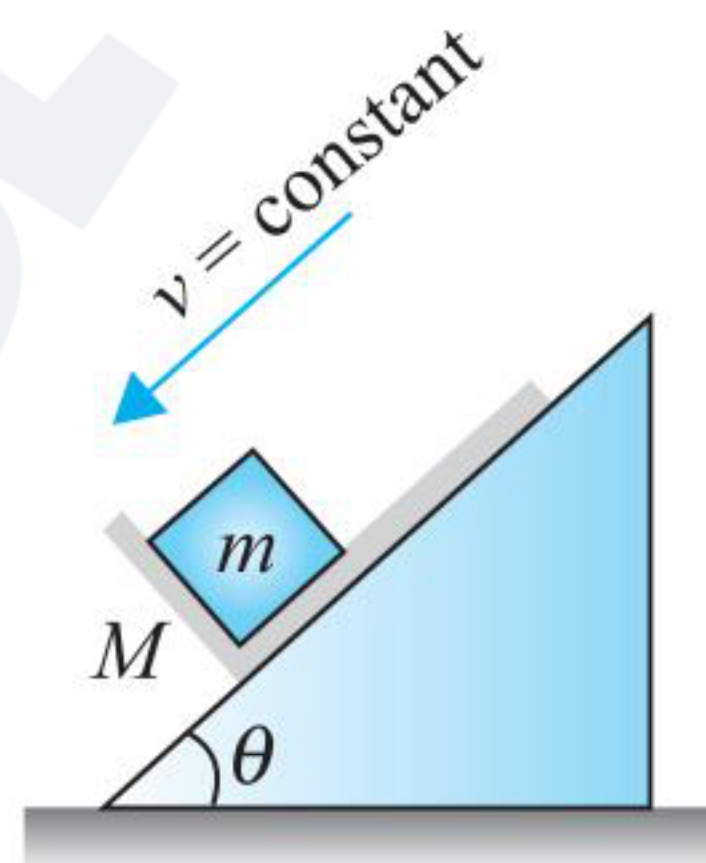
- (1) 2 (2) 1
 (3) 3 (4) 1.5

70. In the system shown, block A is of mass 4.0 kg and blocks B and C are of equal mass each of 3 kg. Find the acceleration in m/s^2 of block C , if the system is set free, [$g = 10 \text{ m/s}^2$]



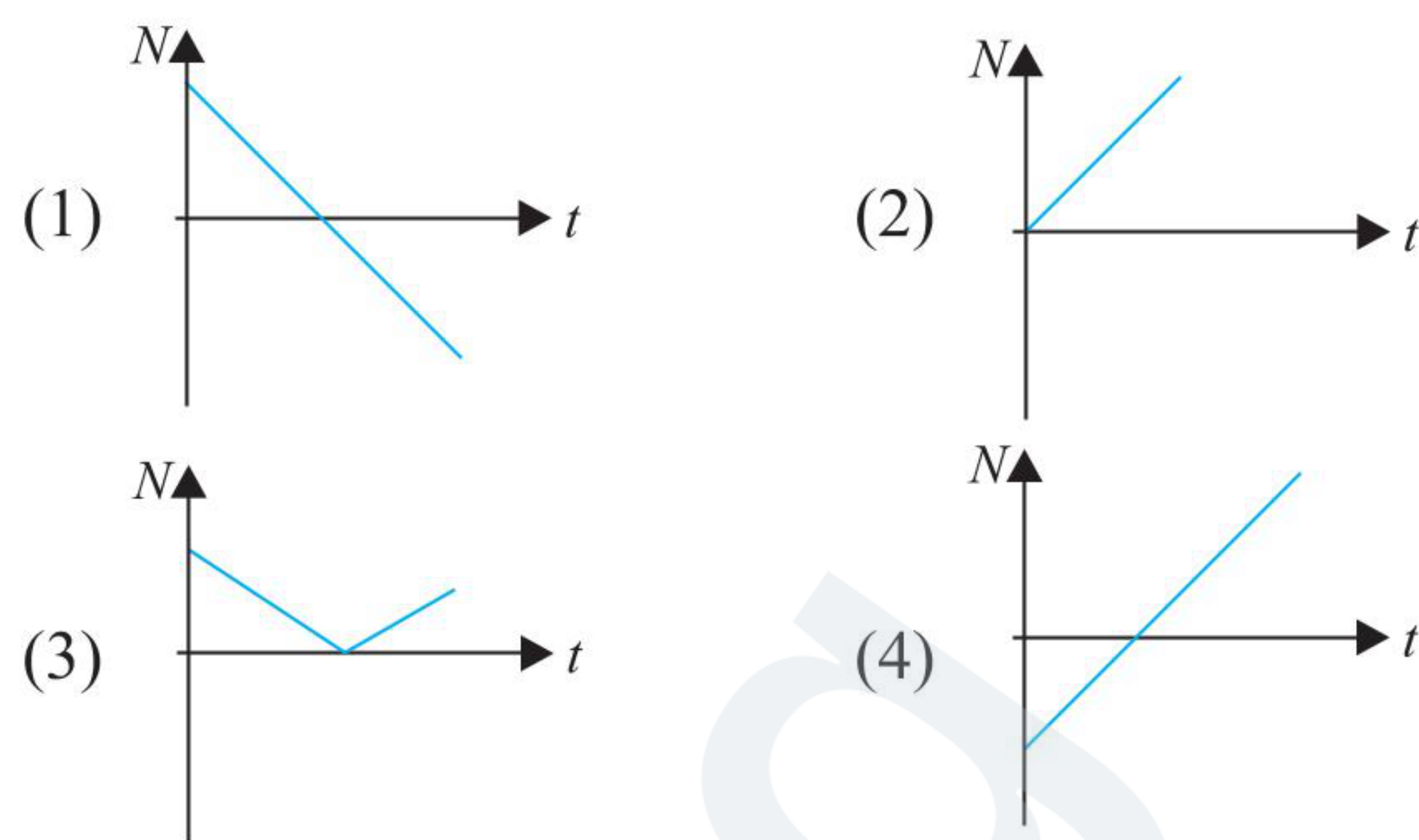
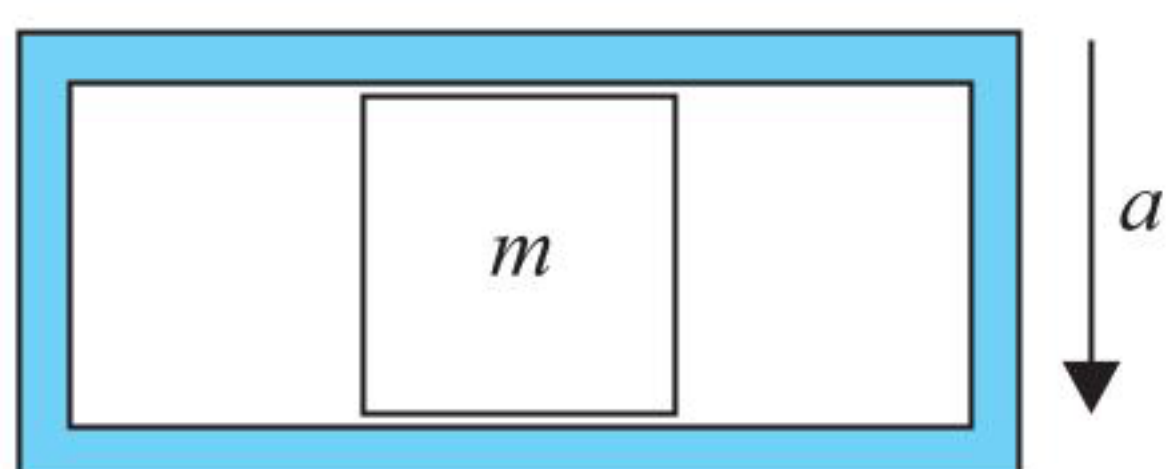
- (1) 1 m/s^2 (2) 2 m/s^2
 (3) 1.5 m/s^2 (4) 3 m/s^2

71. The given figure shows a block of mass m placed on a bracket of mass M . Bracket block system is moved downward with constant velocity on an incline. What is the magnitude of total force of bracket on block?

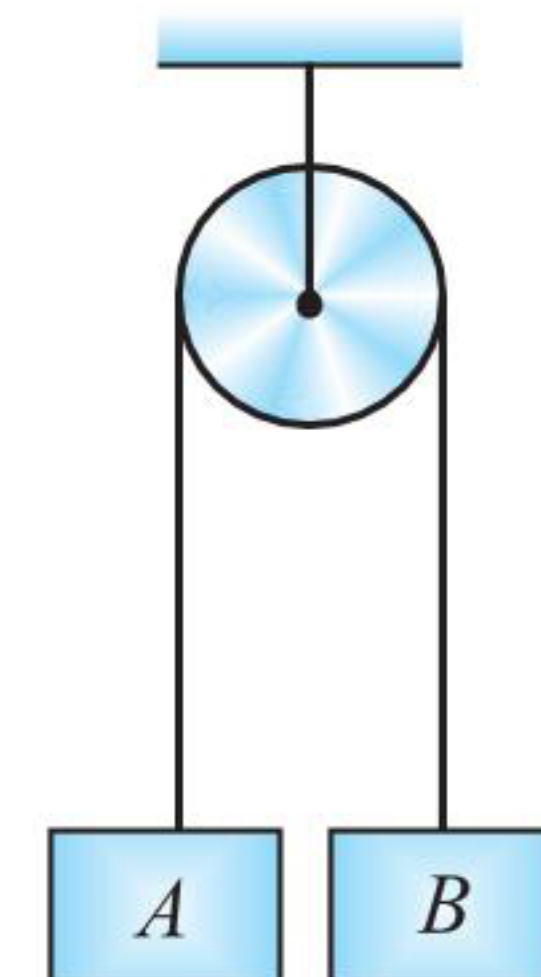


- (1) zero (2) $mg \sin \theta$
 (3) $mg \cos \theta$ (4) mg

72. A block of mass m is placed in a horizontal tube. Block fits the tube with a very small gap. A time dependent force F acts on the system such that the resultant downward acceleration of system is given by its time. Which of the following graphs represents normal reaction of tube on block. Take downward direction as positive for N . Initially block is in contact with lower surface.

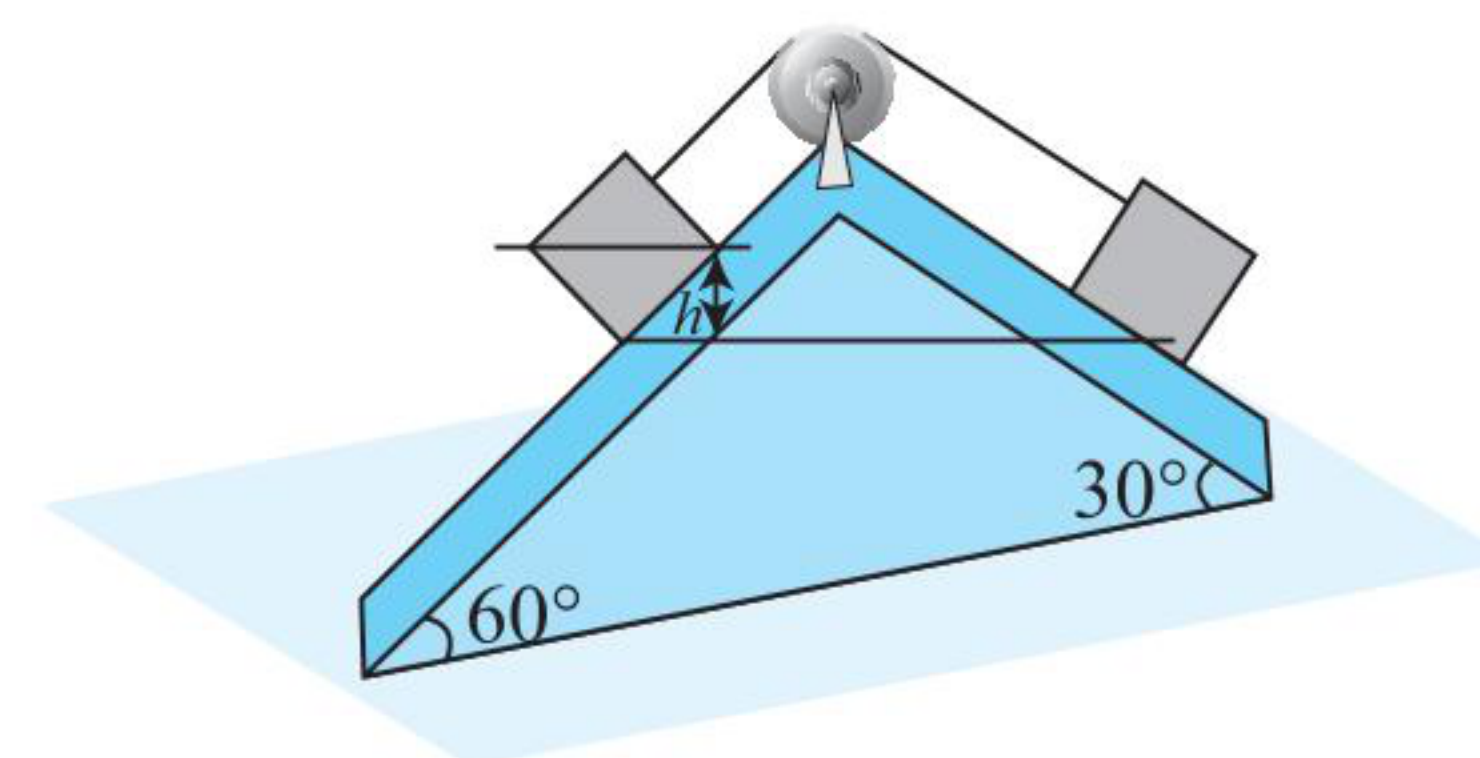


73. In the arrangement shown in the figure, the system is in equilibrium. Mass of the block A is M and that of the insect clinging to block B is m . Pulley and string are light. The insect loses contact with the block B and begins to fall. After how much time the insect and the block B will have a separation L between them.



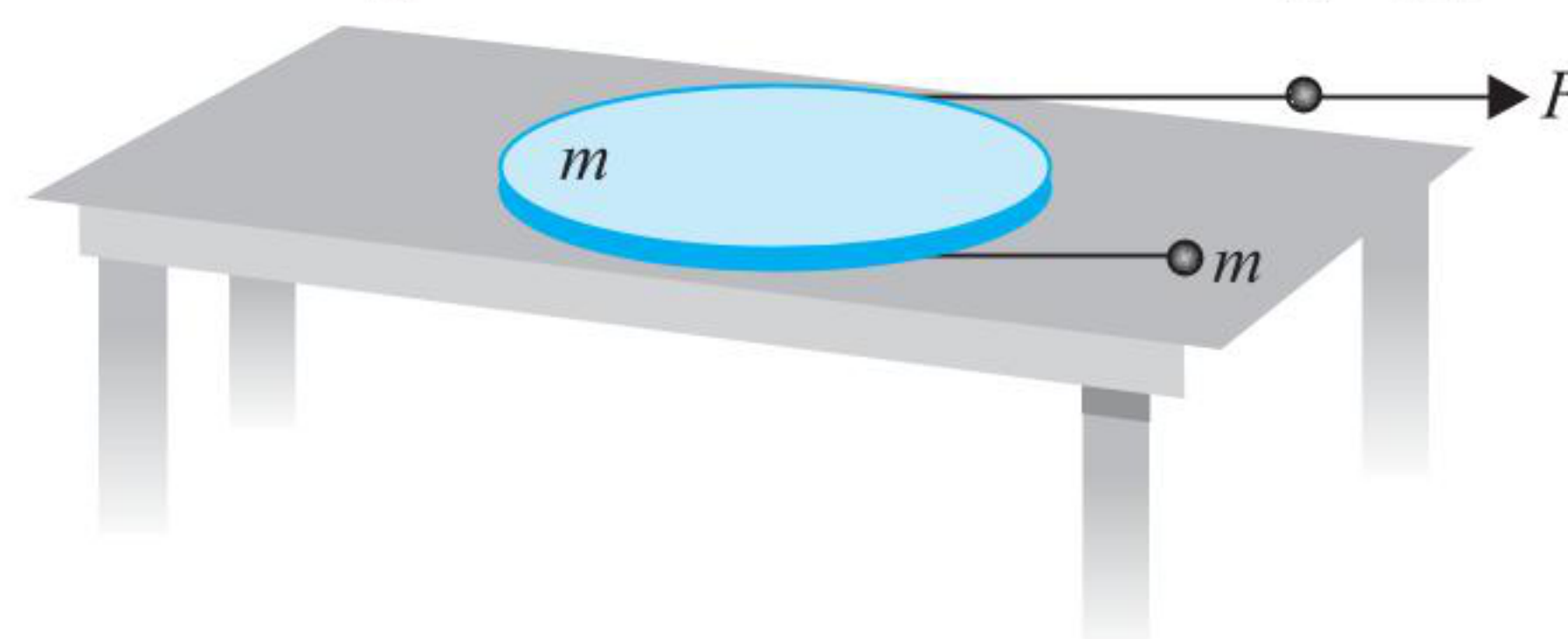
- (1) $\sqrt{\frac{M}{(2M-m)Lg}}$ (2) $\sqrt{\frac{(2M-m)L}{Mg}}$
 (3) $\sqrt{\frac{(2M+m)L}{Mg}}$ (4) $\sqrt{\frac{M}{(2M+m)Lg}}$

74. Two blocks of equal mass have been placed on two faces of a fixed wedge as shown in figure. The blocks are released from position where centre of one block is at a height h above the centre of the other block. Find the time after which the centre of the two blocks will be at same horizontal level. There is no friction anywhere.



- (1) $2\sqrt{2} \sqrt{\frac{h}{g}}$ (2) $2\sqrt{2} \sqrt{\frac{h}{g}}$
 (3) $2\sqrt{2} \sqrt{\frac{h}{g}}$ (4) $2\sqrt{2} \sqrt{\frac{h}{g}}$

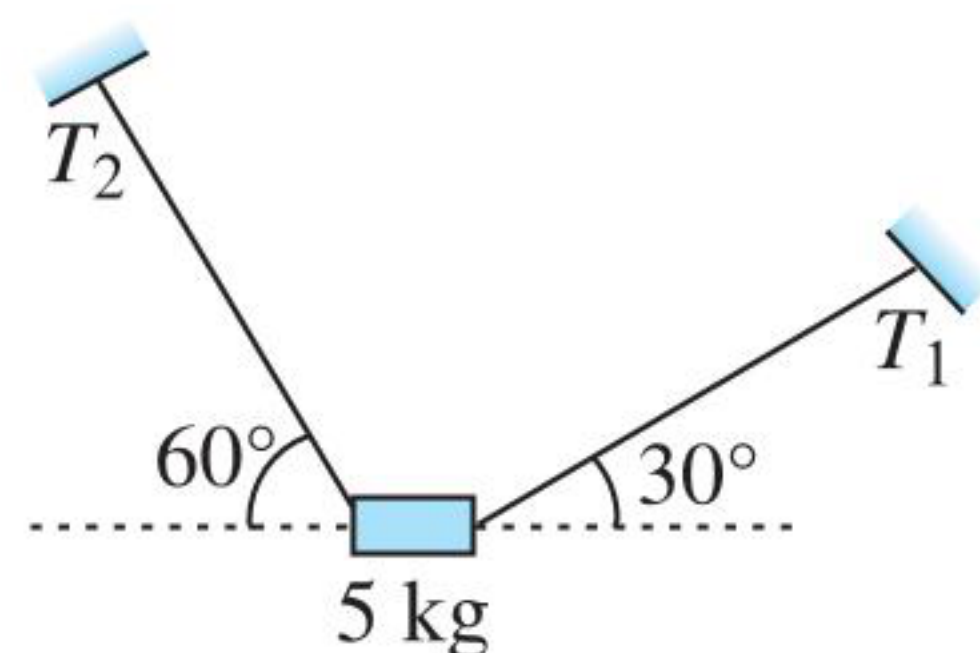
75. A disc of mass m lies flat on a smooth horizontal table. A light string runs halfway around it as shown in figure. One end of the string is attached to a particle of mass m and the other end is being pulled with a force F . There is no friction between the disc and the string. Find acceleration of the end of the string to which force is being applied.



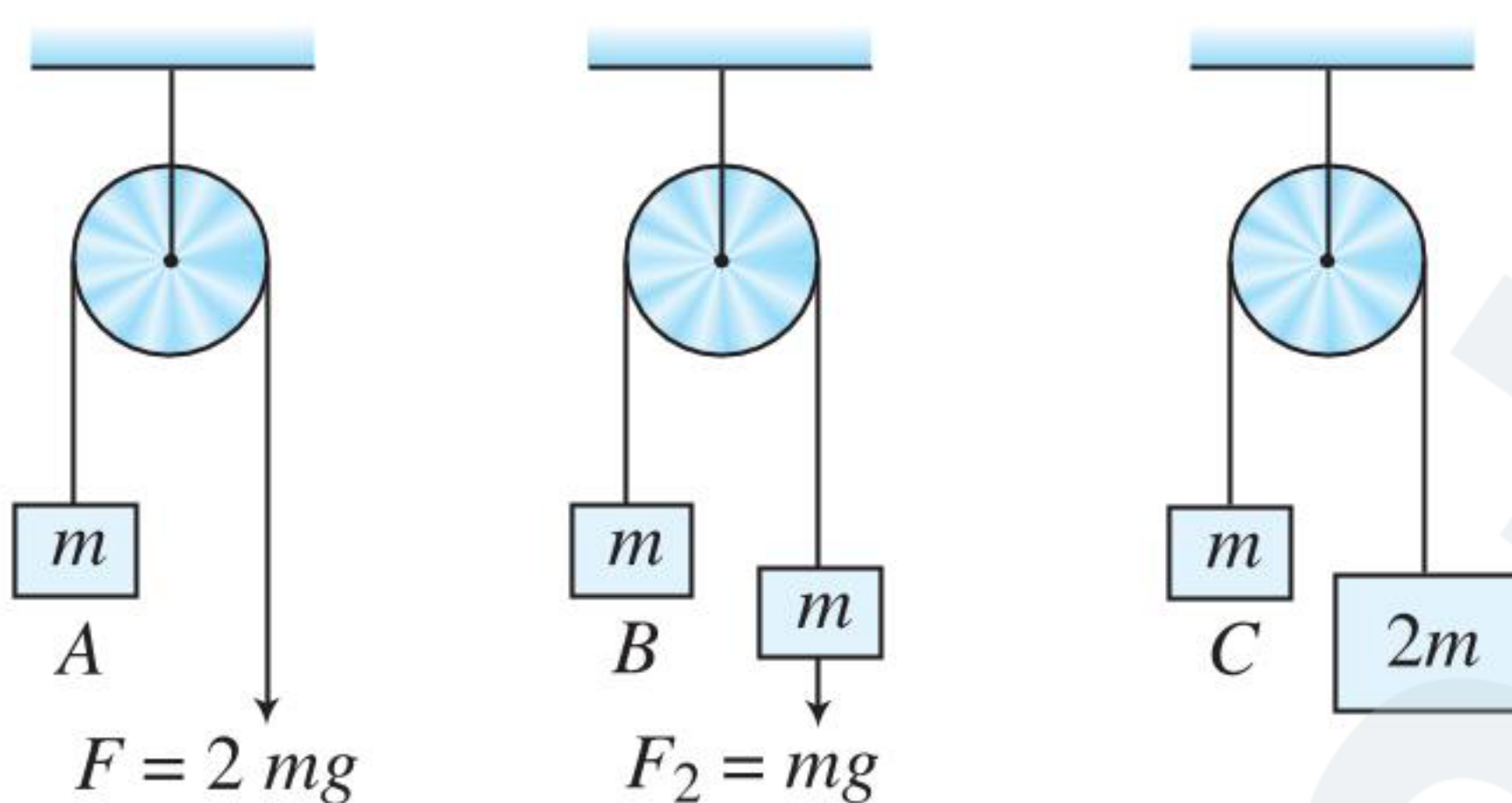
- (1) $\frac{2F}{m}$ (2) $\frac{3F}{m}$
 (3) $\frac{F}{m}$ (4) $\frac{5F}{m}$

Multiple Correct Answers Type

1. A body of mass 5 kg is suspended by the strings making angles 60° and 30° with the horizontal as shown in Figure ($g = 10 \text{ m s}^{-2}$). Then



- (1) $T_1 = 25 \text{ N}$ (2) $T_2 = 25 \text{ N}$
 (3) $T_1 = 25\sqrt{3} \text{ N}$ (4) $T_2 = 25\sqrt{3} \text{ N}$
2. The accelerations of a particle as observed from two different frames S_1 and S_2 have equal magnitudes of 2 m s^{-2} .
 (1) The relative acceleration of the frame may either be zero or 4 m s^{-2} .
 (2) Their relative acceleration may have any value between 0 and 4 m s^{-2} .
 (3) Both the frames may be stationary with respect to earth.
 (4) The frames may be moving with same acceleration in same direction.
3. In the given figure, blocks A, B, and C of mass m each have acceleration $a_1, a_2,$ and a_3 , respectively. F_1 and F_2 are external forces of magnitude $2mg$ and mg , respectively. Then

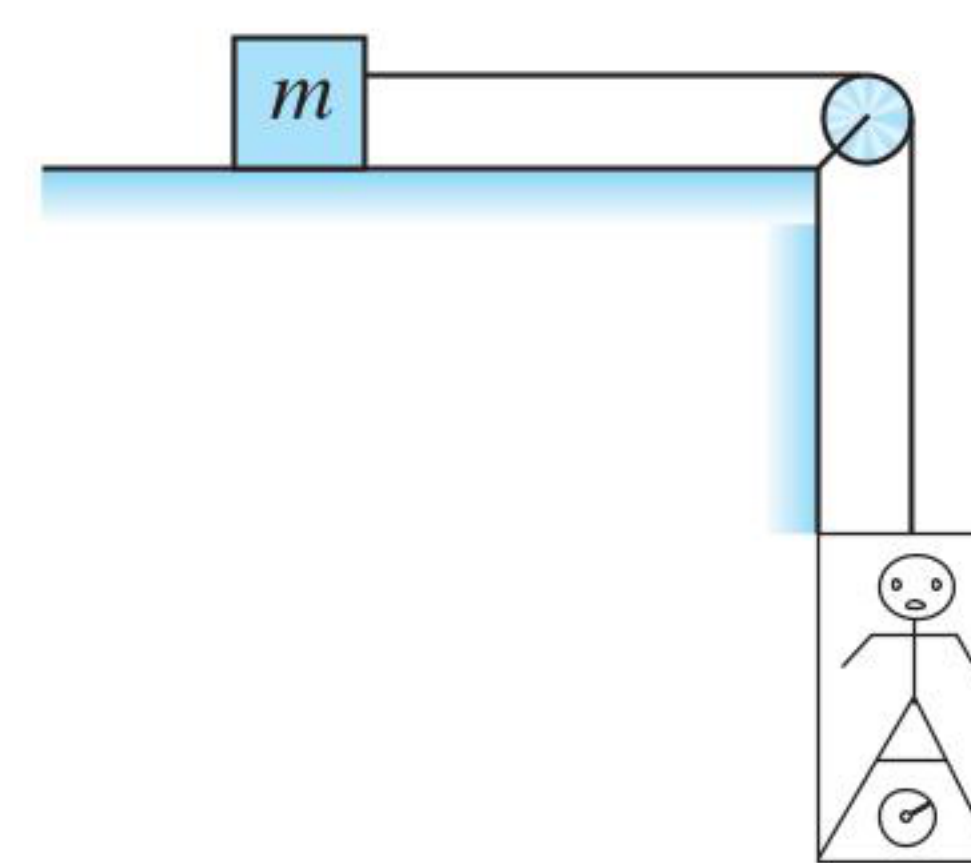


- (1) $a_1 \neq a_2 \neq a_3$ (2) $a_1 = a_2 \neq a_3$
 (3) $a_1 > a_2 > a_3$ (4) $a_1 \neq a_2 = a_3$
4. A block of mass m is placed in contact with one end of a smooth tube of mass M . A horizontal force F acts on the tube in each case (i) and (ii). Then



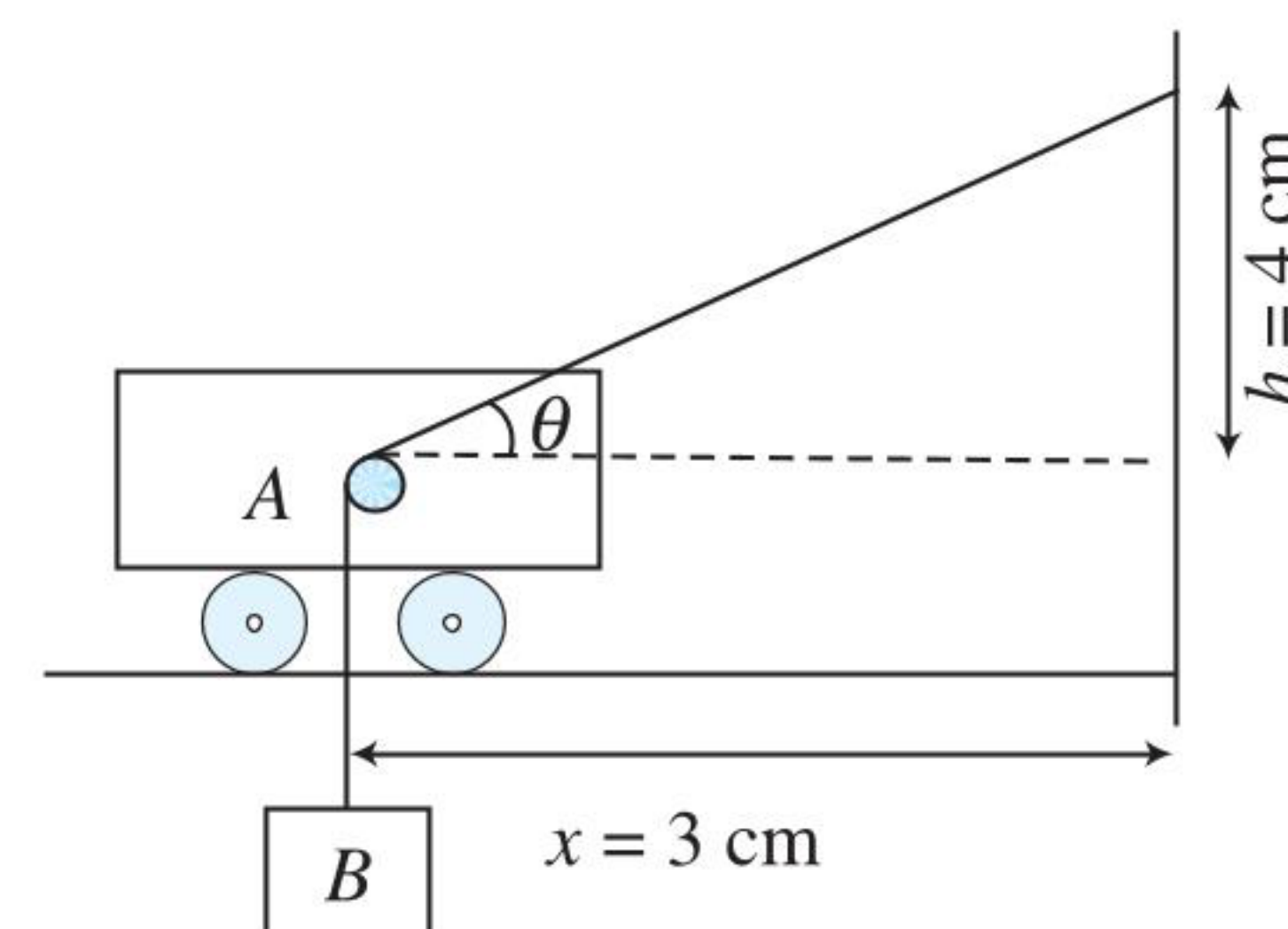
- (1) $a_m = 0$ and $a_M = \frac{F}{M}$ in (i)
 (2) $a_m = a_M = \frac{F}{M+m}$ in (i)
 (3) $a_m = a_M = \frac{F}{M+m}$ in (ii)
 (4) Force on m is $\frac{mF}{M+m}$ in (ii)

5. In the given figure, a man of true mass M is standing on a weighing machine placed in a cabin. The cabin is joined by a string with a body of mass m . Assuming no friction, and negligible mass of cabin and weighing machine, the measured mass of man is (normal force between the man and the machine is proportional to the mass)



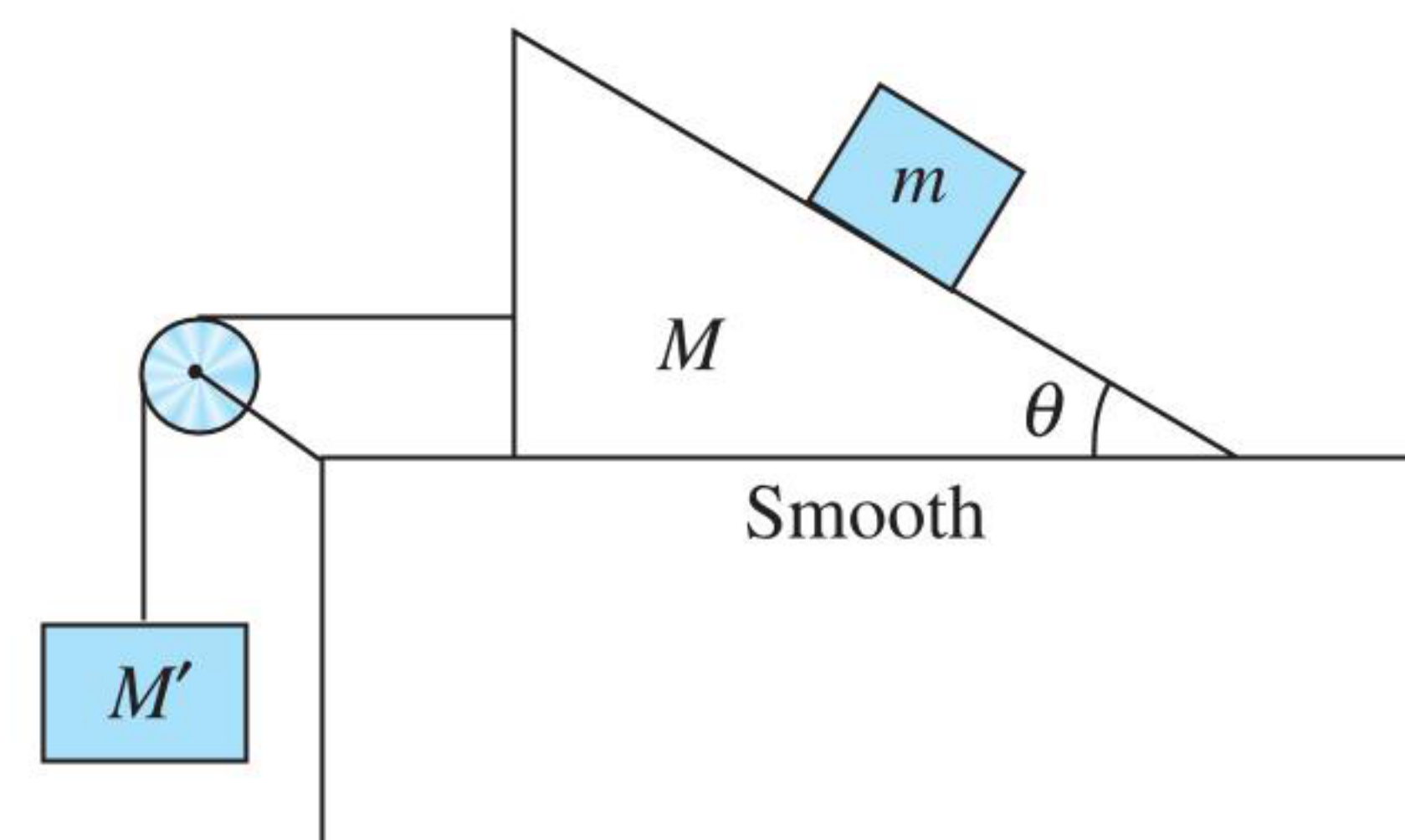
- (1) The measured mass of man is $\frac{Mm}{(M+m)}$.
 (2) The acceleration of man is $\frac{mg}{(M+m)}$.
 (3) The acceleration of man is $\frac{Mg}{(M+m)}$.
 (4) The measured mass of man is M .

6. The string shown in figure is passing over small smooth pulley rigidly attached to trolley A. If the speed of trolley is constant and equal to v_A towards right, speed and magnitude of acceleration of block B at the instant shown in figure are



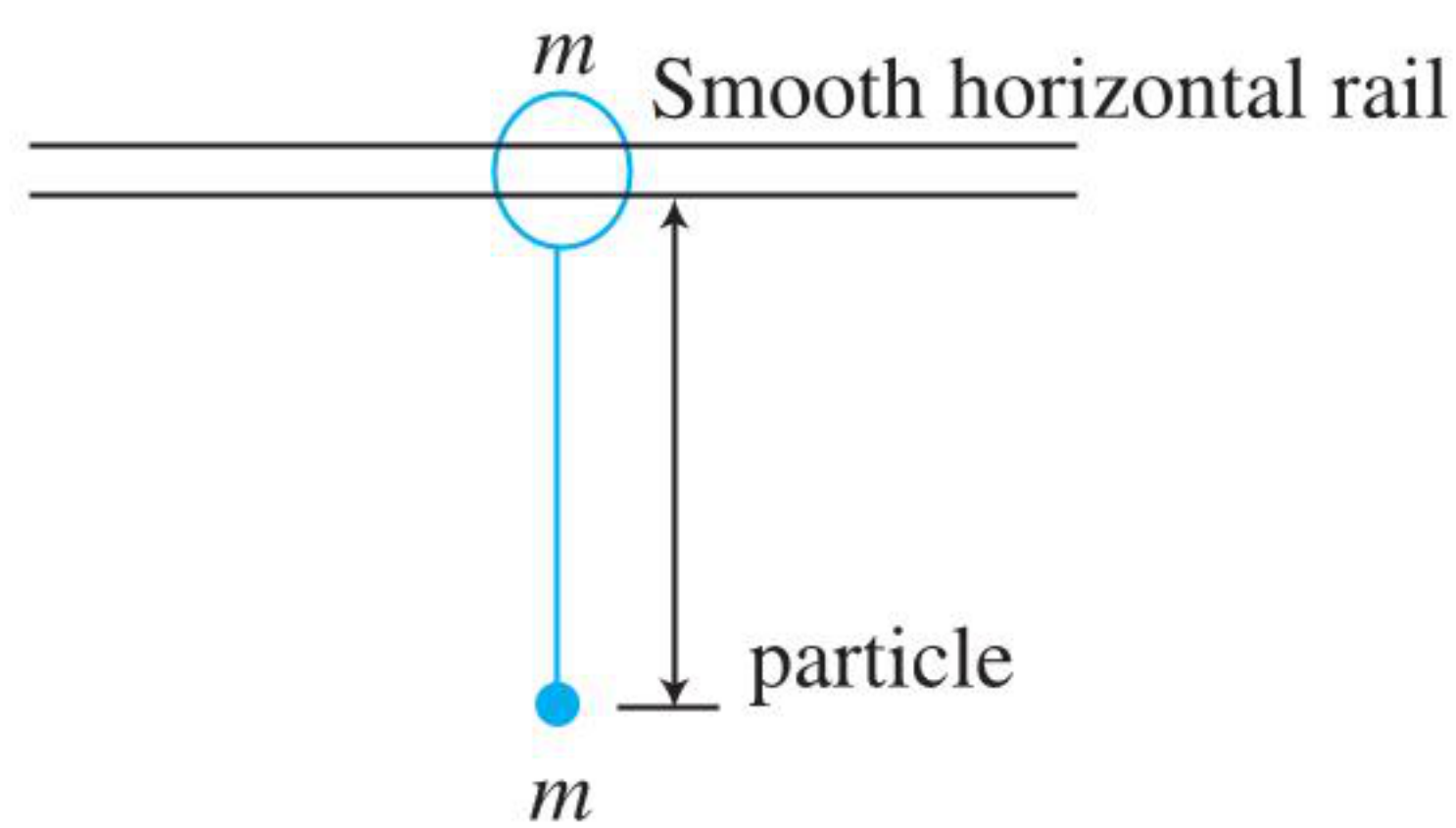
- (1) $v_B = v_A, a_B = 0$ (2) $a_B = 0$
 (3) $v_B = \frac{3}{5} v_A$ (4) $a_B = \frac{16v_A^2}{125}$

7. The given figure shows a block of mass m placed on a smooth wedge of mass M . Calculate the minimum value of M' and tension in the string, so that the block of mass m will move vertically downward with acceleration 10 m s^{-2} .

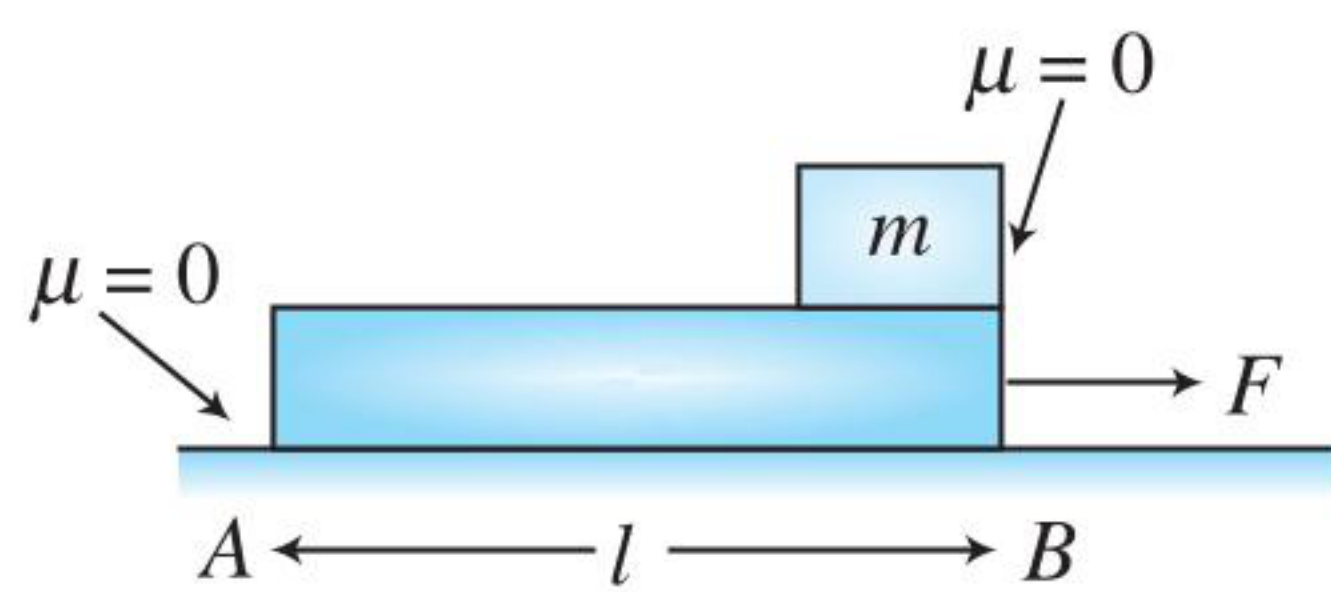


- (1) The value of M' is $\frac{M \cot \theta}{1 - \cot \theta}$.
 (2) The value of M' is $\frac{M \tan \theta}{1 - \tan \theta}$.
 (3) The value of tension in the string is $\frac{Mg}{\tan \theta}$.
 (4) The value of tension is $\frac{Mg}{\cot \theta}$.

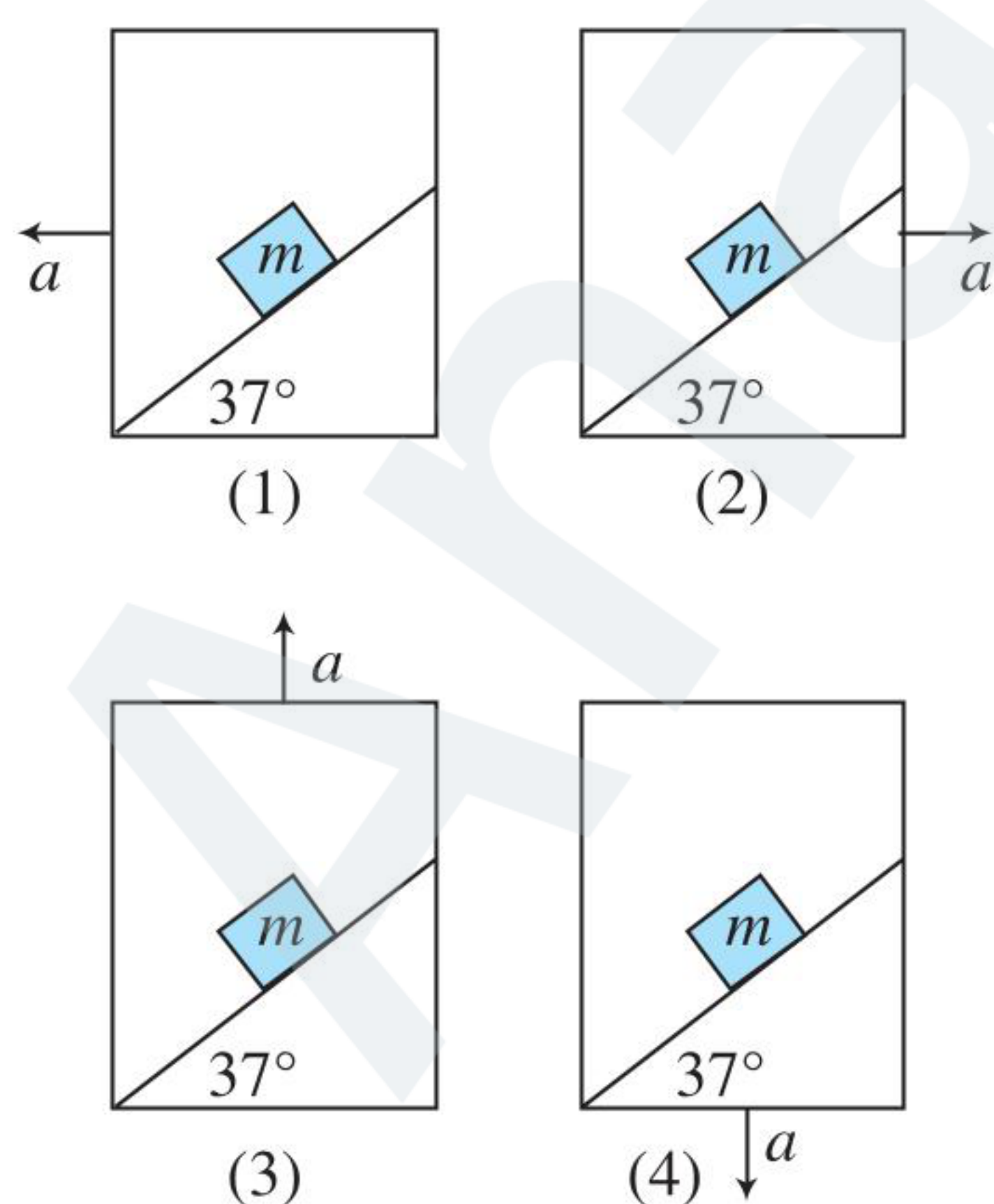
8. The ring shown in figure is given a constant horizontal acceleration ($a_0 = g/\sqrt{3}$). The maximum deflection of the string from the vertical is θ_0 . Then



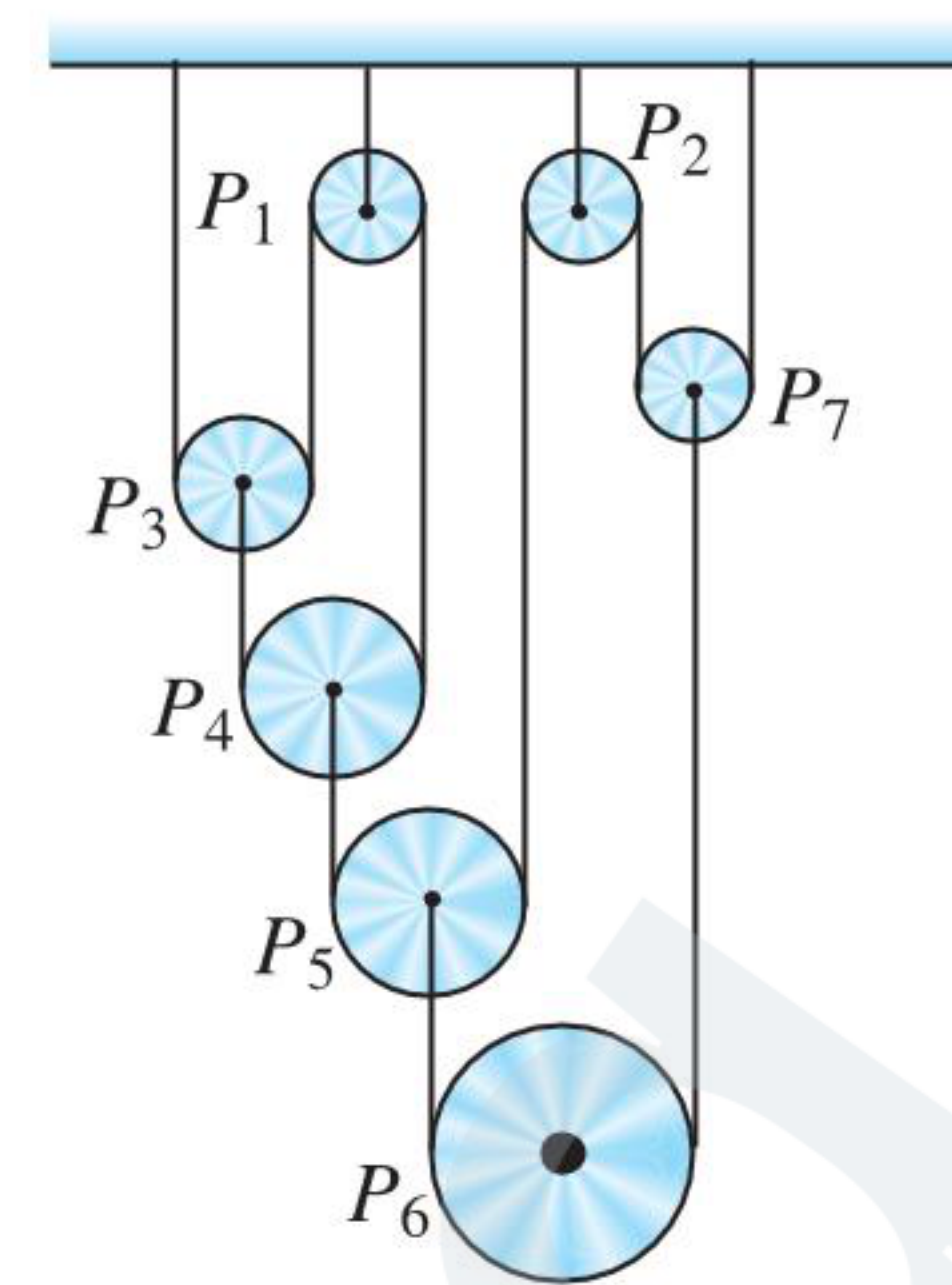
- (1) $\theta_0 = 30^\circ$
 (2) $\theta_0 = 60^\circ$
 (3) At maximum deflection, tension in string is equal to mg .
 (4) At maximum deflection, tension in string is equal to $\frac{2mg}{\sqrt{3}}$.
9. In the given figure, a small block is kept on m . Then



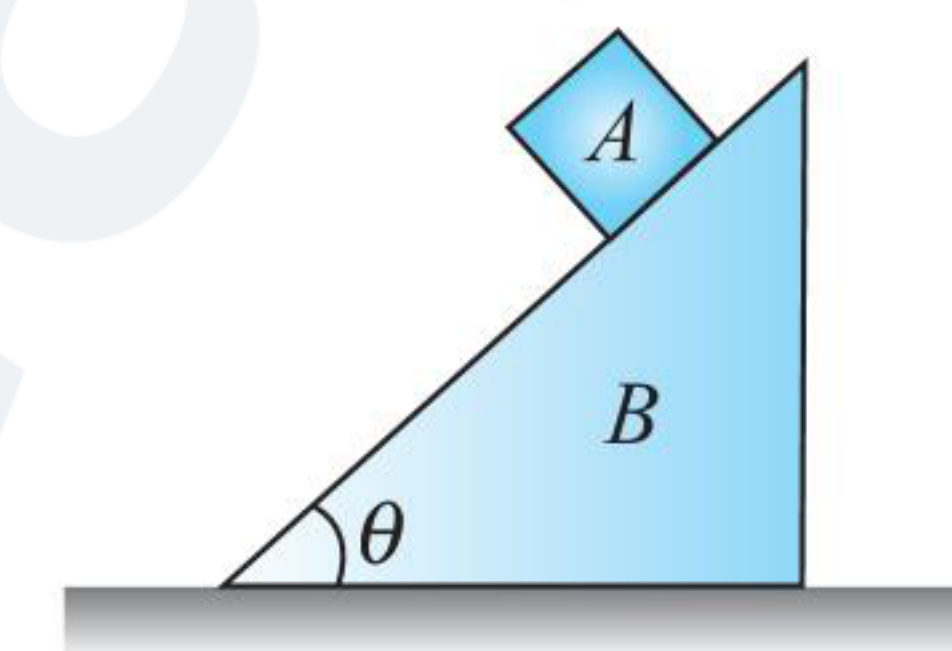
- (1) The acceleration of m w.r.t. ground is F/m .
 (2) The acceleration of m w.r.t. ground is zero.
 (3) The time taken by m to separate from M is $\sqrt{\frac{2lm}{F}}$.
 (4) The time taken by m to separate from M is $\sqrt{\frac{2lM}{F}}$.
10. A block of mass m is placed on a wedge. The wedge can be accelerated in four manners marked as (1), (2), (3), and (4) as shown in figure. If the normal reactions in situations (1), (2), (3), and (4) are N_1 , N_2 , N_3 , and N_4 , respectively, and acceleration with which the block slides on the wedge in the situations are b_1 , b_2 , b_3 , and b_4 , respectively. Then



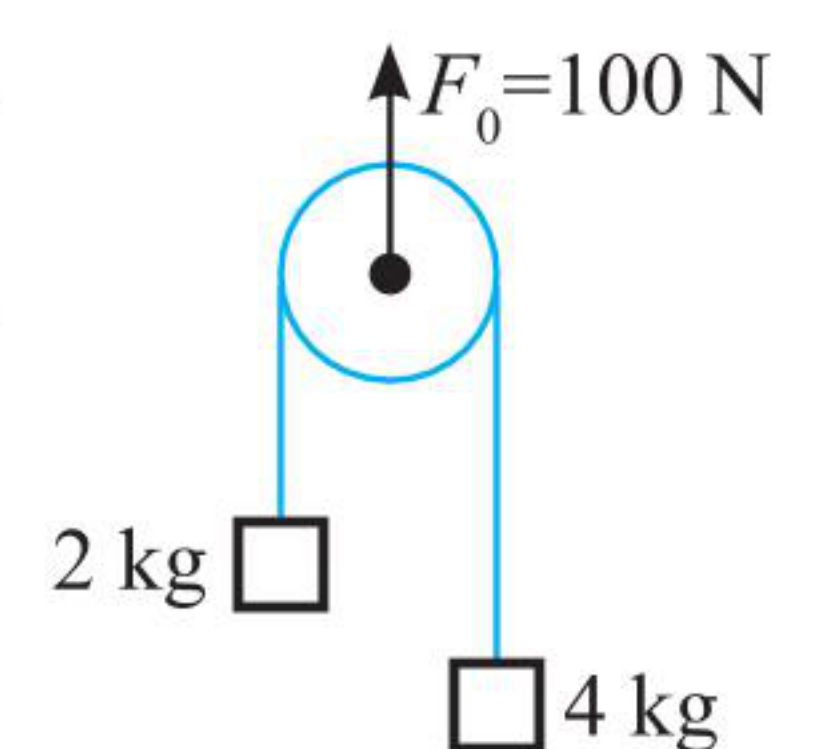
- (1) $N_3 > N_1 > N_2 > N_4$
 (2) $N_4 > N_3 > N_1 > N_2$
 (3) $b_2 > b_3 > b_4 > b_1$
 (4) $b_2 > b_3 > b_1 > b_4$
11. Seven pulleys are connected with the help of three light strings as shown in figure. Consider P_3 , P_4 , P_5 as light pulleys and pulleys P_6 and P_7 have masses m each. For this arrangement, mark the correct statement(s).



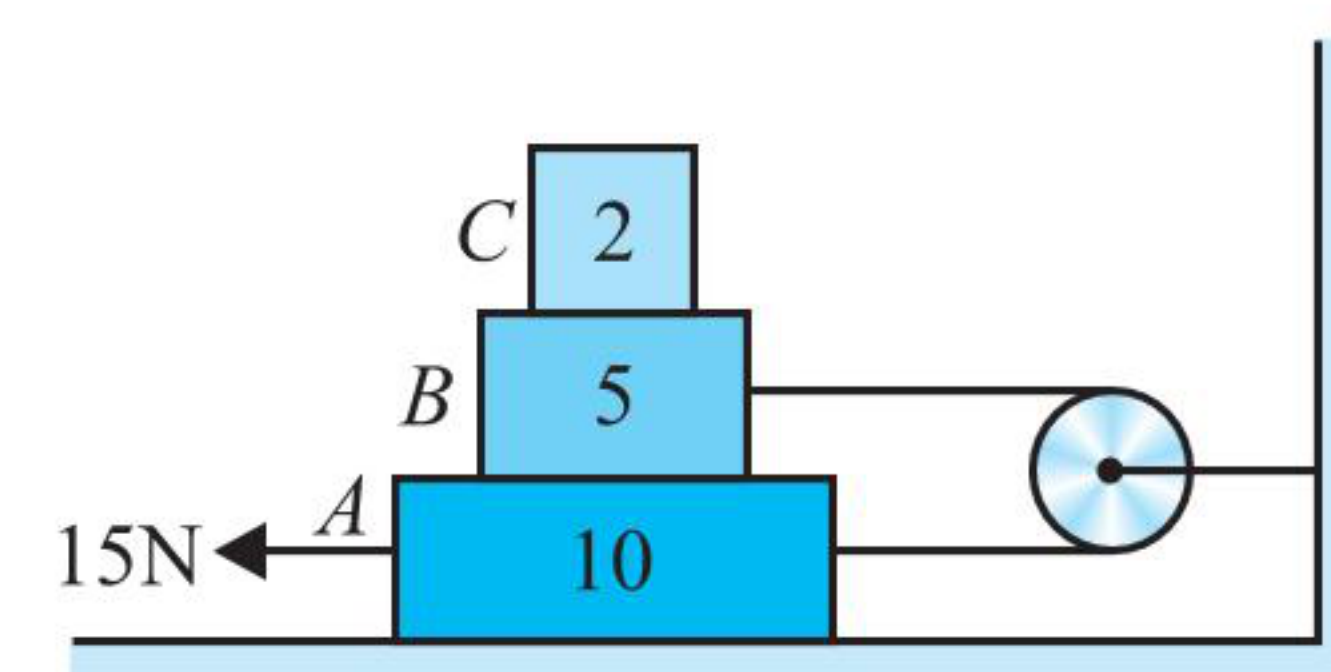
- (1) Tension in the string connecting P_1 , P_3 , and P_4 is zero.
 (2) Tension in the string connecting P_1 , P_3 and P_4 is $mg/3$.
 (3) Tensions in all the three strings are same and equal to zero.
 (4) Acceleration of P_6 is g downwards and that of P_7 is g upwards.
12. In the figure shown, A and B are free to move. All the surfaces are smooth. Then ($0 < \theta < 90^\circ$)



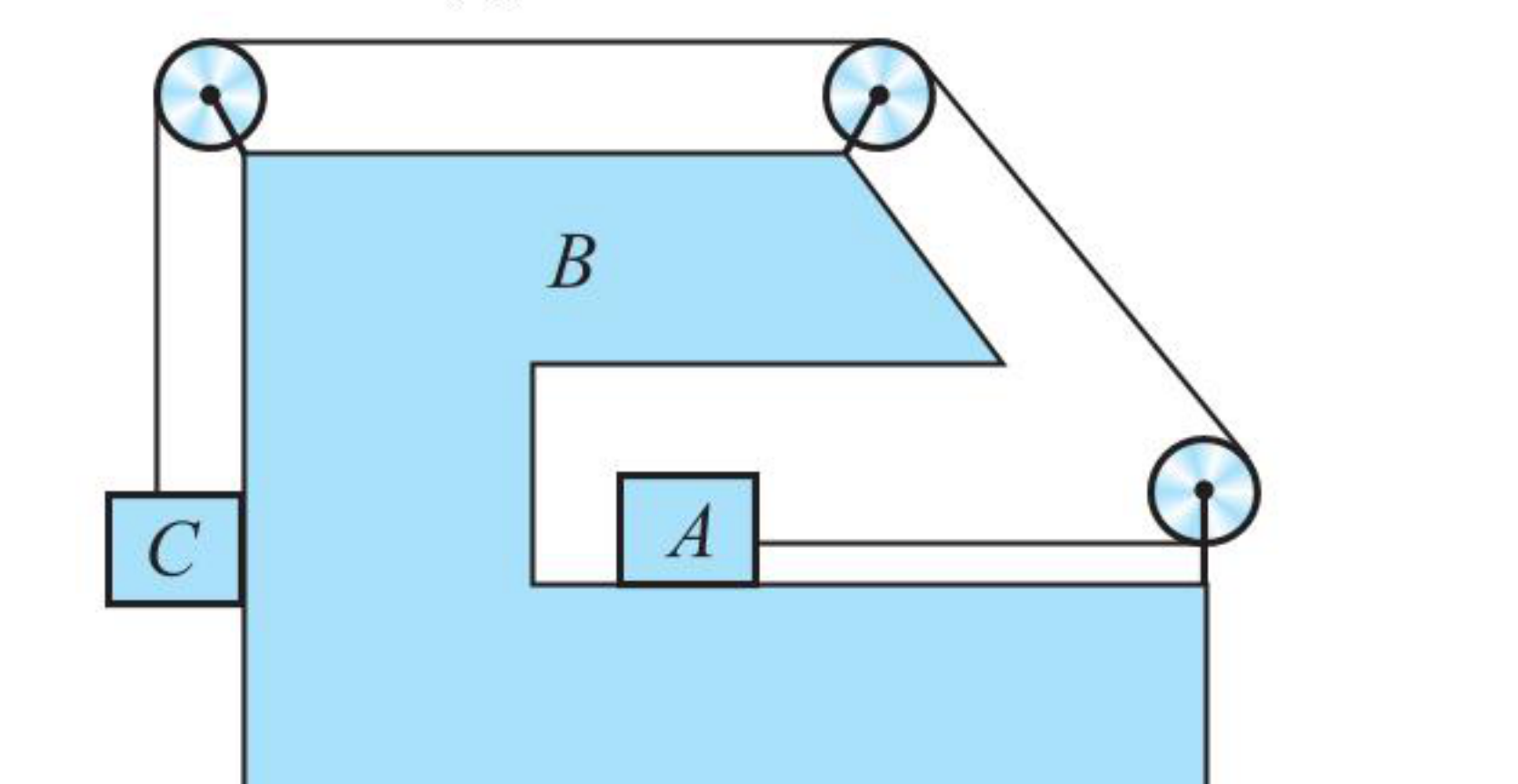
- (1) the acceleration of A will be more than $g \sin \theta$
 (2) the acceleration of A will be less than $g \sin \theta$
 (3) normal force on A due to B will be more than $mg \cos \theta$
 (4) normal force on A due to B will be less than $mg \cos \theta$
13. Two blocks of masses $m_1 = 2$ kg and $m_2 = 4$ kg hang over a massless pulley as shown in the figure. A force $F_0 = 100$ N acting at the axis of the pulley accelerates the system upwards. Then



- (1) acceleration of 2 kg mass is 15 m/s^2
 (2) acceleration of 4 kg mass is 2.5 m/s^2
 (3) acceleration of both the masses is same
 (4) acceleration of both the masses is upward
14. In the figure below, all surfaces are smooth, string is light and pulley is frictionless. Mark the correct statement(s).



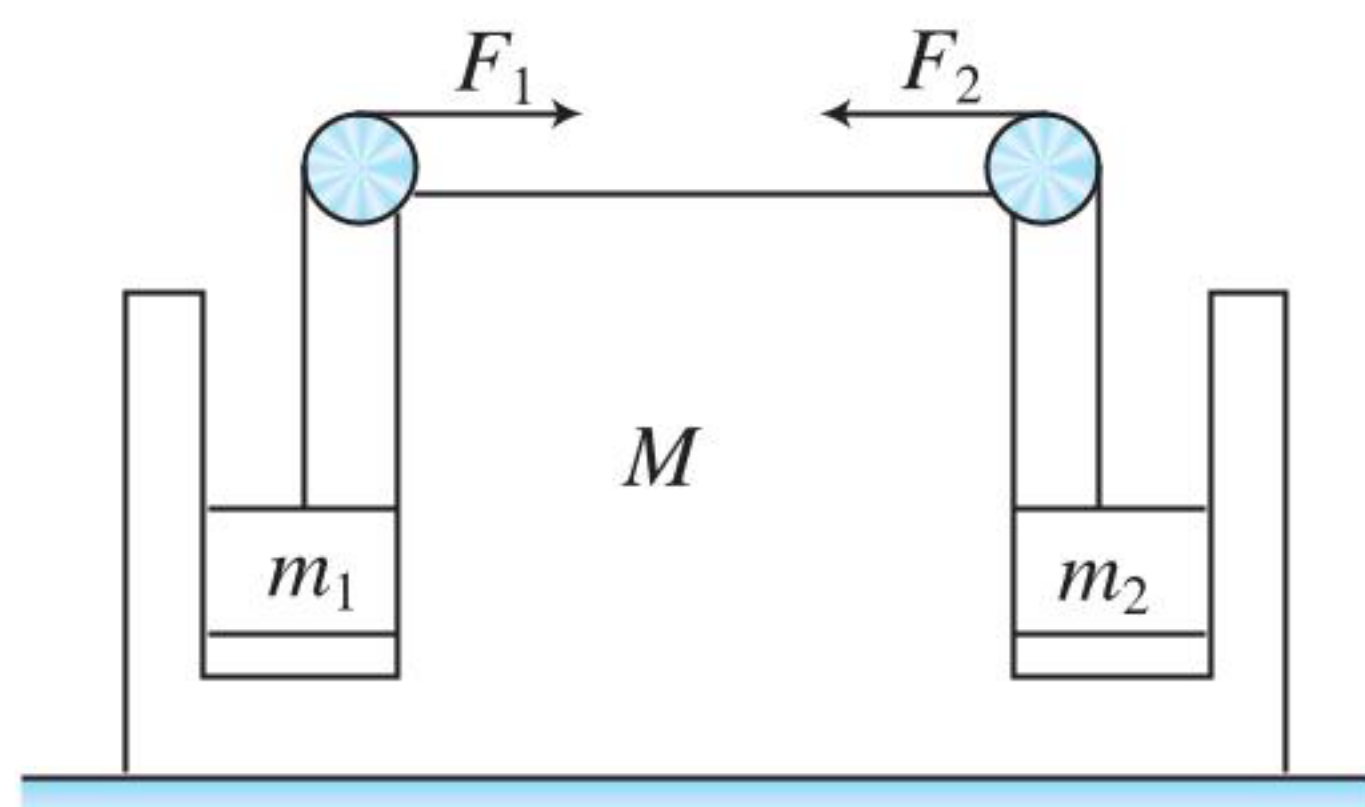
- (1) Acceleration of block A is 1 m/s^2
 (2) Acceleration of block A and B will be same
 (3) Acceleration of block C is zero
 (4) Tension in string between pulley and wall is 10 N .
15. For given figure, m_A is 30 kg, $m_B = m_C = 5$ kg, respectively. All contact surfaces are smooth ($g = 10 \text{ m/s}^2$). Select the correct statement(s).



- Which of the curves in graph correctly gives the acceleration magnitude as a function of the ratio m/M_{tot} (vertical axis is for acceleration)?
 (1) 1 (2) 2
 (3) 3 (4) 4
- Which of them gives the tension in the connecting cord (vertical axis is for tension)?
 (1) 1 (2) 2
 (3) 3 (4) 4

For Problems 3–5

For the system shown in figure, there is no friction anywhere. Masses m_1 and m_2 can move up or down in the slots cut in mass M . Two non-zero horizontal forces F_1 and F_2 are applied as shown. The pulleys are massless and frictionless. Given $m_1 \neq m_2$.



- According to the above passage, which is correct?
 (1) It is not possible for the entire system to be in equilibrium.
 (2) For some values of F_1 and F_2 , it is possible that the entire system is in equilibrium.
 (3) It is possible that F_1 and F_2 are applied in such a way that m_1 and m_2 remain in equilibrium but M does not.
 (4) None of the above.
- Let F_1 and F_2 be applied in such a way that m_1 and m_2 do not move w.r.t. M . Then what is the magnitude of the acceleration of M ? Let $m_1 > m_2$.
 (1) $\frac{(m_1 + m_2)g}{M + m_1 + m_2}$ (2) $\frac{(m_1 - m_2)g}{M}$
 (3) $\frac{(m_1 - m_2)g}{M + m_1 + m_2}$ (4) $\frac{F_1 - F_2}{M}$
- Let F_1 and F_2 be applied in such a way that accelerations of both masses m_1 and m_2 are same both in magnitude and direction. Then
 (1) $\frac{F_1}{m_1} - \frac{m_2 g}{m_1} = \frac{F_2}{m_2} - \frac{m_1 g}{m_2}$ (2) $\frac{F_1}{m_2} = \frac{F_2}{m_1}$
 (3) $\frac{F_1}{m_1} + \frac{m_2 g}{m_1} = \frac{F_2}{m_2} + \frac{m_1 g}{m_2}$ (4) $\frac{F_1}{m_1} = \frac{F_2}{m_2}$

For Problems 6–7

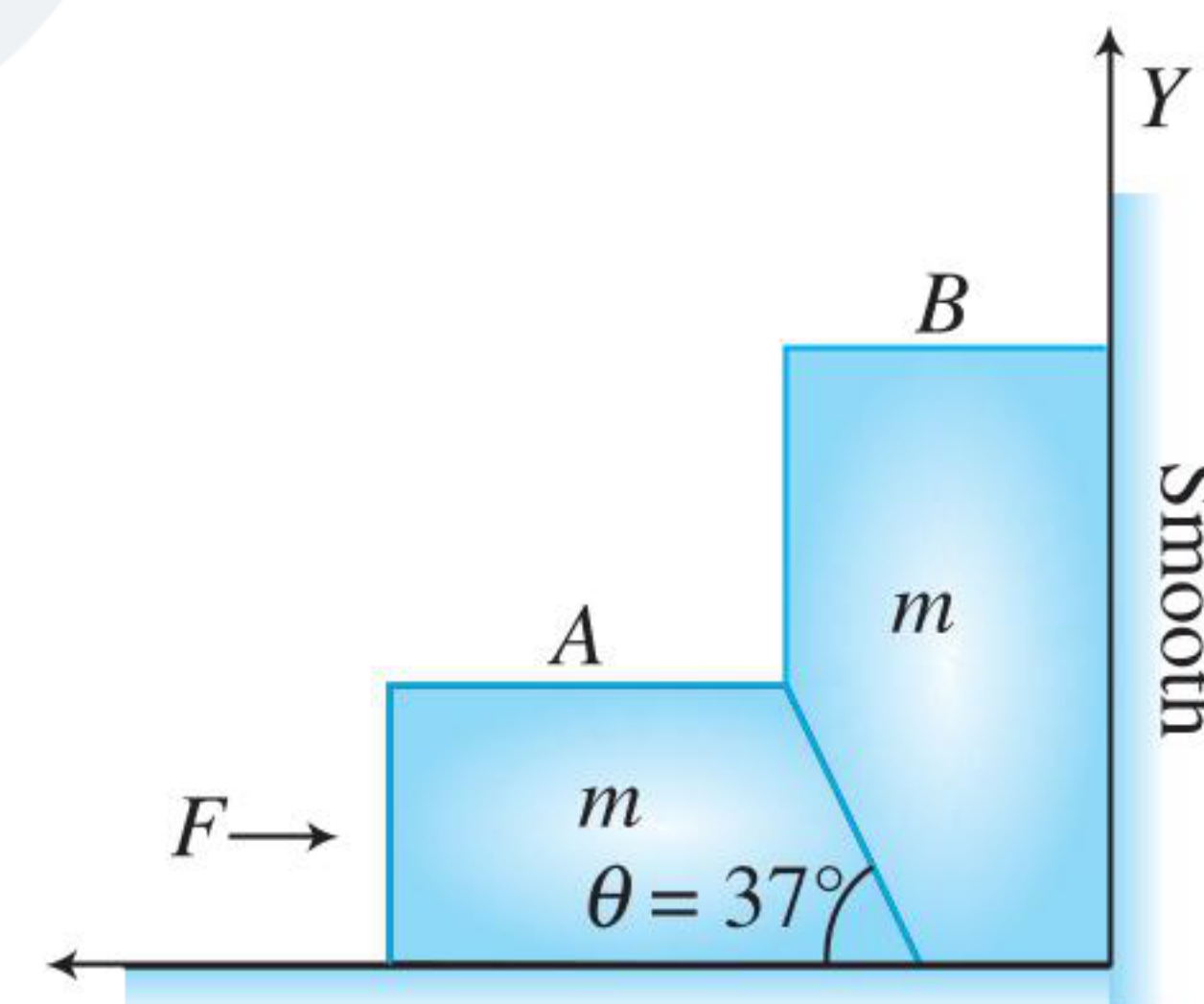
A time-varying force $F = 6t - 2t^2$ N, at $t = 0$ starts acting on a body of mass 2 kg initially at rest, where t is in second. The force is withdrawn just at the instant when the body comes to rest again. We can see that at $t = 0$, the force $F = 0$. Now answer the following:

- Find the duration for which the force acts on the body.
 (1) 2 s (2) 3 s
 (3) 3.5 s (4) 4.5 s

- Find the time when the velocity attained by the body is maximum.
 (1) 2 s (2) 3 s
 (3) 3.5 s (4) 4.5 s
- Mark the correct statement:
 (1) Velocity of the body is maximum when force acting on the body is maximum for the first time.
 (2) The velocity of the body becomes maximum when force acting on the body becomes zero again
 (3) When force becomes zero again, velocity of the body also becomes zero at that instant.
 (4) All of the above

For Problems 9–11

Two smooth blocks are placed at a smooth corner as shown in figure. Both the blocks are having mass m . We apply a force F on the small block m . Block A presses block B in the normal direction, due to which pressing force on vertical wall will increase, and pressing force on the horizontal wall decreases, as we increases F ($\theta = 37^\circ$ with horizontal).

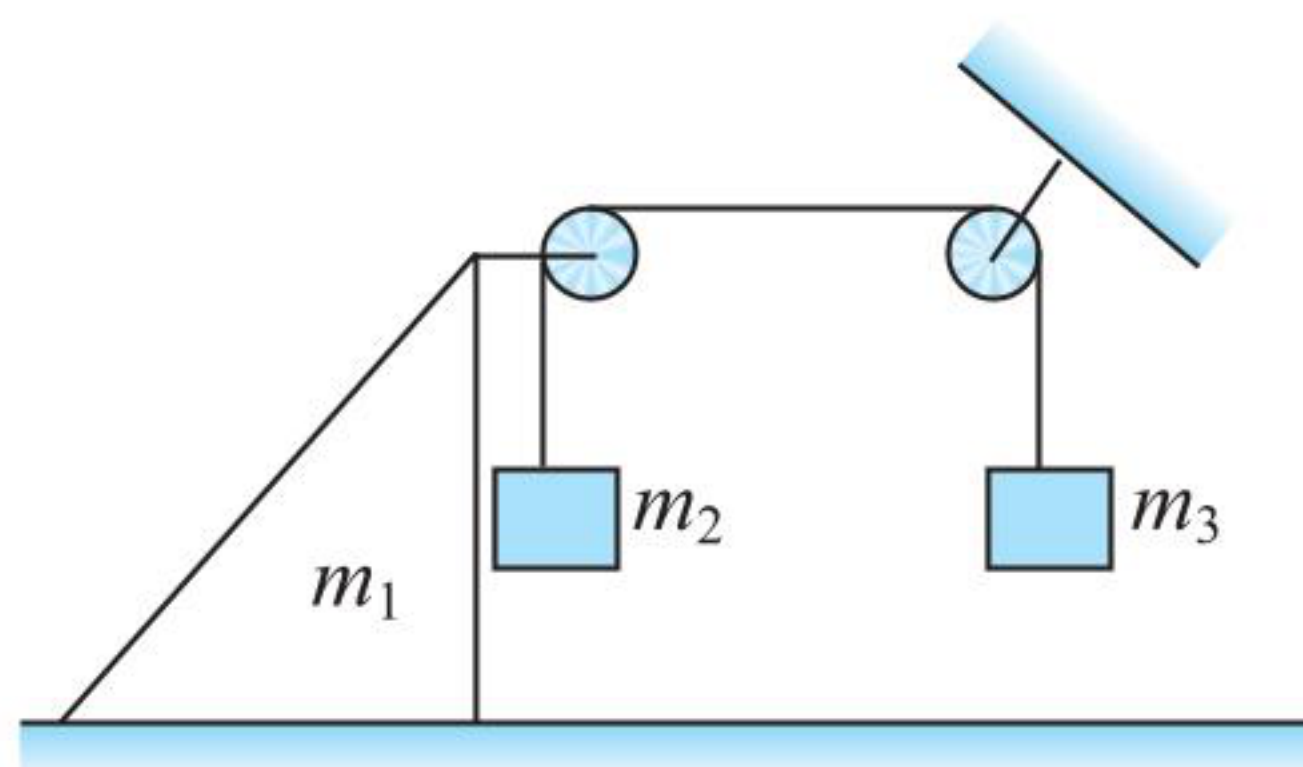


As soon as the pressing force on the horizontal wall by block B becomes zero, it will lose contact with ground. If the value of F further increases, block B will accelerate in the upward direction and simultaneously block A will move towards right.

- What is the minimum value of F to lift block B from ground?
 (1) $\frac{25}{12} mg$ (2) $\frac{5}{3} mg$
 (3) $\frac{3}{4} mg$ (4) $\frac{4}{3} mg$
- If both the blocks are stationary, the force exerted by ground on block A is
 (1) $mg + \frac{3F}{4}$ (2) $mg - \frac{3F}{4}$
 (3) $mg + \frac{4F}{3}$ (4) $mg - \frac{4F}{3}$
- If the acceleration of block A is a rightwards, then the acceleration of block B will be
 (1) $\frac{3a}{4}$, upwards (2) $\frac{4a}{3}$, upwards
 (3) $\frac{3a}{5}$, upwards (4) $\frac{4a}{5}$, upwards

For Problems 12–14

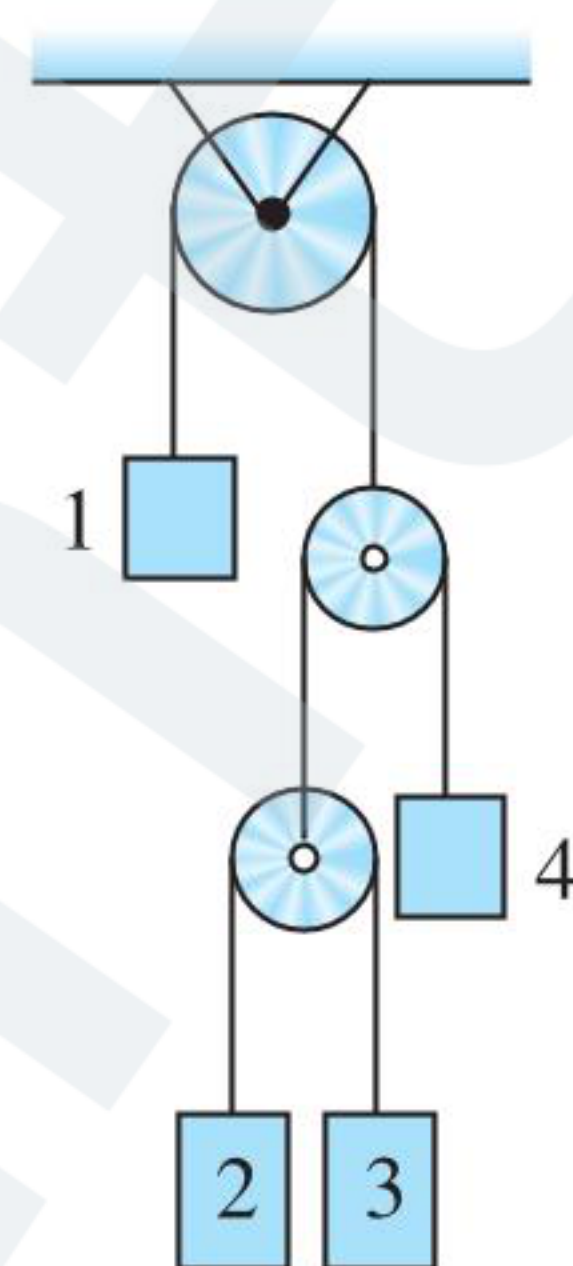
In Figure, both pulleys and the string are massless and all the surfaces are frictionless. Given $m_1 = 1$ kg, $m_2 = 2$ kg, $m_3 = 3$ kg.



12. Find the tension in the string.
- (1) $\frac{120}{7}$ N (2) $\frac{240}{7}$ N
- (3) $\frac{130}{7}$ N (4) None of these
13. The acceleration of m_2 is
- (1) $\frac{40}{7}$ m s⁻² (2) $\frac{30}{7}$ m s⁻²
- (3) $\frac{20}{7}$ m s⁻² (4) $\frac{\sqrt{17}g}{7}$ m s⁻²
14. The acceleration of m_3 is
- (1) $\frac{40}{7}$ m s⁻² (2) $\frac{30}{7}$ m s⁻²
- (3) $\frac{20}{7}$ m s⁻² (4) None of these

For Problems 15–18

In the arrangement shown in figure, all pulleys are smooth and massless. When the system is released from the rest, acceleration of blocks 2 and 3 relative to 1 are 1 m s⁻² downwards and 5 m s⁻² downwards, respectively. Acceleration of block 3 relative to 4 is zero.

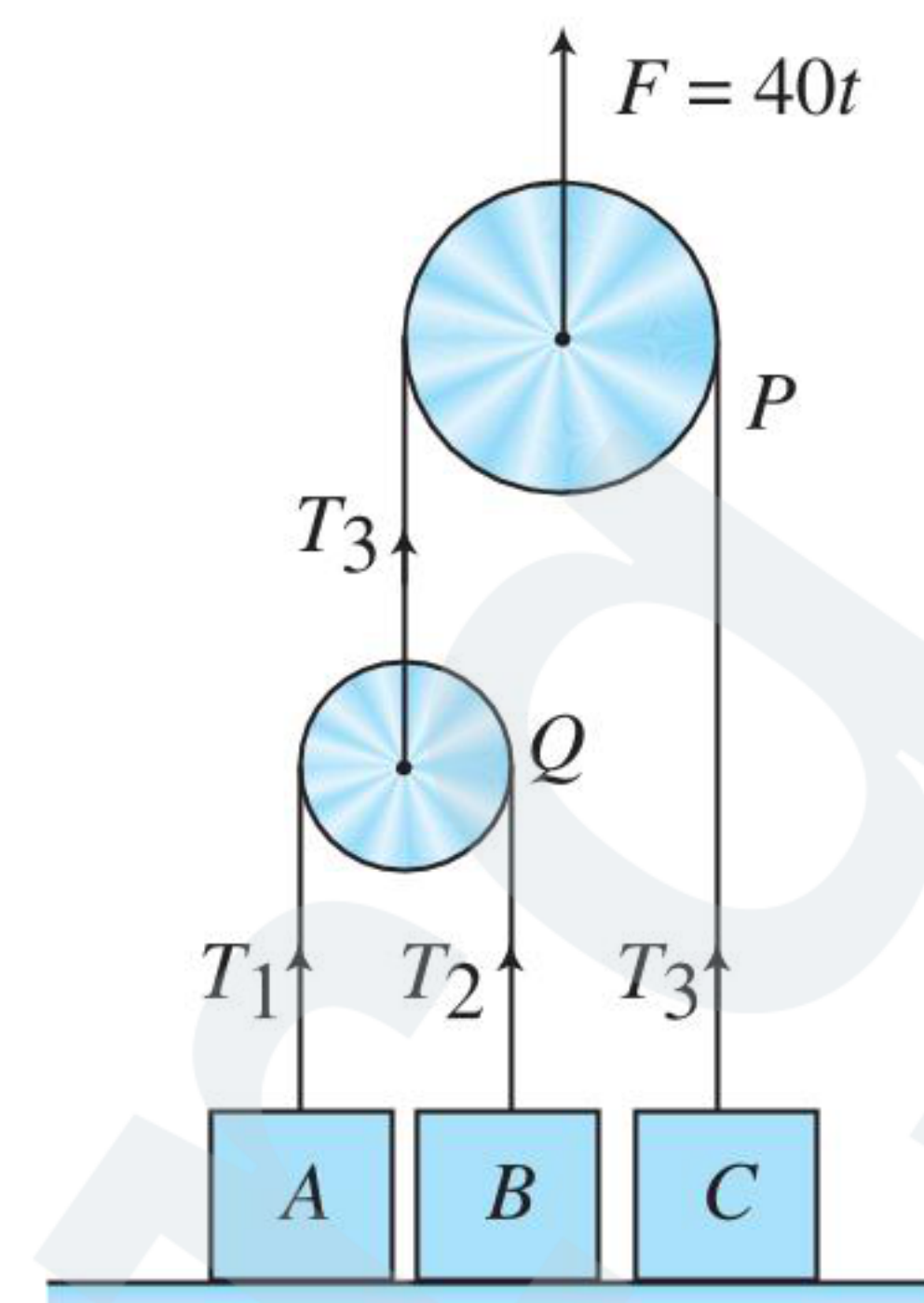


15. Find the absolute acceleration of block 1.
- (1) 2 m s⁻² upwards (2) 1 m s⁻² downwards
- (3) 3 m s⁻² upwards (4) 1.5 m s⁻² downwards
16. Find the absolute acceleration of block 2.
- (1) 2 m s⁻² downwards (2) 1 m s⁻² upwards
- (3) 3 m s⁻² upwards (4) 1.5 m s⁻² downwards
17. Find the absolute acceleration of block 3.
- (1) 2 m s⁻² upwards (2) 1 m s⁻² downwards
- (3) 3 m s⁻² downwards (4) 1.5 m s⁻² upwards
18. Find the absolute acceleration of block 4.
- (1) 2 m s⁻² upwards (2) 1 m s⁻² downwards
- (3) 3 m s⁻² downwards (4) 1.5 m s⁻² upwards

For Problems 19–21

Three blocks A, B, and C having masses 1 kg, 2 kg, and 3 kg, respectively, are arranged as shown in figure. The pulleys P and Q are light and frictionless. All the blocks are resting on a horizontal

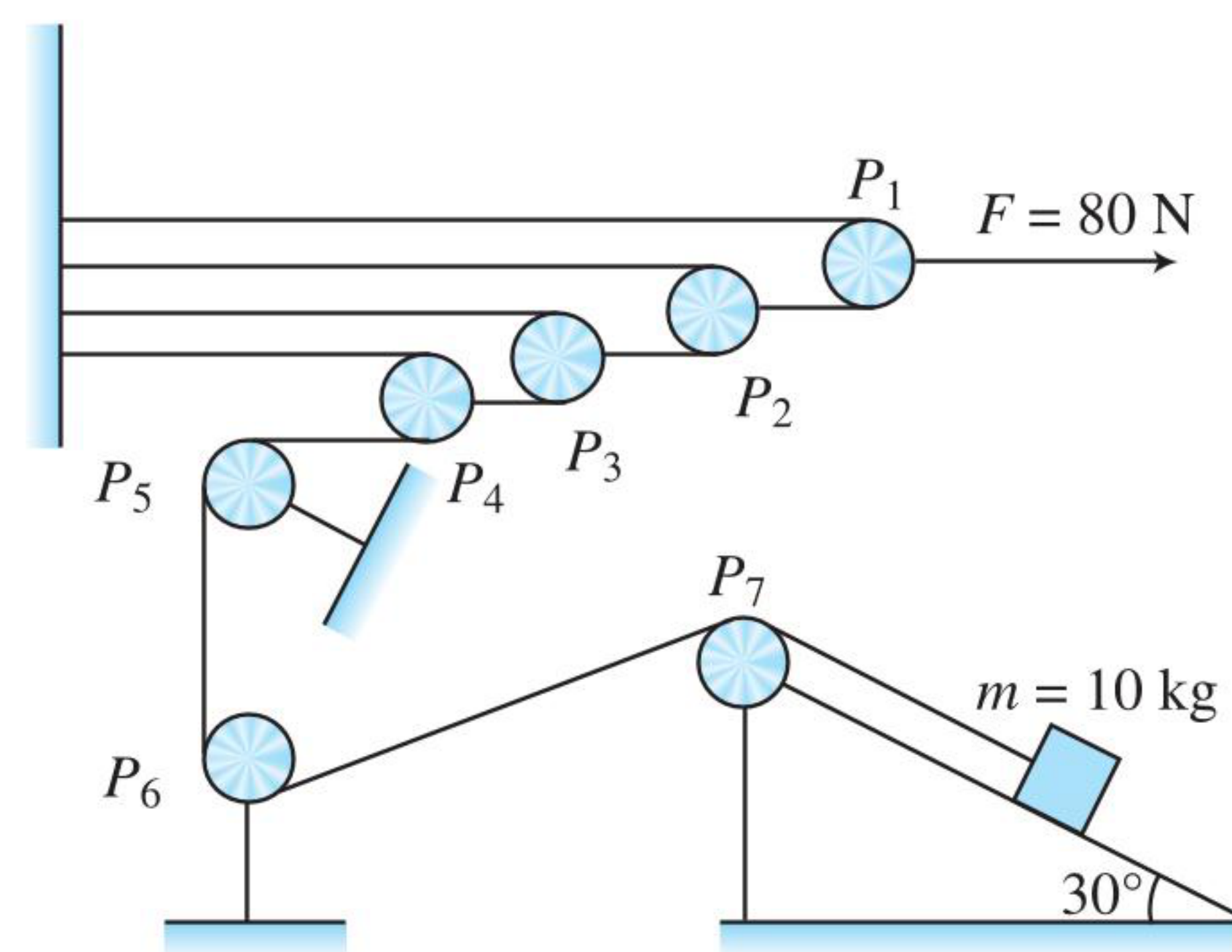
floor and the pulleys are held such that strings remain just taut. At moment $t = 0$, a force $F = 40t$ N starts acting on pulley P along vertically upward direction as shown in the figure. Take $g = 10$ m s⁻².



19. Regarding the times when the blocks lose contact with ground, which is correct?
- (1) A loses contact at $t = 2$ s.
- (2) C loses contact at $t = 1.5$ s.
- (3) A and B lose contact at the same time.
- (4) All three blocks lose contact at the same time.
20. Find the velocity of A when B loses contact with ground.
- (1) 5 m s⁻¹ (2) $5/4$ m s⁻¹
- (3) 4 m s⁻¹ (4) $7/3$ m s⁻¹
21. What is the magnitude of relative acceleration of A with respect to B just after both have lost contact with ground?
- (1) 15 m s⁻¹ (2) 5 m s⁻¹
- (3) 20 m s⁻¹ (4) 10 m s⁻¹

For Problems 22–24

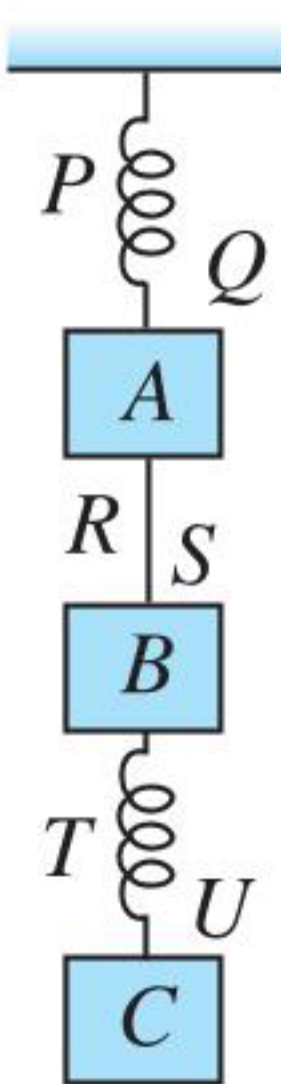
In figure all the pulleys and strings are massless and all the surfaces are frictionless. A small block of mass m is placed on fixed wedge (take $g = 10$ m s⁻²).



22. The tension in the string attached to m is
- (1) 40 N (2) 10 N
- (3) 20 N (4) 5 N
23. The acceleration of m is
- (1) 4.5 m s⁻² down the incline
- (2) 4.5 m s⁻² up the incline
- (3) 5 m s⁻² down the incline
- (4) 5 m s⁻² up the incline
24. The acceleration of pulley p_4 is
- (1) 2.25 m s⁻² towards left (2) 2.25 m s⁻² towards right
- (3) 9 m s⁻² towards left (4) 9 m s⁻² towards right

For Problems 25–27

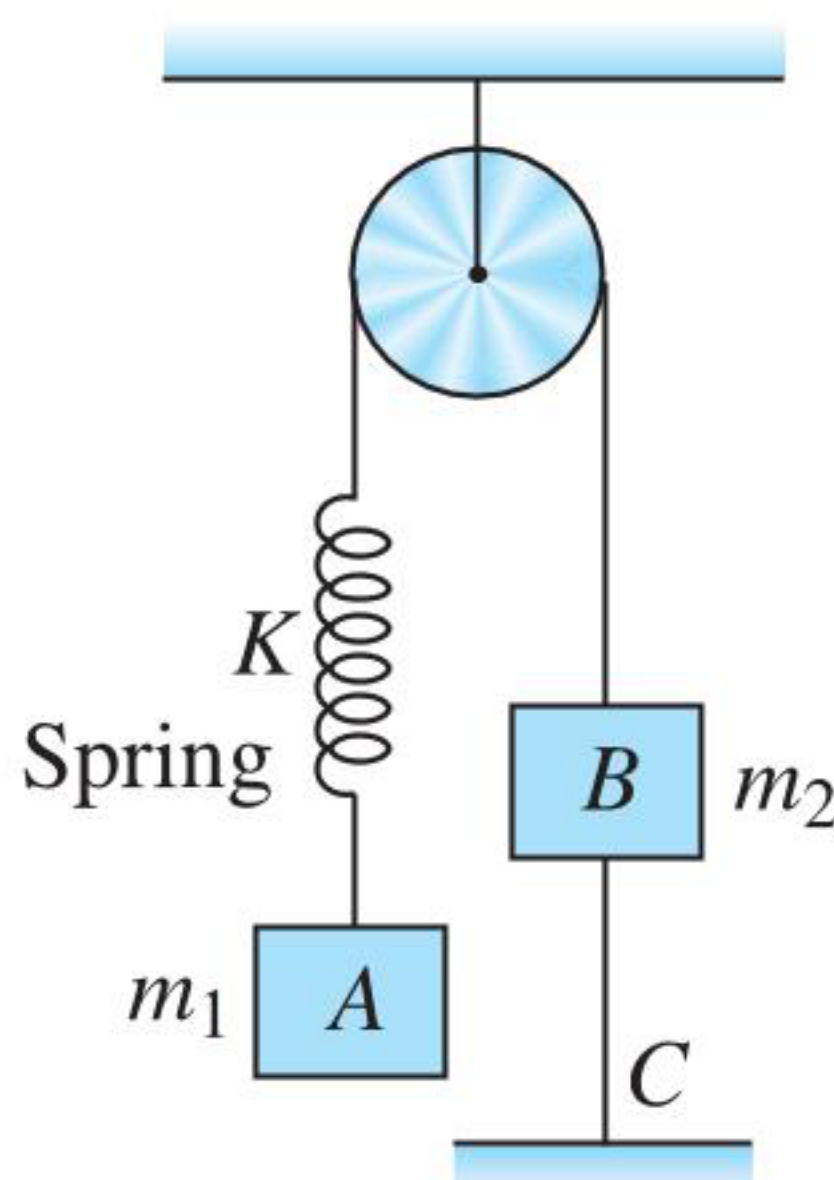
Three blocks A , B , and C of masses $3M$, $2M$, and M are suspended vertically with the help of springs PQ and TU , and a string RS as shown in figure. If the acceleration of blocks A , B , and C is a_1 , a_2 , and a_3 , respectively, then



25. The value of acceleration a_3 at the moment spring PQ is cut is
 (1) g , downward (2) g , upwards
 (3) More than g , downwards (4) Zero
26. The value of acceleration a_1 at the moment string RS is cut is
 (1) g , downward (2) g , upwards
 (3) More than g , downwards (4) Zero
27. The value of acceleration a_2 at the moment spring TU is cut is
 (1) $g/5$, upwards (2) $g/5$, downwards
 (3) $g/3$, upwards (4) Zero

For Problems 28–30

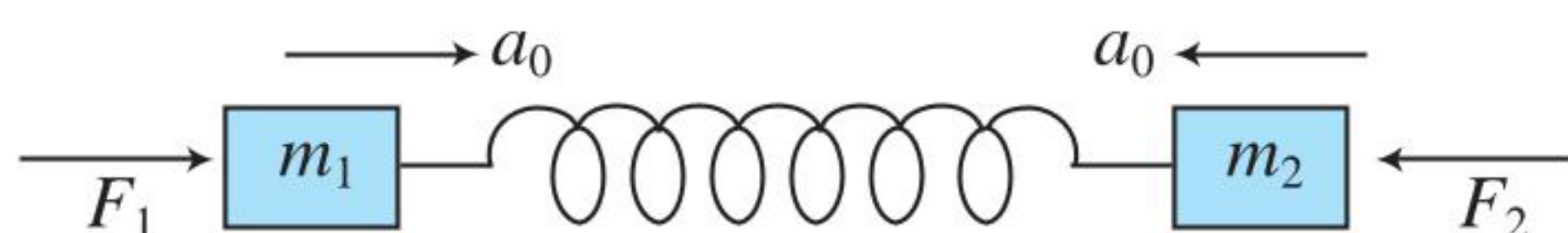
In the system shown in figure, $m_1 > m_2$. The system is held at rest by thread BC . Now thread BC is burnt. Answer the following:



28. Before burning the thread, what are the tensions in spring and thread BC , respectively?
 (1) m_1g , m_2g (2) m_1g , $m_1g - m_2g$
 (3) m_2g , m_1g (4) m_1g , $m_1g + m_2g$
29. Just after burning the thread, what is the tension in the spring?
 (1) m_1g (2) m_2g
 (3) Zero (4) Cannot say
30. Just after burning the thread, what is the acceleration of m_2 ?
 (1) $\left(\frac{m_2 - m_1}{m_2}\right)g$ (2) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$
 (3) Zero (4) $\left(\frac{m_1 - m_2}{m_2}\right)g$

For Problems 31–33

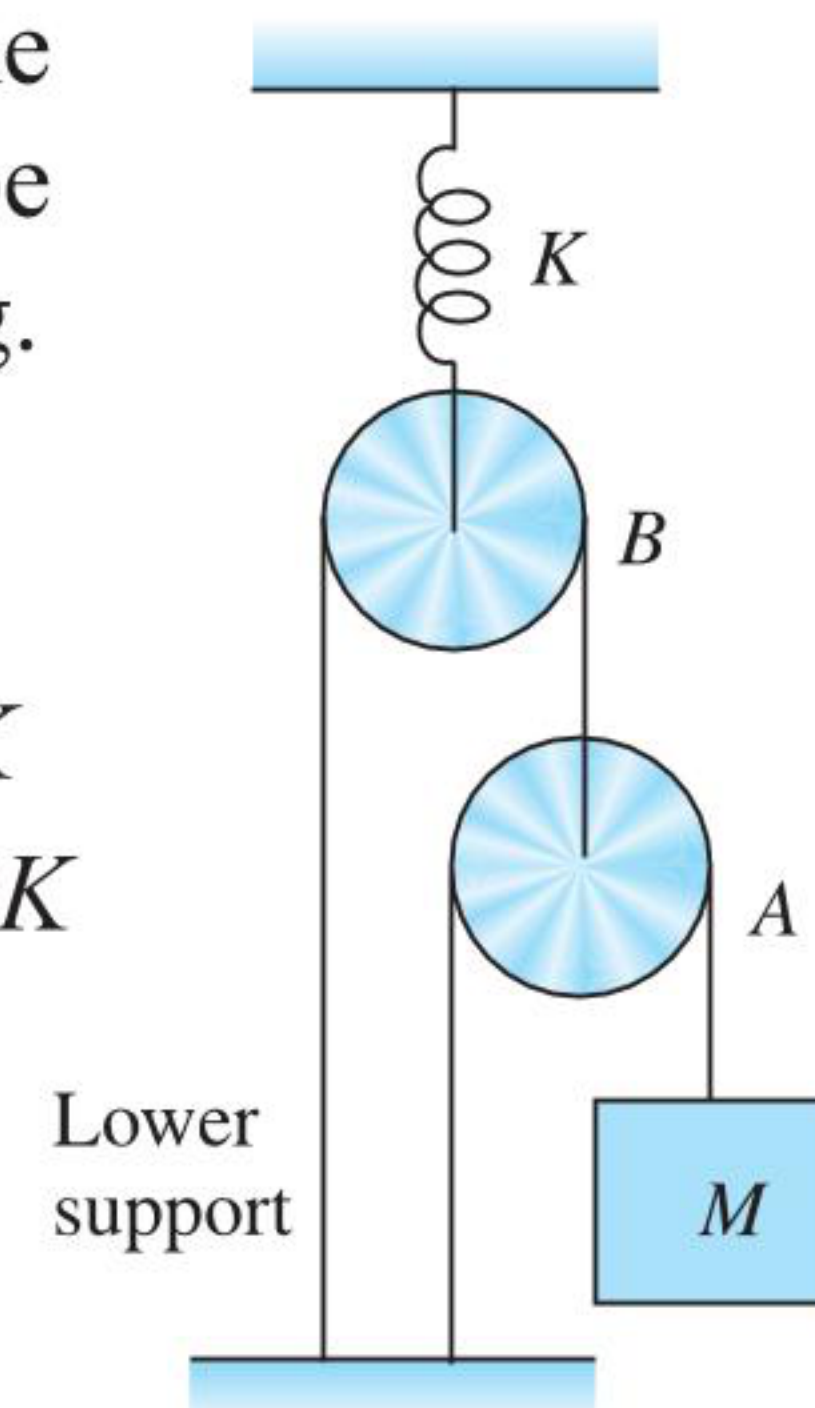
Two blocks of masses m_1 and m_2 are connected with a light spring of force constant k and the whole system is kept on a frictionless horizontal surface. The masses are applied forces F_1 and F_2 as shown in figure. At any time, the blocks have same acceleration a_0 but in opposite directions. Now answer the following:



31. The value of a_0 is
 (1) $\frac{F_1 - F_2}{m_1 + m_2}$ (2) $\frac{F_1 - F_2}{m_1 - m_2}$
 (3) $\frac{F_1 + F_2}{m_1 - m_2}$ (4) $\frac{F_1 + F_2}{m_1 + m_2}$
32. The value of spring force is
 (1) $\frac{m_1 F_2 + F_1 m_2}{m_1 - m_2}$ (2) $\frac{m_1 F_2 - F_1 m_2}{m_1 + m_2}$
 (3) $\frac{m_1 F_2 + F_1 m_2}{m_1 + m_2}$ (4) $\frac{m_1 F_2 - F_1 m_2}{m_1 - m_2}$
33. If F_2 is removed at this moment, then just after this, the acceleration of m_2 is
 (1) $\frac{F_1}{m_2} - a_0$ (2) $a_0 + \frac{F_1}{m_2}$
 (3) $\frac{F_2}{m_2} - a_0$ (4) $a_0 + \frac{F_2}{m_2}$

For Problems 34–36

A mass M is suspended as shown in figure. The system is in equilibrium. Assume pulleys to be massless. K is the force constant of the spring.



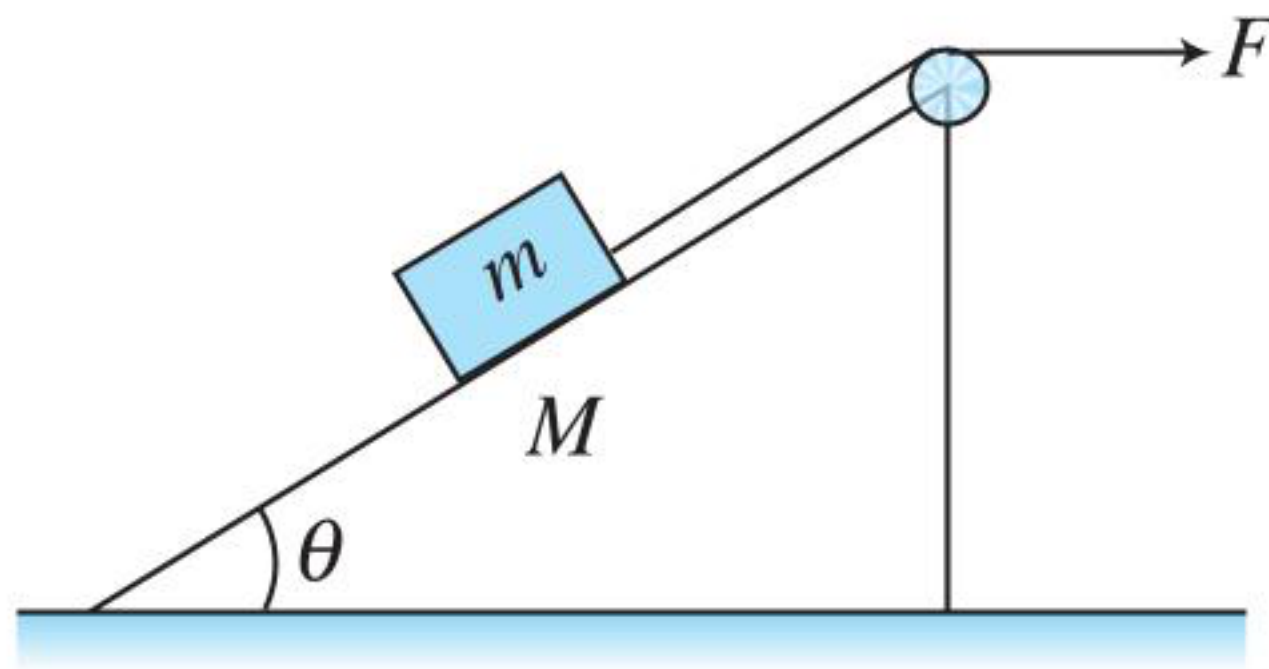
34. The extension produced in the spring is given by
 (1) $4Mg/K$ (2) Mg/K
 (3) $2Mg/K$ (4) $3Mg/K$
35. Find the net tension force acting on the lower support.
 (1) Mg (2) $2Mg$
 (3) $3Mg$ (4) $4Mg$
36. If each of the pulleys A and B has mass M , then find the net tension force acting on the lower support. Assume pulleys to be frictionless.
 (1) $2Mg$ (2) $6Mg$
 (3) $3Mg$ (4) $4Mg$

Matrix Match Type

1. Column I describes the motion of the object and one or more of the entries of Column II may be the cause of motions described in Column I. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. An object is moving towards east.	a. The net force acting on the object must be towards east.
ii. An object is moving towards east with constant acceleration.	b. At least one force must act towards east.
iii. An object is moving towards east with varying acceleration.	c. No force may act towards east.
iv. An object is moving towards east with constant velocity.	d. No force may act on the object.

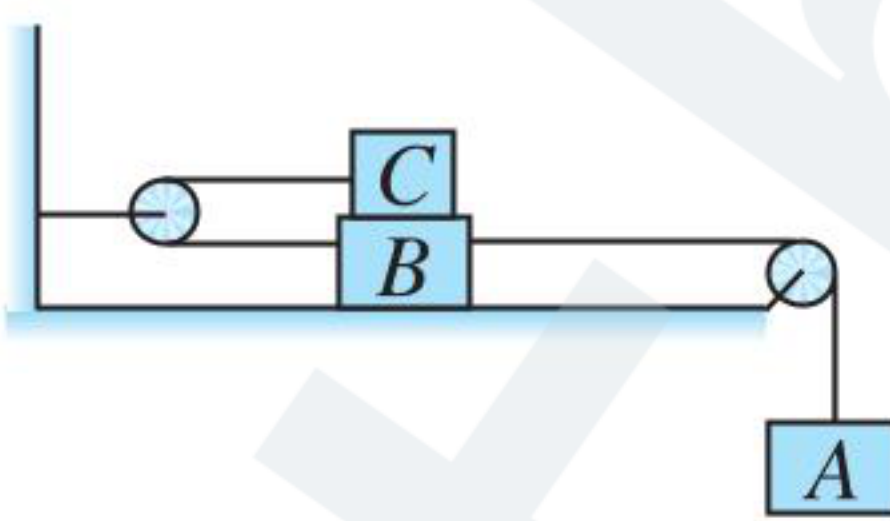
2. There is no friction anywhere in the system shown in figure. The pulley is light. The wedge is free to move on a frictionless surface. A horizontal force F is applied on the system in such a way that m does not slide on M or both move together with some common acceleration. Given $M > \sqrt{2} m$.



Match the entries of Column I with that of Column II.

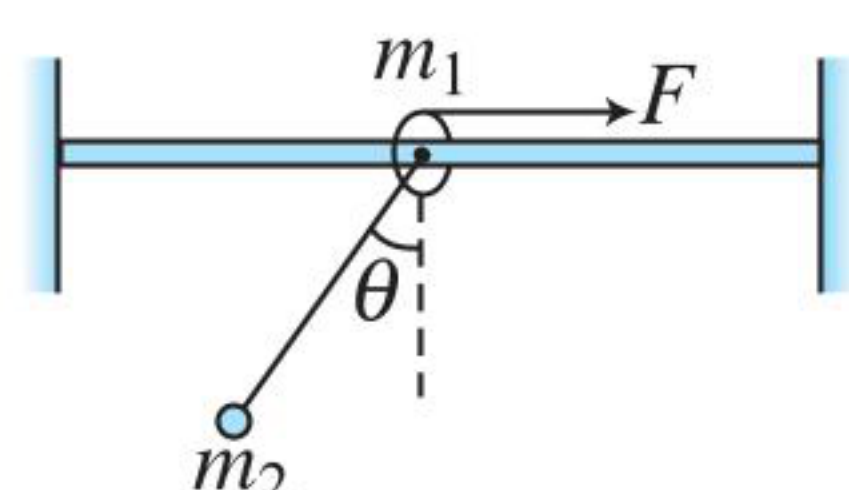
Column I	Column II
i. Pseudo force acting on m as seen from the frame of M is	a. Equal to $\frac{mF}{m+M}$
ii. Pseudo force acting on M as seen from the frame of m is	b. Greater than $\frac{mF}{m+M}$
iii. Normal force (for $\theta = 45^\circ$) between m and M is	c. Less than $mg \sin \theta$
iv. Normal force between ground and M is	d. Greater than $mg \sin \theta$

3. When the system shown in figure is released, A accelerates downwards. Match the entries of Column I with Column II.



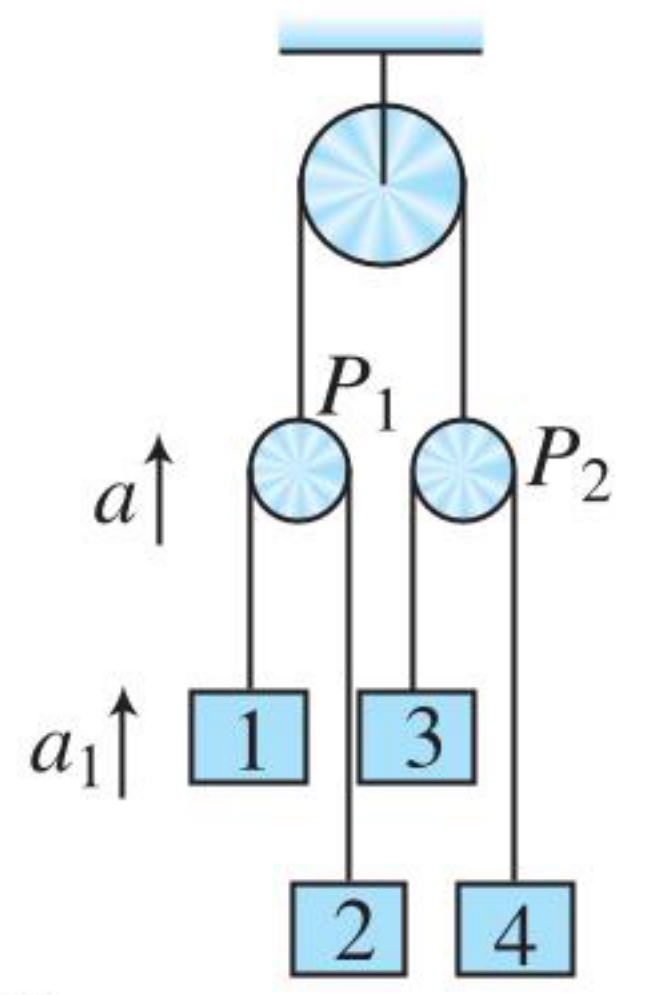
Column I	Column II
i. Acceleration of B	a. Towards left
ii. Acceleration of C w.r.t. B	b. Towards right
iii. Acceleration of A w.r.t. C	c. At some angle θ with horizontal ($0 < \theta < 90^\circ$)
iv. Acceleration of B w.r.t. A	d. At some angle θ with vertical ($0 < \theta < 90^\circ$)

4. A horizontal force F pulls a ring of mass m_1 such that θ remains constant with time. The ring is constrained to move along a smooth rigid horizontal wire. A bob of mass m_2 hangs from m_1 by an inextensible light string. Then match the entries of Column I with that of Column II.



Column I	Column II
i. F	a. $(m_1 + m_2)g$
ii. Force acting on m_2 is	b. $m_2 g \sec \theta$
iii. Tension in the string is	c. $m_2 \frac{F}{m_1 + m_2}$
iv. Force acting on m_1 by the wire is	d. $(m_1 + m_2)g \tan \theta$

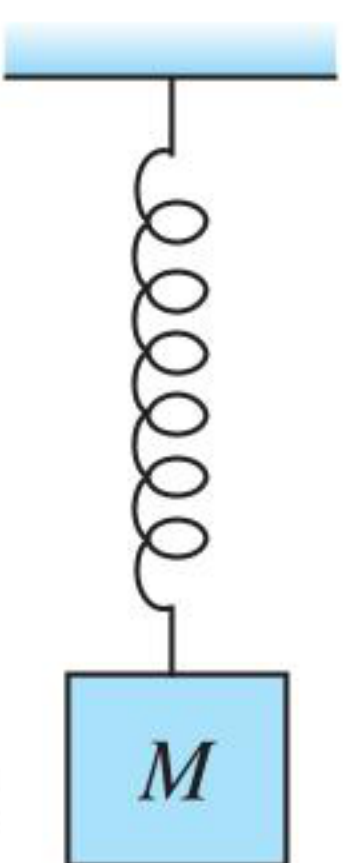
5. In the system shown in figure, masses of the blocks are such that when system is released, the acceleration of pulley P_1 is a upwards and the acceleration of block 1 is a_1 upwards. It is found that the acceleration of block 3 is same as that of 1 both in magnitude and direction.



Given that $a_1 > a > a_1/2$. Match the following:

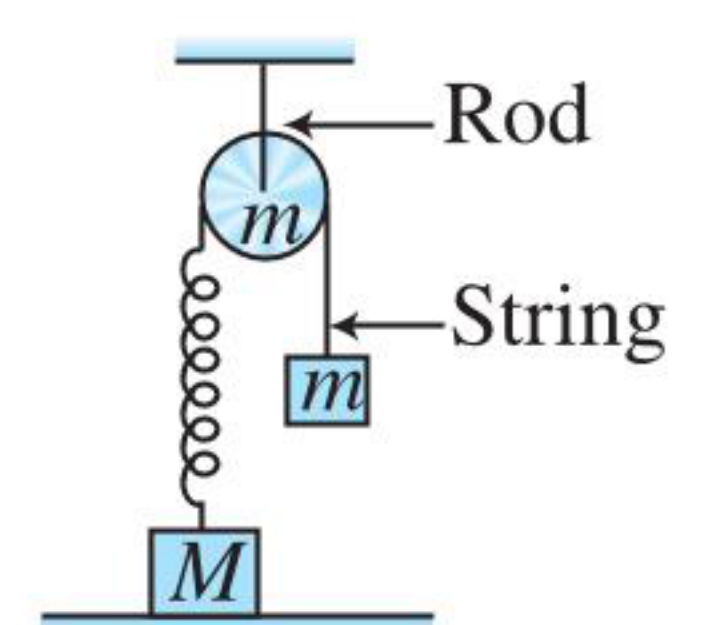
Column I	Column II
i. Acceleration of 2	a. $2a + a_1$
ii. Acceleration of 4	b. $2a - a_1$
iii. Acceleration of 2 w.r.t. 3	c. Upwards
iv. Acceleration of 2 w.r.t. 4	d. Downwards

6. A block is attached to an unstretched vertical spring and released from rest. As a result of this, the block comes down due to its weight, stops momentarily, and then bounces back. Finally, the block starts oscillating up and down. During oscillations, match Column I with Column II:



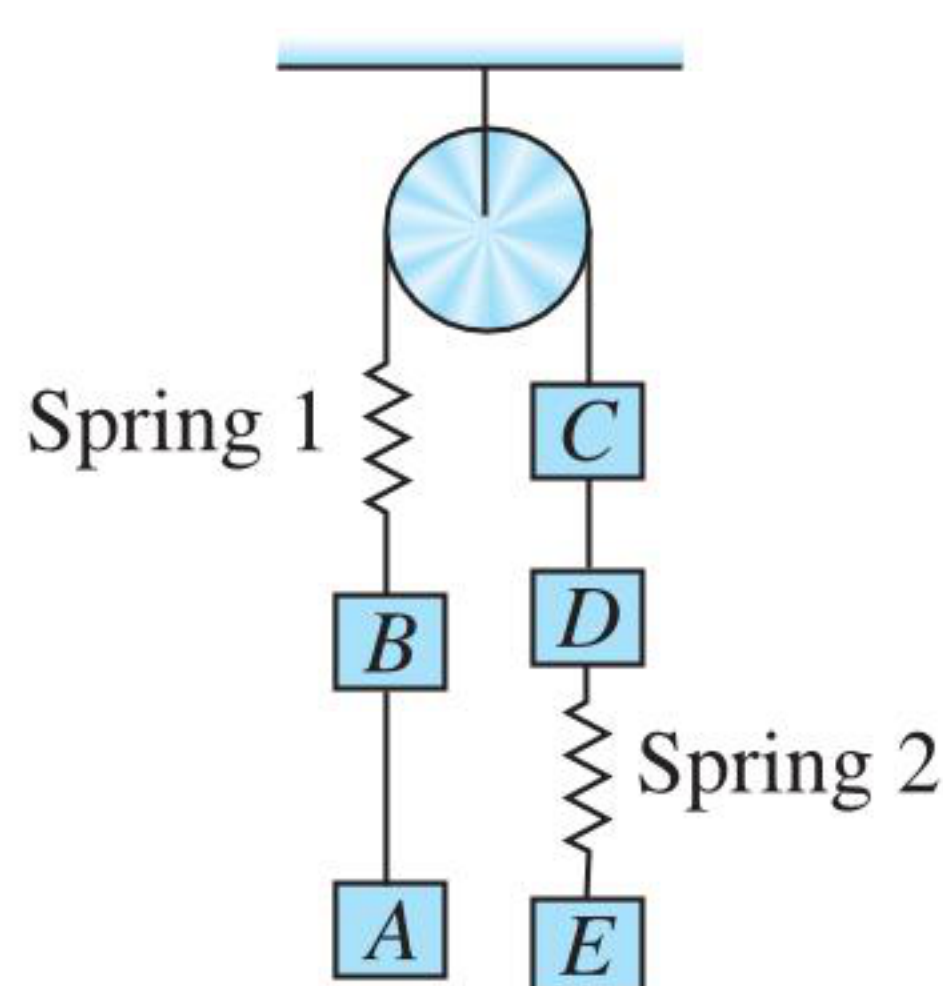
Column I	Column II
i. When the block is at its maximum downward displacement position (may be known as extreme position)	a. Acceleration is in upward direction.
ii. When the block is at its equilibrium position	b. Acceleration is in downward direction.
iii. When the block is somewhere between equilibrium position and downward extreme position	c. Acceleration is zero.
iv. When the block is above equilibrium position but below the initial unstretched position	d. Velocity may be in upward or in downward direction.

7. In figure, a block of mass m is released from rest when spring was in its natural length. The pulley also has mass m but it is frictionless. Suppose the value of m is such that finally it is just able to lift block M up after releasing it.



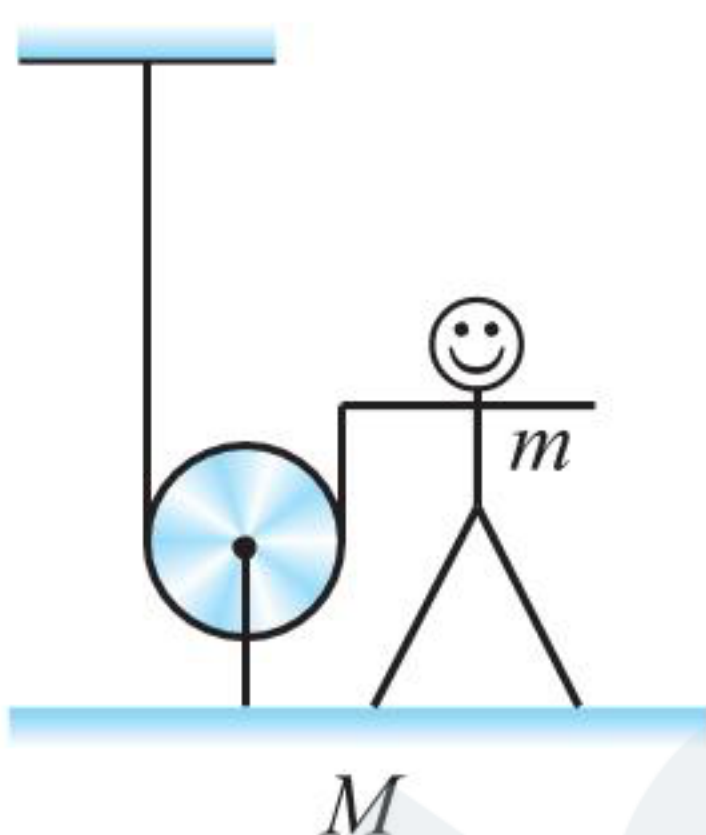
Column I	Column II
i. Weight of m required to just lift M	a. $3\frac{M}{2}g$
ii. Tension in the rod, when m is in equilibrium	b. Mg
iii. Normal force acting on M when m is in equilibrium.	c. $\frac{M}{2}g$
iv. Tension in the string when displacement of m is maximum possible	d. $2mg$

8. The system shown figure is initially in equilibrium. Masses of the blocks A , B , C , D , and E are, respectively, 3 kg, 3 kg, 2 kg, 2 kg and 2 kg. Match the conditions in Column I with the effect in Column II.



Column I	Column II
i. After spring 2 is cut, tension in string AB	a. Increases
ii. After spring 2 is cut, tension in string CD	b. Decreases
iii. After string between C and pulley is cut, tension in string AB	c. Remains constant
iv. After string between C and pulley is cut, tension in string CD	d. Zero

9. A person stands on a platform-and-pulley system, as shown in figure. The masses of the platform and person are M and m respectively. The rope and pulley is massless. The man can pull on the rope so as to produce one of the situations shown in column I. In column II, T is tension & N is normal reaction between the man and the platform. Match the possible situations.



Column I	Column II
i. At equilibrium	a. $N = (2m + M)g$
ii. When the system falls down freely	b. $T = (m + M)g$
iii. When accelerating upwards	c. $N < (2m + M)g$
iv. When accelerating downwards	d. $N > (2m + M)g$
	e. $T < (m + M)g$

- (1) i $\rightarrow$ a,c; ii $\rightarrow$ c,d; iii $\rightarrow$ e; iv $\rightarrow$ c,e
 (2) i $\rightarrow$ a,b; ii $\rightarrow$ c,e; iii $\rightarrow$ d; iv $\rightarrow$ c,e
 (3) i $\rightarrow$ a,d; iii $\rightarrow$ c,e; iii $\rightarrow$ b; iv $\rightarrow$ c,d
 (4) i $\rightarrow$ a,b; iv $\rightarrow$ c,d; iii $\rightarrow$ a; iv $\rightarrow$ c,e
10. A force of 20 N is acting on a block of mass 2 kg kept on a smooth inclined plane. The direction of this force can be any of the four as indicated in column II. Match with the description given in column I.

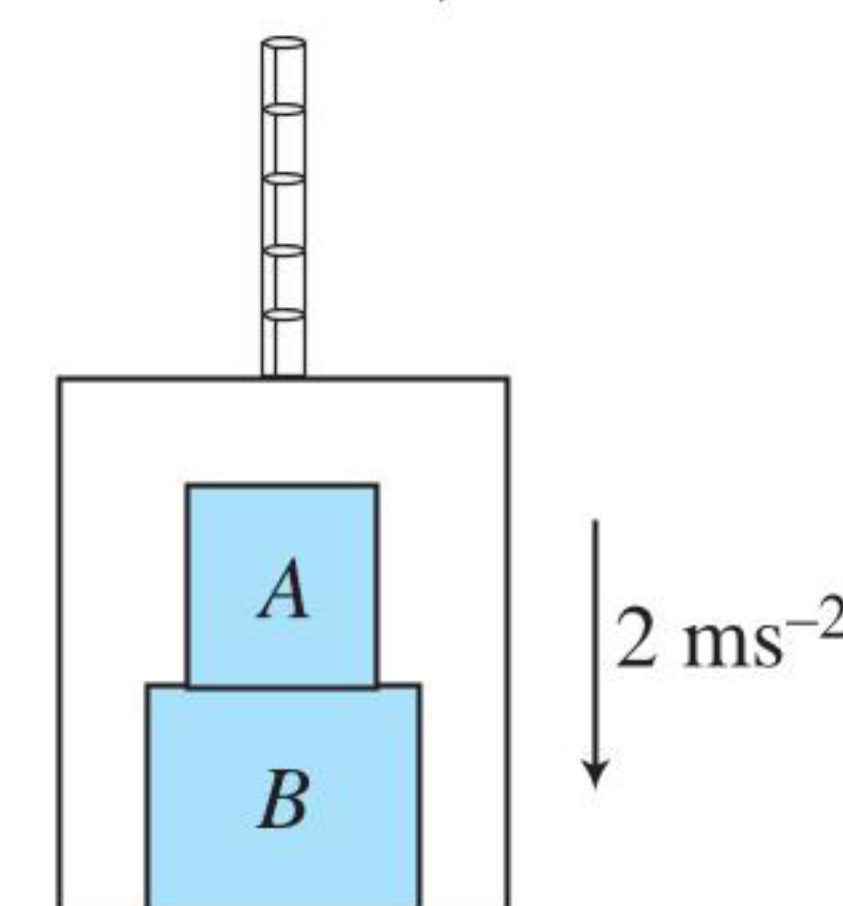
Column I	Column II
i. Normal force cannot be zero	a.
ii. Body slides down the inclined plane.	b.

iii. Acceleration of the body is greater than 10 m/s^2	c.
	d.

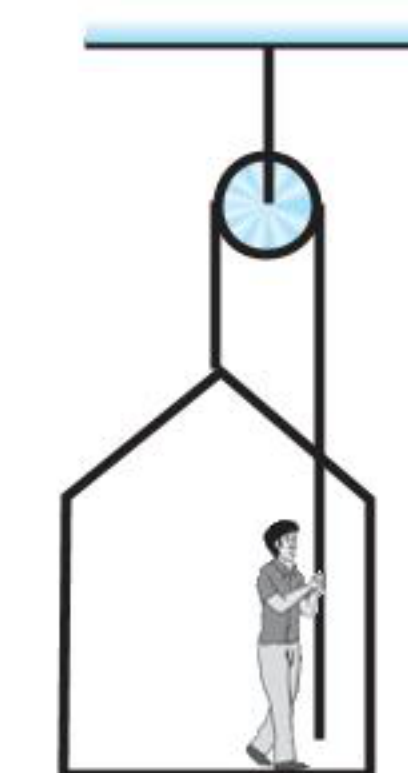
- (1) i $\rightarrow$ a,c,d; ii $\rightarrow$ c,d; iii $\rightarrow$ d
 (2) i $\rightarrow$ a,b,d; ii $\rightarrow$ b,c; iii $\rightarrow$ c
 (3) i $\rightarrow$ c,d; ii $\rightarrow$ b,d; iii $\rightarrow$ b
 (4) i $\rightarrow$ b,c,d; ii $\rightarrow$ c,d; iii $\rightarrow$ d

Numerical Value Type

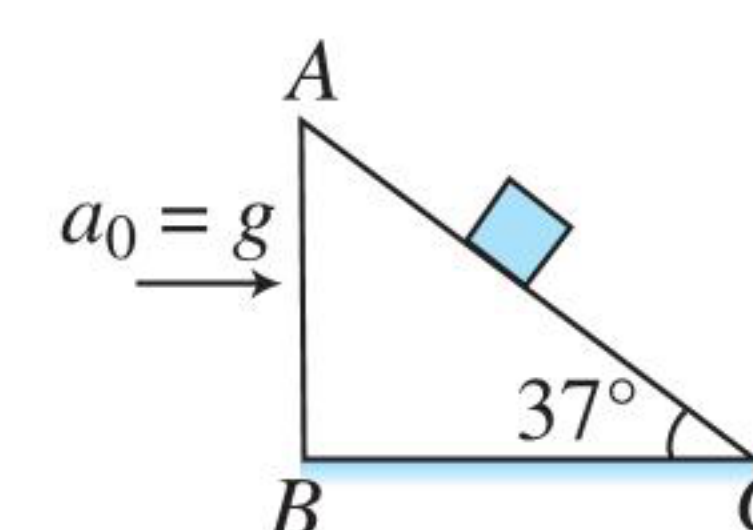
- You are designing an elevator for a hospital. The force exerted on a passenger by the floor of the elevator is not to exceed 1.60 times the passenger's weight. The elevator accelerates upward with constant acceleration for a distance of 3.0 m and then starts to slow down. What is the maximum speed (in ms^{-1}) of the elevator?
- The elevator shown in figure is descending with an acceleration of 2 ms^{-2} . The mass of the block $A = 0.5 \text{ kg}$. Find the force (in Newton) exerted by block A on block B .



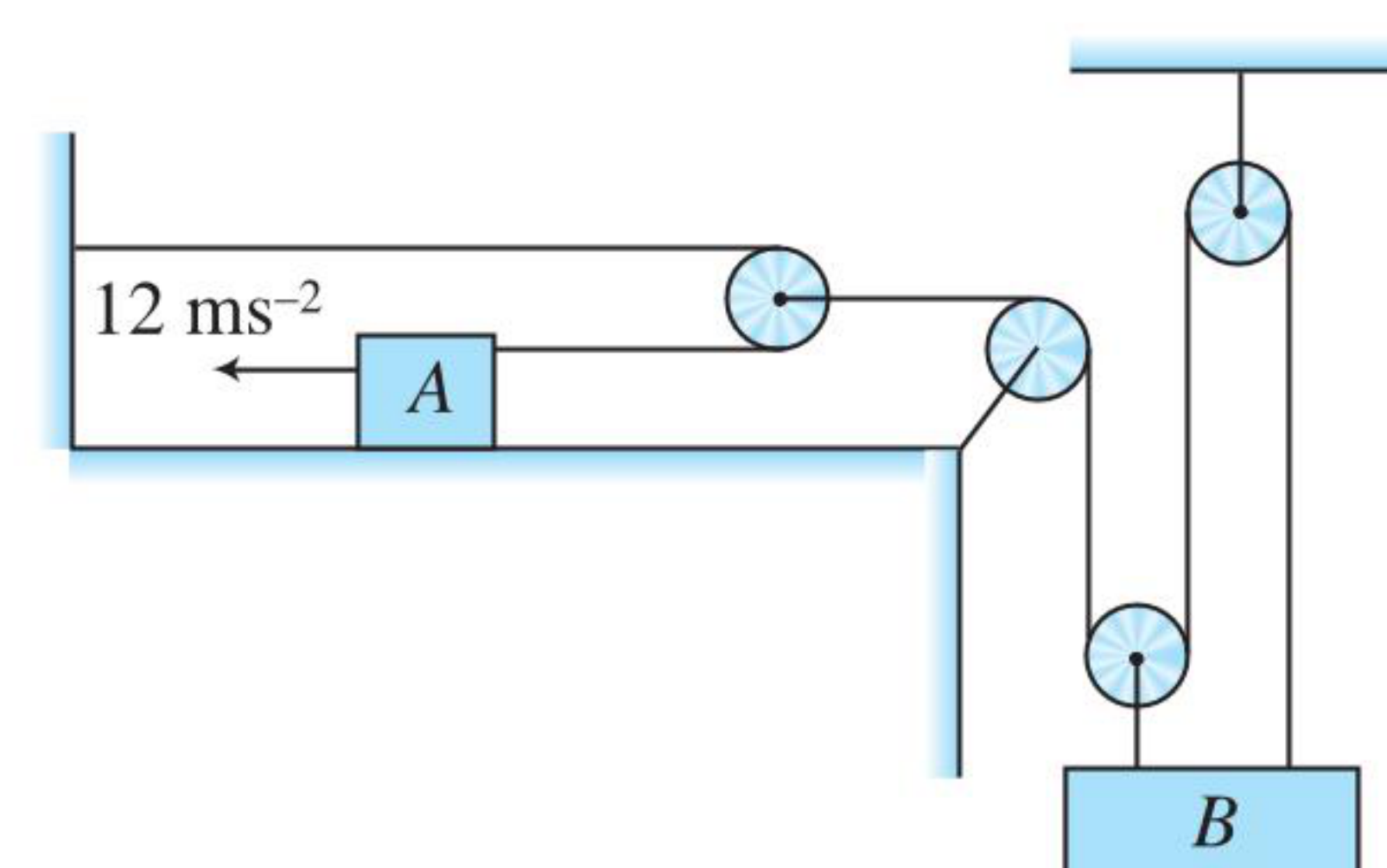
- Figure represents a painter in a crate which hangs alongside a building. When the painter of mass 100 kg pulls the rope, the force exerted by him on the floor of the crate is 450 N. If the crate weighs 25 kg, find the acceleration (in ms^{-2}) of the painter.



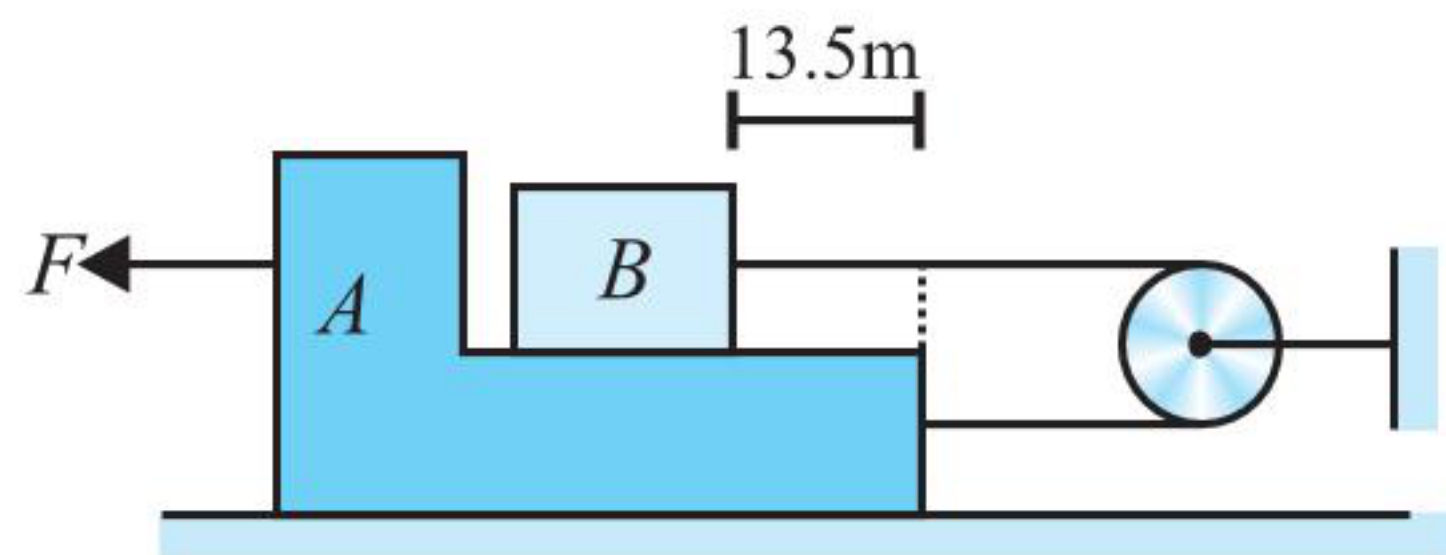
- A block is placed on an inclined plane moving towards right horizontally with an acceleration $a_0 = g$. The length of the plane $AC = 1 \text{ m}$. Friction is absent everywhere. Find the time taken (in seconds) by the block to reach from C to A .



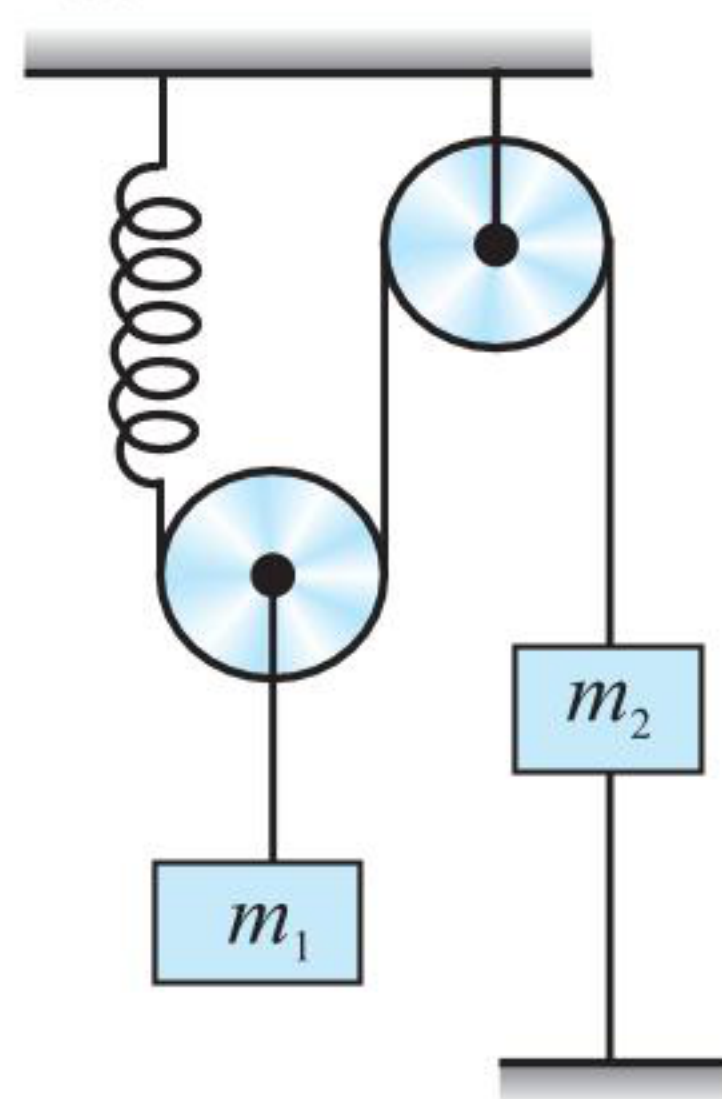
- Block A is given an acceleration 12 ms^{-2} towards left as shown in figure. Assuming block B always remains horizontal, find the acceleration (in ms^{-2}) of B .



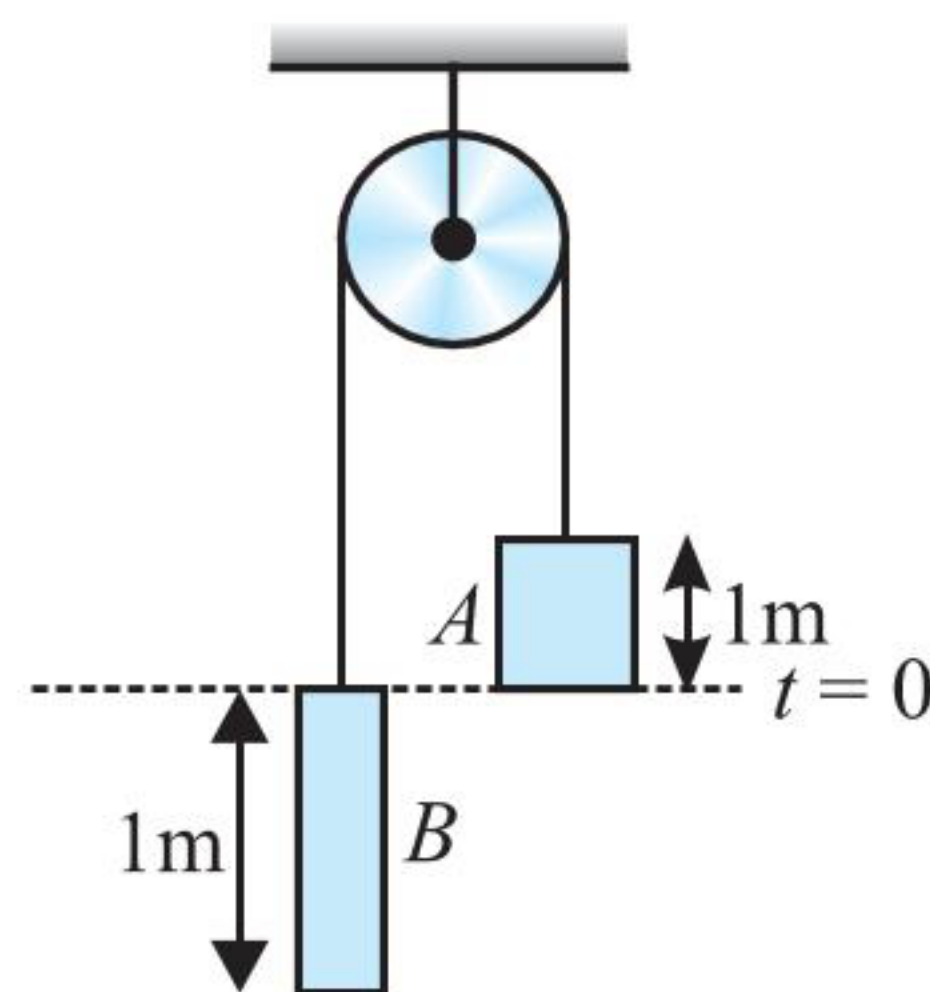
6. A 1-kg block 'B' rests as shown on a frictionless bracket 'A' of same mass. Constant force $F = 3\text{ N}$ starts to act at time $t = 0$, when the distance of block B from the end of bracket is 13.5 m. Find time (in sec), when block B falls off the bracket.



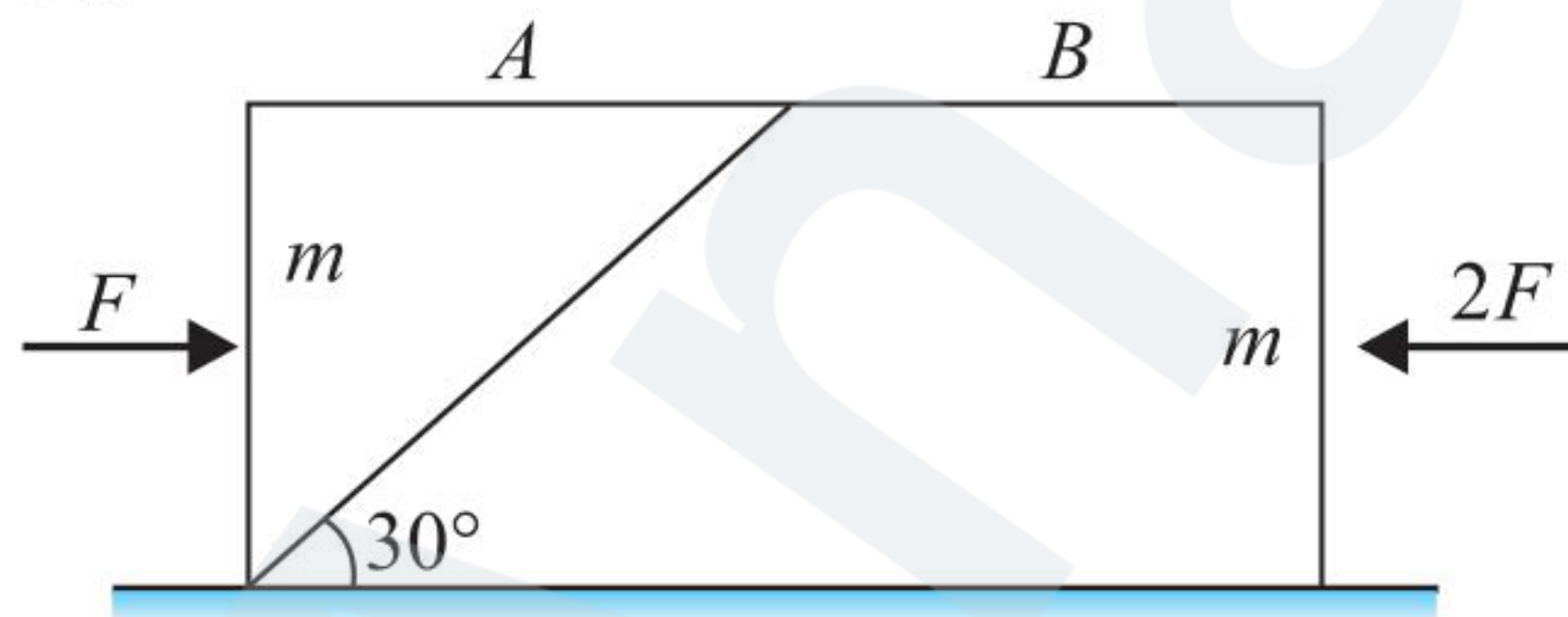
7. In figure shown, pulleys are ideal. Initially the system is in equilibrium and string connecting m_2 to rigid support below is cut. What is the initial acceleration (in m/s^2) of m_2 ? (Given $m_1 = 9\text{ kg}$; $m_2 = 3\text{ kg}$)



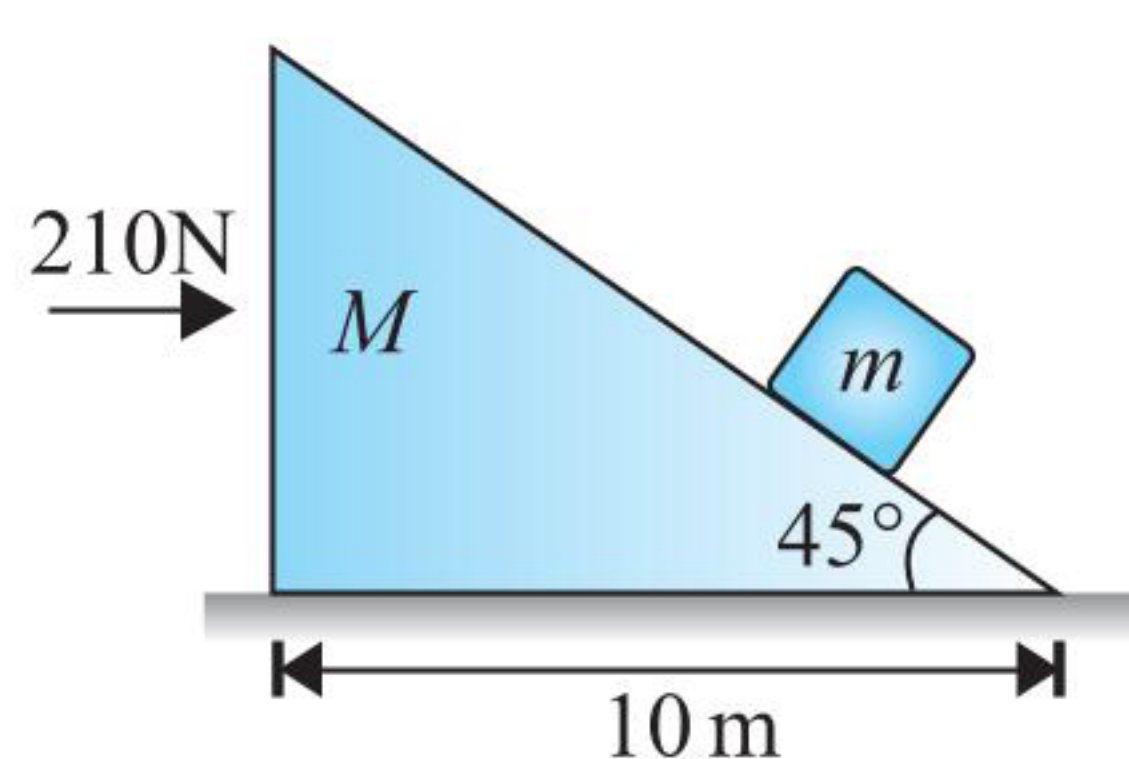
8. At $t = 0$, the lower end of the bar A is just above the upper end of bar B (mass of bar A = 3 kg, mass of bar B = $11/3\text{ kg}$). Find time when upper end of block A just crosses the lower end of B. (Assume the system was released at $t = 0$). [$g = 10\text{ m/s}^2$]



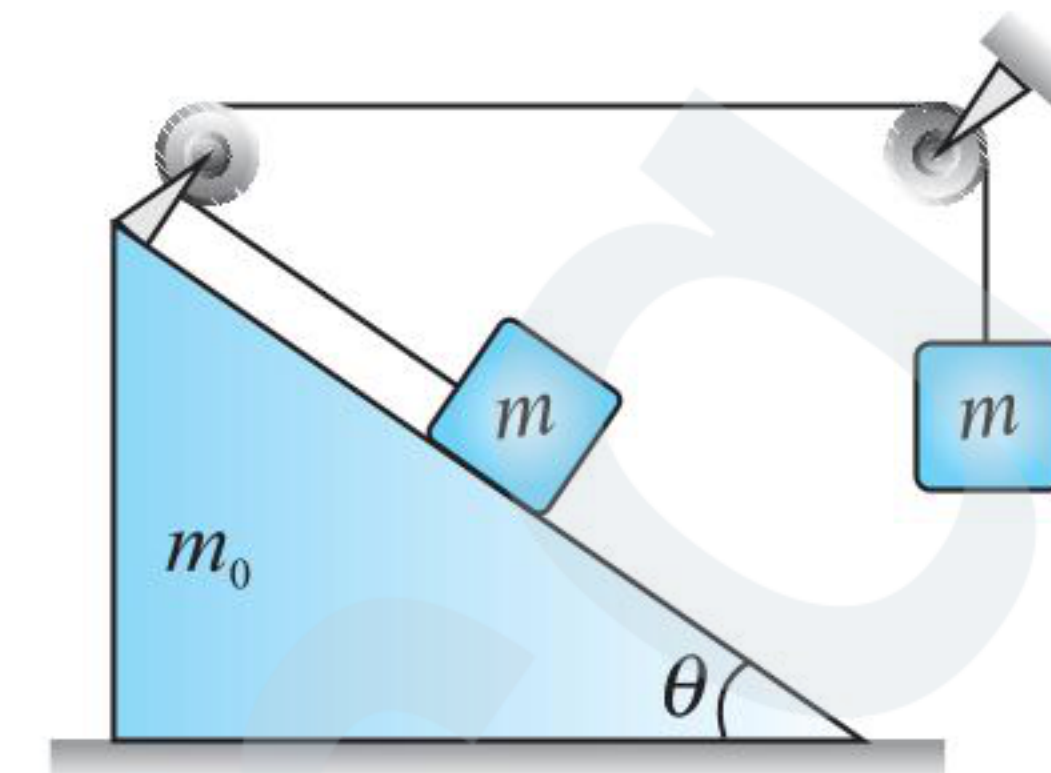
9. Two blocks A and B each of mass m are placed on a smooth horizontal surface. Two horizontal force F and $2F$ are applied on both the blocks A and B, respectively, as shown in figure. The block A does not slide on block B. Find the normal reaction acting between the two blocks (in N) if $F = 10\text{ N}$.



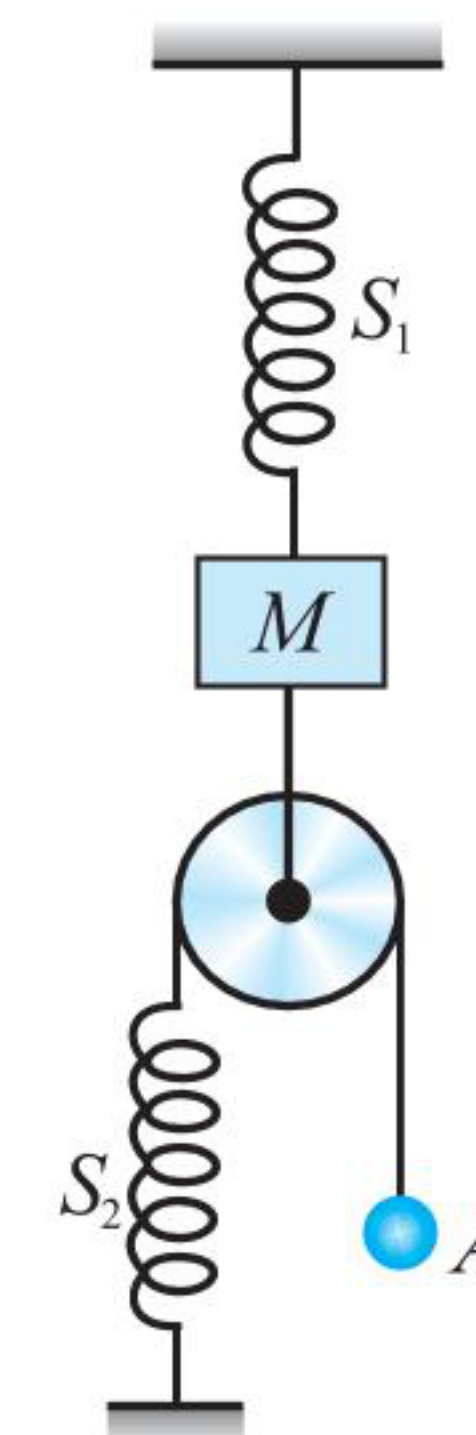
10. A small block of mass $m = 2\text{ kg}$ is kept at rest at the bottom of the incline plane of a wedge of mass $M = 9\text{ kg}$. A force of 210 N is applied on the wedge horizontally as shown in diagram. All the surfaces are smooth. Determine the time taken in seconds by the mass m to reach the highest point of the incline plane of the wedge? ($g = 10\text{ m/s}^2$)



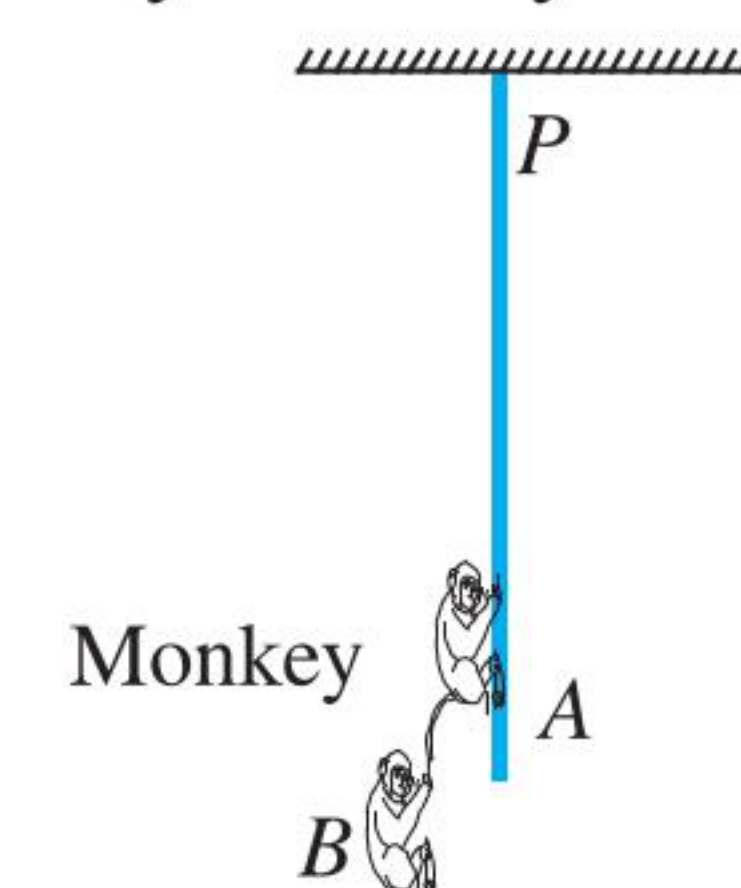
11. In the system shown in figure all surfaces are smooth and string and pulleys are light. Angle of wedge $\theta = \sin^{-1}\left(\frac{3}{5}\right)$. When released from rest it was found that the wedge of mass m_0 does not move. Find $\frac{m}{M}$



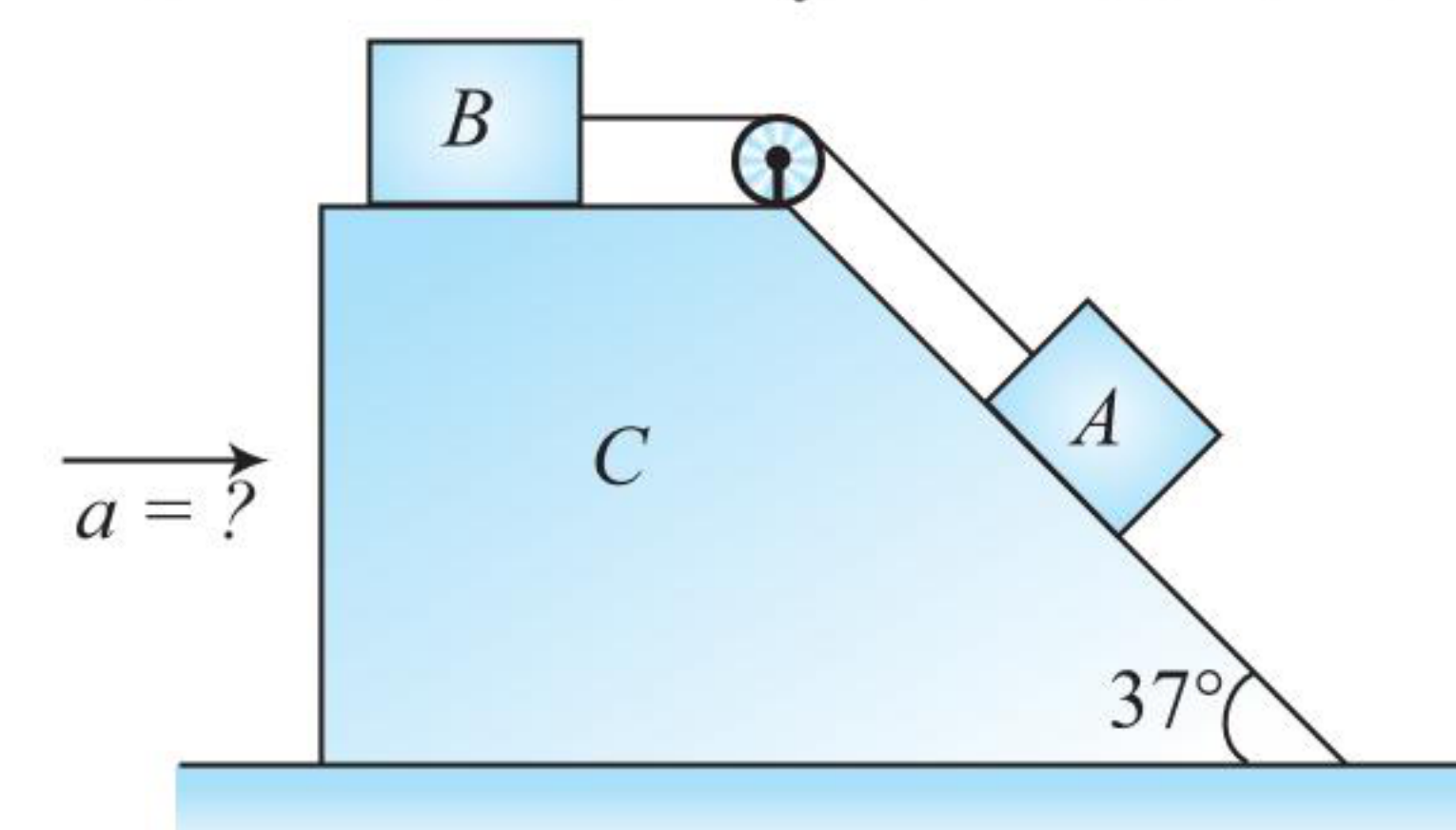
12. In the system shown in figure, the two springs S_1 and S_2 have force constant k each. Pulley, springs and strings are all massless. Initially, the system is in equilibrium with spring S_1 stretched and S_2 relaxed. The end A of the string is pulled down slowly through a distance $L = 10\text{ cm}$. By what distance (in cm) does the block of mass M move?



13. A monkey A (mass = 6 kg) is climbing up a rope tied to a rigid support. The monkey B (mass = 2 kg) is holding on the tail of monkey A. If the tail can tolerate a maximum tension of 30 N, what maximum force should monkey A apply on the rope in order to carry monkey B with it? ($g = 10\text{ m/s}^2$)

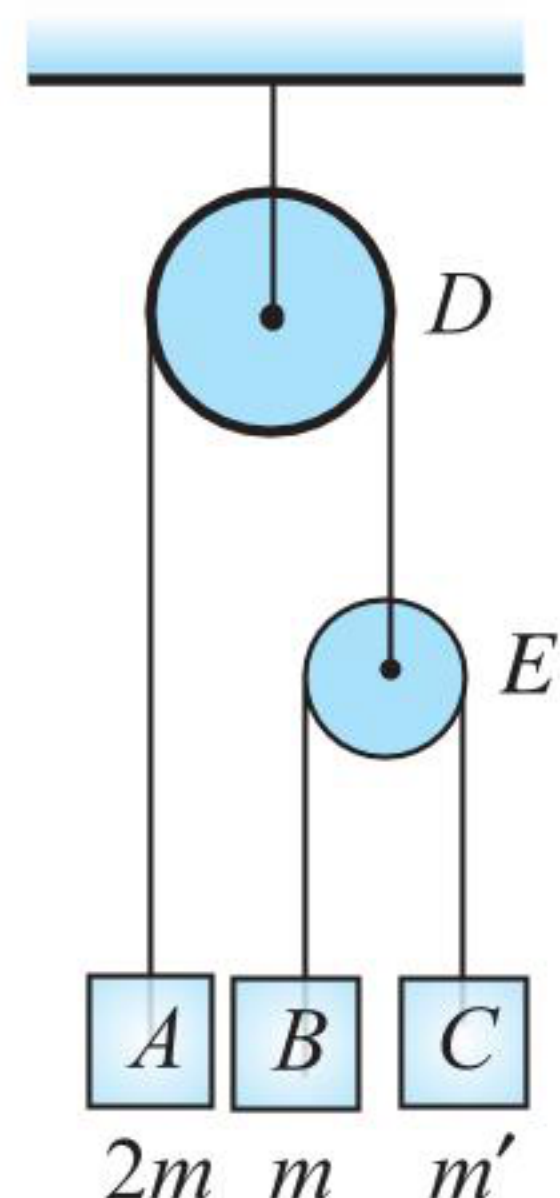


14. The upper surface of block C is horizontal and its right part is inclined to the horizontal at angle 37° . The mass of blocks A and B are $m_1 = 1.4\text{ kg}$ and $m_2 = 5.5\text{ kg}$, respectively. Neglect friction and mass of the pulley. Calculate acceleration a with which block C should be moved to the right so that A and B can remain stationary relative to it.



15. In the arrangement shown in figure, pulley D and E are small and frictionless. They do not rotate but threads slip over them without friction and their masses being 4 kg and

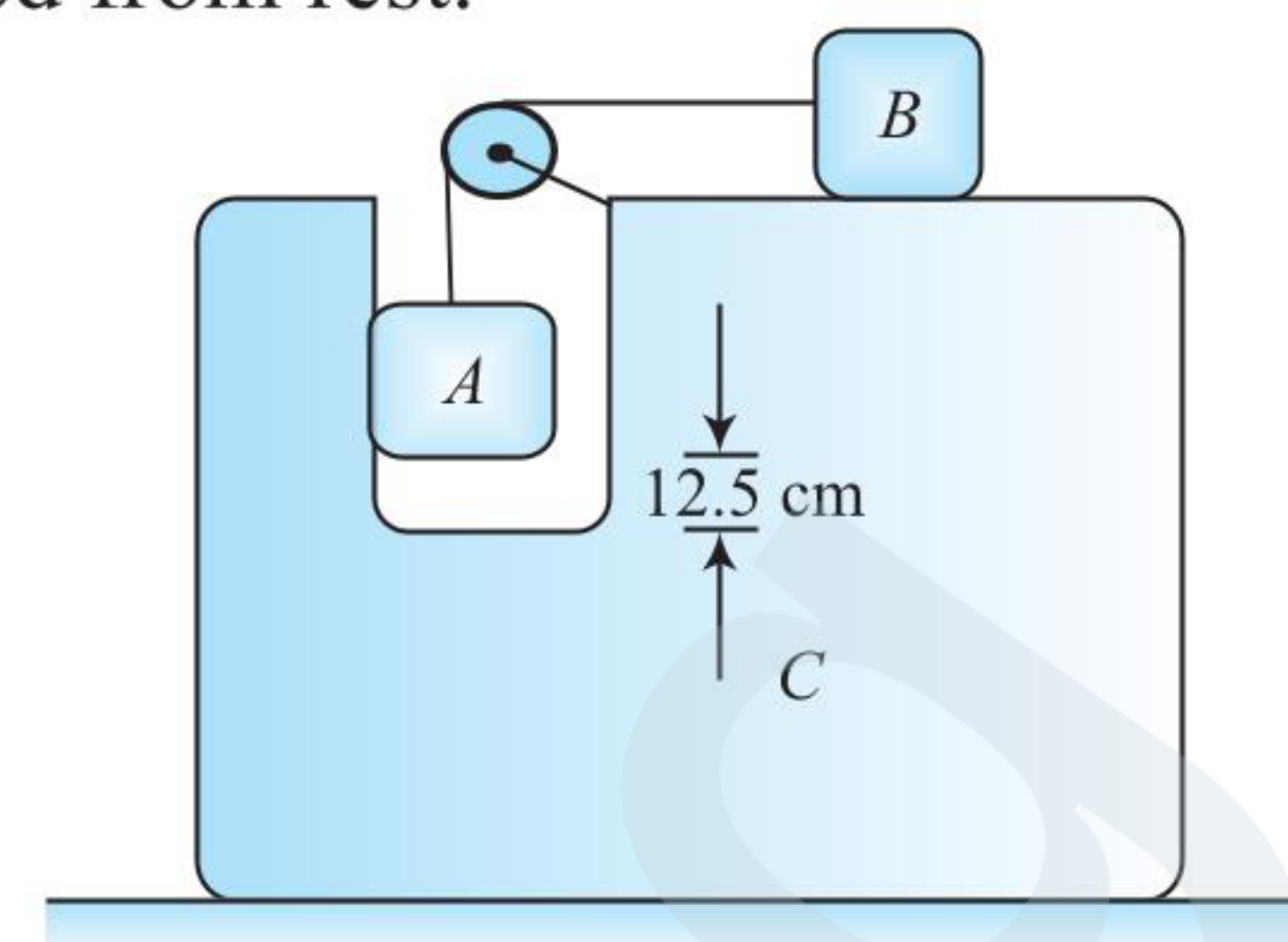
11.25 kg, respectively, while masses of block A , B and C are $2m$, m and m' respectively. When the system is released from rest, downward accelerations of blocks B and C relative to A are found to be 5 ms^{-2} and 3 ms^{-2} respectively.



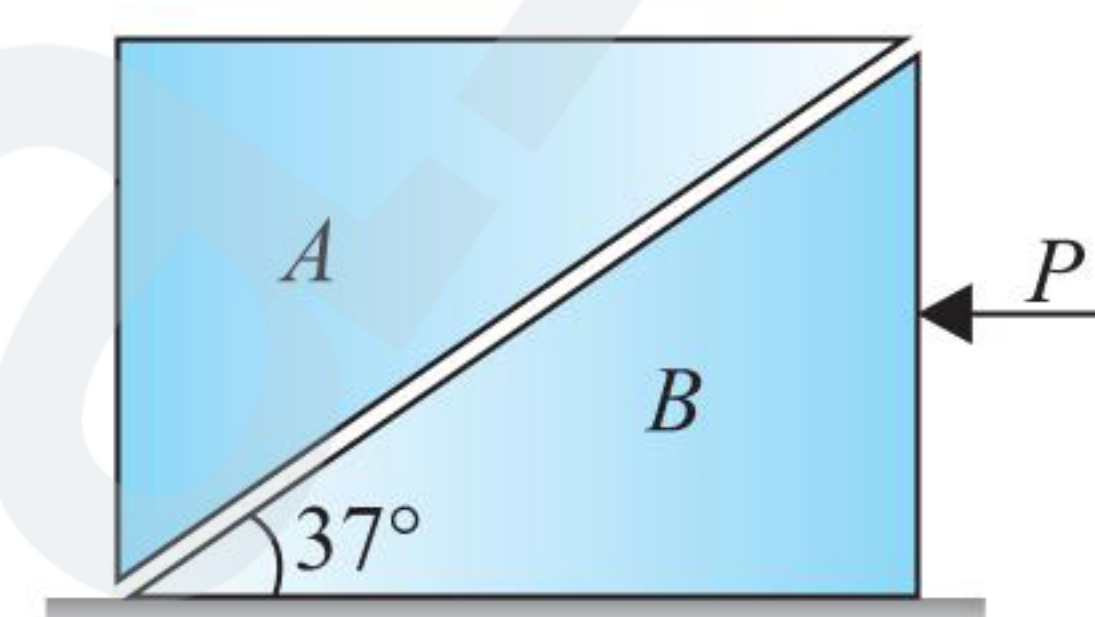
Find the ratio of the acceleration of blocks B and C , relative to the ground.

16. A small, light pulley is attached with a block C of mass 4 kg as shown in figure. Block B of mass 1.5 kg is placed on the top horizontal surface of C . Another block A of mass 2 kg is hanging from a string, attached with B and passing

over the pulley. Taking $g = 10 \text{ ms}^{-2}$ and neglecting friction, find the acceleration of block B (in m/s^2) when the system is released from rest.



17. Blocks A and B each have same mass $m = 1 \text{ kg}$. Determine the largest horizontal force P (in newton) which can be applied to B so that A will not slip up on B . Neglect any friction.



Archives

JEE ADVANCED

Single Correct Answer Type

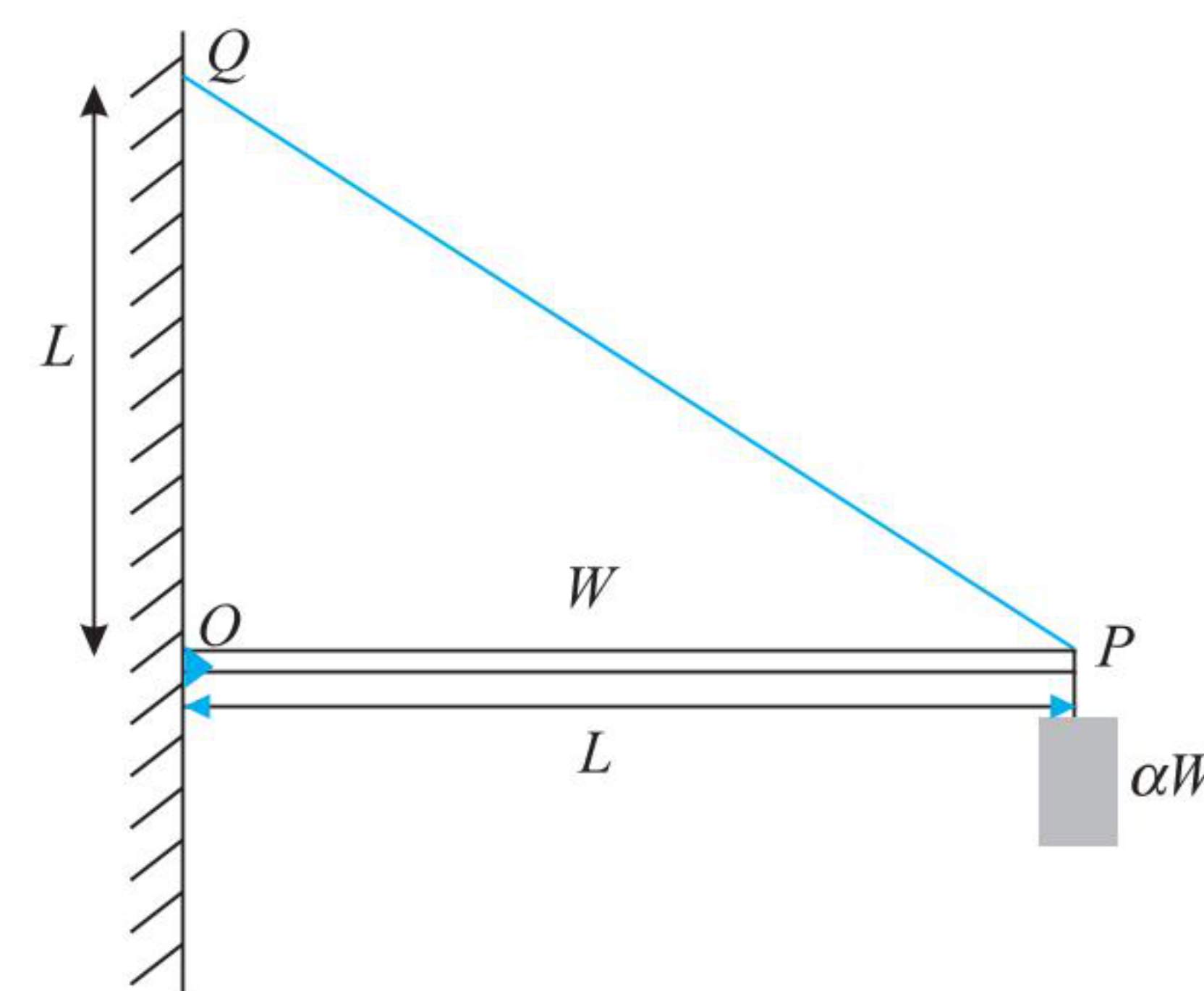
1. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y -axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x -axis with a constant acceleration a . The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y -axis is

- | | |
|---------------------|---------------------|
| (1) $\frac{a}{gk}$ | (2) $\frac{a}{2gk}$ |
| (3) $\frac{2a}{gk}$ | (4) $\frac{a}{4gk}$ |

(IIT-JEE 2009)

Multiple Correct Answers Type

1. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q , at a height L above the hinge at point O . A block of weight αW is attached at the point P of the beam, as shown in the figure $(2\sqrt{2})W$. Which of the following statement(s) is(are) correct?



- (1) The vertical component of reaction force at O does not depend on α
- (2) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$
- (3) The tension in the rope is $2W$ for $\alpha = 0.5$
- (4) The rope breaks if $\alpha > 1.5$.

(JEE Advanced 2021)

Answers Key

EXERCISES

Single Correct Answer Type

1. (3)

6. (3)

11. (3)

16. (4)

21. (1)

26. (4)

31. (3)

36. (3)

41. (4)

46. (2)

51. (3)

56. (4)

61. (2)

66. (3)

71. (4)
2. (1)

7. (2)

12. (2)

17. (2)

22. (2)

27. (1)

32. (1)

37. (1)

42. (4)

47. (2)

52. (2)

57. (2)

62. (2)

67. (1)

72. (4)
3. (1)

8. (4)

13. (4)

18. (3)

23. (4)

28. (3)

33. (4)

38. (4)

43. (3)

48. (1)

53. (2)

58. (3)

63. (4)

68. (4)

73. (2)
4. (2)

9. (3)

14. (4)

19. (4)

24. (3)

29. (1)

34. (3)

39. (2)

44. (3)

49. (3)

54. (4)

59. (3)

64. (4)

69. (1)

74. (1)
5. (2)

10. (3)

15. (4)

20. (2)

25. (4)

30. (1)

35. (3)

40. (3)

45. (2)

50. (1)

55. (2)

60. (4)

65. (1)

70. (2)

75. (4)

Multiple Correct Answers Type

1. (1),(4)

4. (1),(3),(4)

7. (1),(3)

10. (1),(3)

13. (1),(2),(4)

16. (1),(3),(4)

19. (1),(2),(3),(4)
2. (2),(3),(4)

5. (1),(3)

8. (1),(4)

11. (1),(3)

14. (1),(3),(4)

17. (1),(4)

20. (1),(3)
3. (1),(3)

6. (3),(4)

9. (1),(4)

12. (1),(4)

15. (1),(3),(4)

18. (2),(3)

Linked Comprehension Type

1. (1)

6. (4)

11. (1)
2. (4)

7. (2)

12. (1)
3. (1)

8. (2)

13. (4)
4. (3)

9. (3)

14. (2)
5. (4)

10. (3)

15. (1)

16. (2)

21. (4)

26. (2)

31. (2)

36. (4)
17. (3)

22. (4)

27. (1)

32. (4)
18. (3)

23. (1)

28. (2)

33. (3)
19. (2)

24. (1)

29. (1)

34. (1)
20. (1)

25. (4)

30. (4)

35. (3)

Matrix Match Type

1. $i \rightarrow c, d; ii \rightarrow a, c; iii \rightarrow a, c; iv \rightarrow c, d.$

2. $i \rightarrow a, c; ii \rightarrow b, c; iii \rightarrow b, c; iv \rightarrow b, d.$

3. $i \rightarrow b; ii \rightarrow a; iii \rightarrow c, d; iv \rightarrow c, d.$

4. $i \rightarrow d; ii \rightarrow c; iii \rightarrow b; iv \rightarrow a.$

5. $i \rightarrow b, c; ii \rightarrow a, d; iii \rightarrow d; iv \rightarrow c.$

6. $i \rightarrow a; ii \rightarrow c, d; iii \rightarrow a, d; iv \rightarrow b, d.$

7. $i \rightarrow c; ii \rightarrow c; iii \rightarrow c; iv \rightarrow b, d.$

8. $i \rightarrow c; ii \rightarrow b; iii \rightarrow b, d; iv \rightarrow b.$

9. (2)

10. (4)

Numerical Value Type

1. (6)

6. (3)

11. (5)

16. (5)
2. (4)

7. (5)

12. (4)

17. (15)
3. (2)

8. (2)

13. (120)
4. (1)

9. (30)

14. (1.82)
5. (2)

10. (2)

15. (3)

ARCHIVES

JEE Advanced

Single Correct Answer Type

1. (2)

Multiple Correct Answers Type

1. (1),(2),(4)

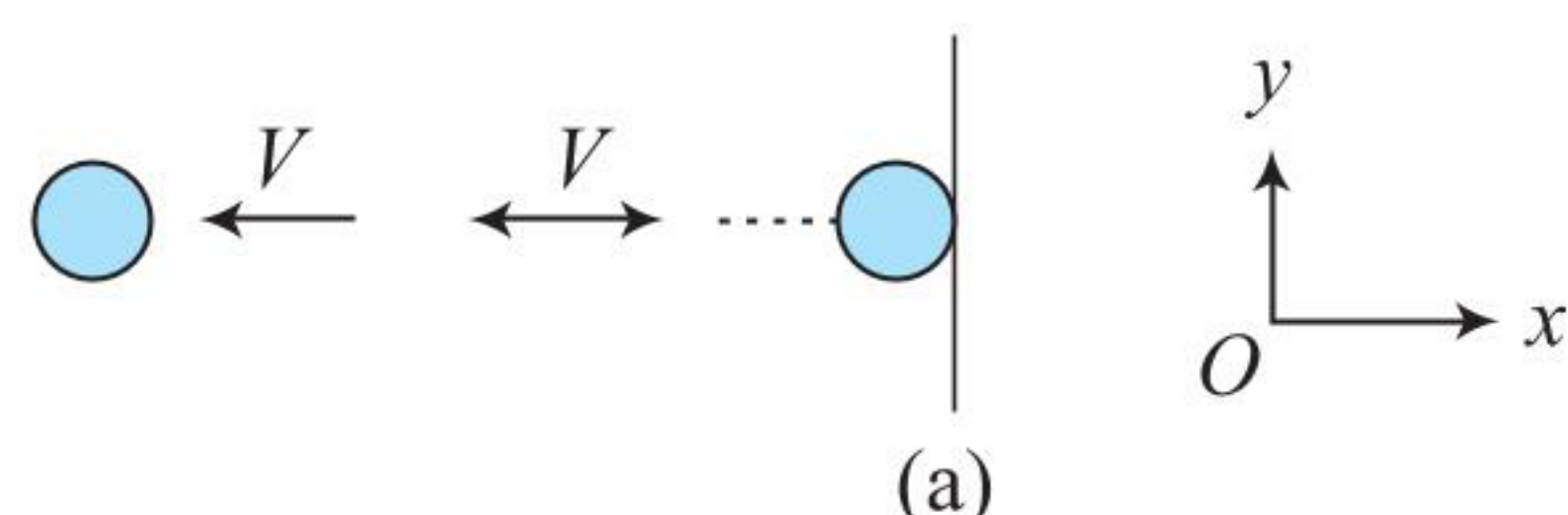
Chapter 6

Concept Application Exercises

Exercise 6.1

1. (a) Force on a body acts in a direction of change in its momentum. In both cases change in momentum of ball is along XO . Hence, in both the cases force will be along x -direction on the wall.

(b) Case (a)



Momentum of the ball before reflection = mv (along OX)

Momentum after reflection = $-mv$ (along XO)

Now,

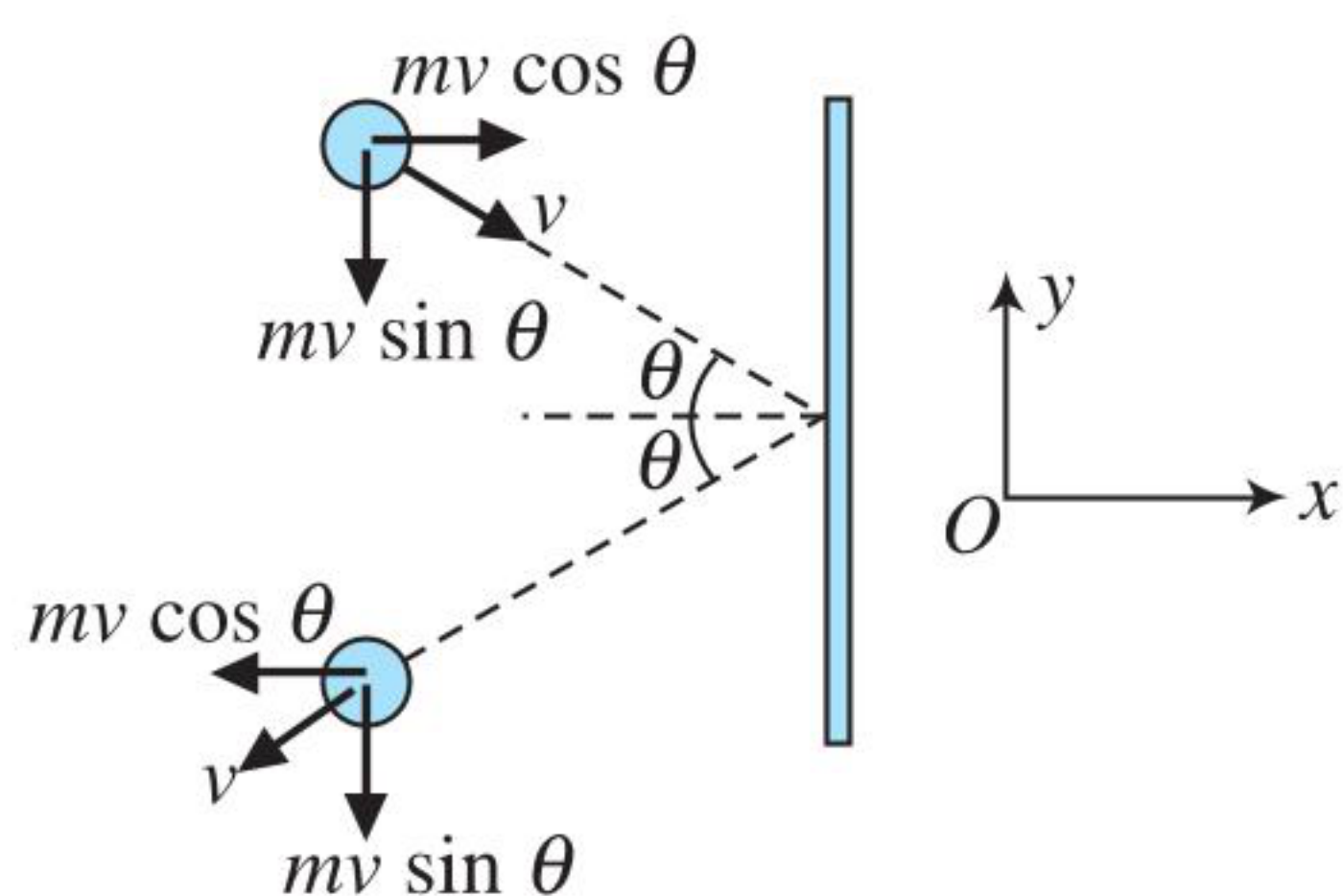
I = change in momentum of ball

$$= -mv - mv = -2mv \text{ (along } OX) = 2mv \text{ (along } x)$$

Hence, impulse on wall will be $2mv$ (along ox)

Case (b)

Similarly $I' = -mv \cos \theta - mv \cos \theta = -2mv \cos \theta$



$$\text{Ratio} = \frac{I}{I'} = \frac{2mv}{-2mv \cos \theta}; \frac{I}{I'} = \frac{1}{\cos \theta}$$

2. Just before $t = 2$ s, the velocity of the particle is

$$u = \frac{2-0}{2-0} = 1 \text{ cm s}^{-1} = 0.01 \text{ m s}^{-1}$$

Just after $t = 2$ s, the velocity of the particle is

$$v = \frac{0-2}{4-2} = -1 \text{ cm s}^{-1} = -0.01 \text{ m s}^{-1}$$

The magnitude of impulse, $J = |m(v - u)|$

$$= |0.04(-0.01 - 0.01)| = 8 \times 10^{-4} \text{ N s}$$

The given $x-t$ graph may represent the repeated rebounding of a particle between two elastic walls at $x = 0$ and $x = 2$ cm. The particle will get an impulse of 8×10^{-4} N s after every 2 s.

3. Velocity of the ball just before collision: $v^2 = 0 + 2gh$

$$v = v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m s}^{-1}$$

Therefore, momentum of the ball before collision,

$$\begin{aligned} \vec{P}_i &= m\vec{v}_i \\ &= 0.050 \times (-10)\hat{j} \text{ N s} = -0.50\hat{j} \text{ N s} \end{aligned}$$

Velocity of the ball just after collision,

$$v_f = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.25} = 5.0 \text{ m s}^{-1}$$

Therefore, momentum of the ball just after collision,

$$\vec{P}_f = m\vec{v}_f = 0.050 \times (5.0\hat{j}) = 0.25\hat{j} \text{ N s}$$

Now impulse imparted by the ground on the ball

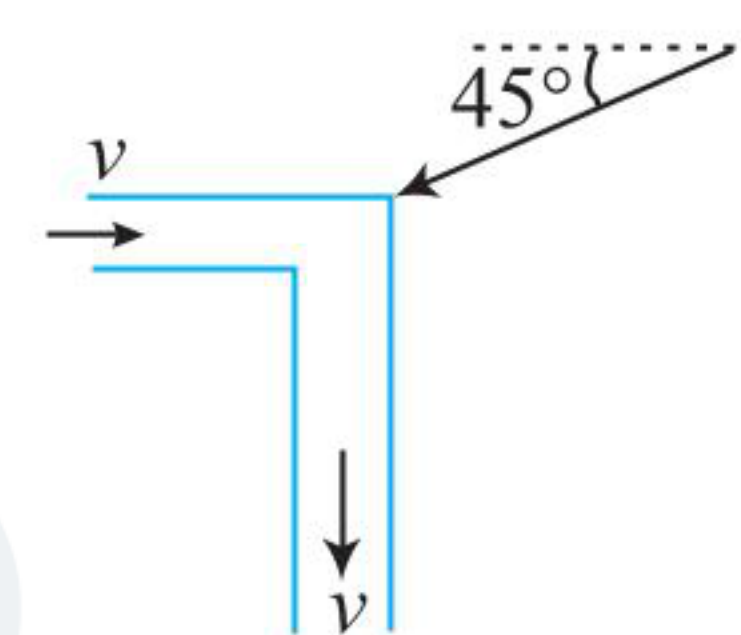
$$\vec{I} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i = 0.25\hat{j} - (-0.50\hat{j}) = 0.75\hat{j} \text{ N s}$$

$$\text{Required force, } F = \frac{\Delta P}{\Delta t} = \frac{0.75}{0.1} = 7.5 \text{ N}$$

4. Impulse, $I = 2mu \cos 30^\circ$

$$= 2 \times 3 \times 10 \times \frac{\sqrt{3}}{2} = 30\sqrt{3} \text{ N s}$$

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{30\sqrt{3}}{0.2} = 150\sqrt{3} \text{ N}$$



5. (a) The impulse is to the right and equal to the area under the $F-t$ graph
 $I = \{[5+1]/2\} \times 4 = 12 \text{ N s}$

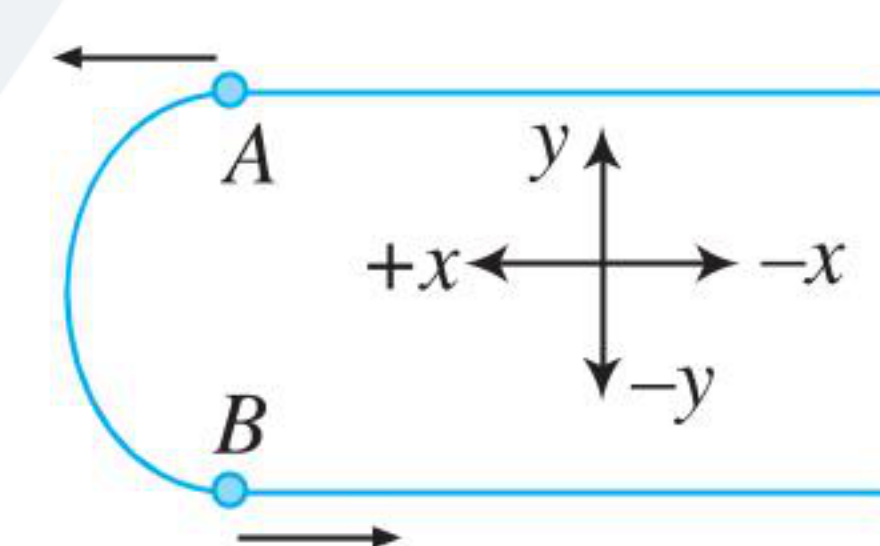
$$(b) m\vec{v}_i + \vec{F}t = m\vec{v}_f$$

$$\Rightarrow 2.5 \times 0 + 12 = (2.5)v \Rightarrow v = 4.8 \text{ m s}^{-1}$$

- (c) From the same equation,

$$\Rightarrow 2.5 \times (-2) + 12 = (2.5)v \Rightarrow v = 2.8 \text{ m s}^{-1}$$

- (d) $F_{\text{avg}} \Delta t = 12.0 \Rightarrow F_{\text{avg}} = 2.40 \text{ N}$



6. Force applied by hail stone,

$$\begin{aligned} |F| &= \frac{\Delta p}{\Delta t} = \left| \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} \right| = \left| \frac{0 - m\vec{v}_i}{\Delta t} \right| = \frac{mv_i}{\Delta t} \\ |F| &= \frac{2000 \times 10 \times 10 \times \frac{4}{3} \pi \left(\frac{1}{2 \times 100} \right)^3 \times 900 \times 20}{1} \\ &= 1900 \text{ N} \end{aligned}$$

7. Choosing the positive X - Y axis as shown in the figure, the momentum of the bead at A is $\vec{p}_i = +m\vec{v}$. The momentum of the bead at B is $\vec{p}_f = -m\vec{v}$.

Therefore, the magnitude of the change in momentum between A and B is

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -2m\vec{v}$$

i.e. $\Delta p = 2mv$ along positive X -axis.

The time interval taken by the bead to reach from A to B is

$$\Delta t = \frac{\pi \cdot d/2}{v} = \frac{\pi d}{2v}$$

Therefore, the average force exerted by the bead on the wire is

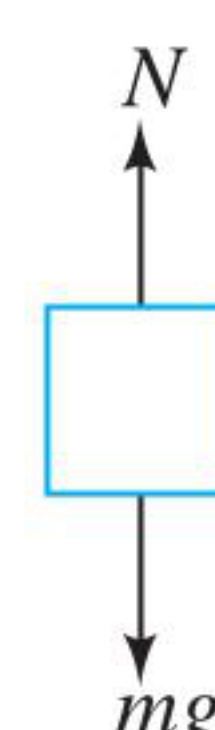
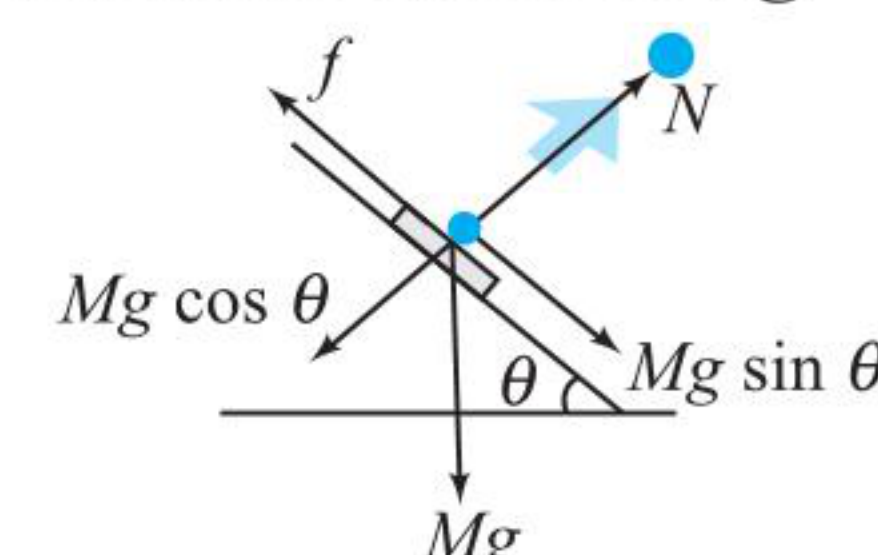
$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{\pi d}{2v}} = \frac{4mv^2}{\pi d}$$

8. Force exerted by the air on the wall is

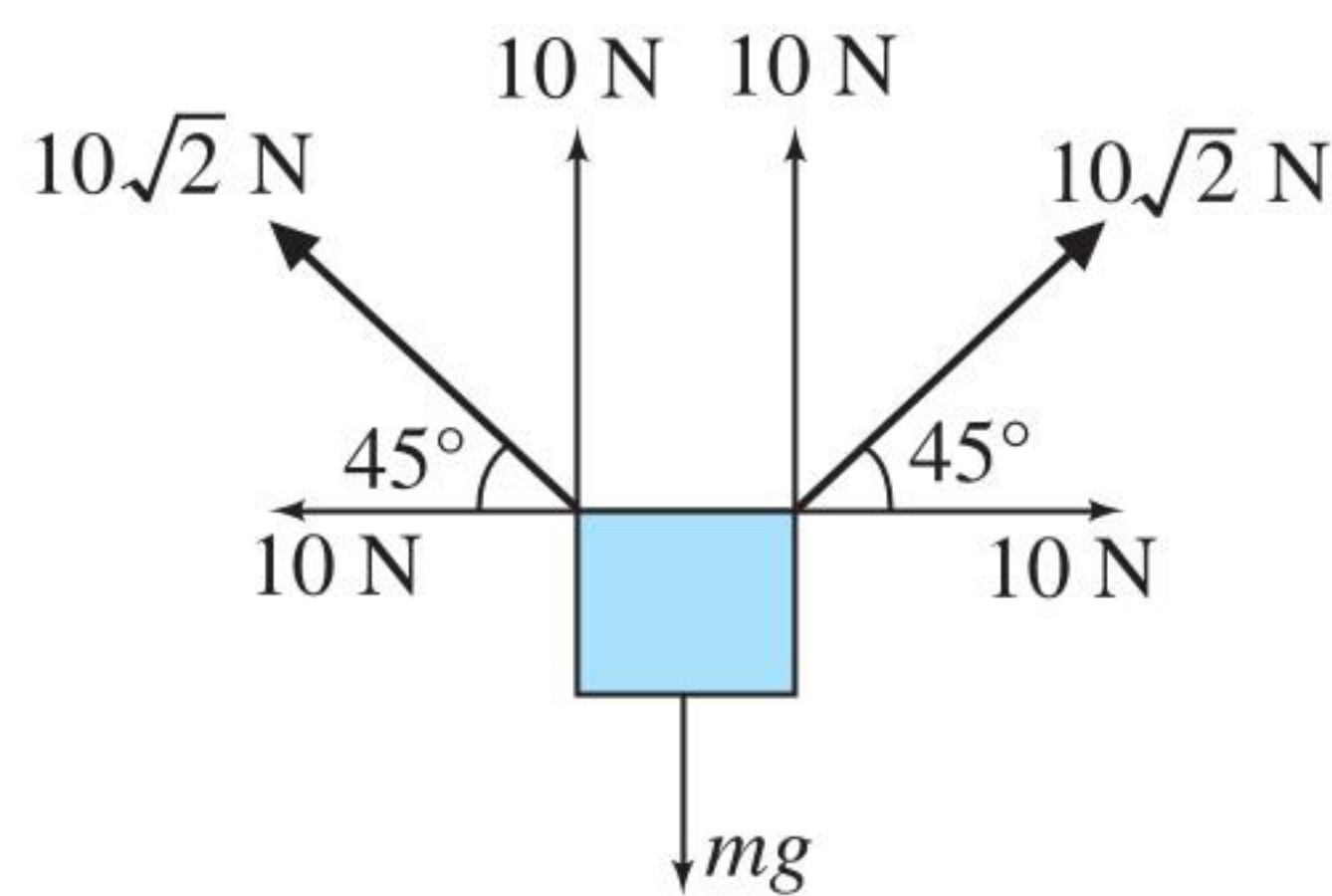
$$Av^2 \rho = 108 \times \left(\frac{100 \times 1000}{3600} \right)^2 \times 1.2 \approx 10^5 \text{ N}$$

Exercise 6.2

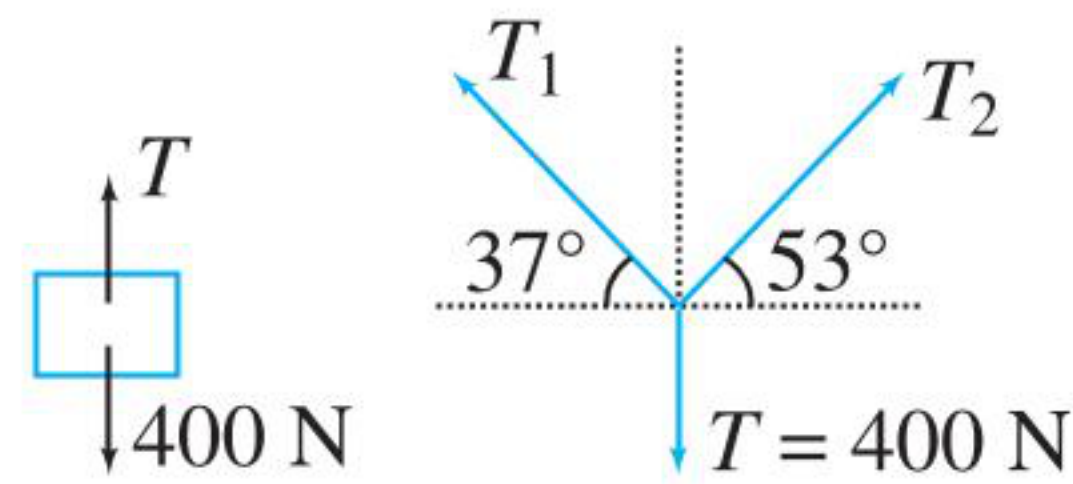
- Incorrect. F is not acting on M .
- No force is acting in horizontal direction on m , as there is no friction between blocks. No relative motion of m . Acceleration of m is zero.
- Weight shown by machine is $N = Mg \cos \theta$. So the mass shown by the machine will be $N/g = M \cos \theta$



$$4. 10 + 10 = mg \Rightarrow m = 2 \text{ kg}$$



5. From the FBD of the block it is clear that tension in lower cord is 400 N. Figure shows the junction where the three cords meet.



As the system is in equilibrium, net sum of all the forces at the junction must be zero. In horizontal direction, we get

$$\cos 53^\circ T_2 - \cos 37^\circ T_1 = 0 \quad \dots(i)$$

In vertical direction, we get

$$\sin 37^\circ T_1 + \sin 53^\circ T_2 - 400 = 0 \quad \dots(ii)$$

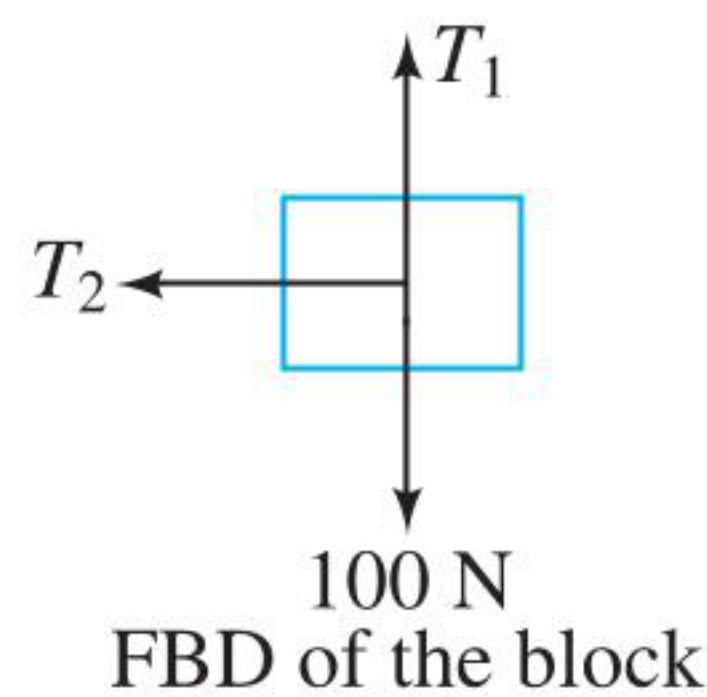
On solving the above equations, we get

$$T_1 = 240 \text{ N and } T_2 = 320 \text{ N}$$

6. A horizontal string cannot balance a vertical weight.

$$\sum F_y = 0 \Rightarrow T_1 = 100 \text{ N,}$$

$$\sum F_x = 0 \Rightarrow T_2 = 0$$

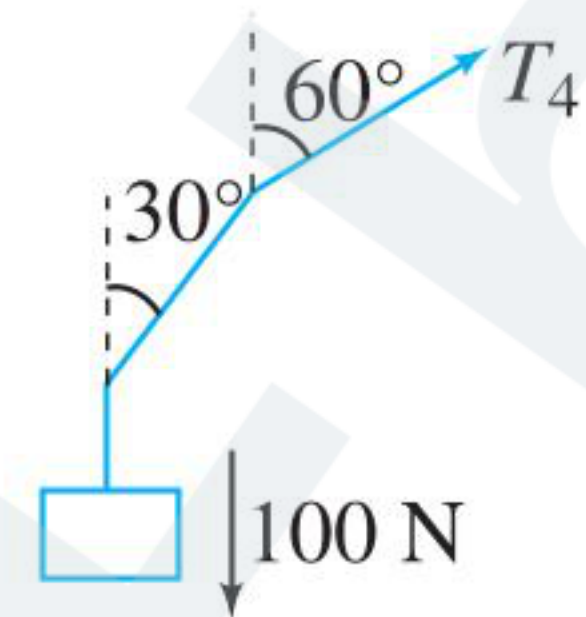


7. The free body diagram is drawn so that only the string T_4 is cut, as shown in Figure

$$\sum F_y = 0$$

$$\Rightarrow T_4 \cos 60^\circ = 100$$

$$\Rightarrow T_4 = 200 \text{ N}$$



8. The FBD for the two cases is shown in Figure.

In first case, let the force exerted by the man on the floor is N_1 . we have

$$N_1 = T + 50g$$

Block is to be raised without acceleration, so $T = 25g$

$$\therefore N_1 = 25g + 50g = 75g = 75 \times 10 = 750 \text{ N}$$

In second case, let the force exerted by the man on the floor is N_2 . We have $N_2 = 50g - T$ and $T = 25g$

$$\therefore N_2 = 50g - 25g = 25g = 25 \times 10 = 250 \text{ N}$$

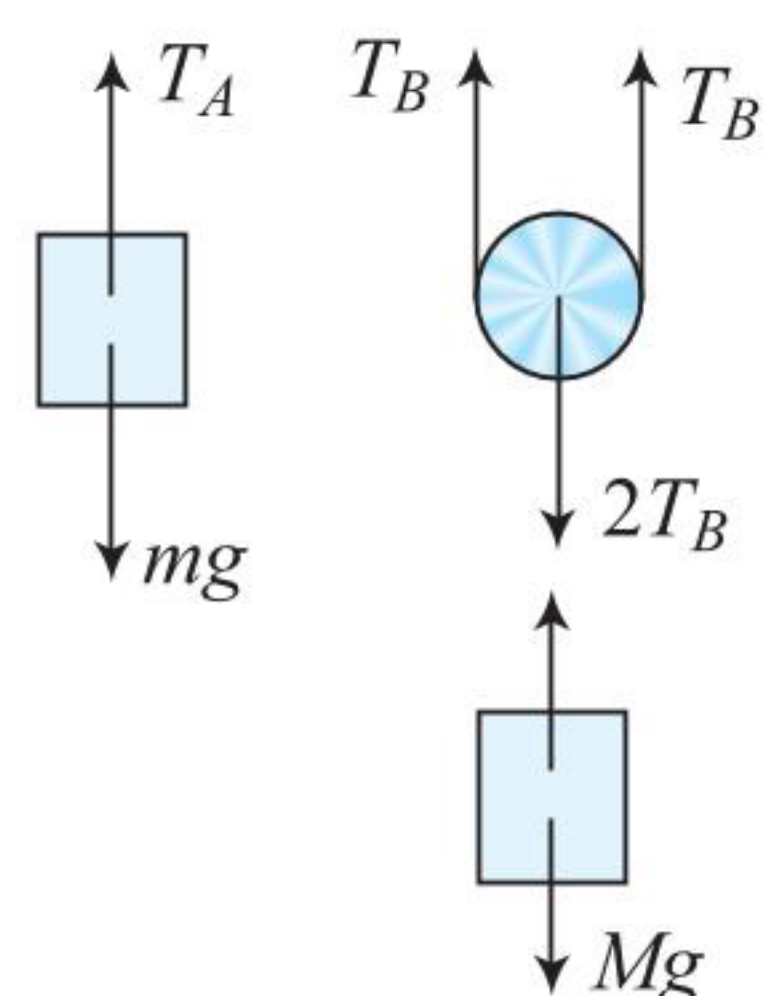
As the floor yields to a downward force of 700 N, so the man should adopt the second case.

9. From diagram it is clear

$$T_A = Mg \quad \dots(i)$$

$$\text{and } 2T_B = Mg \quad \dots(ii)$$

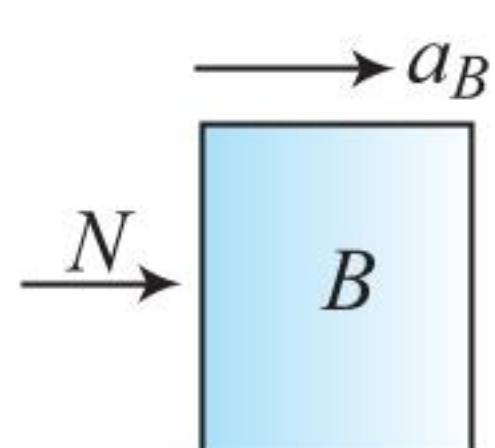
$$\text{Hence, } \frac{T_A}{T_B} = 2$$



Exercise 6.3

1. Block C will not move and blocks A and B will move with common acceleration

$$a_A = a_B = \frac{50}{(1+2)} = \frac{50}{3} \text{ ms}^{-2}$$



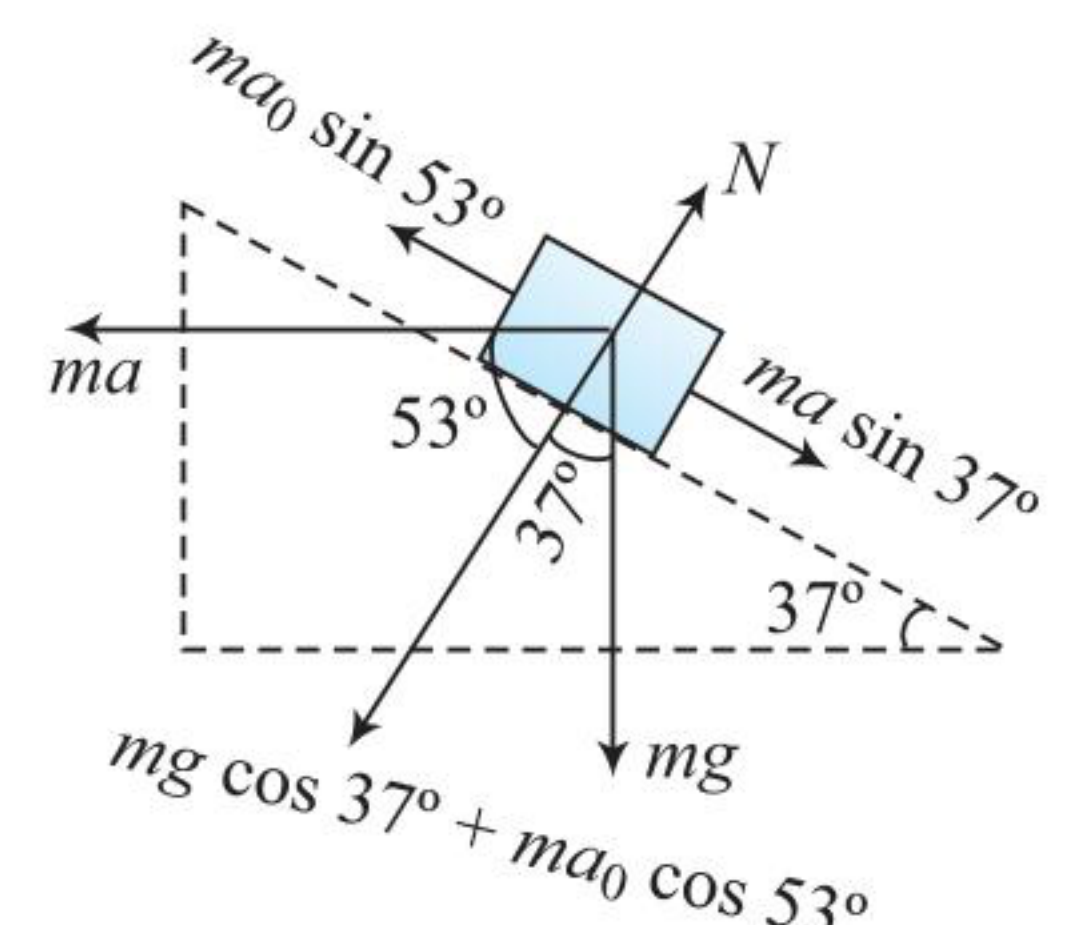
From free body diagram of B

$$N = m_B a_B = 2 \times \frac{50}{3} = \frac{100}{3} \text{ N}$$

2. (a) Acceleration of dominoes does not depend upon the number of blocks as acceleration $a = \frac{F}{\text{Total mass}} = \frac{10}{7} \text{ ms}^{-2}$
- (b) To maximize the force on column C due to column B , the mass of column C should be maximum. But at least one block should be present in each column. Hence, 5 blocks should be on column C and one each on column A and B .
- (c) Similarly, to maximize the force on column B , maximum mass should be on column B . Hence column A —one, column B —five, and column C —one.
- (d) To maximize force on column B due to column A , only one block should be on column A and remaining six can be placed in any arrangement.

3. For discussion of the block inclined plane is taken as reference frame. From the force diagram,

$$\begin{aligned} N &= mg \cos 37^\circ + ma_0 \cos 53^\circ \\ &= 5 \times 10 \times 0.8 + 5 \times 5 \times 0.6 \\ &= 40 + 15 = 55 \text{ N} \end{aligned}$$



4. When there is no sliding at any contact surface, we may take complete system as a single body.

Considering motion of the system

$$F = (M + m)a$$

From FBD of m

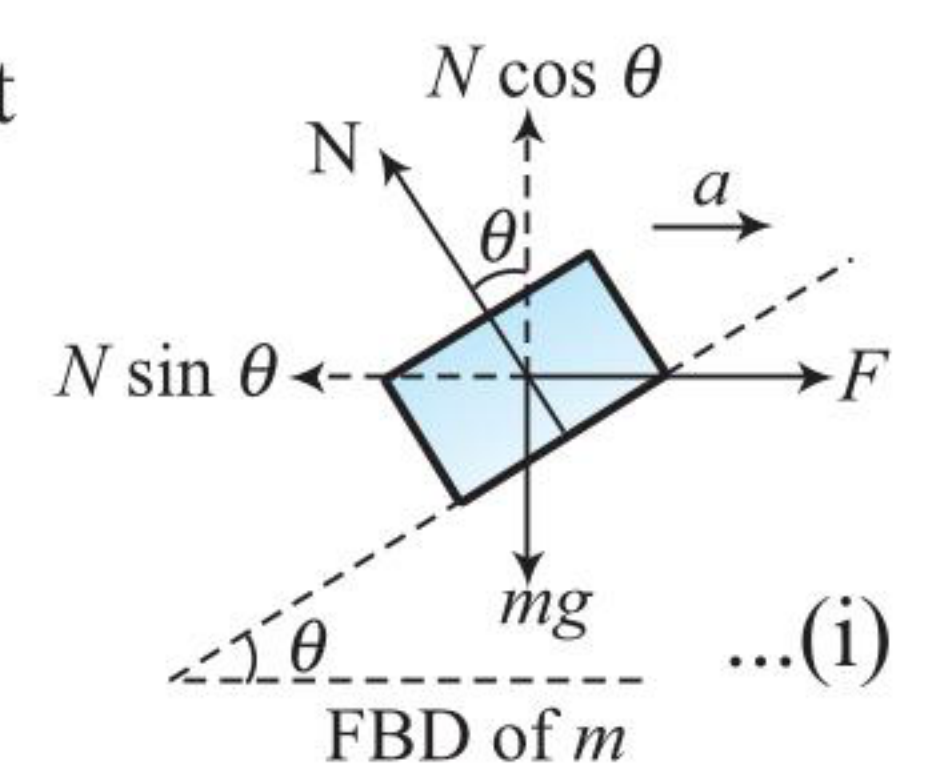
$$N \cos \theta = mg$$

$$\text{and } F - N \sin \theta = ma$$

$$\text{From Eq. (ii) } N = \frac{mg}{\cos \theta}$$

Solving Eqs. (i), (ii) and (iii), we get

$$F = \frac{mg}{M} (m + M) \tan \theta$$



5. Normal to inclined the object is at rest

$$N = mg \cos 37^\circ - ma \cos 37^\circ$$

$$= m(g - a) \cos 37^\circ$$

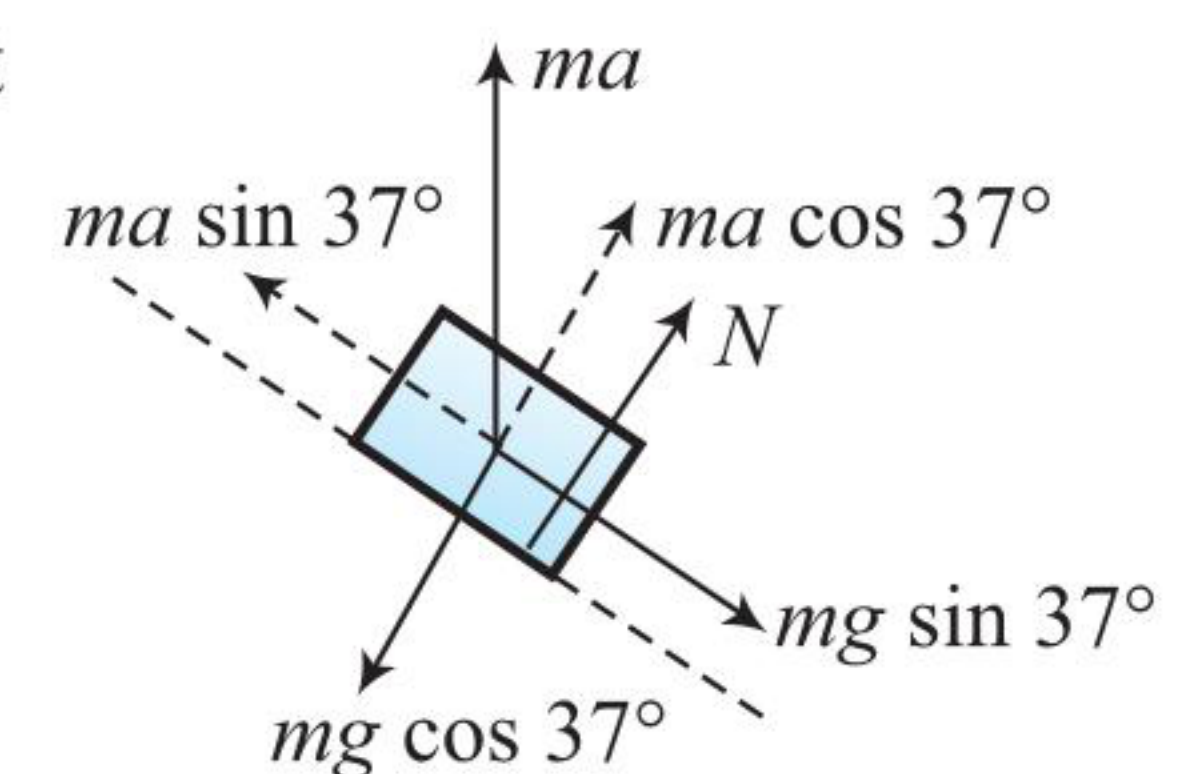
$$= 1 \times 7 \times \frac{4}{5} = \frac{28}{5} \text{ N}$$

$$mg \sin 37^\circ - ma \sin 37^\circ = ma_{\text{rel}}$$

$$a_{\text{rel}} = (g - a) \sin 37^\circ$$

$$= (10 - 3) \times \frac{3}{5} = \frac{21}{5} \text{ ms}^{-2}$$

$$s_{\text{rel}} = u_{\text{rel}} t + \frac{1}{2} a_{\text{rel}} t^2 \Rightarrow 2.1 = 0 + \frac{1}{2} \left(\frac{21}{5} \right) t^2 \Rightarrow t = 1 \text{ s}$$



6. From $t = 0$ to 2 s, the lift has a uniform acceleration

$$a_1 = \frac{\Delta v}{\Delta t} = \frac{3.6 - 0}{2 - 0} = 1.8 \text{ ms}^{-2}$$

From $t = 2$ to 10 s, the lift has a constant speed and hence acceleration a_2 during the period is 0 ms^{-2} .

From $t = 10$ s to 12 s, the lift has a uniform acceleration. If a_3 is the acceleration during the period, then

$$a_3 = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{0 - 3.6}{12 - 10} = -1.8 \text{ ms}^{-2}$$

(a) Here $m = 1500 \text{ kg}$, $g = 9.8 \text{ ms}^{-2}$

(i) At $t = 1 \text{ s}$, $a = a_1 = 1.8 \text{ ms}^{-2}$

$$\text{Tension } T_1 = m(g + a) = 1500(10 + 1.8) = 17700 \text{ N}$$

(ii) At $t = 6 \text{ s}$, $a = a_2 = 0$

$$\text{Tension } T_2 = m(g + a_2) = mg = 1500 \times 10 = 15000 \text{ N}$$

(iii) At $t = 11 \text{ s}$ $a = a_3 = -1.8 \text{ ms}^{-2}$

$$\text{Tension } T_3 = m(g + a_3) = 1500(10 - 1.8) = 12300 \text{ N}$$

(b) Height reached by the lift

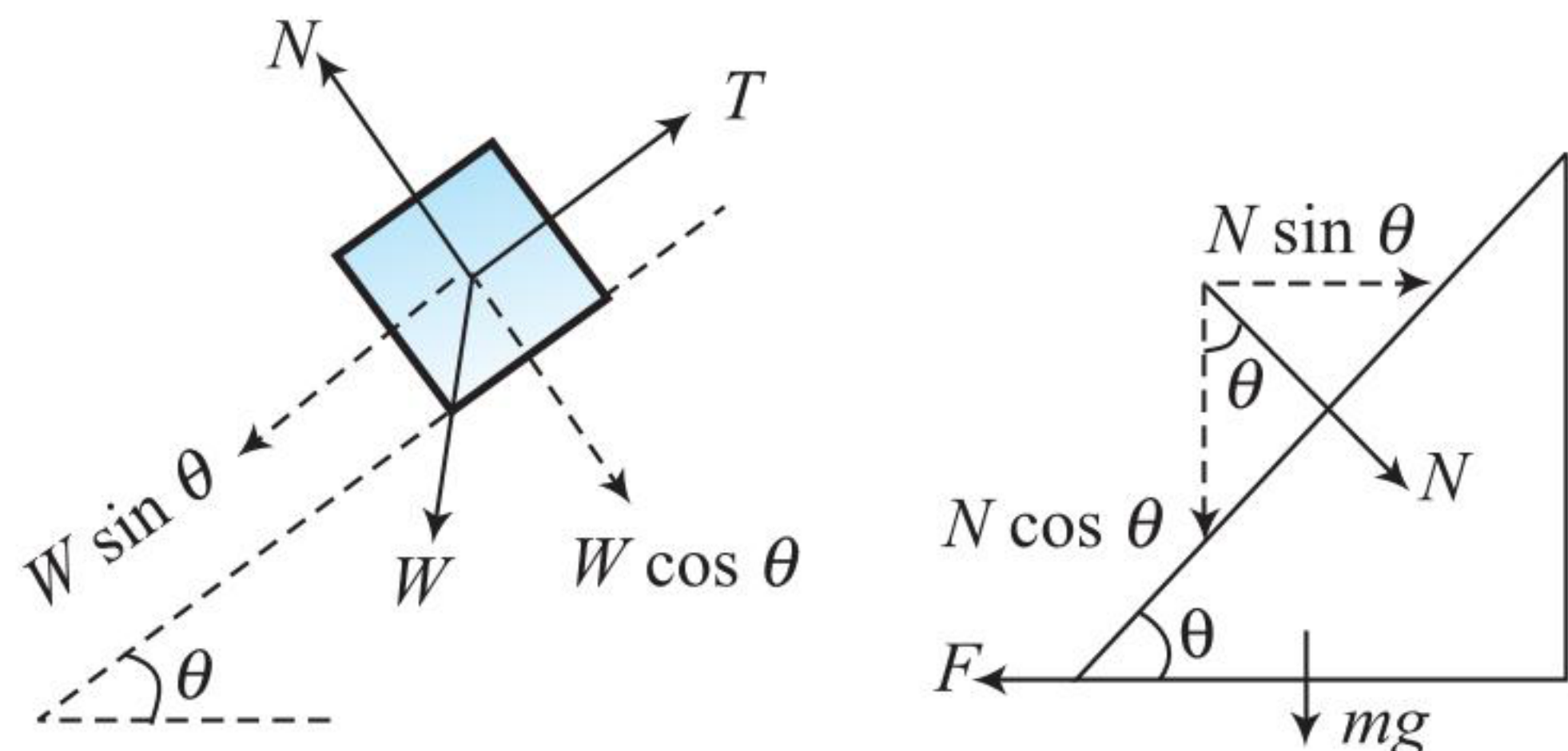
= Area of enclosed by $v - t$ curve and t -axis

= Area of quadrilateral $OABC = 36 \text{ m}$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{26}{12} = 3 \text{ ms}^{-1}$$

$$\begin{aligned} \text{Average acceleration} &= \frac{\text{Net change in velocity}}{\text{Total time}} \\ &= \frac{0}{12} = 0 \text{ ms}^{-2} \end{aligned}$$

7.



$$N = W \cos \theta \text{ and } N \sin \theta = F$$

$$\Rightarrow W \cos \theta \sin \theta = F$$

$$\text{or } F = W \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \Rightarrow F = \frac{\sqrt{3}W}{4}$$

Exercise 6.4

1. By Newton's second law, we have

$$1g - T_1 = 1a \quad \dots(i)$$

$$(T_1 + 2g) - T_2 = 2a \quad \dots(ii)$$

$$\text{and } T_2 - 2g = 2a \quad \dots(iii)$$

Solving Eqs. (i), (ii), and (iii),

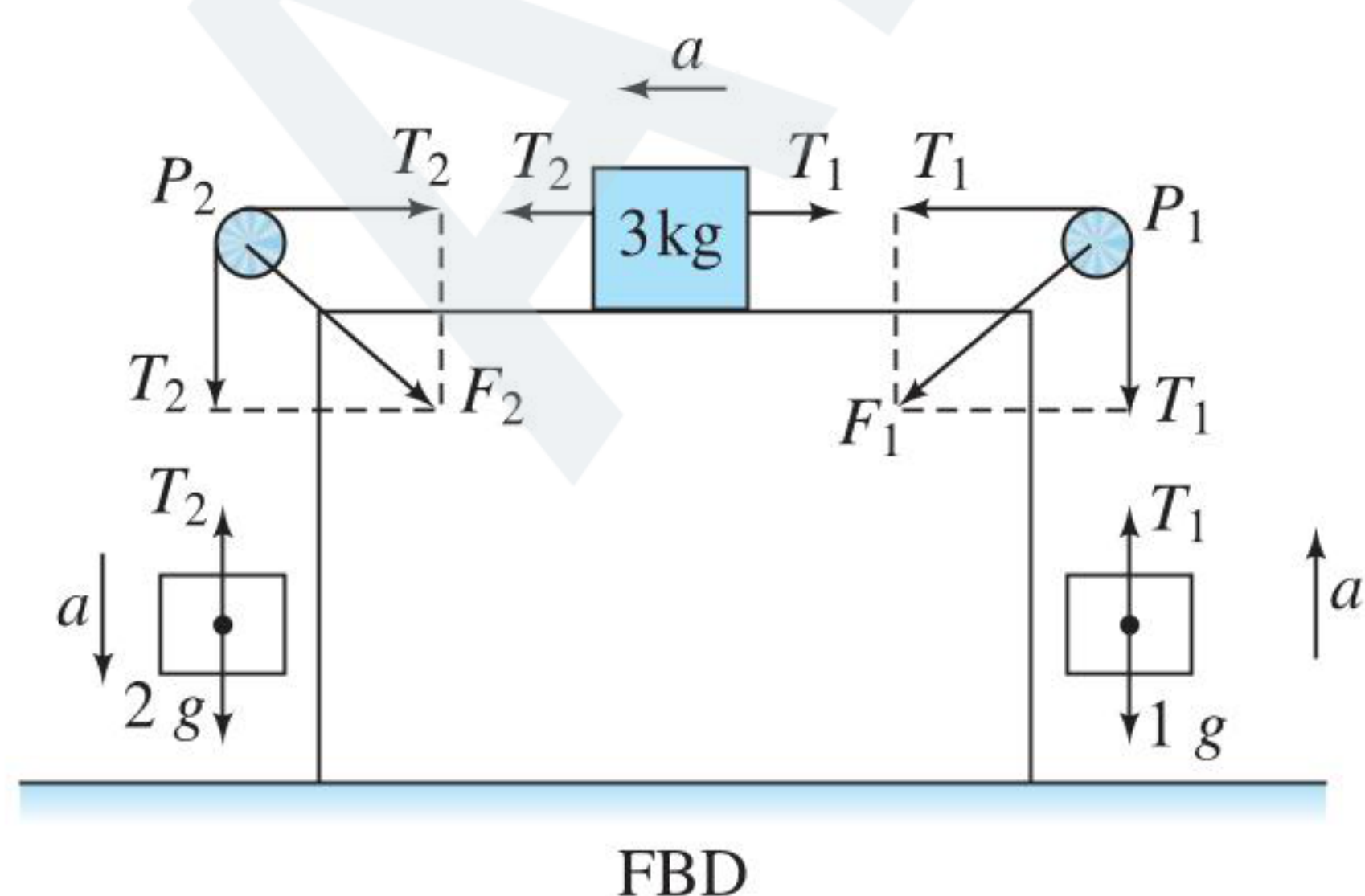
we get $a = 2 \text{ ms}^{-2}$, $T_1 = 8 \text{ N}$, and $T_2 = 24 \text{ N}$

By shortcut method:

$$a = \frac{[(2+1)-2]g}{(1+2+2)} = 2 \text{ ms}^{-2}$$

$$T_1 = m(g - a) = 1(10 - 2) = 8 \text{ N}$$

2. **Note:** As the motion of the block is on smooth horizontal surface, so no need to mark the forces along vertical direction.



By Newton's second law, we have

$$T_1 - 1g = 1a \quad \dots(i)$$

$$T_2 - T_1 = 3a \quad \dots(ii)$$

$$\text{and } 2g - T_2 = 2a \quad \dots(iii)$$

Solving above equations, we get

$$a = \frac{g}{6} \text{ ms}^{-2}, \quad T_1 = \frac{7g}{6} \text{ N}, \quad \text{and} \quad T_2 = \frac{5g}{3} \text{ N}$$

$$\text{Force on pulley } P_1, F_1 = \sqrt{T_1^2 + T_1^2} = \sqrt{2} T_1$$

$$\text{Force on pulley } P_2, F_2 = \sqrt{T_2^2 + T_2^2} = \sqrt{2} T_2$$

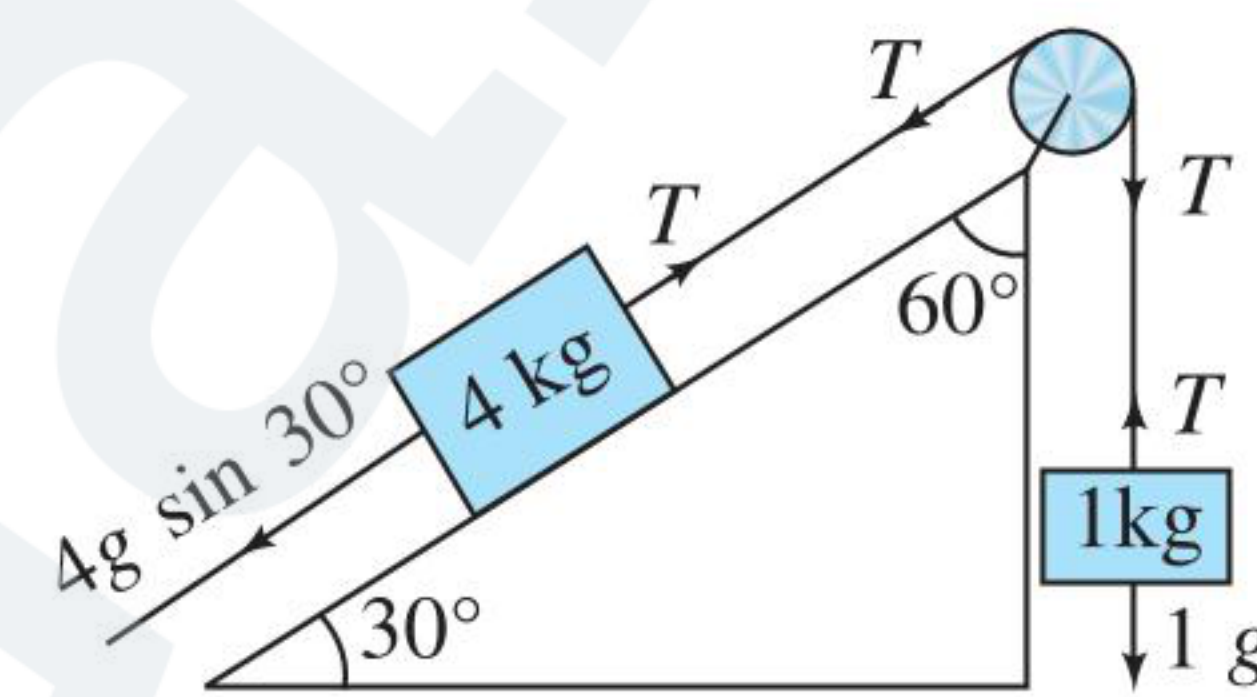
By shortcut method:

$$a = \frac{\text{Unbalanced load}}{\text{Total mass}} = \frac{(2-1)g}{(1+3+2)} = \frac{g}{6} \text{ ms}^{-2}$$

$$T_1 = m_{\text{up}}(g + a) = 1 \left(g + \frac{g}{6} \right) = \frac{7g}{6} \text{ N}$$

$$T_2 = m_{\text{down}}(g - a) = 2 \left(g - \frac{g}{6} \right) = \frac{5g}{3} \text{ N}$$

3. By Newton's second law



$$4g \sin 30 - T = 4a \quad \dots(i)$$

$$\text{and } T - 1g = 1a \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$a = 2 \text{ ms}^{-2} \text{ and } T = 12 \text{ N.}$$

By shortcut method:

$$a = \frac{\text{Unbalanced load}}{\text{Total mass}} = \frac{4g \sin 30^\circ - 1g}{4+1} = 2 \text{ ms}^{-2}$$

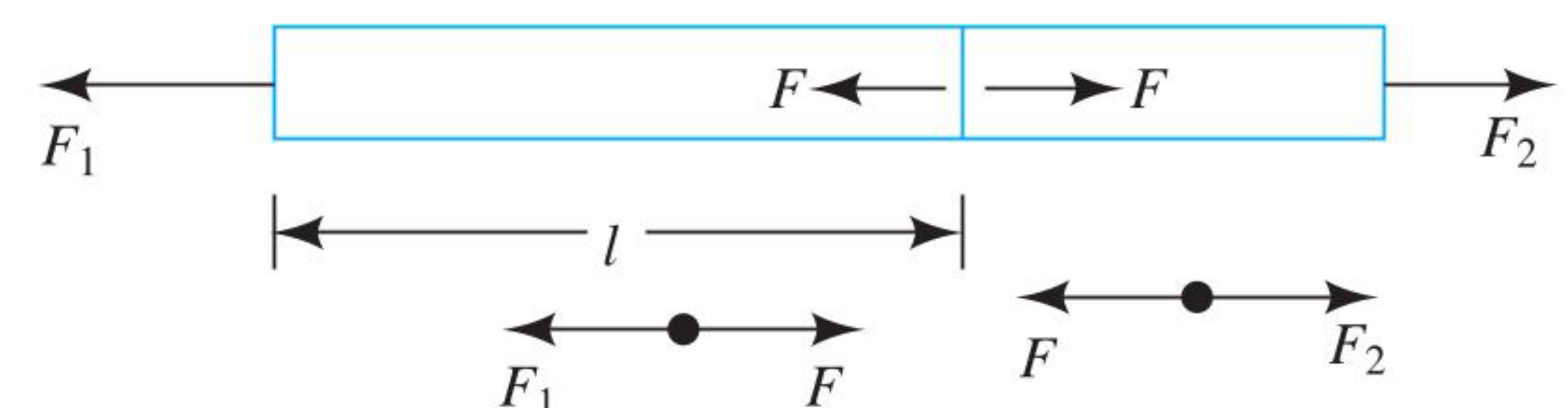
$$T = M_{\text{up}}(g + a) = 1(10 + 2) = 12 \text{ N}$$

4. Let tension in the rope is T . By Newton's second law, we have

$$T - (10g + 8g) = 10 \times 2 + 8 \times 0$$

$$\text{or } T = 18g + 20 = 200 \text{ N}$$

5. Suppose $F_2 > F_1$. Then the motion of the rod will take place in the direction of the force F_2 with acceleration, say, a . Considering the motion of the entire rod, we have $F_2 - F_1 = \text{net force on the rod} = mL \times a$ where m is the mass of the rod per unit length of it.



Considering the motion of the length l from the first end $F - F_1 = mla$. Dividing, we get

$$\frac{F - F_1}{F_2 - F_1} = \frac{l}{L} \quad \text{or} \quad F = \frac{(F_2 - F_1)l}{L} + F_1$$

6. The point to this problem is that the monkey and the bananas have the same weight, and the tension in the string is the same at the point where the bananas are suspended and where the monkey is pulling. In all the cases, the monkey and the bananas will have the same net force and hence the same acceleration, direction, and magnitude.

(a) The bananas move up.

(b) The monkey and bananas always move at the same velocity, so the distance between them stays the same.

(c) Both the monkey and the bananas are in free fall, and as they have the same initial velocity, the distance between them does not change.

(d) The bananas will slow down at the same rate as the monkey; if the monkey comes to a stop, so will the bananas.

7. Equation of motion for M :

Since M is stationary, $T - Mg = 0$

$$\Rightarrow T = Mg \quad \dots(i)$$

Since the boy moves up with an acceleration a ,

$$T - mg = ma$$

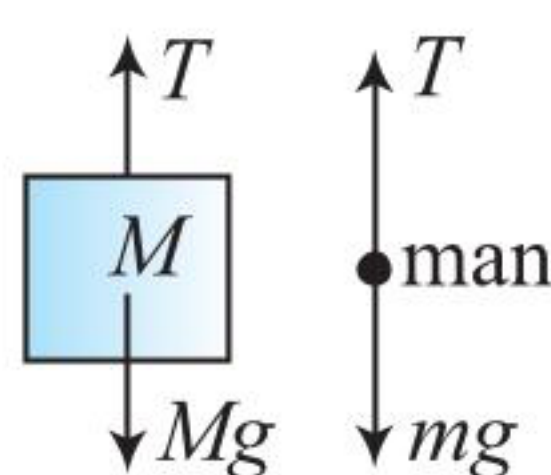
$$\Rightarrow T = m(g + a) \quad \dots(ii)$$

Equating (i) and (ii), we obtain

$$Mg = m(g + a)$$

$$\Rightarrow a = \left(\frac{M}{m} - 1 \right) g$$

That means, if $a > \left(\frac{M}{m} - 1 \right) g$, block M can be lifted from floor.



8. From force diagram of pulley,

$$P + P \cos \theta = T$$

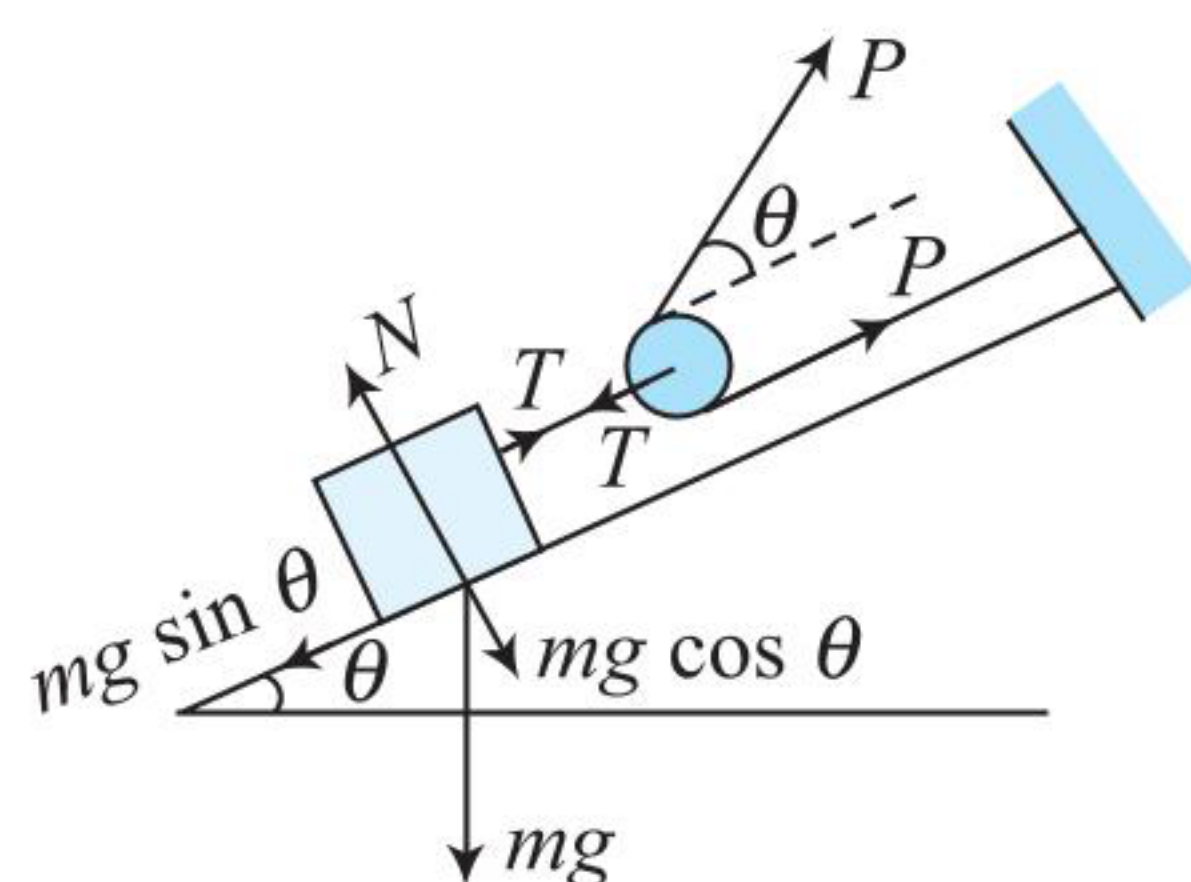
From force diagram of block

$$T > mg \sin \theta$$

$$\text{or } P + P \cos \theta > mg \sin \theta$$

$$\text{or } P > \frac{mg \sin \theta}{1 + \cos \theta}$$

$$\therefore P_{\min} = \frac{mg \sin \theta}{1 + \cos \theta}$$

9. Here, $T_0 = 8g = 80 \text{ N}$

Also, $T_0 = 2T$

$$\therefore T = 40 \text{ N}$$

From force diagram of 5 kg block,

$$50 - T = 5a$$

$$50 - 40 = 5a$$

$$10 = 5a$$

$$a = 2 \text{ m s}^{-2}$$

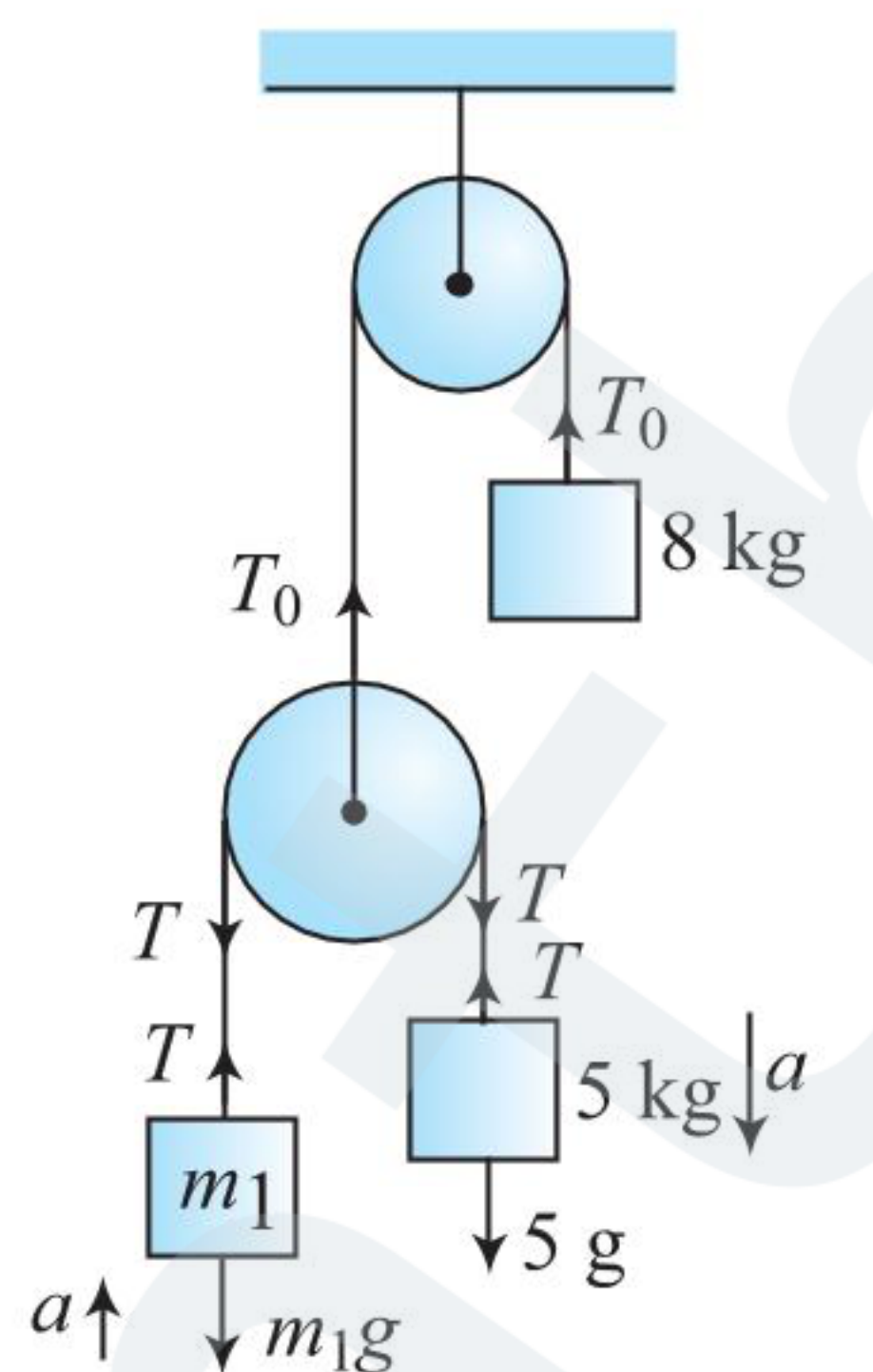
From force diagram of m_1 ,

$$T - m_1 g = m_1 a$$

$$\text{or } 40 - m_1 g = m_1 \times 2$$

$$\text{or } 40 = 10m_1 + 2m_1 = 12m_1$$

$$\therefore m_1 = \frac{40}{12} = \frac{10}{3} \text{ kg}$$

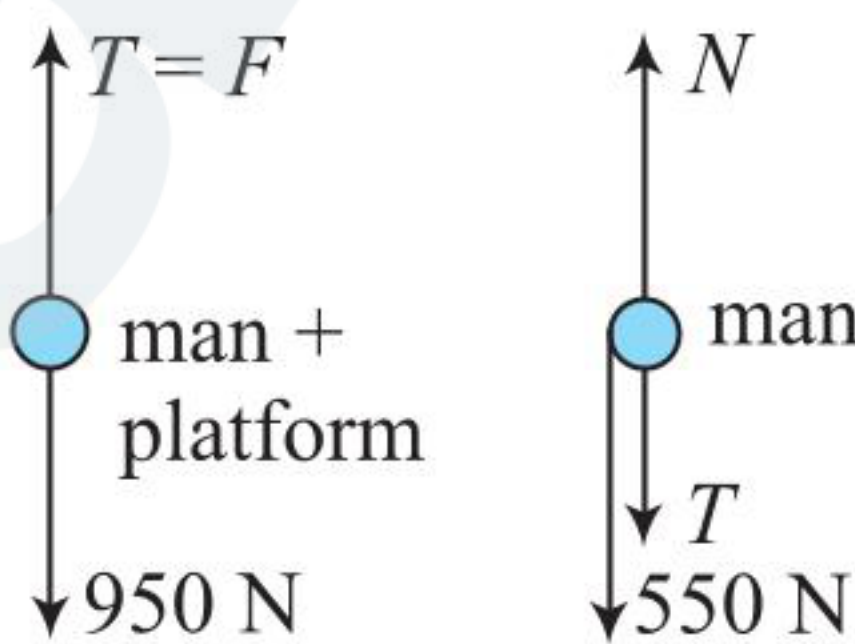


10. Taking man and platform together on system with constant velocity.

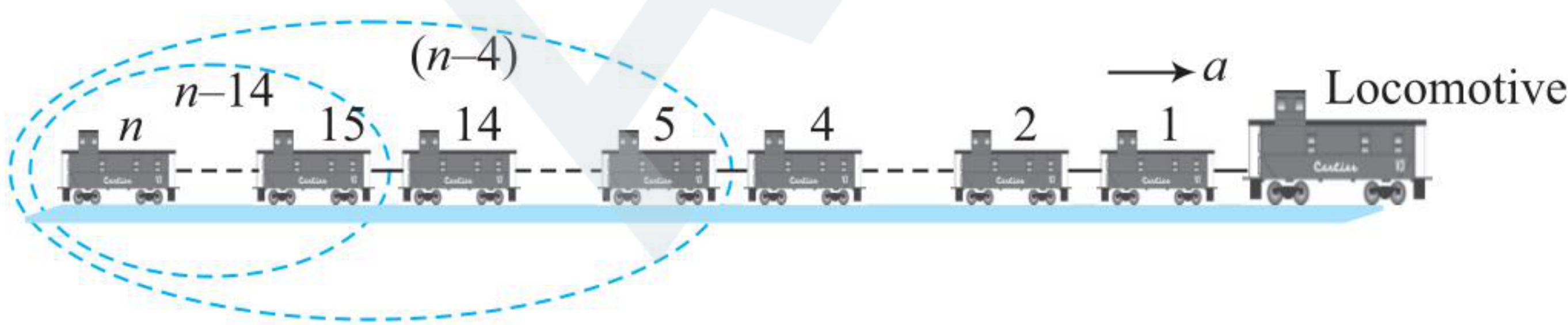
$$T = 950 \text{ N} = \text{force applied by man}$$

From FBD of man only,

$$N = T + 550 = 950 + 550 = 1500 \text{ N}$$



11. Acceleration of each cart will be same



Tension in connection between 4 and 5 cart

$$T_{4,5} = m(n - 4)a \quad \dots(i)$$

Tension in connection between 14 and 15

$$T_{14,15} = m(n - 14)a \quad \dots(ii)$$

Given, $T_{4,5} = 3T_{14,15}$

$$M(n - 4)a = 3m(n - 14)a \Rightarrow n = 19$$

12. Let the tension in the string be T at any angular position θ . The acceleration of each ball along x and y axes be a and a_1 , respectively. Writing the equation of motion of m , we obtain

$$\Sigma F_x = ma \Rightarrow T \cos \theta = ma \quad \dots(i)$$

$$\Sigma F_y = ma_1 \Rightarrow T \sin \theta = ma_1 \quad \dots(ii)$$

At point P , as it is accelerating with an acceleration a , therefore $F - 2T \cos \theta = m_p a$

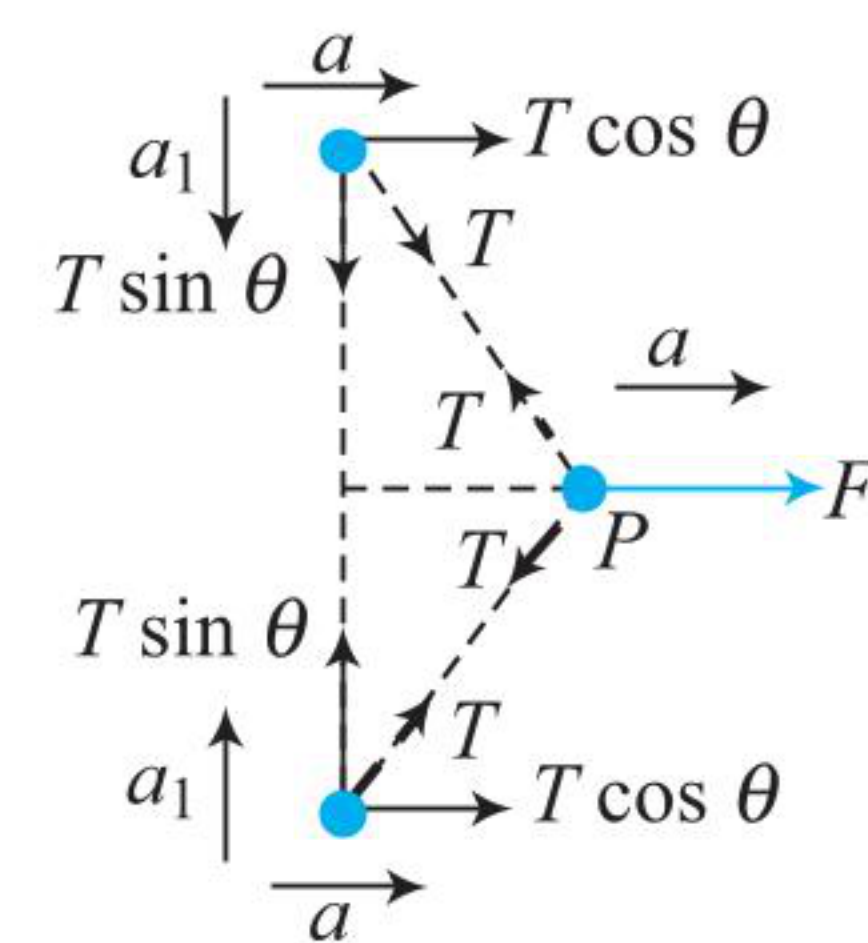
where m_p = mass of the string at the point $P \approx 0$

$$\Rightarrow F = 2T \cos \theta \quad \dots(iii)$$

$$(ii) \div (i) \Rightarrow \tan \theta = \frac{a_1}{a}$$

$$\Rightarrow a_1 = a \tan \theta, \text{ where } a = \frac{T \cos \theta}{m} \text{ from (i).}$$

$$\text{Putting } T = \frac{F}{2 \cos \theta} \text{ from (3), we obtain } a_1 = \frac{F}{2m} \tan \theta$$



13. (a) When the lift moves upward with an acceleration, then the apparent weight is

$$W' = W + ma \text{ or } W' = W + \frac{W}{g}a = W \left(1 + \frac{a}{g} \right)$$

Here $W' = 50 \text{ N}$, $a = 2.45 \text{ m s}^{-2}$, $g = 9.8 \text{ m s}^{-2}$

$$50 = \left(1 + \frac{2.45}{9.8} \right) W \Rightarrow W = \frac{4}{5} \times 50 = 40 \text{ N}$$

(b) Again if a is upward acceleration, then

$$W' = W \left(1 + \frac{a}{g} \right) \text{ gives } 30 = 40 \left(1 + \frac{a}{g} \right)$$

$$\Rightarrow \frac{a}{g} = \frac{30}{40} - 1 = -\frac{1}{4} \Rightarrow a = -\frac{g}{4}$$

i.e., lift has downward with acceleration $g/4$.

(c) When the elevator cable breaks, the lift falls freely with acceleration g .

$$\text{Therefore, apparent weight } W' = W \left(1 - \frac{g}{g} \right) = 0 \text{ N}$$

Exercise 6.5

1. Pseudo force = ma in backward direction.

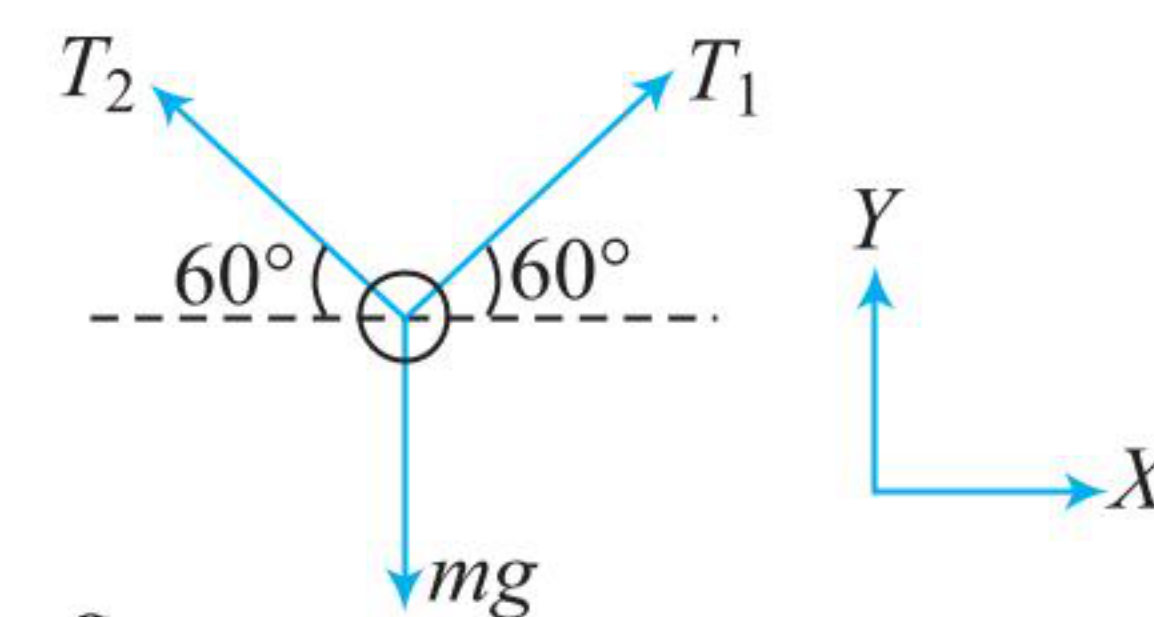
$$2. T_1 \cos 60^\circ - T_2 \cos 60^\circ = ma$$

$$T_1 \sin 60^\circ + T_2 \sin 60^\circ = mg$$

$$T_1 = 2T_2$$

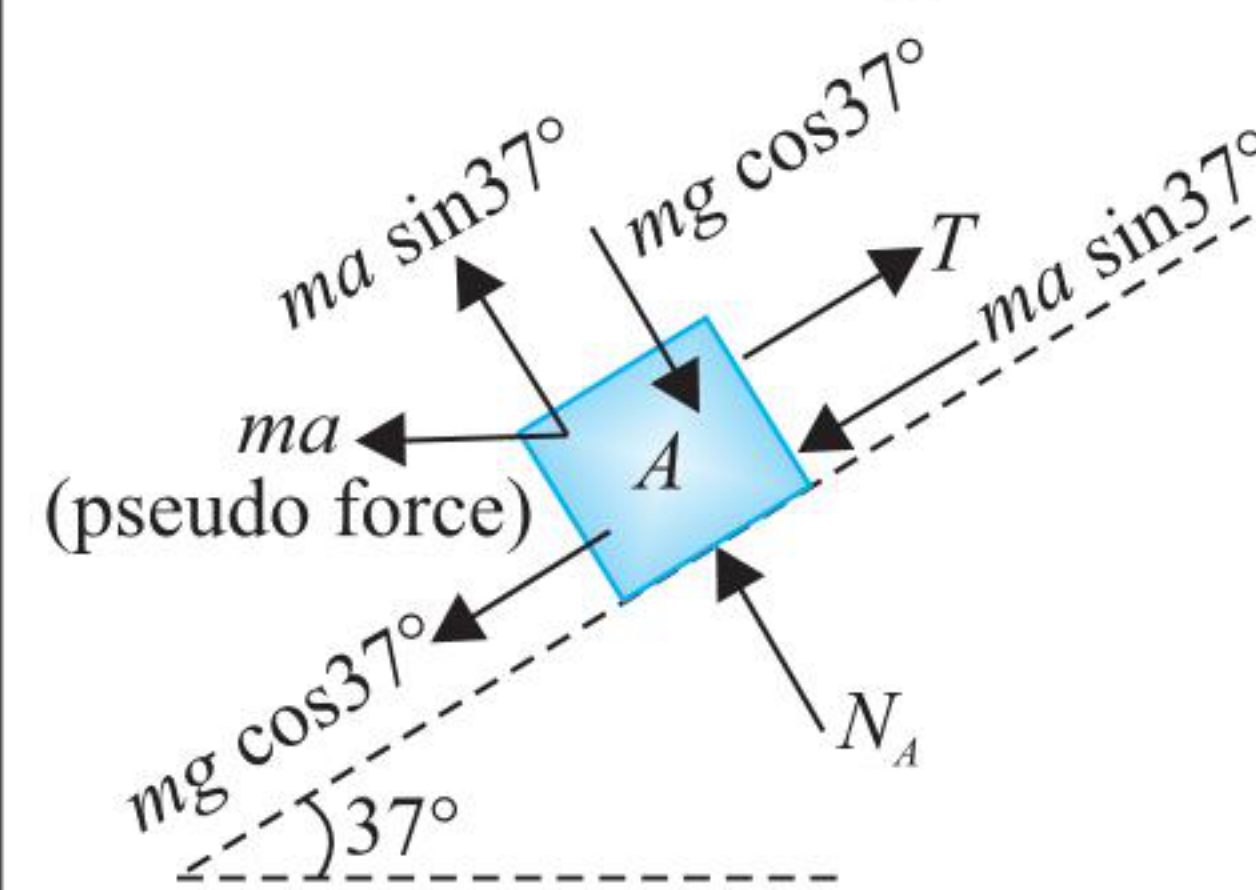
On solving equations, we get

$$T_1 = \frac{4mg}{3\sqrt{3}}, T_2 = \frac{2mg}{3\sqrt{3}}, \text{ and } a = \frac{g}{3\sqrt{3}}$$

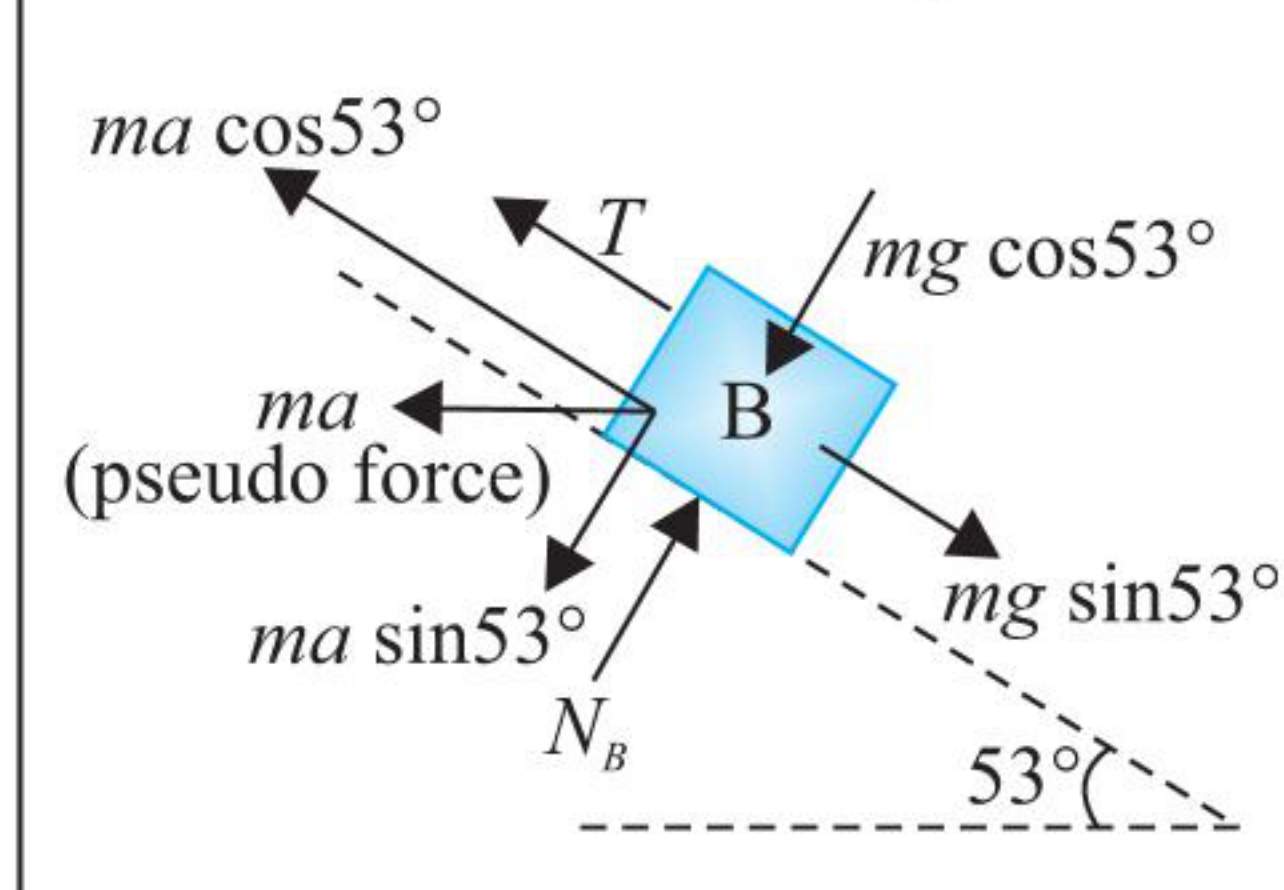


3. Let us analyse the blocks w.r.t. wedge

F.B.D. of A w.r.t wedge



F.B.D. of B w.r.t wedge



(i) In frame of reference of the wedge, the block A is not moving normal to the surface of inclined plane. Hence from F.B.D of A

$$\Rightarrow N_A = mg \cos 37^\circ - ma \sin 37^\circ$$

$$\Rightarrow N_A = mg \times \frac{4}{5} - ma \times \frac{3}{5} = \frac{m}{5}(4g - 3a)$$

(ii) Similarly analyzing block B, w.r.t wedge

$$N_B = mg \cos 53^\circ + ma \sin 53^\circ = mg \times \frac{3}{5} + ma \times \frac{4}{5}$$

$$\Rightarrow N_B = \frac{m}{5}(3g + 4a)$$

(iii) Let the magnitude of the acceleration of the blocks w.r.t wedge be a' . Writing equation of motion of the blocks parallel to slopping surface.

$$\text{For A: } mg \sin 37^\circ + ma \cos 37^\circ - T = ma' \quad \dots(i)$$

$$\text{For B: } T + ma \cos 53^\circ - mg \sin 53^\circ = ma' \quad \dots(ii)$$

$$\text{From (i) and (ii), we get } a' = \frac{1}{10}(7a - g)$$

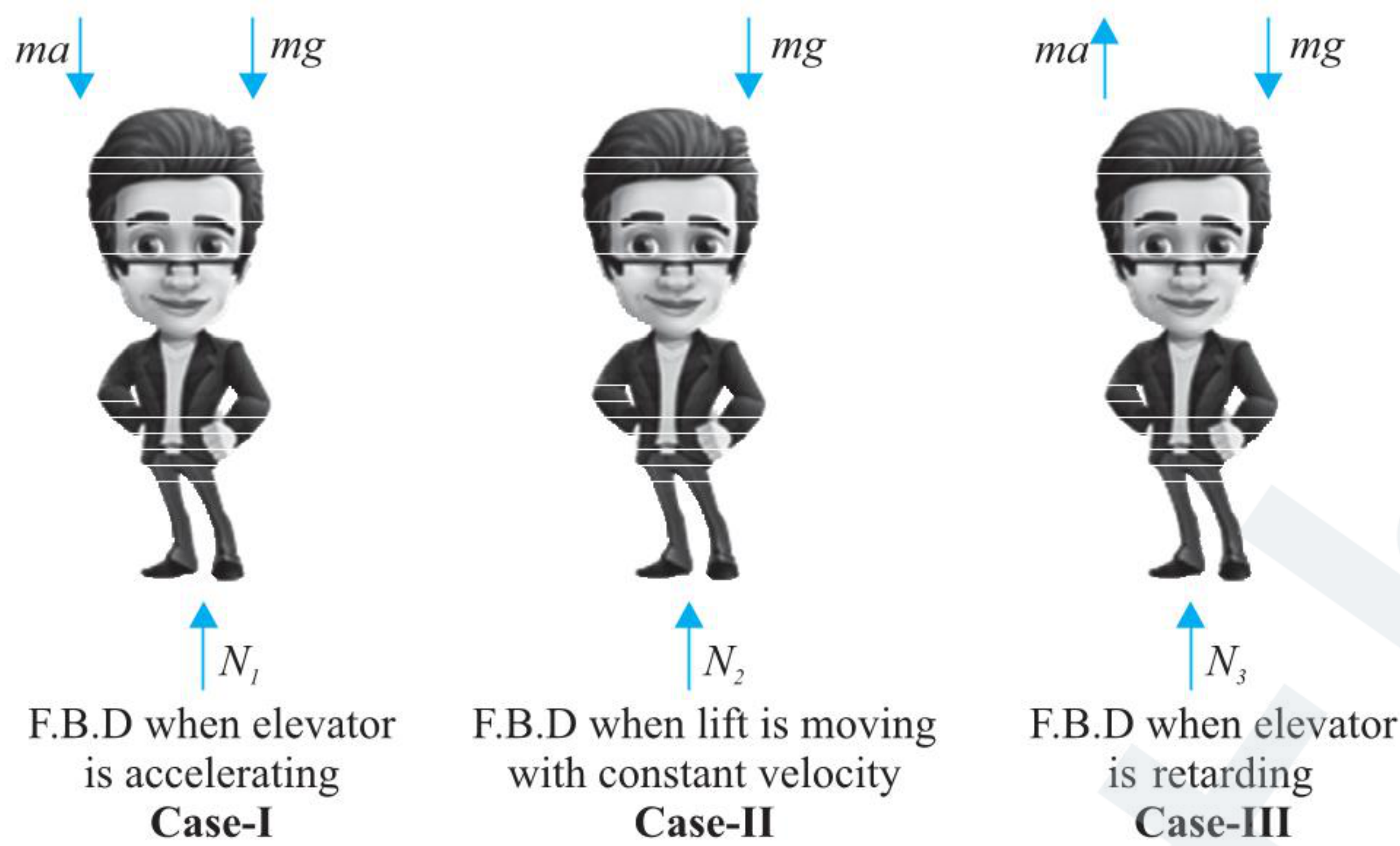
(iv) For maximum acceleration of the wedge for normal reaction on the block A to be non zero,

$$N_A > 0$$

$$\frac{m}{5}(4g - 3a) > 0 \quad \text{or} \quad a > \frac{4}{3}g$$

Hence the maximum value of acceleration of the wedge is $\frac{4}{3}g$.

4. Drawing F.B.D of the person w.r.t elevator



Equation of motion,

For Case-I:

$$N_1 = mg + ma = 72 \times 10 \text{ N} \quad \dots(i)$$

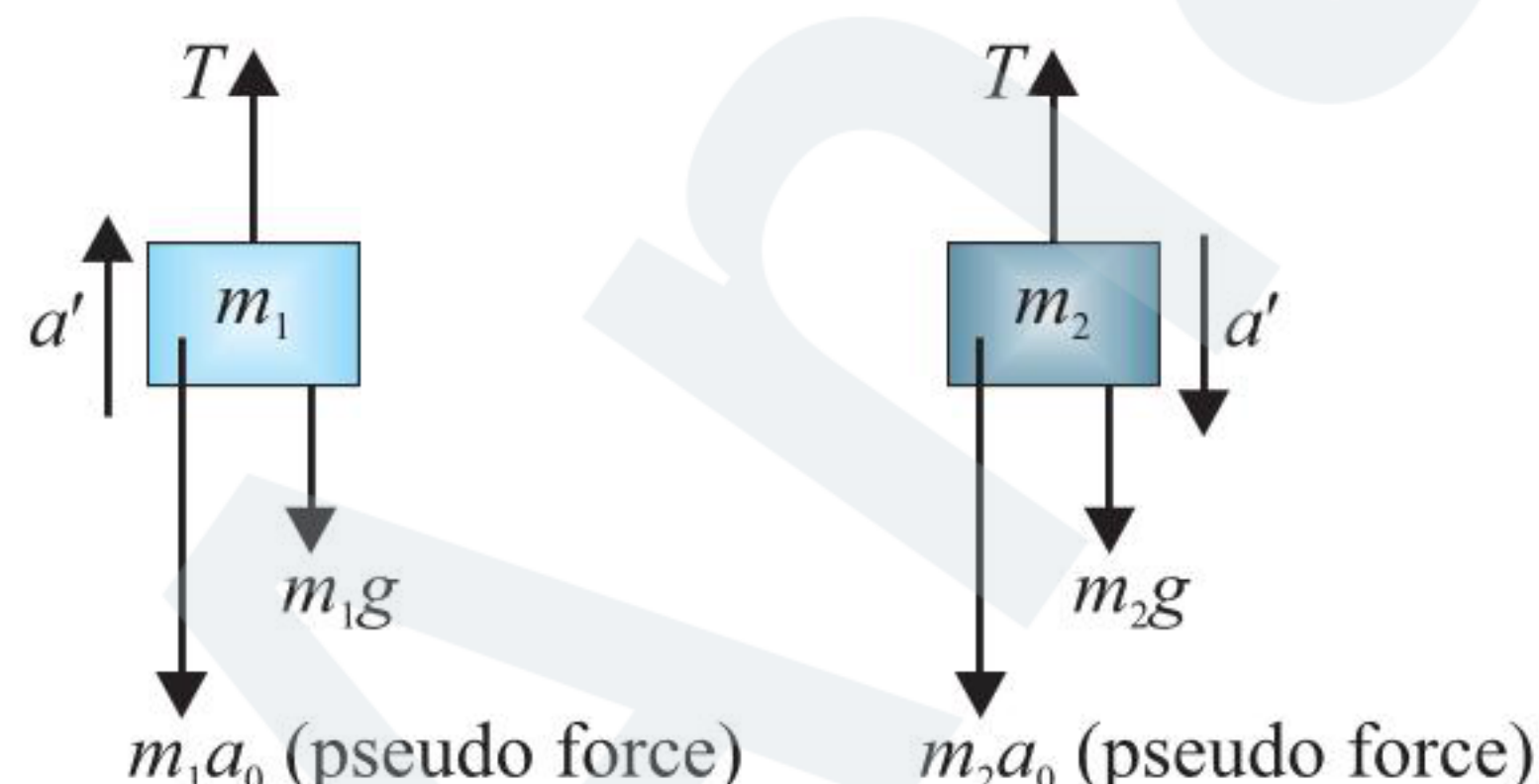
$$\text{For Case-III: } N_3 = mg - ma = 60 \times 10 \text{ N} \quad \dots(ii)$$

From (i) and (ii) $mg = \text{true weight} = 660 \text{ N}$

or true weight recorded by machine will be 66 kg.

$$\text{Again from (i), } a = \frac{720 - 660}{66} = \frac{10}{11} \text{ m/s}^2$$

5. Analysing the block w.r.t elevator



Writing equation of motion w.r.t elevator

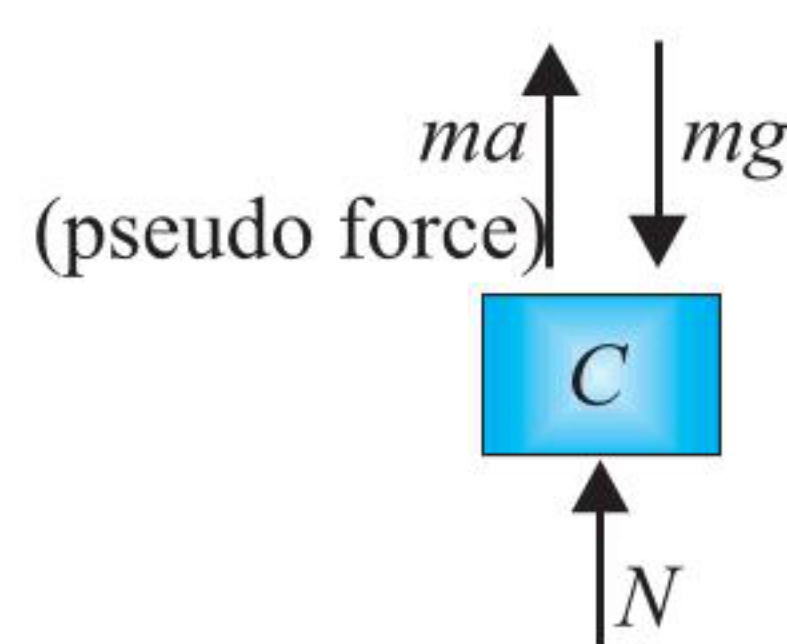
$$\text{For } m_1: T - m_1a_0 - m_1g = m_1a \quad \dots(i)$$

$$\text{For } m_2: m_2g - m_2a_0 - T = m_2a \quad \dots(ii)$$

$$\text{From (i) and (ii) we get } a = \frac{(m_2 - m_1)}{(m_2 + m_1)}(g - a_0)$$

6. When the block C is placed on the block B, the system will start accelerating with acceleration.

$$a = \frac{[(M + m) - M]}{[(M + m) + M]}g = \frac{mg}{(2M + m)}$$



The block 'B + C' combined will move down with acceleration ' a '.

Now analyzing the block C w.r.t block B.

As the block C is at rest w.r.t block B

$$\text{Hence } N = mg - ma$$

$$\Rightarrow N = mg - m \left(\frac{mg}{2M + m} \right) = \frac{2Mmg}{(2M + m)}$$

Exercise 6.6

1. Sum of change in length of segments of string (1)

$$\text{Should be zero} \Rightarrow (-2x_A) + (2x_{p1}) = 0$$

$$\text{or } x_A = x_{p1} \Rightarrow a_A = a_{p1} \quad \dots(i)$$

Also sum of the change in segment length should also be zero

$$(-x_{p1} + x_B) + (x_B) = 0$$

$$2x_B - x_{p1} = 0 \Rightarrow 2a_B = a_{p1} \quad \dots(ii)$$

$$\text{From (1) and (2) } a_A = 2a_B$$

$$\Rightarrow a_B = \frac{a_A}{2} = 1 \text{ m/s}^2$$

2. $v_A = v_{p2} = 10 \text{ m/s}$

Using shortcut method

$$\text{For pulley } P_2$$

$$v_{P2} = \frac{V_B + (-V_C)}{2}$$

$$10 = \frac{V_B - 2}{2} \Rightarrow v_B = 22 \text{ m/s}$$

3. Sum of segment lengths should be zero.

$$(-x_A) + (2x_B) + x_C = 0$$

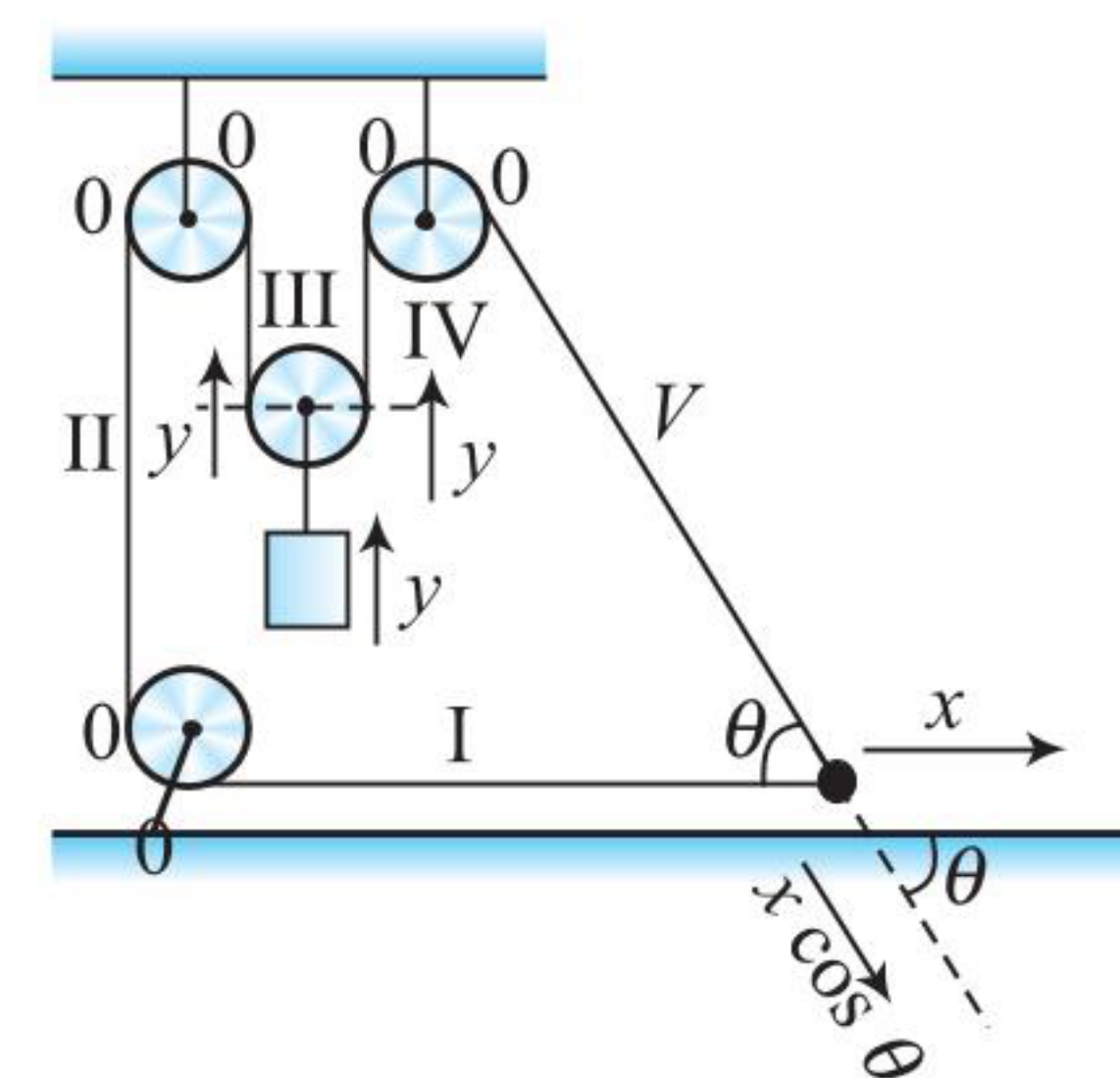
$$\text{or } x_A = 2x_B + x_C$$

$$\text{or } a_A = 2a_B + a_C$$

$$5 = 2 \times 2 + a_C$$

$$\Rightarrow a_C = 1 \text{ m/s}^2$$

4. Sum of total change in segment lengths should be zero.



$$(+x) + (0) + (-y) + (-y) + (0 + x \cos \theta) = 0$$

$$2y = x + x \cos \theta$$

$$y = \frac{x}{2}(1 + \cos 60^\circ) = \frac{x}{2} \left(1 + \frac{1}{2} \right)$$

$$y = \frac{3x}{4} \Rightarrow v_{\text{block}} = \frac{3}{4} v_{\text{truck}}$$

5. Speed of C and B will be same as shown. Let it be v . $|x_B| = |x_C|$

Using constraint for string passing over C:

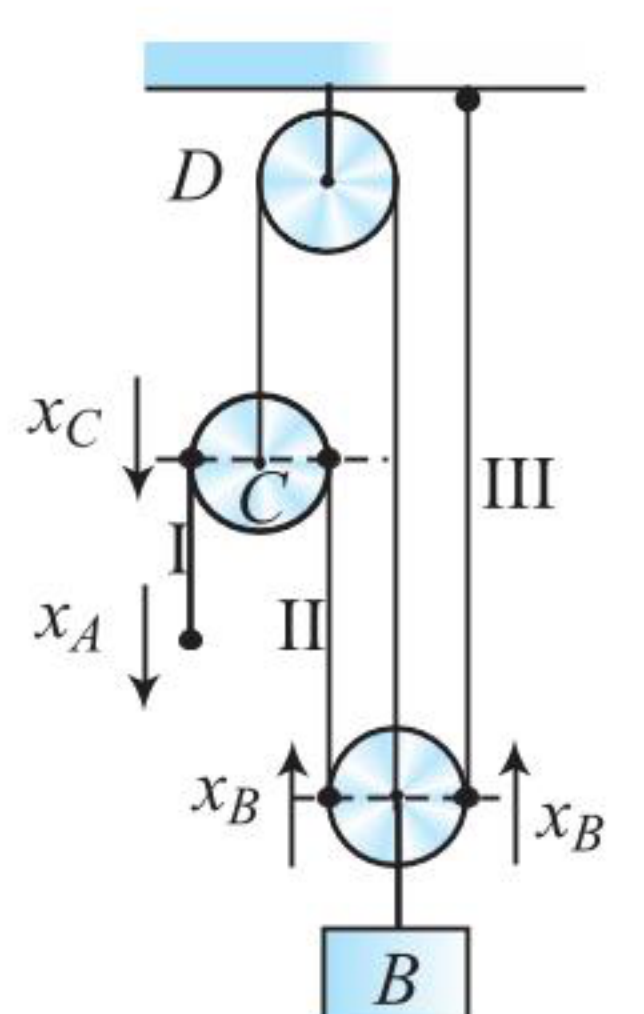
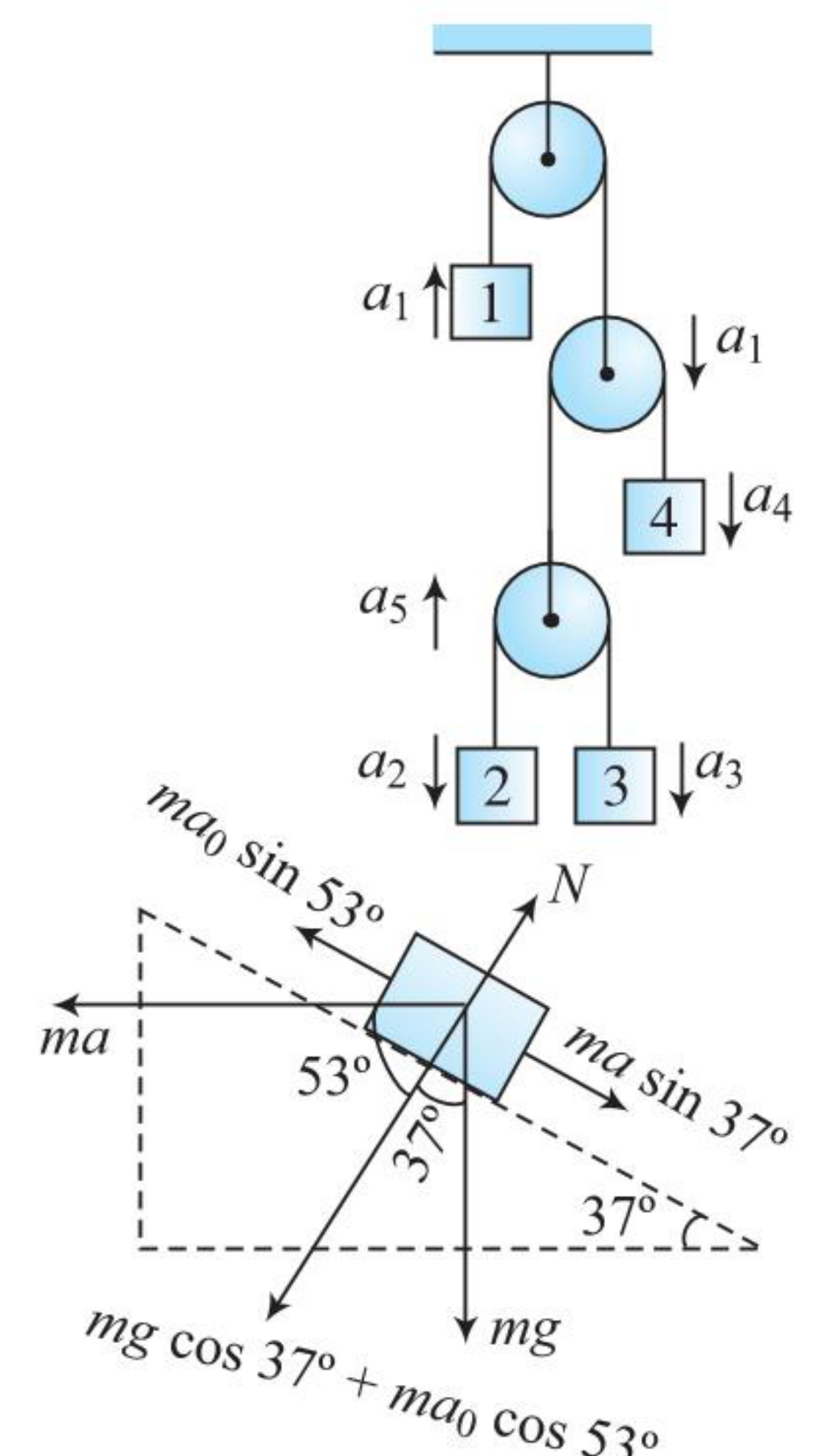
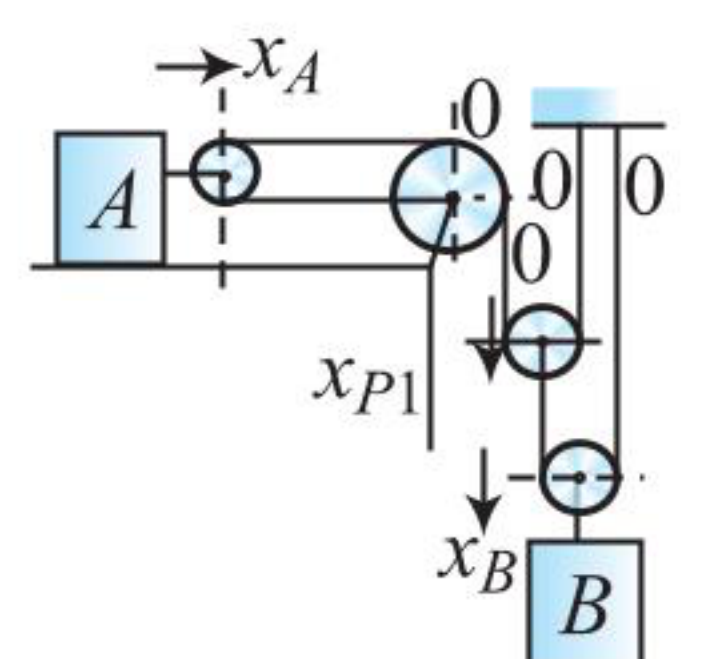
$$(x_A - x_C) + (x_C - x_B) + (-x_B) = 0$$

$$x_A - 2x_C - 2x_B = 0$$

$$\text{But } x_B = x_C \Rightarrow x_A - 4x_B = 0$$

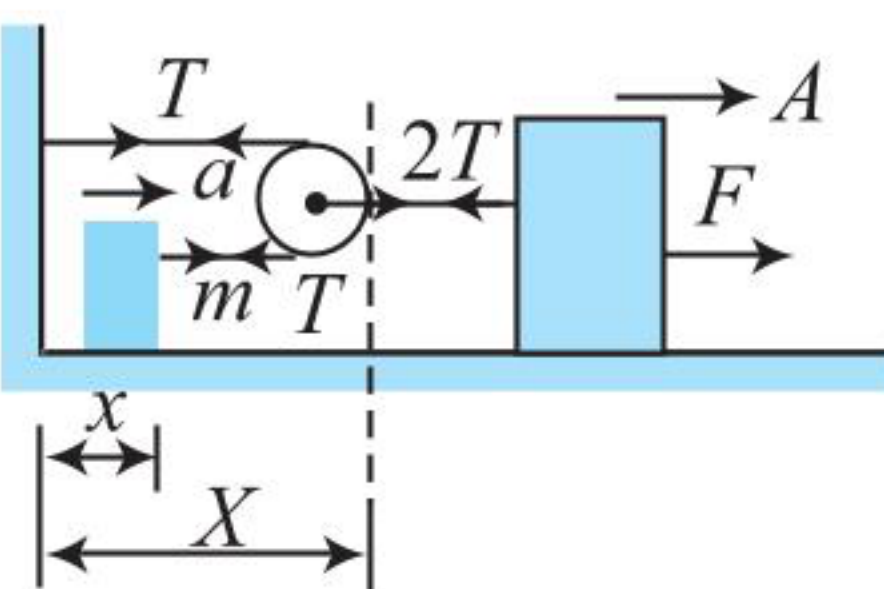
$$\text{or } v_A = 4v_B$$

$$\text{Hence, } v_B = \frac{v_A}{4} = \frac{2}{4} = 0.5 \text{ m/s}$$



6. Let A be the acceleration of block M and pulley, and a be the acceleration of m .

Let x, X be the coordinates of m and pulley as shown.



Length of string passing over the pulley is

$$L = X + (X - x) \Rightarrow L = 2X - x$$

Differentiating twice with respect to time,

$$0 = 2A - a \Rightarrow a = 2A \quad \dots(i)$$

From force diagrams of m and M , we have

$$T = ma \quad \dots(ii)$$

and $F - 2T = MA$

$$\dots(iii)$$

Comparing (i), (ii), and (iii), we get $A = \frac{F}{M + 4m}$ and $a = \frac{2F}{(M + 4m)}$

7. Let in a given time displacement of A is x_1 , of B is x_2 , and that of y is x_3 in downward direction.

As the length of string will remain constant, so we can write

$$x_1 + (x_1 - x_3) + x_3 + (x_2 - x_3) + x_2 = 0$$

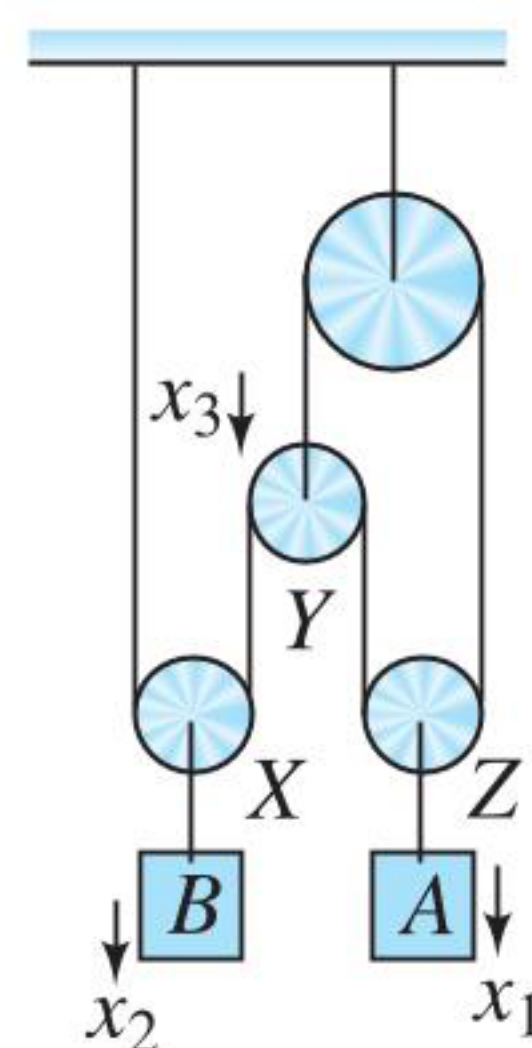
$$\Rightarrow 2x_1 + 2x_2 - x_3 = 0$$

- (a) Differentiating, we get $2v_1 + 2v_2 - v_3 = 0$

$$\Rightarrow 2v_1 + 2(-1) - (-2) = 0 \Rightarrow v_1 = 0$$

- (b) Again differentiating, we get $2a_1 + 2a_2 - a_3 = 0$

$$\Rightarrow 2a_1 + 2(-3) - 1 = 0 \Rightarrow a_1 = 7/2 \text{ ms}^{-2}$$



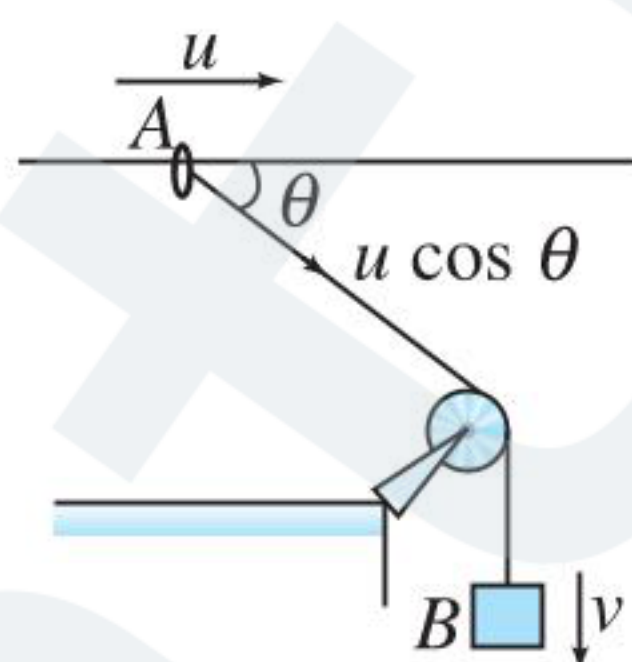
8. (a) $V_P = \frac{V_A + V_B}{2} \Rightarrow 5 = \frac{V_A + 10}{2} \Rightarrow V_A = 0$

(b) $5 = \frac{V_A - 20}{2} \Rightarrow V_A = 30 \text{ ms}^{-1}$

9. Let the speed of the ring is u , then $u \cos \theta = v$

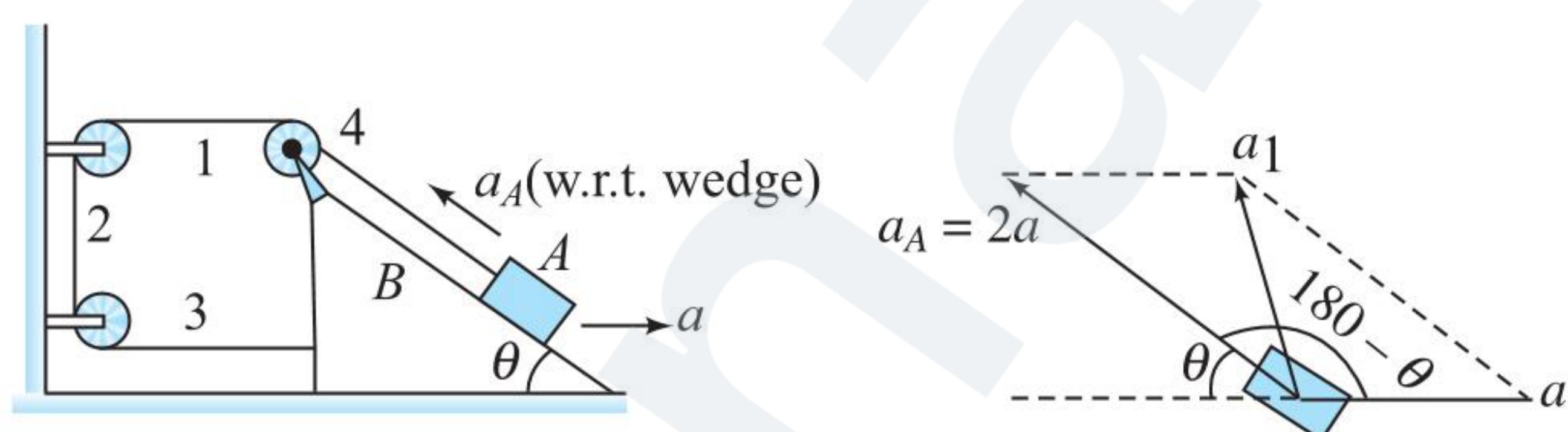
Because components of velocity along the string should be same.

$$\Rightarrow u = v/\cos \theta = v \sec \theta$$



10. (a) Length of (1) and (3) will increase at the cost of part 4 (Fig). Hence decrease in length of (4) will be twice of increase in lengths of either 1 or 3.

$$\text{So, } a_A = 2a$$



- (b) Acceleration of block w.r.t. ground :

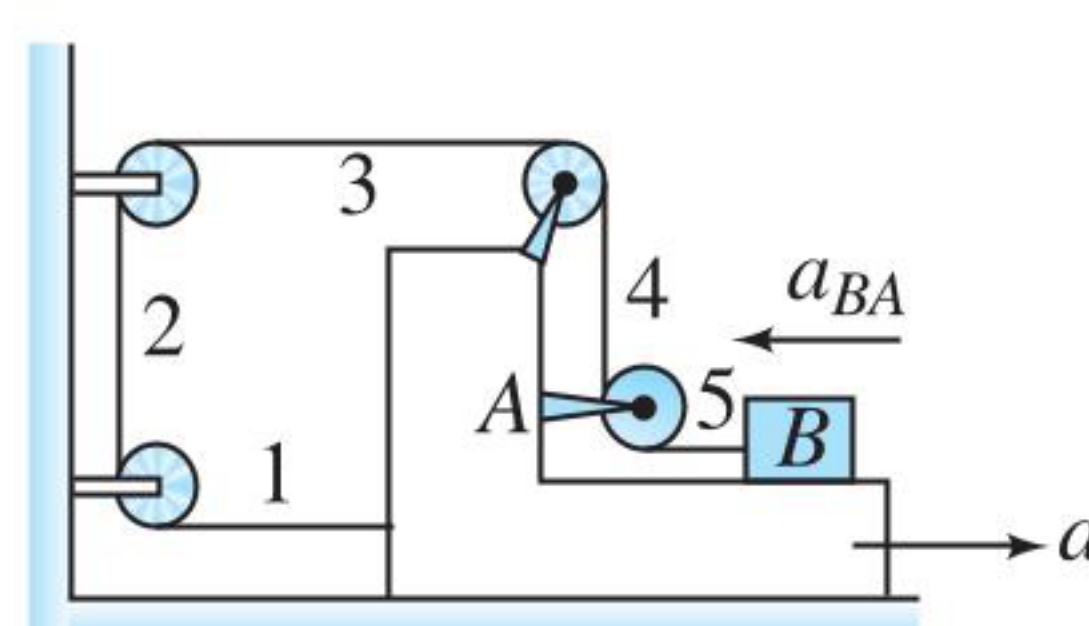
$$a_1 = \sqrt{a^2 + a_A^2 + 2a \cdot a_A \cos(180 - \theta)} = a\sqrt{5 - 4\cos \theta}$$

11. (a) (i) Length of (5) will decrease at the cost of increase in length of (1) and (3).

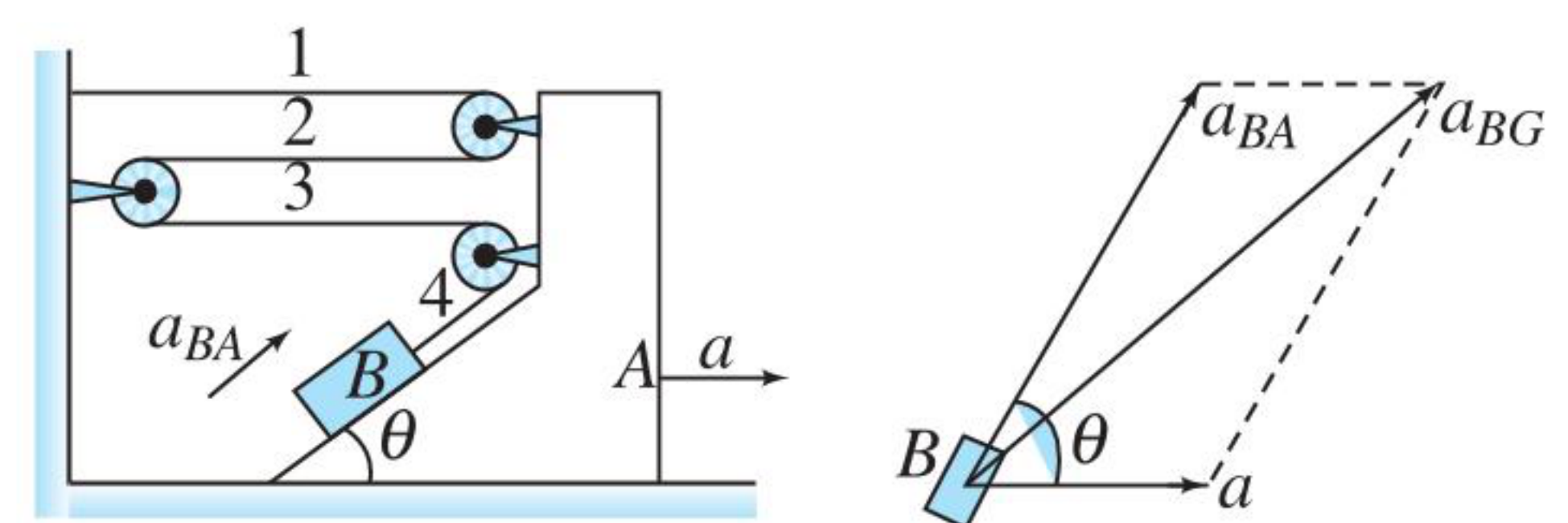
Hence, acceleration of B w.r.t. A : $a_{BA} = 2a$

- (ii) Acceleration of B w.r.t. ground

$$a_{BA} = a_B - a_A \Rightarrow -2a = a_B - a \Rightarrow a_B = -a$$



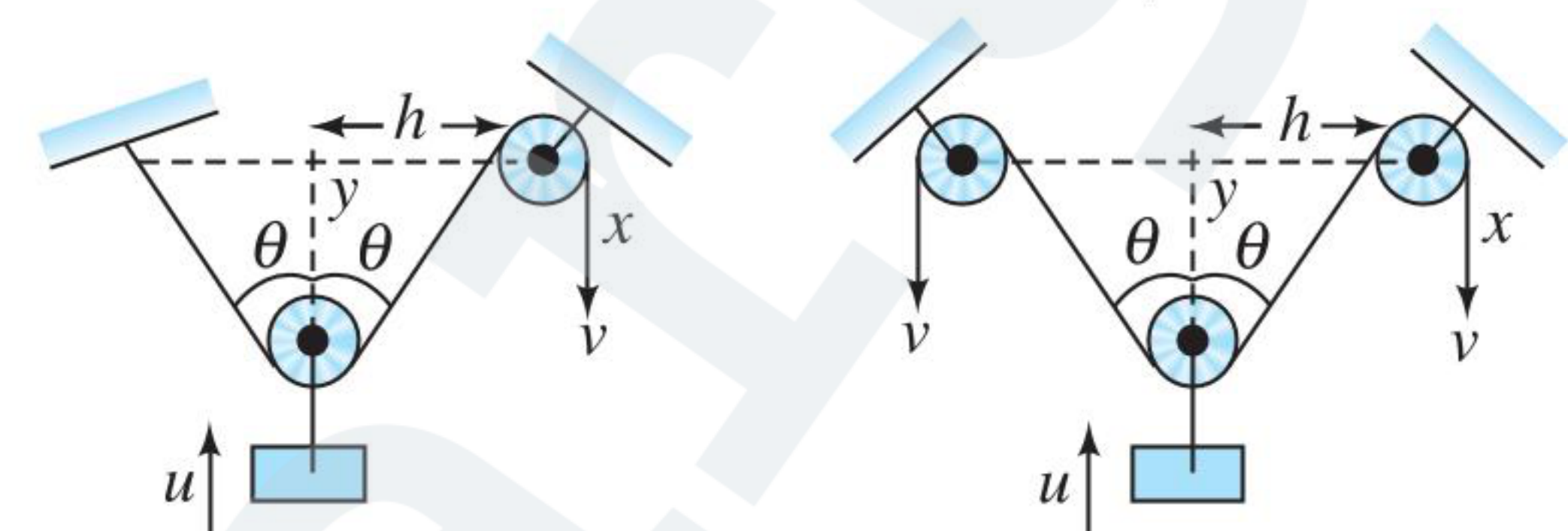
- (b) (i) Length of 4 will decrease at the the cost of 1, 2 an 3, hence $a_{BA} = 3a$



$$(ii) a_{BG} = \sqrt{a^2 + a_{BA}^2 + 2aa_{BA} \cos \theta} = a\sqrt{10 + 6\cos \theta}$$

12. (a) Length of the string: $l = x + 2\sqrt{h^2 + y^2}$

$$\text{Differentiating, we get } \frac{dl}{dt} = \frac{dx}{dt} + 2 \frac{2y}{2\sqrt{h^2 + y^2}} \frac{dy}{dt}$$



$$\Rightarrow 0 = v + \frac{2y}{\sqrt{h^2 + y^2}} (-u)$$

$$\Rightarrow 0 = v - 2u \cos \theta \Rightarrow u = v/(2\cos \theta)$$

- (b) Length of string, $l = 2x + 2\sqrt{h^2 + y^2}$

Again differentiate to get $u = v/\cos \theta$

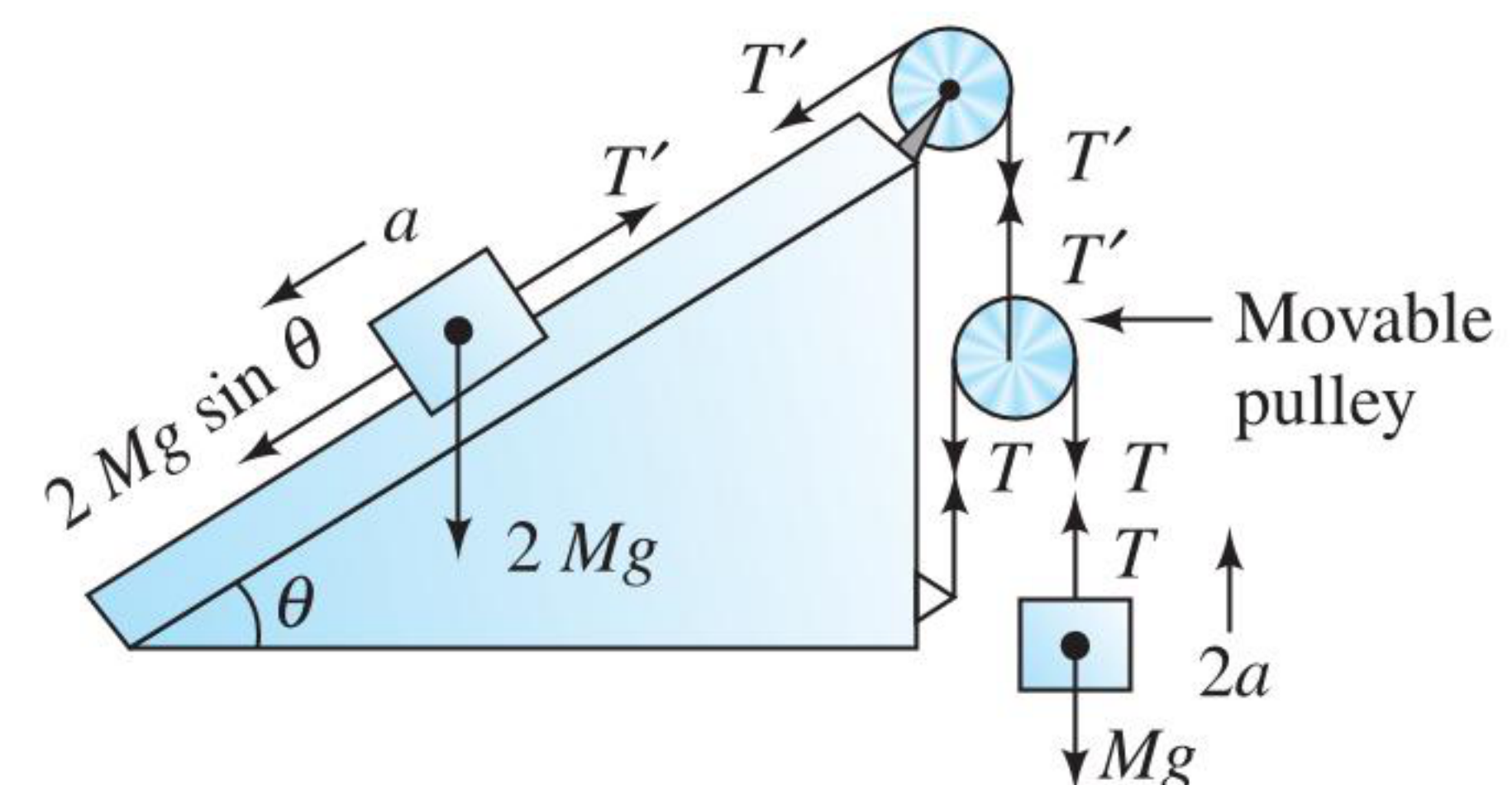
13. (a) For block $2M$, $2T = 2Ma$... (i)

For block M , $Mg - T = M2a$... (ii)

Solving equations (i) and (ii), we get $a = g/3$.

- (b) For block 2,

$$2Mg \sin \theta - T' = 2Ma \quad \dots(i)$$



For movable pulley,

$$T' - 2T = 0 \times a \Rightarrow T' = 2T \quad \dots(ii)$$

For block M , $T - Mg = M2a$... (iii)

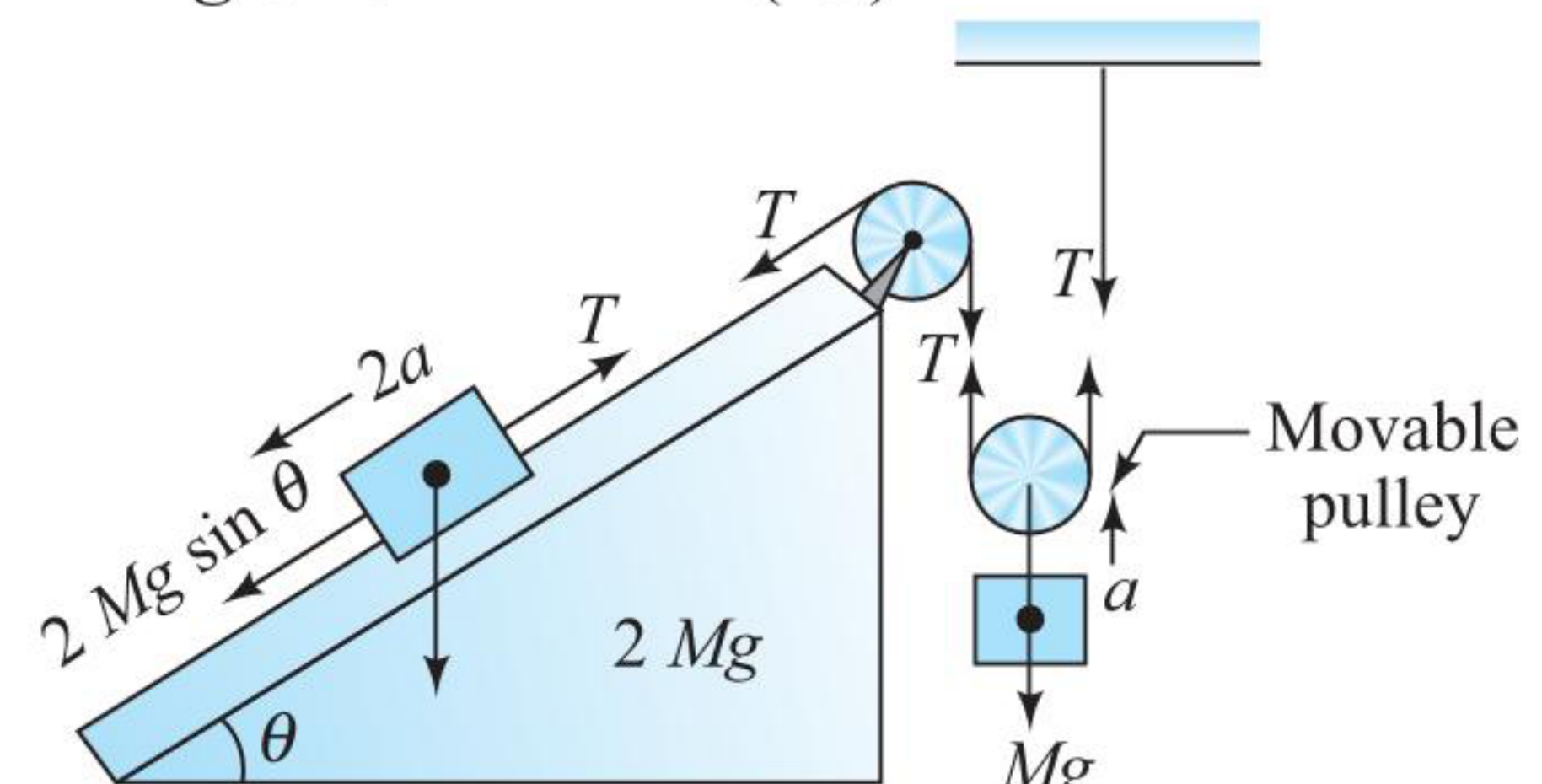
$$\text{Solving Eqs. (i), (ii), and (iii), we get } a = \frac{g[\sin \theta - 1]}{3}$$

This is negative, hence directions of accelerations will be opposite to those shown in figure.

Note: Here we assume that the block $2M$ is moving down the plane. You may assume that it is moving up the plane. The magnitude of the acceleration found will remain the same.

- (c) For block $2M$,

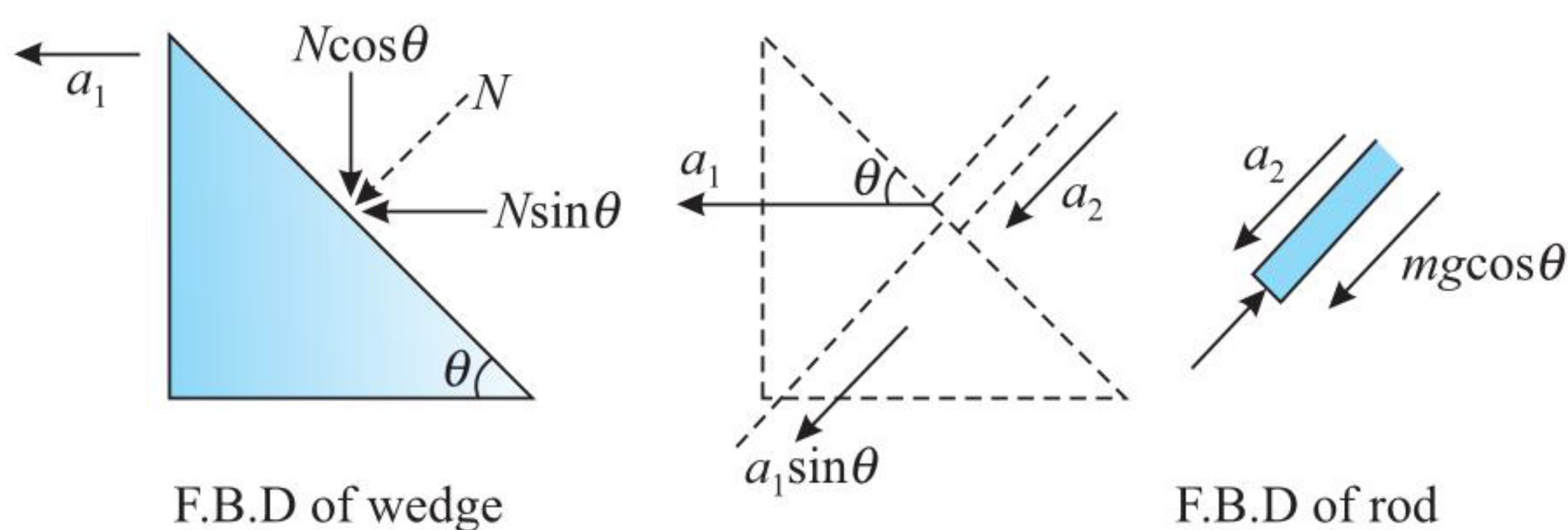
$$2Mg \sin \theta - T = 2M(2a) \quad \dots(i)$$



For block M , $2T - Mg = Ma$... (ii)

$$\text{Solving Eqs. (i) and (ii), we get } a = \frac{g[4\sin \theta - 1]}{9}$$

14. Components of acceleration of M and m perpendicular to the incline should be same as both always remain in contact.



F.B.D of wedge

F.B.D of rod

For this $a_2 = a_1 \sin \theta$ For M : $N \sin \theta = Ma_1$ For m : $mg \cos \theta - N = ma_2$

Solving equations (i), (ii), and (iii), we get

$$a_1 = \frac{mg \sin \theta \cos \theta}{M + m \sin^2 \theta}, \quad a_2 = \frac{mg \sin^2 \theta \cos \theta}{M + m \sin^2 \theta}$$

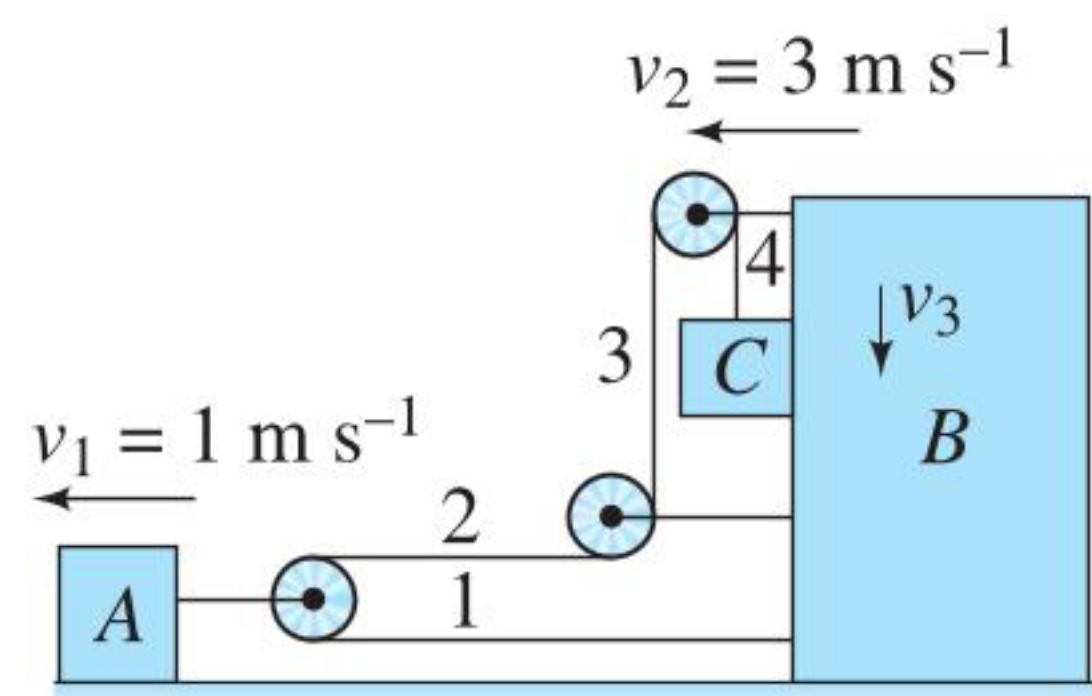
15. $v_1 = 1 \text{ m s}^{-1}$, $v_2 = 3 \text{ m s}^{-1}$

Let the velocity of C is v_3 downward w.r.t. B . Then

$$(v_1 - v_2) + (v_1 - v_2) + v_3 = 0$$

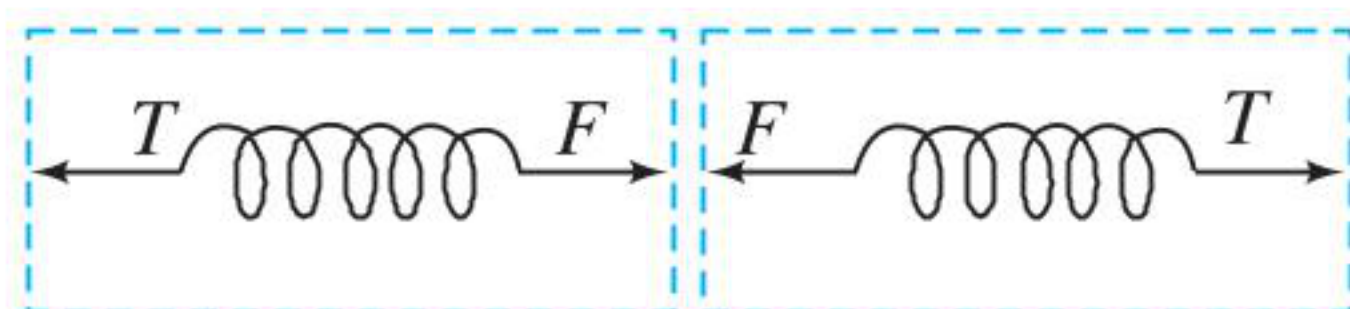
$$\Rightarrow v_3 = 2(v_2 - v_1) = 2(3 - 1) = 4 \text{ m s}^{-1}$$

$$\text{Net velocity of } C = \sqrt{v_2^2 + v_3^2} = \sqrt{3^2 + 4^2} = 5 \text{ m s}^{-1}$$

**Exercise 6.7**

1. In all the three cases the spring balance reads 10 kg.

Let us cut a section inside the spring as shown in figure.

As each part of the spring is at rest, so $F = T$. As the block is stationary, so $T = F = 100 \text{ N}$.

2. Let T be the reading of the spring balance. Then

$$\text{For 20-kg block: } 20g - T = 20a \quad \dots(i)$$

$$\text{For 10-kg block: } T - 10g = 10a \quad \dots(ii)$$

$$\text{Solving Eqs. (i) and (ii), we get } a = \frac{g}{3}, \quad T = \frac{40g}{3}$$

So the spring balance reading is $\frac{40}{3} \text{ kg}$.

3. The spring force on all springs will be same as springs are ideal. As spring constants of all three springs are same; hence, stretch of all three springs will be equal.

4. Given spring constant $k = 20 \text{ N m}^{-1}$

Initial length = 0.25 m

Final length = 0.30 m

$$\Delta x = 0.30 - 0.25 = 0.05 \text{ m } (\rightarrow)$$

Mass of the block = m

$$\vec{F} = -k\vec{x} \Rightarrow ma = -k\Delta x$$

$$a = -\frac{k\Delta x}{m} = \frac{-20 \times 0.05}{m} = -\frac{1}{m}$$

Acceleration = $1/m$ ($\leftarrow$)

5. When blocks A and B are in equilibrium position

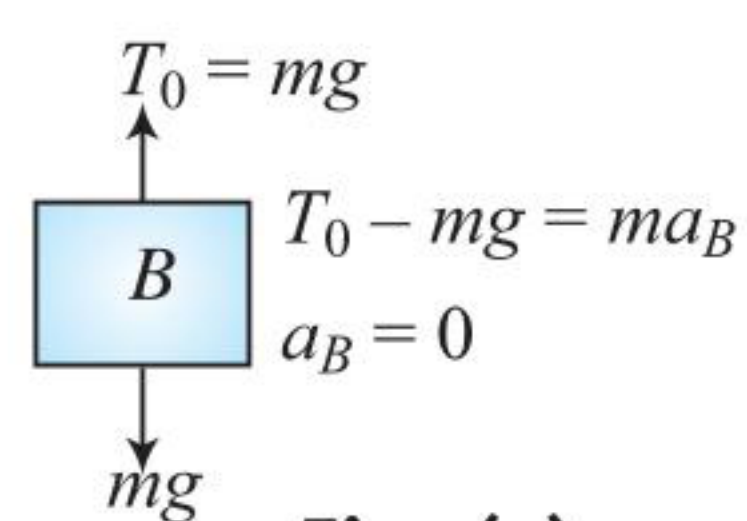
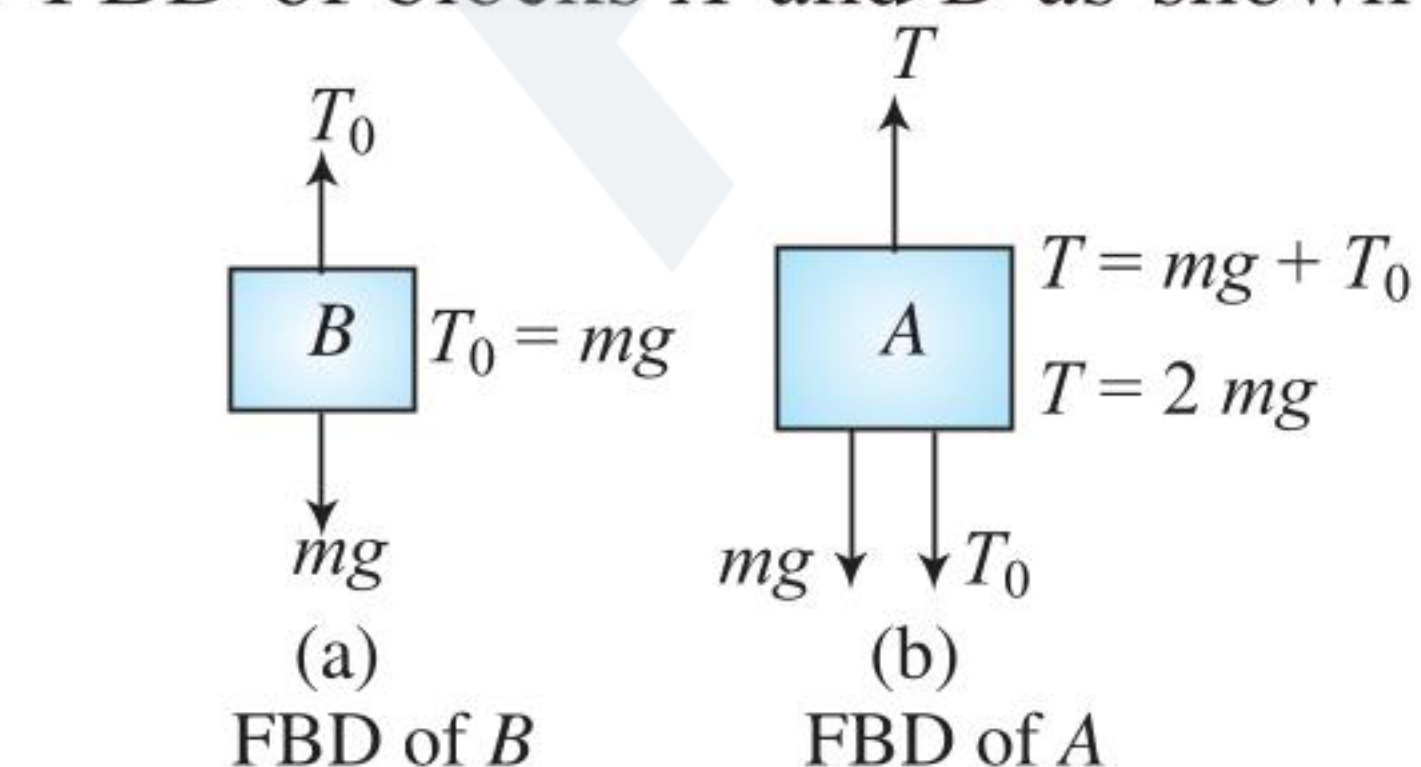
When string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B , again draw FBD of blocks A and B as shown in Fig.

Fig. (a)

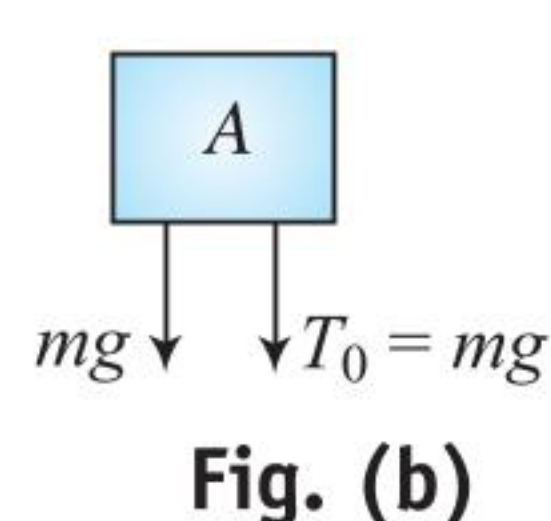


Fig. (b)

$$mg + T_0 = ma_A$$

$$2mg = ma_A$$

$$a_A = 2g \text{ (downwards)}$$

As forces acting on block B are not changing, hence, block B will remain at equilibrium. It means $a_B = 0$.

6. For calculating the reading, first we draw FBD of 20-kg block.

F.B.D of 20 kg.

$$mg - T = 0$$

$$T = 20g = 200 \text{ N}$$

Since both balances are light so, both the scales will read 20 kg.

7. (a) Extension in spring = 1 m.

$$F_{\text{spring}} = K \times x = 200 \times 1 = 200 \text{ N}$$

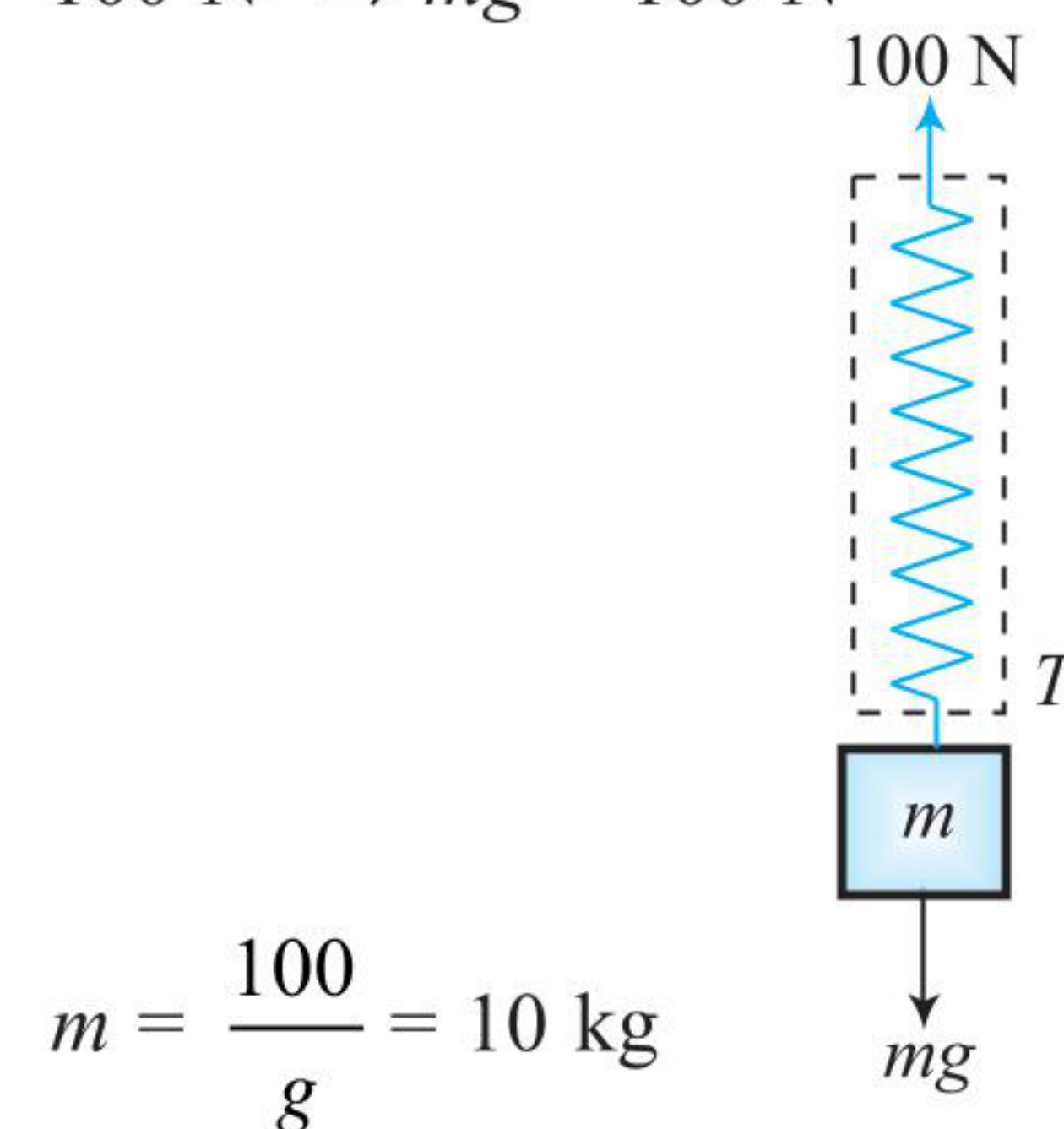
- (b) Same Extension same spring force in both directions,

$$F_{\text{spring}} = 200 \text{ N}$$

- (c) Both displacements of A or B are compressing the spring total compressing = $0.75 + 0.25 = 1 \text{ m}$

$$\therefore F_{\text{spring}} = kx = 200 \times 1 = 200 \text{ N}$$

8. $T = 100 \text{ N} \Rightarrow mg = 100 \text{ N}$



$$m = \frac{100}{g} = 10 \text{ kg}$$

9. When string is not cut:

FBD of A block

$$kx = mg + T$$

FBD of B block

$$T = mg$$

When string is cut:

FBD of A block

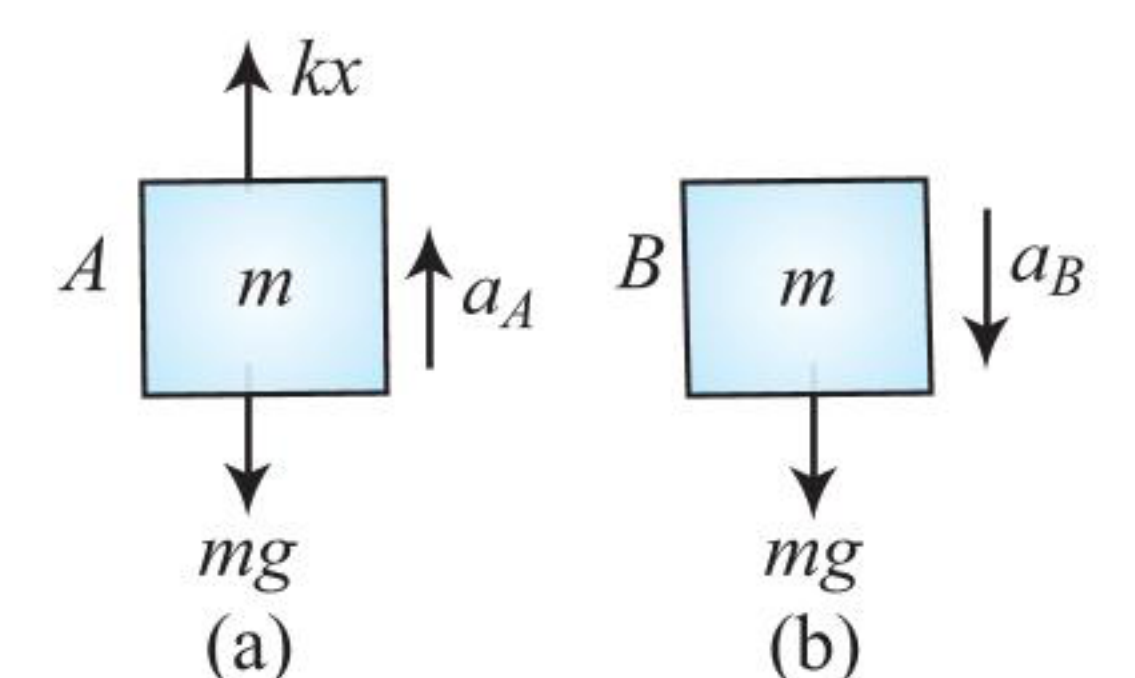
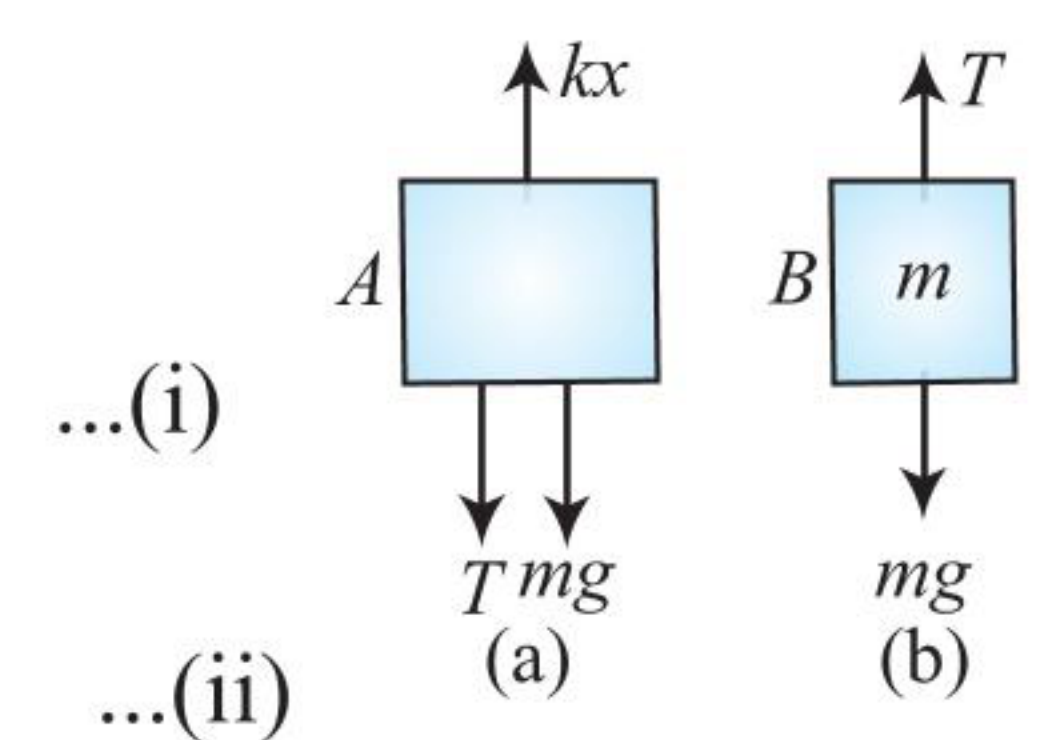
$$kx - mg = ma_A$$

$$2mg - mg = ma_A \\ = a_A = g \text{ (upwards)}$$

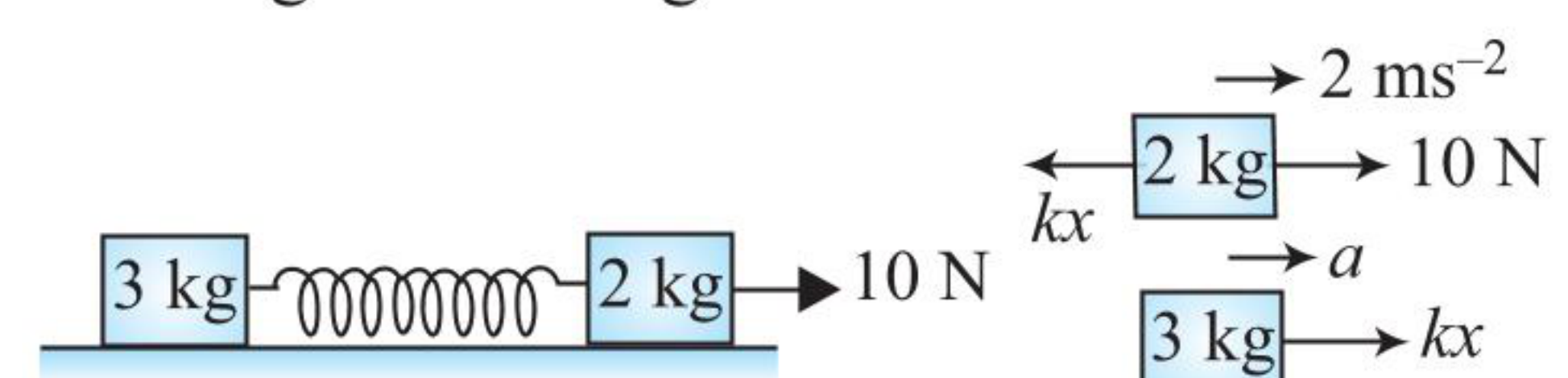
FBD of B block

$$ma_B = mg$$

$$a_B = g \text{ (downwards)}$$



10. From force diagram of 2-kg block:



$$10 - kx = 2a = 4$$

$$\therefore kx = 6$$

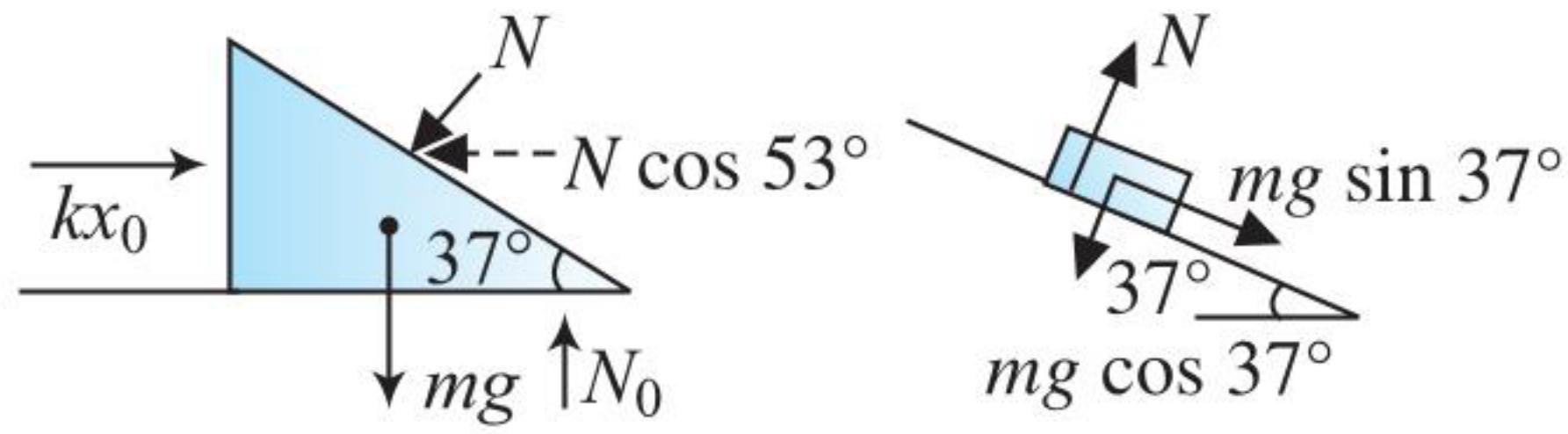
From force diagram of 3kg block,

$$kx = 3a$$

$$\text{or } 6 = 3a$$

$$\therefore a = \frac{6}{3} = 2 \text{ m s}^{-2}$$

11.



From force diagram of wedge,

$$N \cos 53^\circ = kx_0$$

...(i)

From force diagram of smaller block:

$$N = mg \cos 37^\circ$$

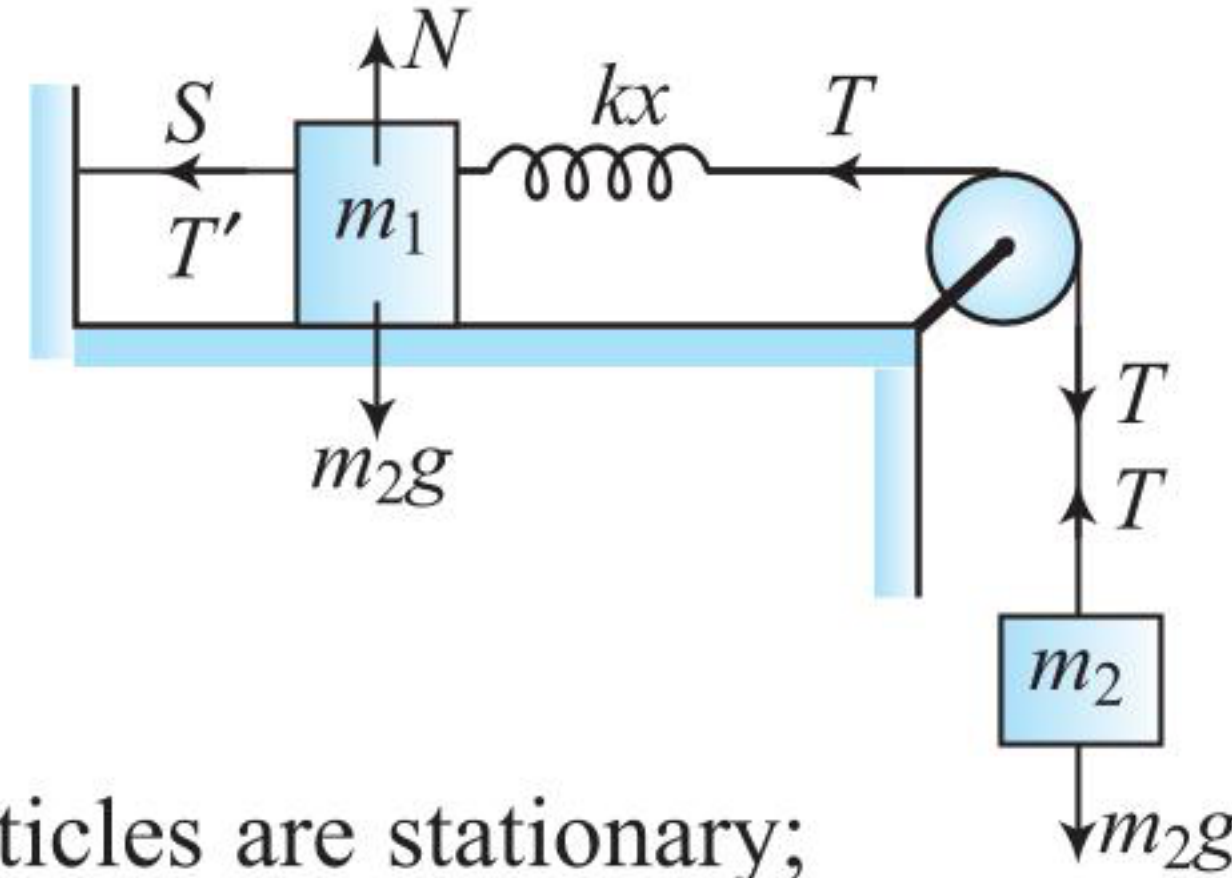
$$kx_0 = N \cos 53^\circ$$

$$kx_0 = mg \cos 37^\circ \cos 53^\circ$$

$$kx_0 = 2.5 \times 10 \times 0.8 \times 0.6 = 12 \text{ N}$$

Hence, reading is $kx_0 = 12 \text{ N}$

12. FBD: Let the spring forces be $F = kx$ just after cutting the spring. Hence, at that instant, the forces acting on m_1 are $T = kx \rightarrow m_1g \downarrow$ and $N \uparrow$; on m_2 the forces are $m_2g \downarrow$ and $T \uparrow$.



Force equation: Initially, all the particles are stationary; hence, $a_1 = a_2 = 0$. Applying Newton's second law,

$$\text{For } m_1: \sum F = T' - kx = 0 \quad \dots(i)$$

$$\text{For } m_2: \sum F = T - m_2g = 0 \quad \dots(ii)$$

$$\text{For spring: } \sum F = T - kx = 0 \quad \dots(iii)$$

Solving Eqs. (i), (ii) and (iii), we have

$$T' = T = kx = m_2g$$

Just after the string S is cut, tension T' vanishes immediately, but the spring cannot regain its shape and size instantaneously. Therefore, the spring force remains as it is. Then, just after cutting the string, the net force acting on m_1 is equal to kx , whereas the net force acting on m_2 is equal to $T - m_2g$. Hence, the acceleration of m_1 and m_2 just after cutting the string is given by

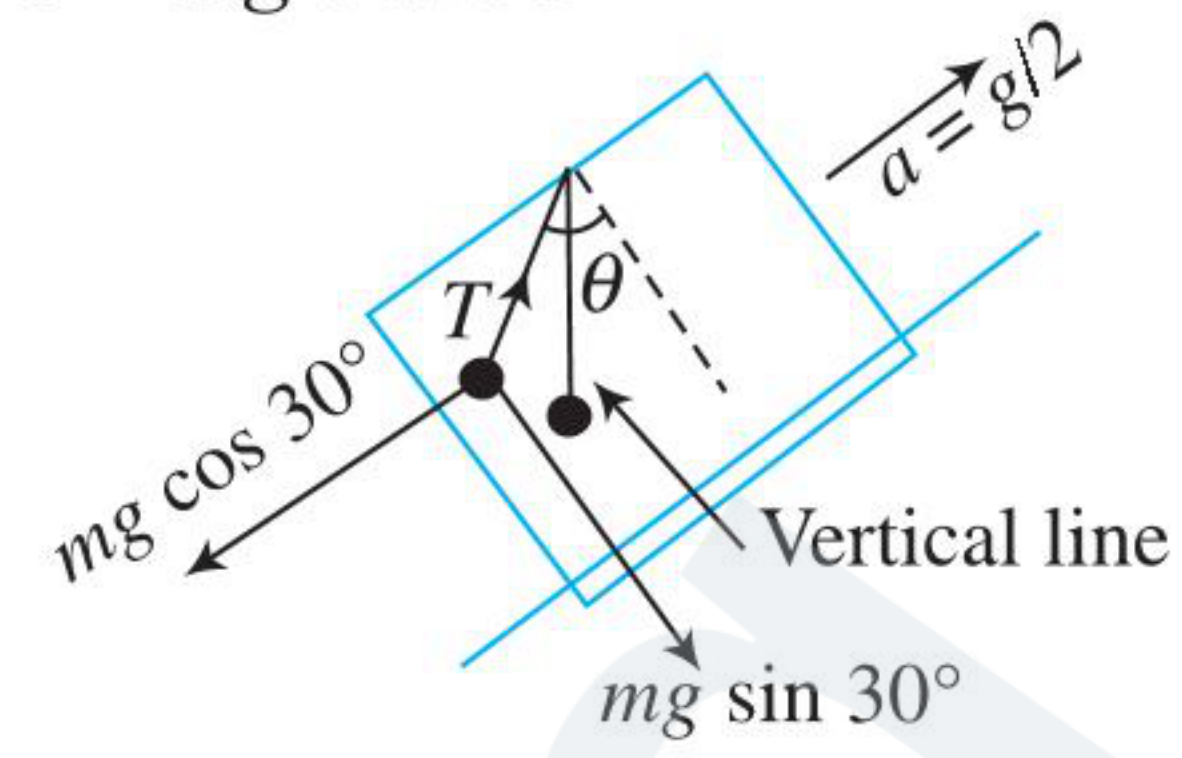
$$\sum F = m_1a_1 = kx = m_2g, \quad \sum F = m_2a_2 = T - m_2g = 0$$

$$\text{This gives } a_1 = \left(\frac{m_2}{m_1}\right)g \text{ and } a_2 = 0$$

$$4. (2) T \sin \theta - mg \sin 30^\circ = ma$$

$$\Rightarrow T \sin \theta = mg \sin 30^\circ + mg/2 \quad \dots(i)$$

$$T \cos \theta = mg \cos 30^\circ \quad \dots(ii)$$



Dividing Eqs. (i) by (ii), we get

$$\tan \theta = \frac{2}{\sqrt{3}}$$

5. (2) Acceleration of cylinder down the plane is

$$a = (g \sin 30^\circ)(\sin 30^\circ)$$

$$= 10 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) = 2.5 \text{ m s}^{-2}$$

$$\text{Time taken } t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 5}{2.5}} = 2 \text{ s}$$

6. (3) Given ($V = 10 \text{ m s}^{-1}$)

$$\text{After 2s: } V_x = \frac{10}{\sqrt{2}} - \frac{10}{\sqrt{2}} \times 2 \Rightarrow V_x = -\frac{10}{\sqrt{2}} \text{ m s}^{-1}$$

$$V_x = -\frac{10}{\sqrt{2}} \text{ m s}^{-1} \text{ and } V_y = -\frac{10}{\sqrt{2}} \text{ m s}^{-1}$$

$$V = \sqrt{\frac{100}{2} + \frac{100}{2}} = 10 \sqrt{\frac{1}{2} + \frac{1}{2}} = 10 \text{ m s}^{-1}$$

7. (2) $\vec{a}_{b,l} = \vec{a}_b - \vec{a}_l = (g - a) \downarrow \Rightarrow \vec{a}_b = g \downarrow$

8. (4) $\vec{a} = \frac{\vec{F}}{m} = -10 \hat{j} (\text{ms}^{-1})^2$

Displacement in y-direction

$$y = ut + \frac{1}{2}at^2 \Rightarrow 0 = 4 \times t \times -\frac{1}{2} \times 10 \times t^2$$

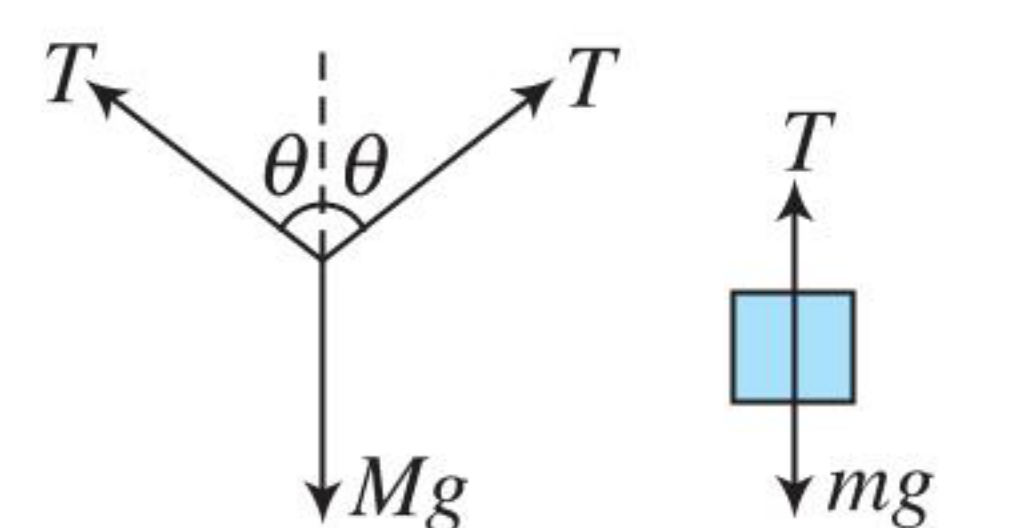
$$t = \frac{4}{5} \text{ s} \Rightarrow x = 4t = 4 \times \frac{4}{5} = 3.2 \text{ m}$$

9. (3) At equation $2T \cos \theta = Mg$, $T = mg$

$$\cos \theta = \frac{Mg}{2mg} = \frac{M}{2m}$$

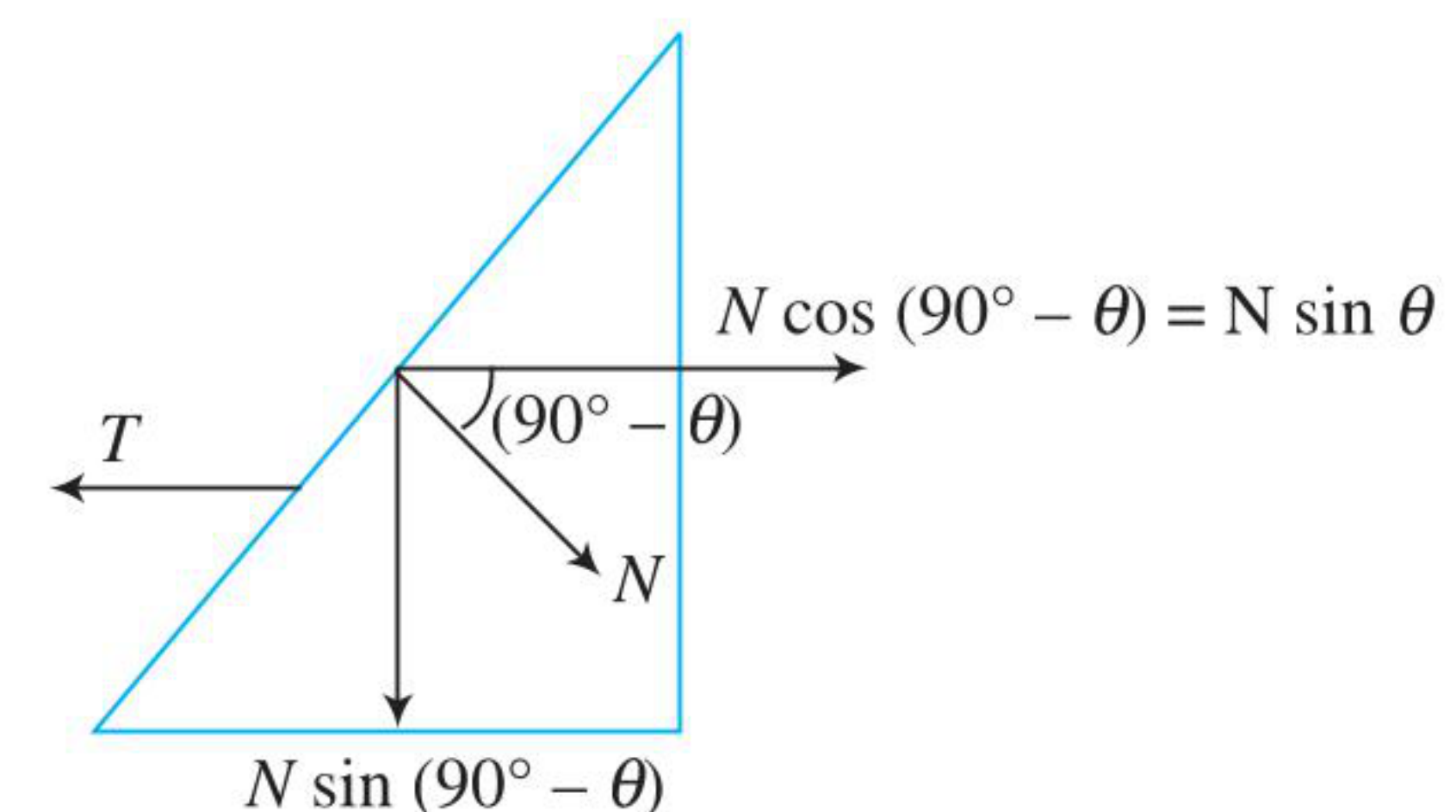
$$\Rightarrow \cos \theta < 1$$

$$\frac{M}{2m} < 1 \Rightarrow M < 2m$$

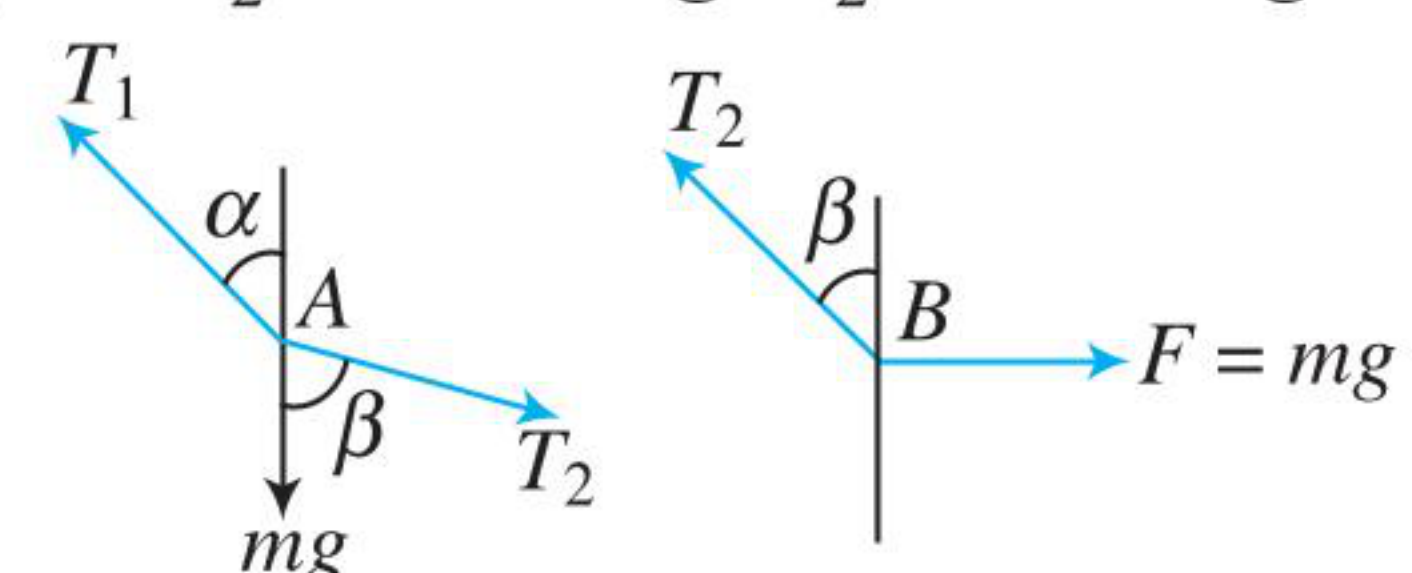


10. (3) $T = N \sin \theta$ and $N = mg \cos \theta$

$$T = mg \cos \theta \sin \theta = \frac{mg}{2} \sin 2\theta$$



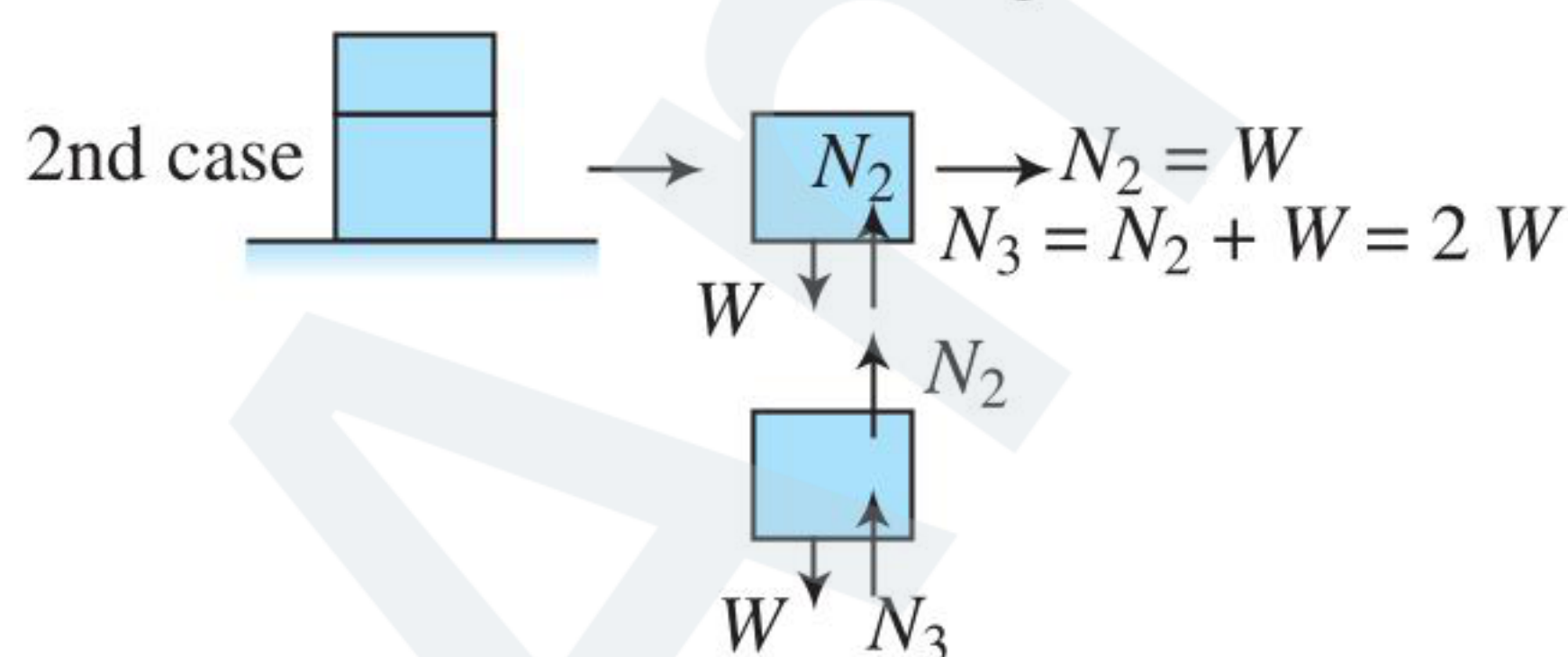
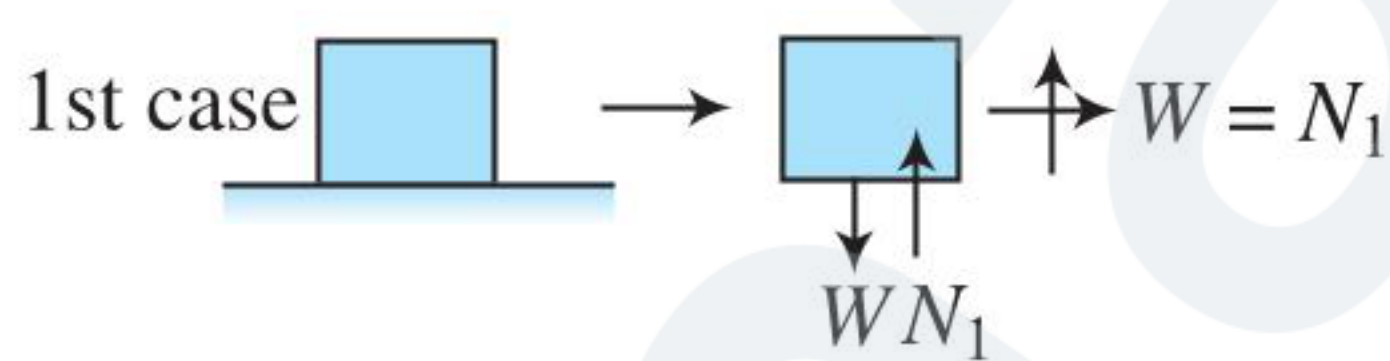
11. (3) From figure, $T_2 \cos \theta = mg$, $T_2 \sin \theta = mg$



Exercises

Single Correct Answer Type

1. (3) Let the weight of each block be W (Figure)



$$\text{So, } N_1 = N_2 = \frac{N_3}{2}$$

$$2. (1) a = \frac{\sqrt{R_1^2 + R_2^2}}{m} = \frac{R_3}{m} \quad \left[\because R_3 = \sqrt{R_1^2 + R_2^2} \right]$$

3. (1) Acceleration of the skaters will be in the ratio

$$\frac{F}{4} : \frac{F}{5} \quad \text{or} \quad 5 : 4$$

Now according to the problem, $s = 0 + \frac{1}{2}at^2$, we get

$$\frac{s_1}{s_2} = \frac{a_1}{a_2} = \frac{5}{4}$$

$$T_1 \sin \alpha = mg \sqrt{2} \sin 45^\circ$$

$$T_1 \sin \alpha = mg \Rightarrow \frac{\cos \alpha}{\sin \alpha} = \frac{2mg}{mg}$$

$$\tan \alpha = \frac{1}{2} \Rightarrow \cos \alpha = \frac{2}{\sqrt{5}}$$

$$T_1 \frac{2}{\sqrt{5}} = 2mg \Rightarrow T_1 = mg\sqrt{5}$$

$$\frac{T_1}{T_2} = \frac{\sqrt{5}}{\sqrt{2}} \Rightarrow \sqrt{2} T_1 = \sqrt{5} T_2$$

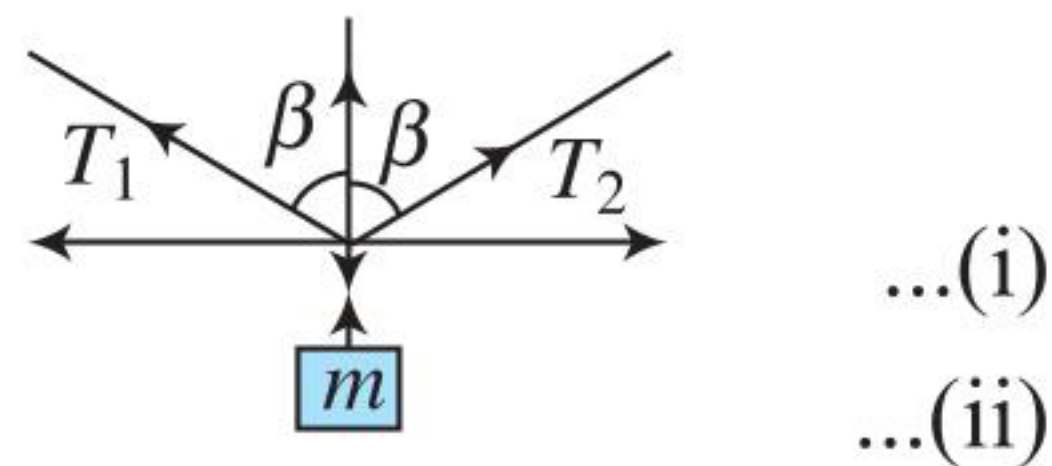
$$12. (2) \tan \beta = \frac{12}{5} \therefore \cos \beta = \frac{5}{13}$$

$$T_1 \cos \beta + T_2 \cos \beta = mg$$

$$T_1 \sin \beta = T_2 \sin \beta$$

$$\therefore T_1 = T_2 = T$$

$$\therefore 2T \cos \beta = mg \Rightarrow T = \frac{mg}{2 \cos \beta} \Rightarrow T = \frac{13}{10} mg$$



$$13. (4) \text{ Speed, } v = \sqrt{v_x^2 + v_y^2}$$

Rate of change of speed,

$$\frac{dv}{dt} = \frac{2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt}}{2\sqrt{v_x^2 + v_y^2}} = \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}}$$

$$14. (4) \text{ Let at any time, their velocities are } v_1 \text{ and } v_2, \text{ respectively. Then } v_1 = v_2 \cos \theta$$

$$\text{Differentiating: } a_1 = a_2 \cos \theta - v_2 \sin \theta \frac{d\theta}{dt}$$

Hence, none of them is correct.

[Note: Option (1) is correct initially, because initially $v_2 = 0$.]

$$15. (4) \text{ The acceleration of block-rope system is } a = \frac{F}{(M+m)},$$

where M is the mass of block and m is the mass of rope.

So the tension in the middle of the rope will be

$$T = \{M + (m/2)\} a = \frac{M + (m/2)F}{M + m}$$

Given that $m = M/2$

$$\therefore T = \left[\frac{M + (M/4)}{M + (M/2)} \right] F = \frac{5F}{6}$$

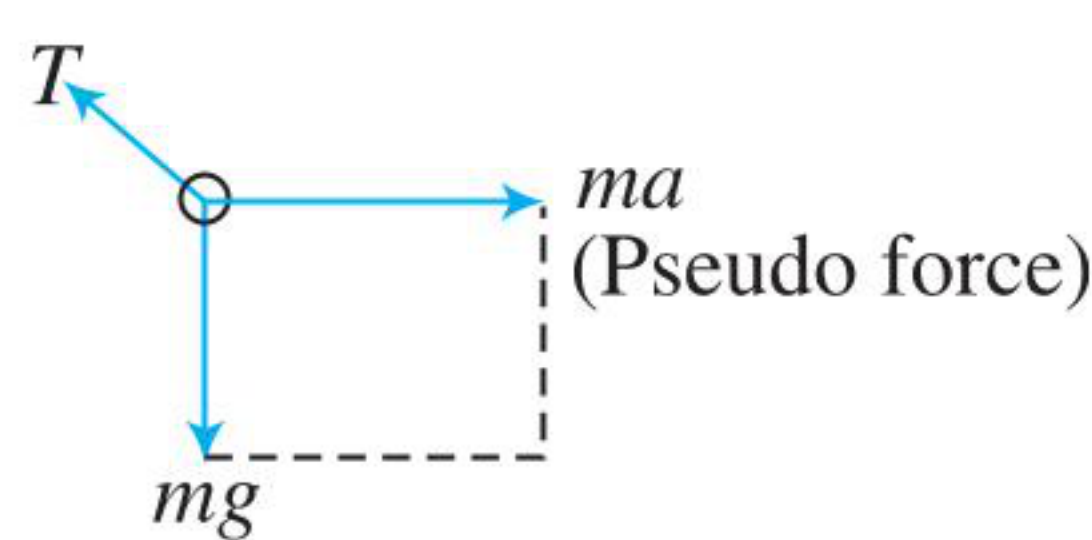
$$16. (4) \text{ The reading of the spring scale is the normal reaction between man and spring scale.}$$

As the reading is decreasing, it means normal reaction is decreasing. Firstly, the lift must be moving upwards with constant velocity and decelerated to rest.

$$17. (2) \text{ Acceleration of box} = 10 \text{ m s}^{-2}$$

Inside the box forces acting on bob (see figure)

$$T = \sqrt{(mg)^2 + (ma)^2} = 10\sqrt{2} \text{ N}$$



$$18. (3) \text{ Let } T \text{ be the tension in the rope and } a \text{ the acceleration of rope.}$$

The absolute acceleration of man is, therefore, $\left(\frac{5g}{4} - a\right)$. Equations of motion for mass and man gives:

$$T - 100g = 100a \quad \dots(i)$$

$$T - 60g = 60\left(\frac{5g}{4} - a\right) \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get $T = \frac{4875}{4} \text{ N}$

$$19. (4) \text{ Let spring does not get elongated, then net pulling force on the system is } Mg + mg - mg \text{ or simply } Mg. \text{ Total mass being pulled is } M + 2m. \text{ Hence, acceleration of the system is}$$

$$a = \frac{Mg}{M + 2m}$$

Now since $a < g$, there should be an upward force on M so that its acceleration becomes less than g . It means there is some tension developed in the string. Hence, for any value of M spring will be elongated.

$$20. (2) \text{ Suppose } F = \text{upthrust due to buoyancy}$$

Then while descending, we find

$$Mg - F = M\alpha \quad \dots(i)$$

When ascending, we have

$$F - (M - m)g = (M - m)\alpha \quad \dots(ii)$$

$$\text{Solving Eqs. (i) and (ii), we get } m = \left[\frac{2\alpha}{\alpha + g} \right] M$$

$$21. (1) \text{ Acceleration of the system:}$$

$$a = \frac{P}{M + m} \quad \dots(i)$$

The FBD of mass m is shown in the figure.

$$R \sin \beta = ma \quad \dots(ii)$$

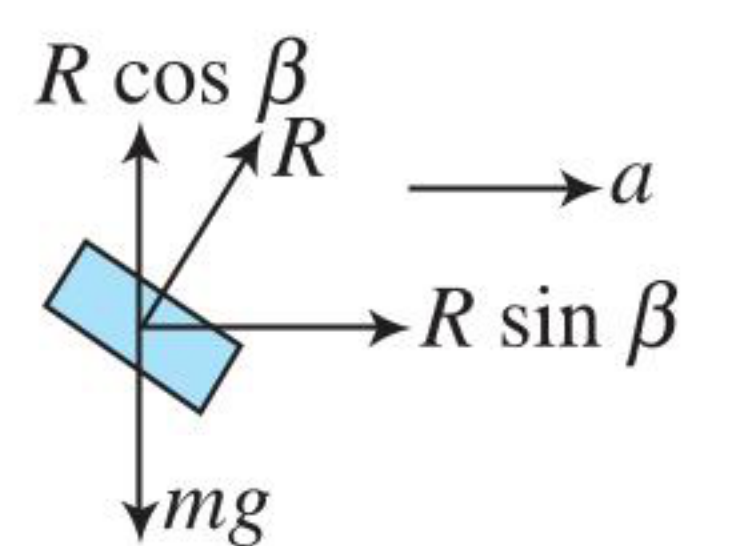
$$R \cos \beta = mg \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$a = g \tan \beta$$

Putting the value of a in (i), we get

$$P = (M + m)g \tan \beta$$



$$22. (2)$$

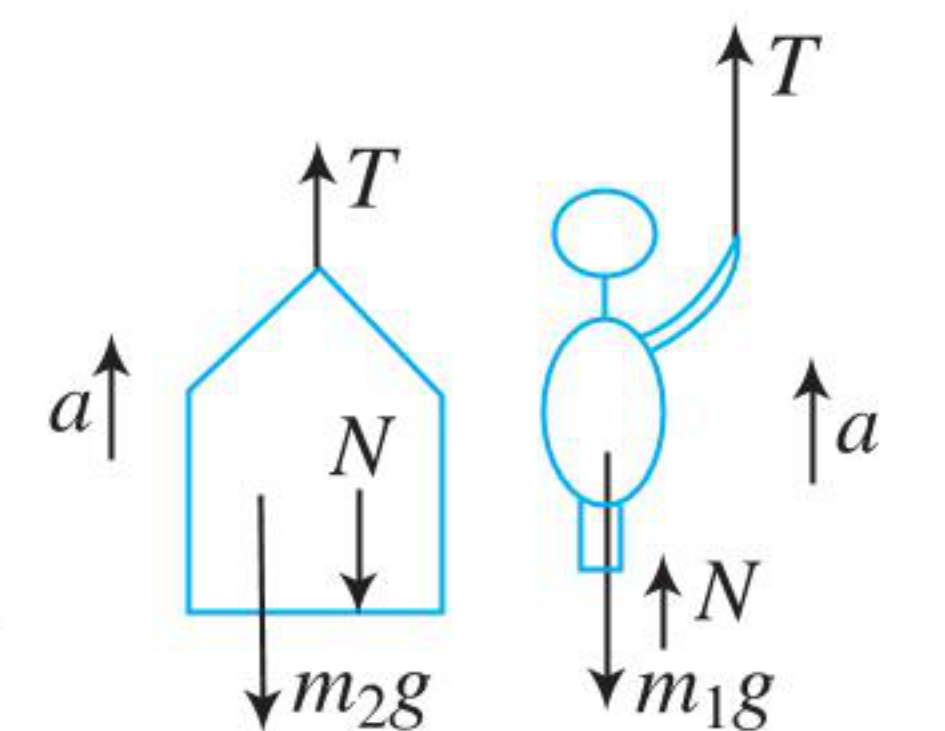
$$23. (4) \text{ Same solution for both}$$

$$m_1 = 100 \text{ kg, } m_2 = 50 \text{ kg, } a = 5 \text{ m s}^{-2}$$

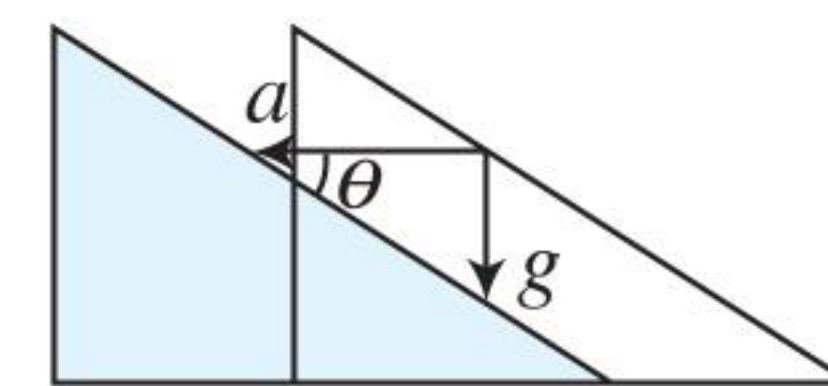
$$T + N - m_1 g = m_1 a,$$

$$T - N - m_2 g = m_2 a$$

Solving these: $T = 1125 \text{ N}$ and $N = 375 \text{ N}$



$$24. (3) \tan \theta = \frac{g}{a} \Rightarrow a = g \cot \theta$$



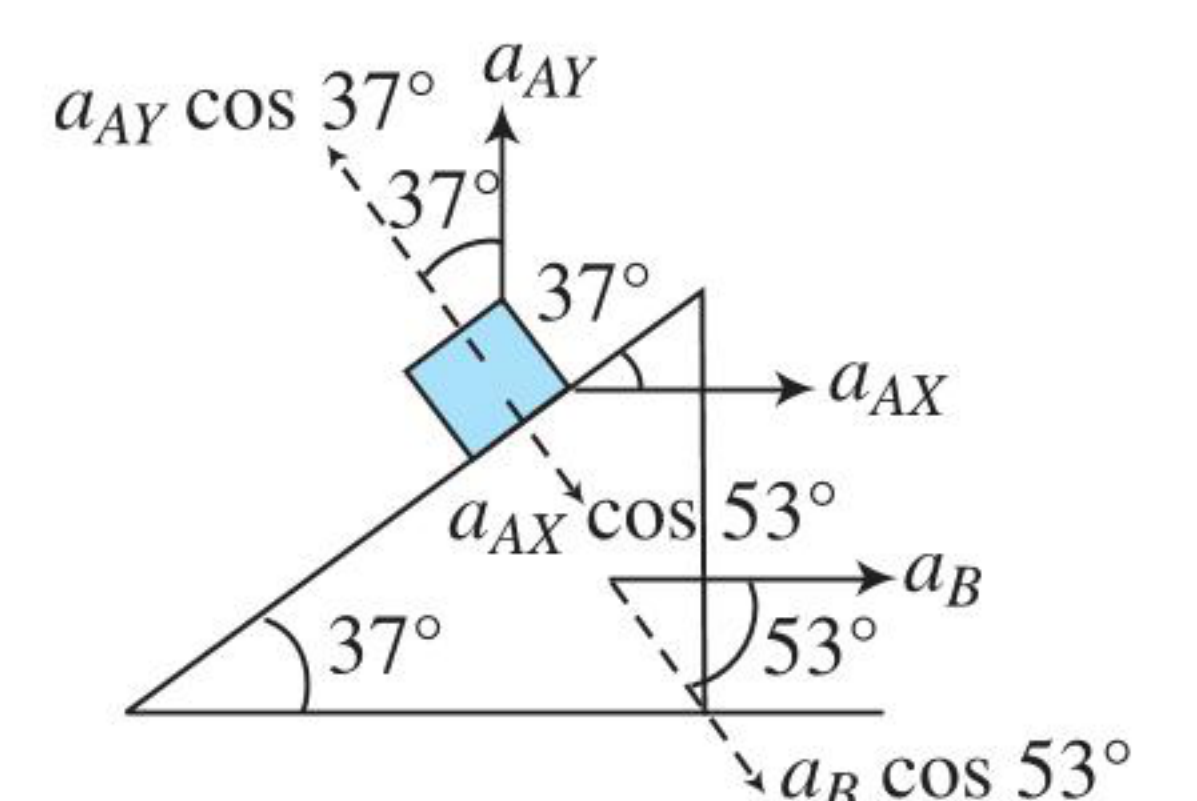
$$25. (4) \text{ From constraint, the acceleration of both block and wedge should be same in a direction perpendicular to the inclined plane as shown in Fig.}$$

$$(a_A)_\perp = (a_B)_\perp,$$

$$a_{AX} = 15, a_{AY} = 15$$

$$a_{AX} \cos 53^\circ - a_{AY} \cos 37^\circ = a_B \cos 53^\circ$$

$$\text{or } a_B = -5 \text{ m s}^{-1} \text{ or } \vec{a}_B = -5\hat{i}$$



$$26. (4) \text{ From constraint relations we can see that the acceleration of block B in upward direction is}$$

$$a_B = \left(\frac{a_C + a_A}{2} \right) \text{ with proper signs}$$

$$\text{So } a_B = \left(\frac{3 - 12t}{2} \right) = 1.5 - 6t$$

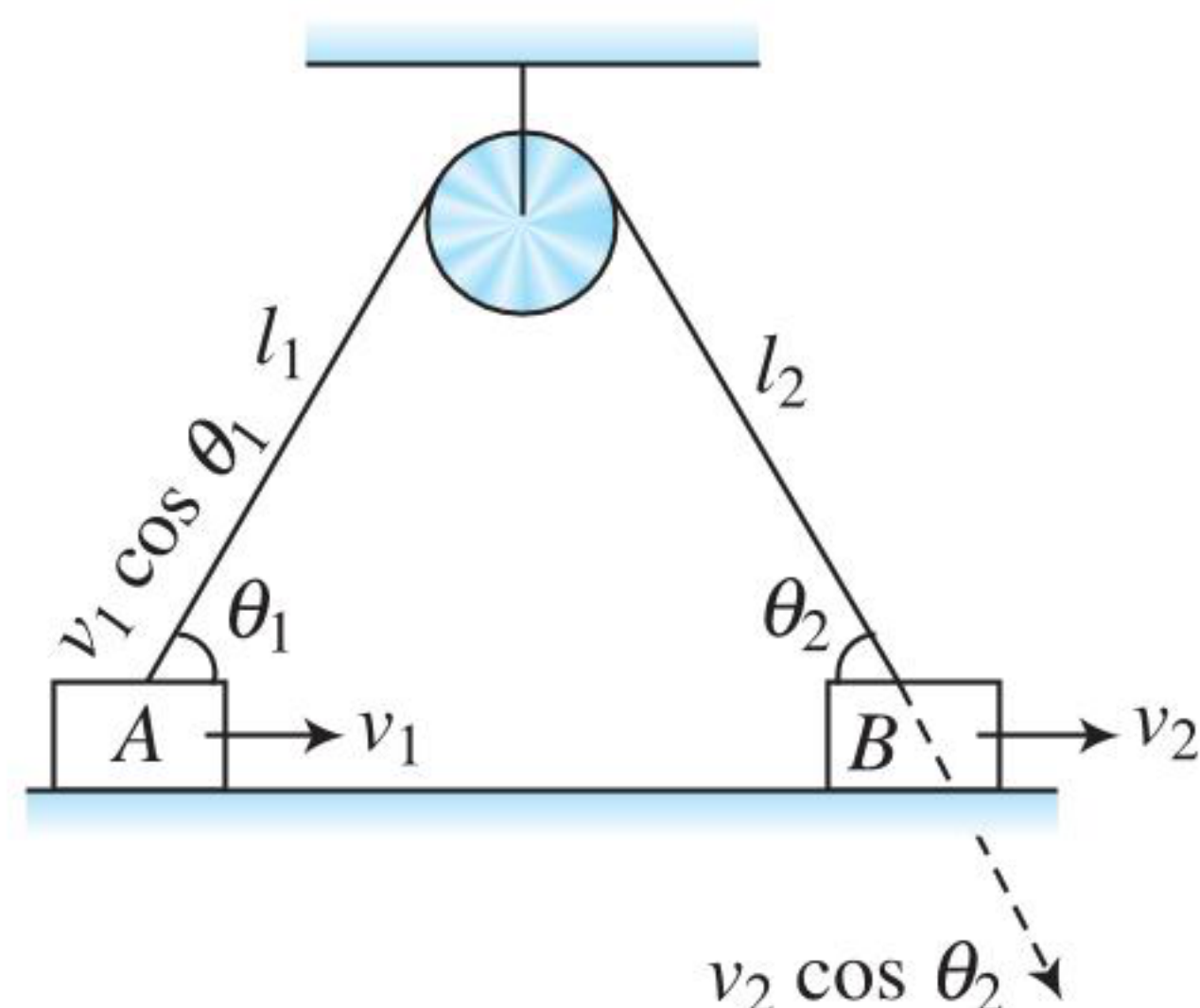
$$\text{or } \frac{dv_B}{dt} = 1.5 - 6t \quad \text{or } \int_0^{v_B} dv_B = \int_0^1 (1.5 - 6t) dt$$

$$\text{or } v_B = 1.5t - 3t^2 \quad \text{or } v_B = 0 \text{ at } t = 1/2 \text{ s}$$

27. (1) As m_2 moves with constant velocity, there is no acceleration in the centre of mass. Net force should be zero. For this

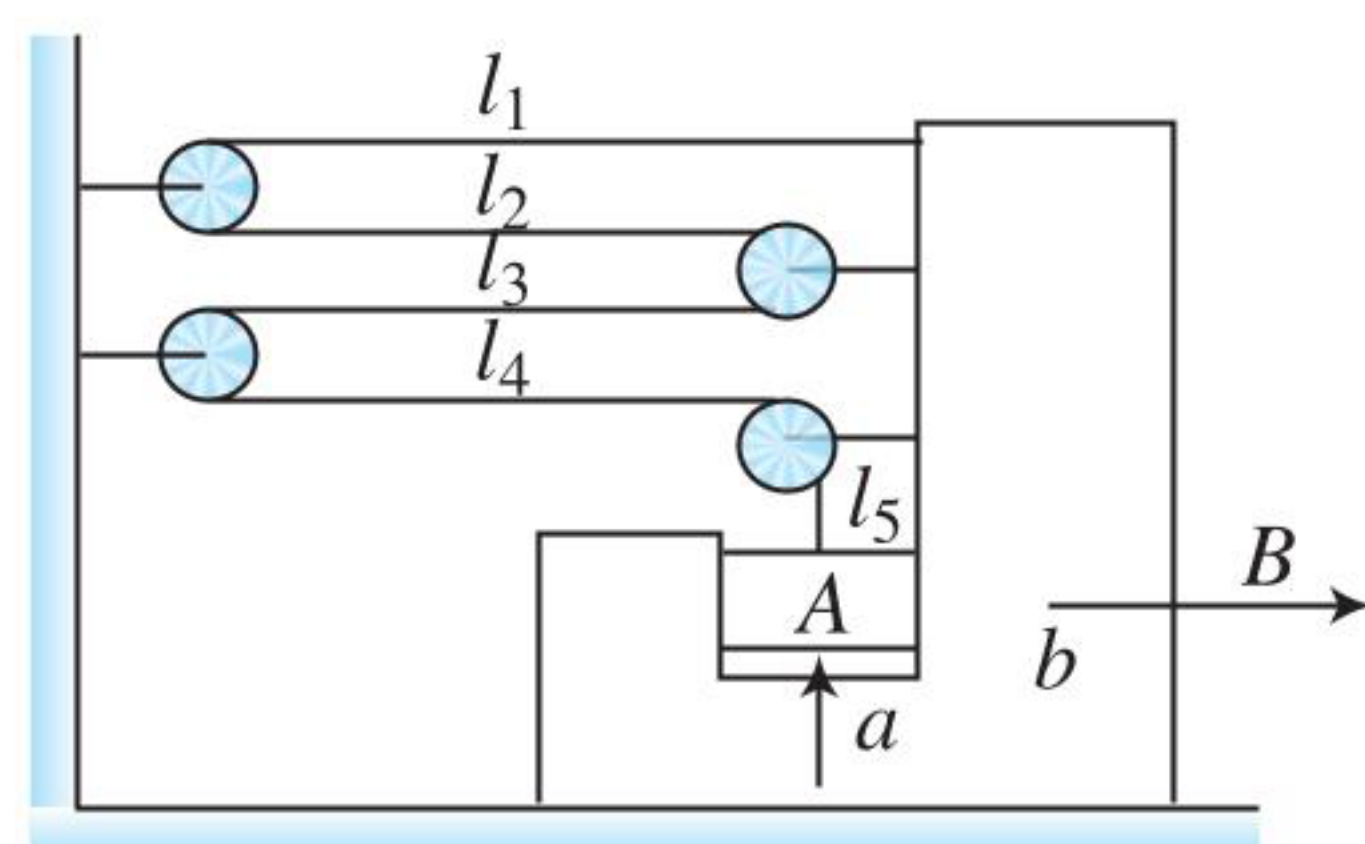
$$N = m_1 g + m_2 g.$$

28. (3) From Figure, $l_1 + l_2 = C$ or $\frac{dl_1}{dt} + \frac{dl_2}{dt} = 0$



$$-v_1 \cos \theta_1 + v_2 \cos \theta_2 = 0 \quad \text{or} \quad \frac{v_1}{v_2} = \frac{\cos \theta_2}{\cos \theta_1}$$

29. (1)

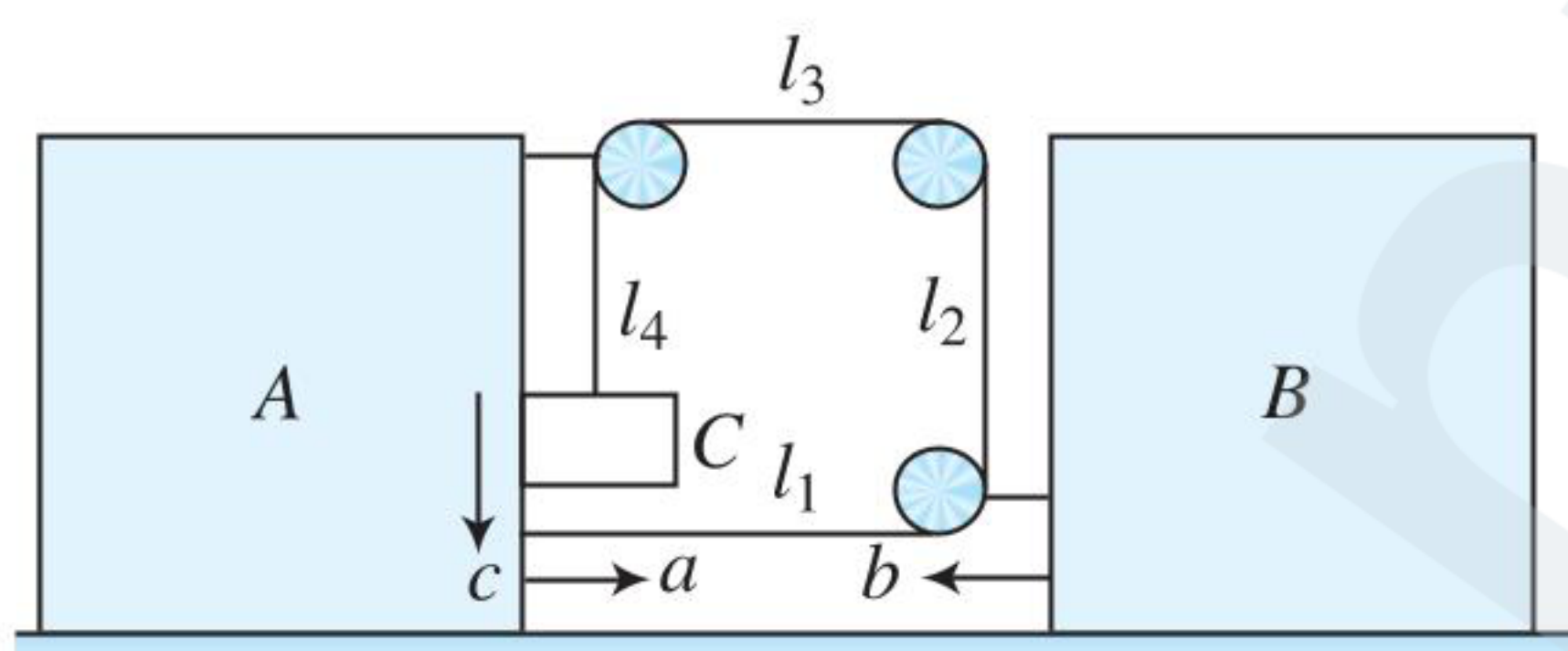


Acceleration of A in horizontal direction = the acceleration of B = b rightwards.

Acceleration of A in vertical direction = the acceleration of A with respect to b in upwards direction = $a = 4b$.

Hence, net acceleration of A = $\hat{b}i + 4b\hat{j}$.

30. (1)



From length constraint $l_1 + l_2 + l_3 + l_4 = C$

$$l_1' + l_2' + l_3' + l_4' = 0$$

$$(-a - b) + 0 + (-a - b) + c = 0$$

$$c = 2a + 2b$$

From wedge constraints, acceleration of C is right side is a.
acceleration of C w.r.t. ground = $\hat{a}i - 2(a + b)\hat{j}$

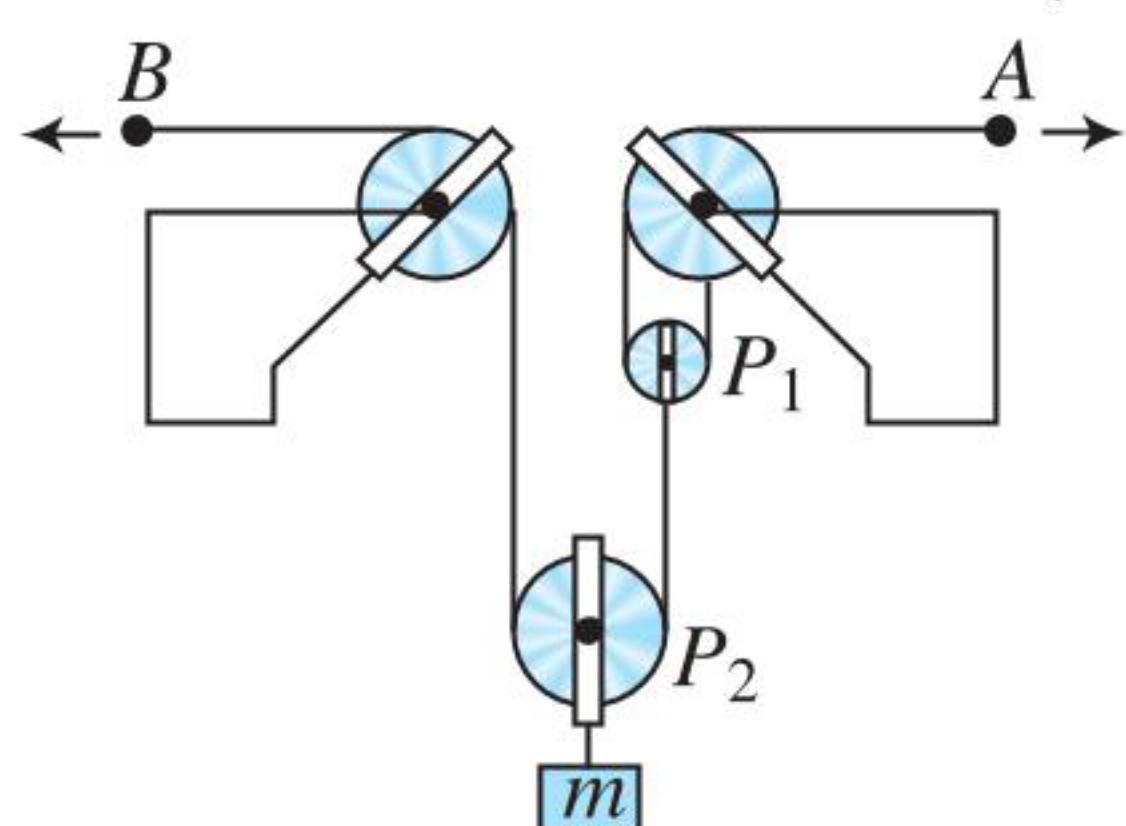
31. (3) $Tx(\text{Hanged part}) = 2Tx'(\text{Sliding part})$

$$\therefore x = 3x' \Rightarrow x = 3 \times 0.6 = 1.8 \text{ ms}^{-1}$$

32. (1) $V_A = 2 \text{ ms}^{-1}$ (towards right)

$$\therefore V_{P_1} = \frac{V_A}{2} = 1 \text{ ms}^{-1} \quad (\text{upwards})$$

$$V_B = 2 \text{ ms}^{-1} \quad (\text{towards left})$$



$$\text{Now, } 2V_{P_2} = V_B + V_{P_1}$$

$$\therefore V_{P_2} = \frac{V_B + V_{P_1}}{2} = \frac{2 + 1}{2} = 1.5 \text{ ms}^{-1}$$

33. (4) For A: $5g - T = 5(2C)$

$$\text{For C: } 2T - 8g = 8C \Rightarrow C = \frac{10}{14} = \frac{5}{7} \text{ ms}^{-2}$$

34. (3) If acceleration of M is 2 ms^{-2} , then acceleration of m w.r.t. M will be 2 ms^{-2}



Acceleration of m w.r.t. ground

$$a_m = \sqrt{2^2 + 2^2 + 2 \times 2 \times 2 \cos(180 - 37^\circ)} = \sqrt{8/5} \text{ ms}^{-2}$$

35. (3) From 0 to 2 s: at any time t, $F = 10t$

$$\Rightarrow a = F/m = 10t/m$$

$$\Rightarrow \int_0^v dv = \int_0^t \frac{10t}{m} dt \Rightarrow v = \frac{5t^2}{m}$$

$$\text{Momentum: } P = mv = 5t^2$$

$$\text{At } t = 2 \text{ s, } P = 5(2)^2 = 20 \text{ kg ms}^{-1}, v = 20/m$$

$$\text{From 2 to 4 s: } F = 40 - 10t$$

$$\int_{20/m}^v dv = \int_2^t \frac{40 - 10t}{m} dt \Rightarrow v = \frac{1}{m} [40t - 40 - 5t^2]$$

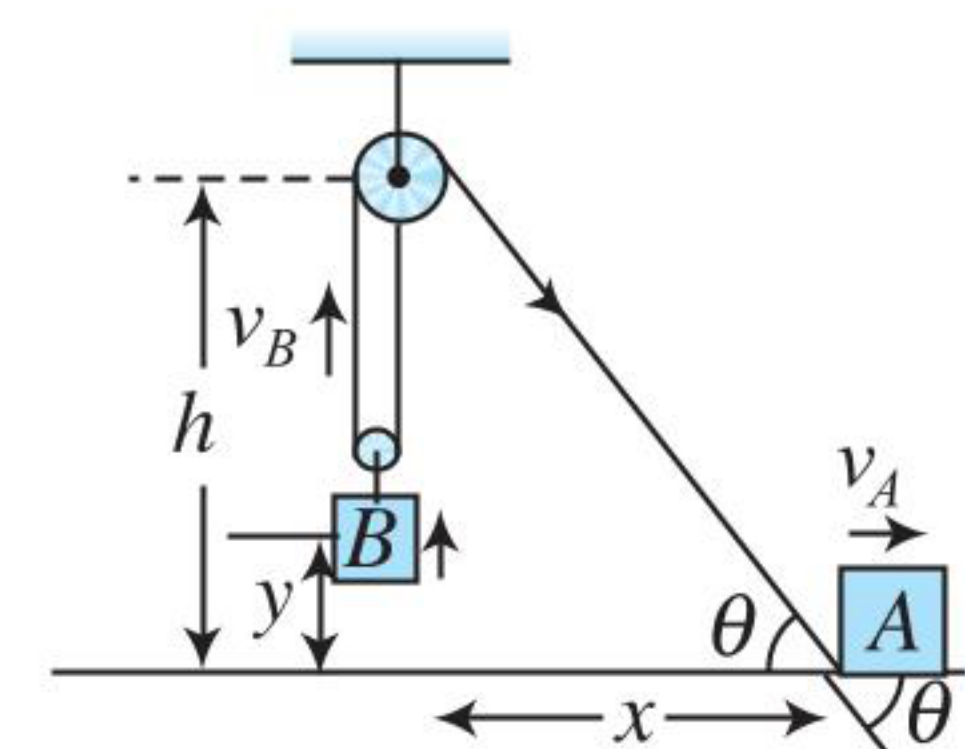
$$P = mv = 40t - 40 - 5t^2$$

36. (3) From figure, $L = 2h - 2y + \sqrt{x^2 + h^2}$

Differentiating the equation, we get

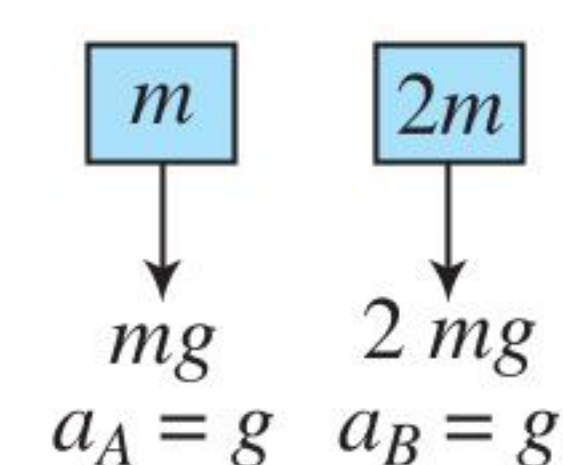
$$\frac{dy}{dt} = \frac{x}{2\sqrt{h^2 + x^2}} \frac{dx}{dt}$$

$$\Rightarrow v_B = \frac{xv_A}{2\sqrt{h^2 + x^2}}$$



37. (1) In this case, spring force is zero initially. FBD of A and B are shown in figure.

Tension in the string and spring will be zero just after release.



38. (4) $a_1 = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$ and $a_2 = \frac{(m_2 - m_1)g}{m_1}$

$$\text{Hence } \frac{a_1}{a_2} = \frac{m_1}{(m_1 + m_2)} = \frac{1}{\left(1 + \frac{m_2}{m_1}\right)} \Rightarrow \frac{a_1}{a_2} < 1$$

$$\text{As } T_1 = \frac{2m_1m_2g}{(m_1 + m_2)} \text{ and } T_2 = m_2g$$

$$\text{Hence, } \frac{T_1}{T_2} = \frac{2m_1}{(m_1 + m_2)} = \frac{2}{\left(1 + \frac{m_2}{m_1}\right)}$$

Therefore, T_1/T_2 will depend upon the values of m_1 and m_2 .

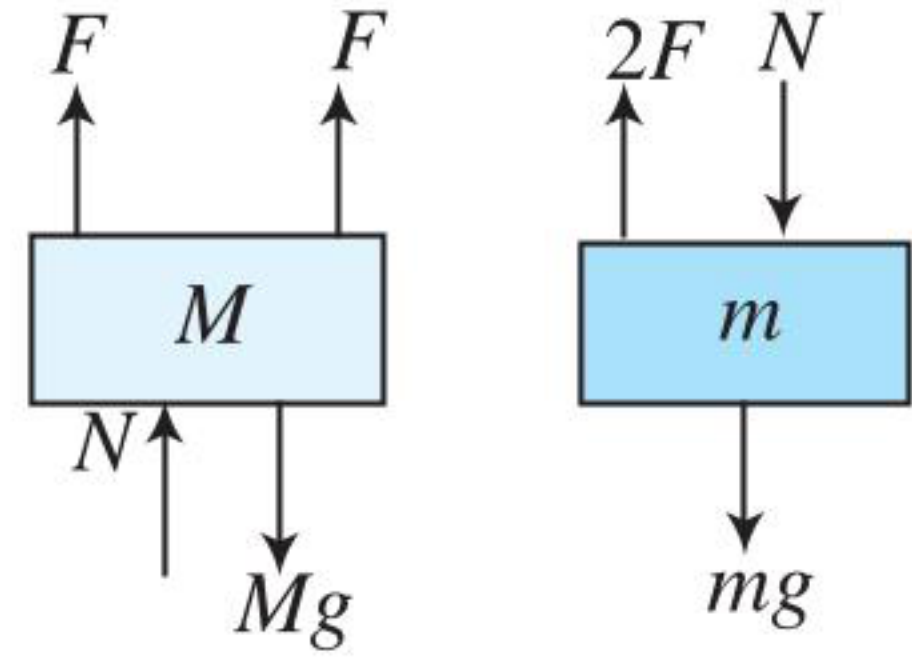
$$N_1 = 2T_1, N_2 = 2T_2$$

So the relation of N_1/N_2 will be same as T_1/T_2 .

39. (2) $a = \frac{3T - mg \sin \theta}{m}$
- $$= \frac{3 \times 250 - 100 \times 10 \times \sin 30^\circ}{100} = 2.5 \text{ ms}^{-2}$$

40. (3) From figure,

$$\begin{aligned} 2F + N - Mg &= Ma \\ 2F - mg - N &= ma \\ 4F - (M + m)g &= (M + m)a \\ a &= \frac{4F - (M + m)g}{M + m} \end{aligned}$$



41. (4) Extension in the spring is

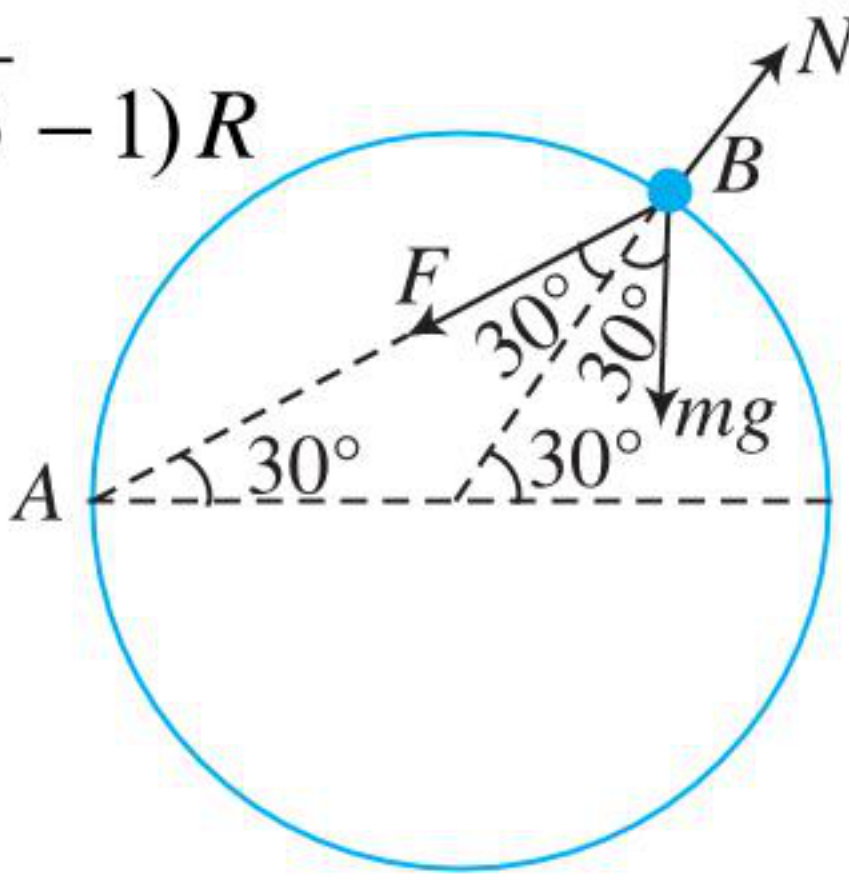
$$x = AB - R = 2R \cos 30^\circ - R = (\sqrt{3} - 1)R$$

Spring force:

$$F = kx = \frac{(\sqrt{3} + 1)mg}{R} \times (\sqrt{3} - 1)R = 2mg$$

From the figure, we have

$$N = (F + mg) \cos 30^\circ = \frac{3\sqrt{3}mg}{2}$$

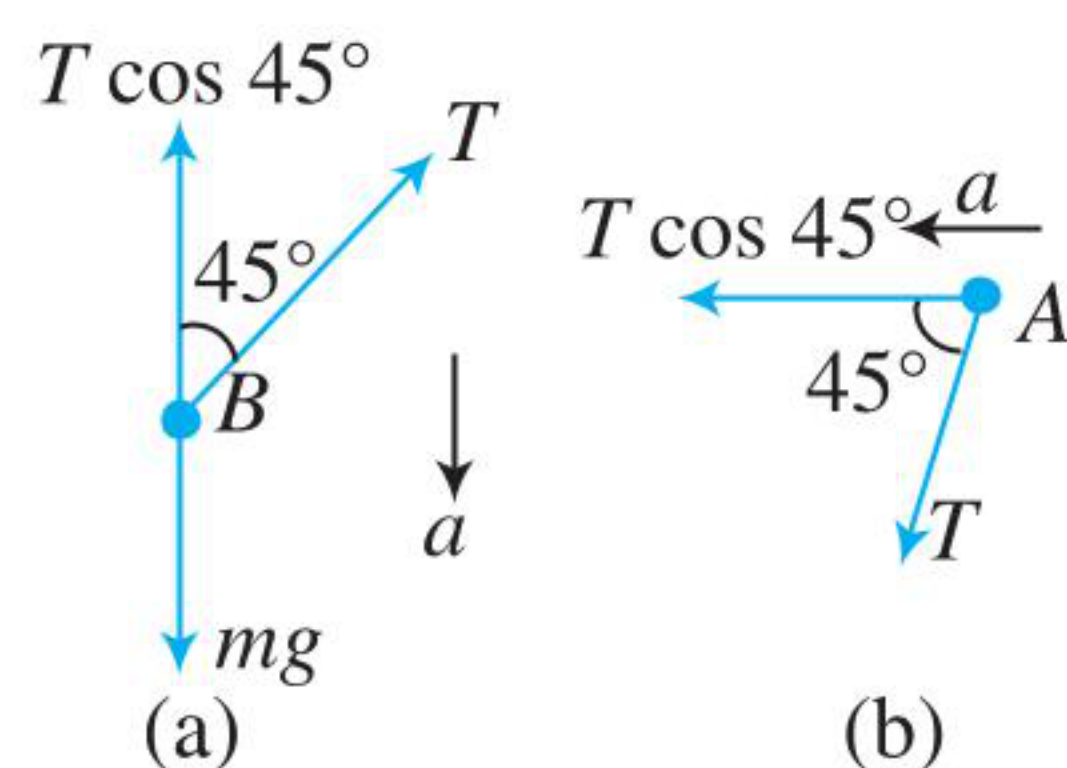


42. (4) As shown in figure (a and b) from FBD of A,

$$T \cos 45^\circ = ma \quad \dots(i)$$

From FBD of B:

$$Mg - T \cos 45^\circ = ma \quad \dots(ii)$$



From (i) and (ii), we get $T = mg/\sqrt{2}$

43. (3) By virtual work method:

Acceleration of B w.r.t. A will be 10 m s^{-2} downward.

Apart from this, B also has an acceleration 5 m s^{-2} in horizontal direction along with A, so net acceleration of B is

$$\sqrt{10^2 + 5^2} = \sqrt{100 + 25} = \sqrt{125} = 5\sqrt{5} \text{ m s}^{-2}$$

44. (3) In equilibrium, if θ is the required angle,

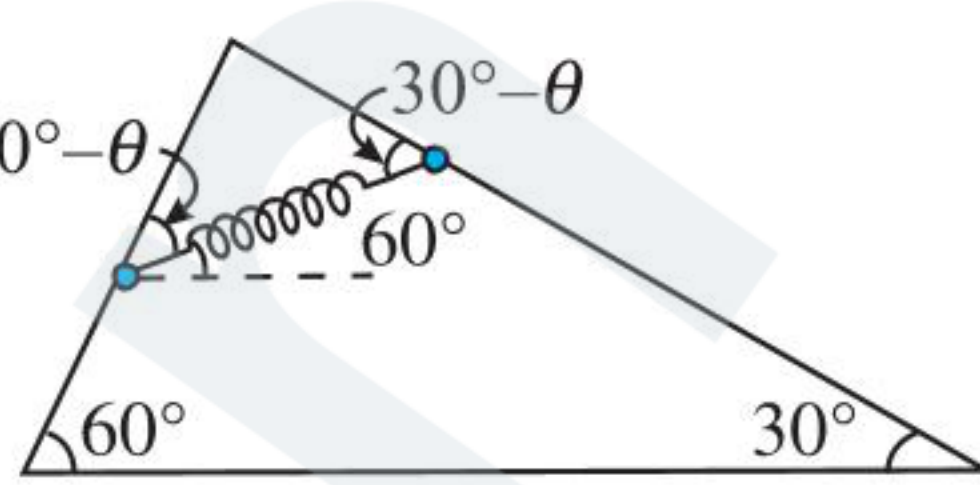
Cylinder A:

$$mg \sin 60^\circ = kx \cos (60^\circ - \theta)$$

Cylinder B:

$$mg \sin 30^\circ = kx \cos (30^\circ + \theta)$$

$$= kx \sin(60^\circ - \theta) \text{ on solving } \theta = 30^\circ.$$

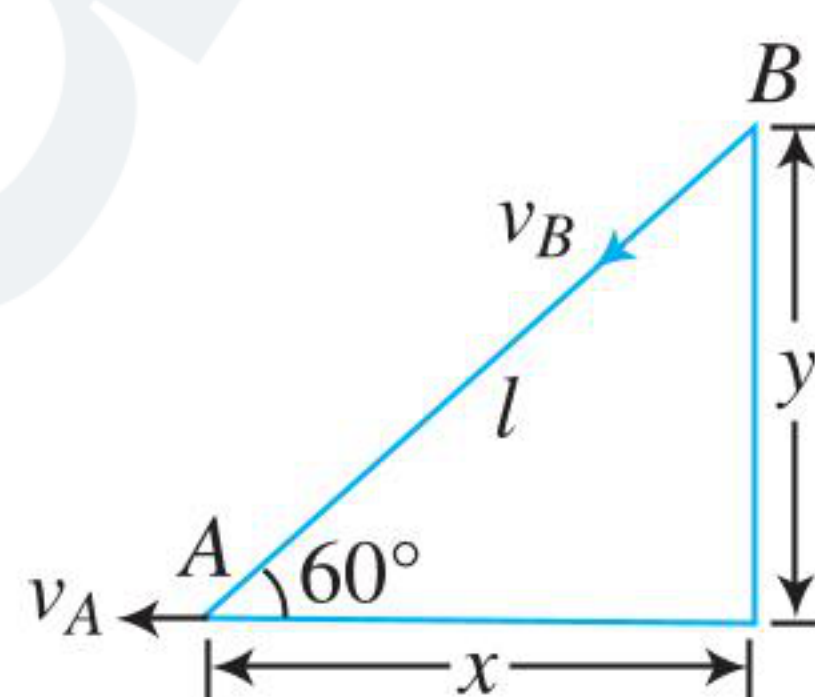


45. (2) Here y is constant $\frac{dl}{dt} = v_B$

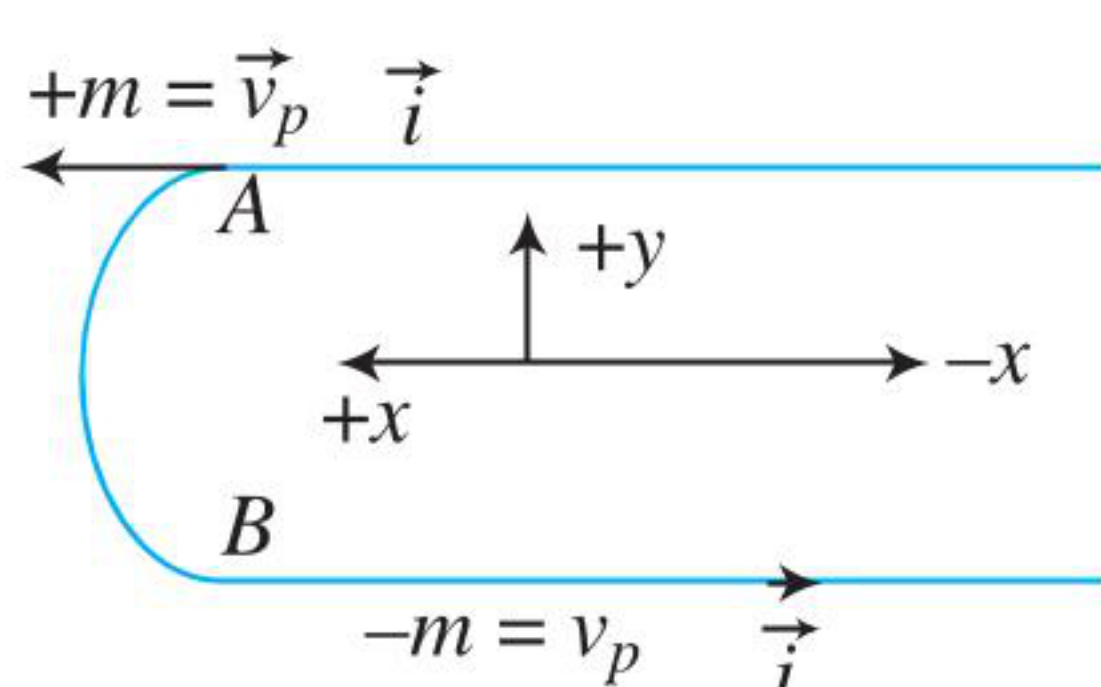
$$l^2 = y^2 + x^2$$

$$\Rightarrow 2l \frac{dl}{dt} = 2x \frac{dx}{dt}$$

$$\Rightarrow v_B = \frac{x}{l} v_A = v_A \cos 60^\circ = 1 \text{ m s}^{-1}$$



46. (2) Choosing the positive x - y axis as shown in the figure, the momentum of the bead at A is $\vec{p}_i = +m\vec{v}$. The momentum of the bead at B is $\vec{p}_f = -m\vec{v}$



Therefore, the magnitude of the change in momentum between A and B is $\Delta \vec{p} = \vec{p}_f - \vec{p}_i = -2m\vec{v}$, i.e., $\Delta p = 2mv$ along the positive x -axis.

The time taken by the bead to reach from A to B is

$$\Delta t = \frac{\pi \cdot d/2}{v} = \frac{\pi d}{2v}$$

Therefore, the average force exerted by the bead on the wire is

$$F_{av} = \frac{\Delta p}{\Delta t} = \left(2mv / \frac{\pi d}{2v} \right) = \frac{4mv^2}{\pi d}$$

47. (2) Equation of motion for M:

Since M is stationary

$$T - Mg = 0 \Rightarrow T = Mg$$

Since the boy moves up with acceleration a ,

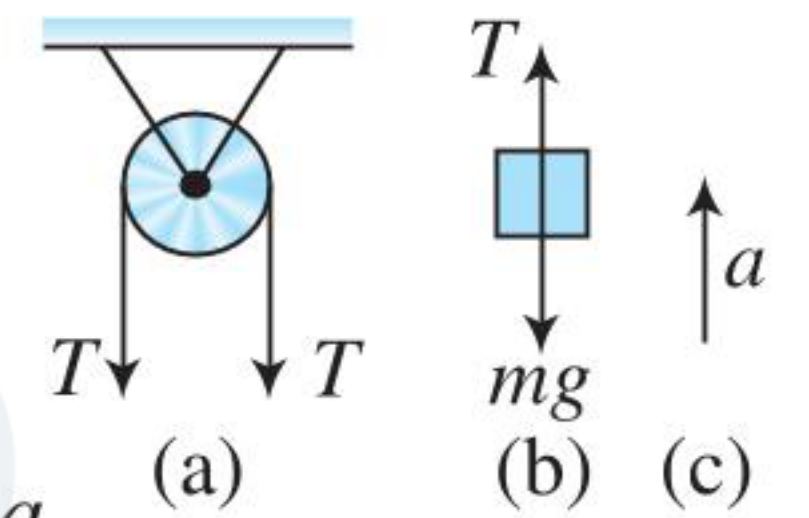
$$\therefore T - mg = ma$$

$$\Rightarrow T = m(g + a)$$

Equating Eqs. (i) and (ii), we obtain

$$Mg = m(g + a)$$

$$\Rightarrow a = \left(\frac{M}{m} - 1 \right) g, \text{ the block } M \text{ can be lifted}$$

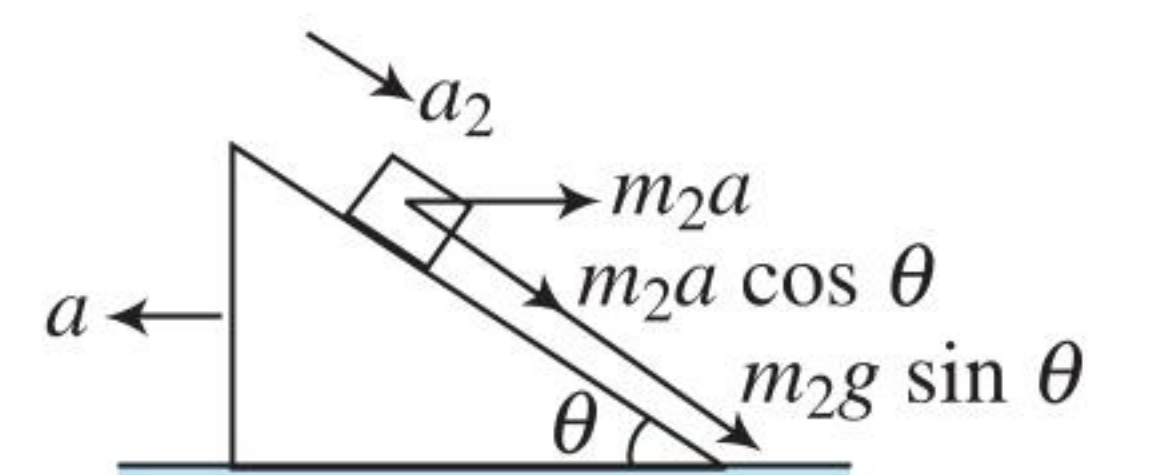


48. (1) In first case, acceleration of m_1 will be $a_1 = g \sin \theta$ down the inclined plane. In second case, acceleration of m_2 w.r.t. incline is

$$a_2 = \frac{m_2 g \sin \theta + m_2 a \cos \theta}{m_2}$$

$$\Rightarrow a_2 = g \sin \theta + a \cos \theta$$

Since $a_2 > a_1$, so $T_2 < T_1$



49. (3) (Check figure in the frame of the car)

Applying Newton's law perpendicular to string,

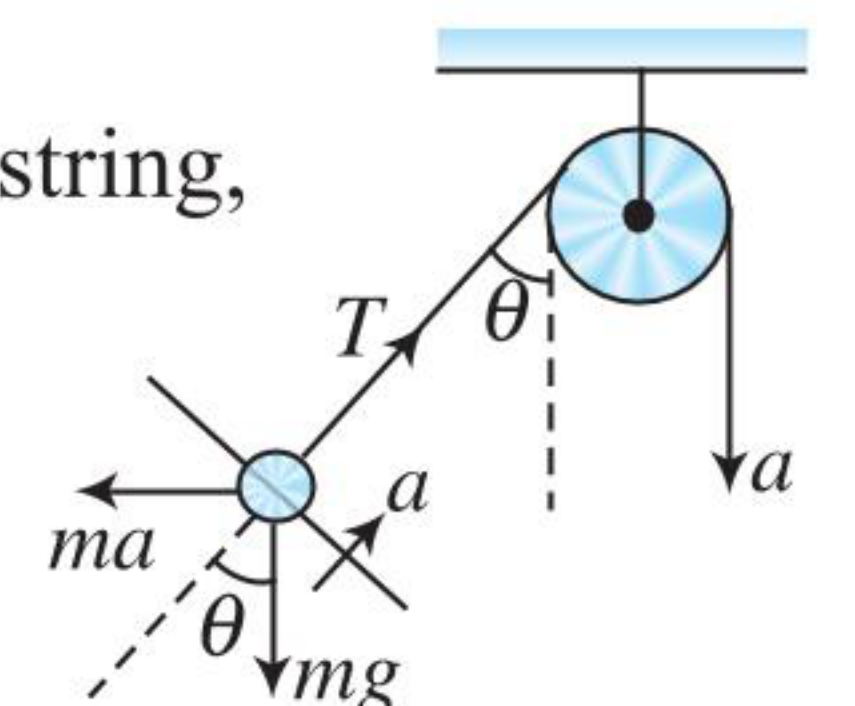
$$mg \sin \theta = ma \cos \theta$$

$$\Rightarrow \tan \theta = \frac{a}{g}$$

Applying Newton's law along string,

$$T - mg \cos \theta - ma \sin \theta = ma$$

$$\Rightarrow T = m\sqrt{g^2 + a^2} + ma$$

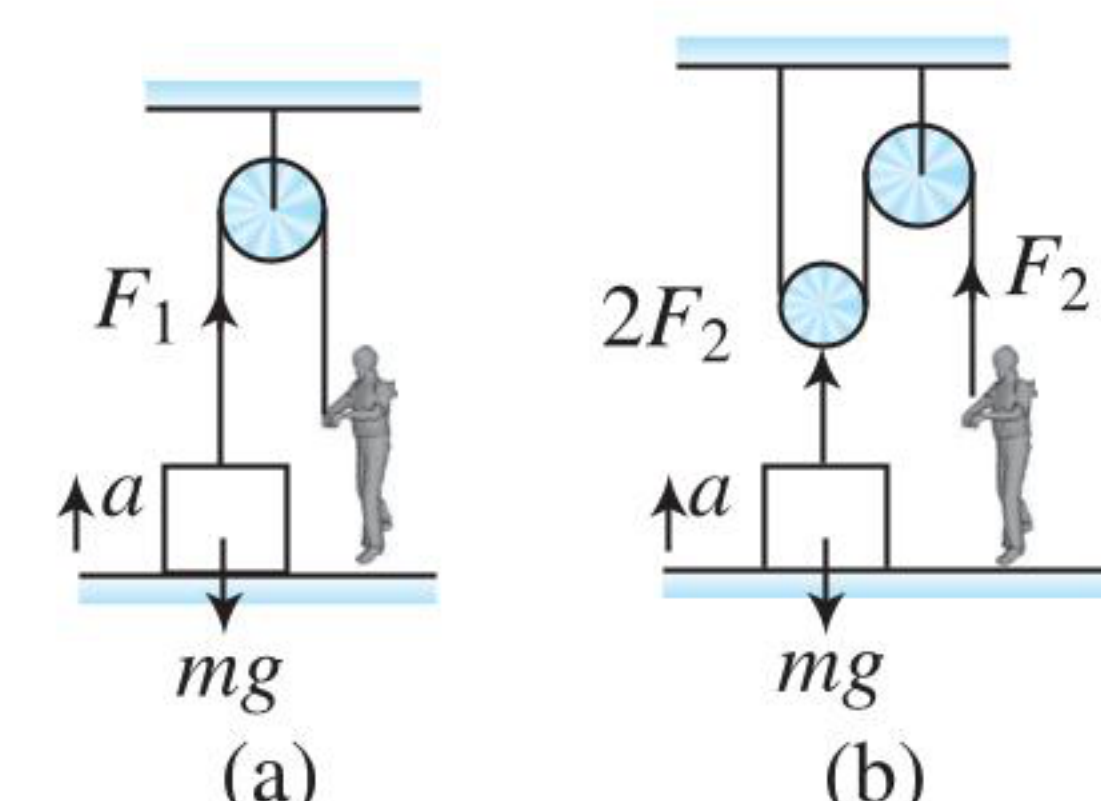


50. (1) Since, $h = \frac{1}{2}at^2$, a should be same in both cases, because h and t are same in both cases as given.

In figure (a), $F_1 - mg = ma \Rightarrow F_1 = mg + ma$

In figure (b), $2F_2 - mg = ma \Rightarrow F_2 = \frac{mg + ma}{2}$

$$\therefore F_1 > F_2$$



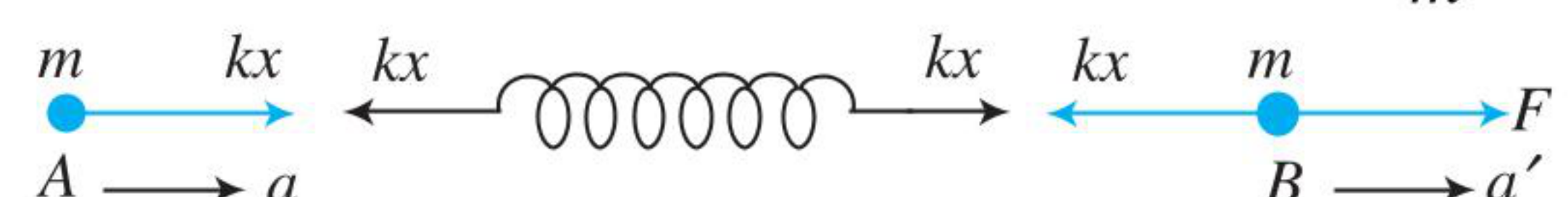
51. (3) Equation of motion for A (figure),

$$kx = ma \Rightarrow a = \frac{kx}{m}$$

For B: $F - T = ma'$

$$\Rightarrow a' = \frac{F - kx}{m}$$

Thus, relative acceleration $= a_r = |a' - a| = \frac{F - 2kx}{m}$



52. (2) As the springs have natural length initially, if one spring is compressed, the other must be expanded. Hence, the compression will be negative.

The free-body diagram of m_2 [Figure]

$$T + F_2 = 80 \text{ N} \quad \text{and}$$

$$F_2 = 70 \times 0.5 = 35 \text{ N}$$

$$\therefore T = 80 - 35 = 45 \text{ N}$$

FBD of m_1 (figure)

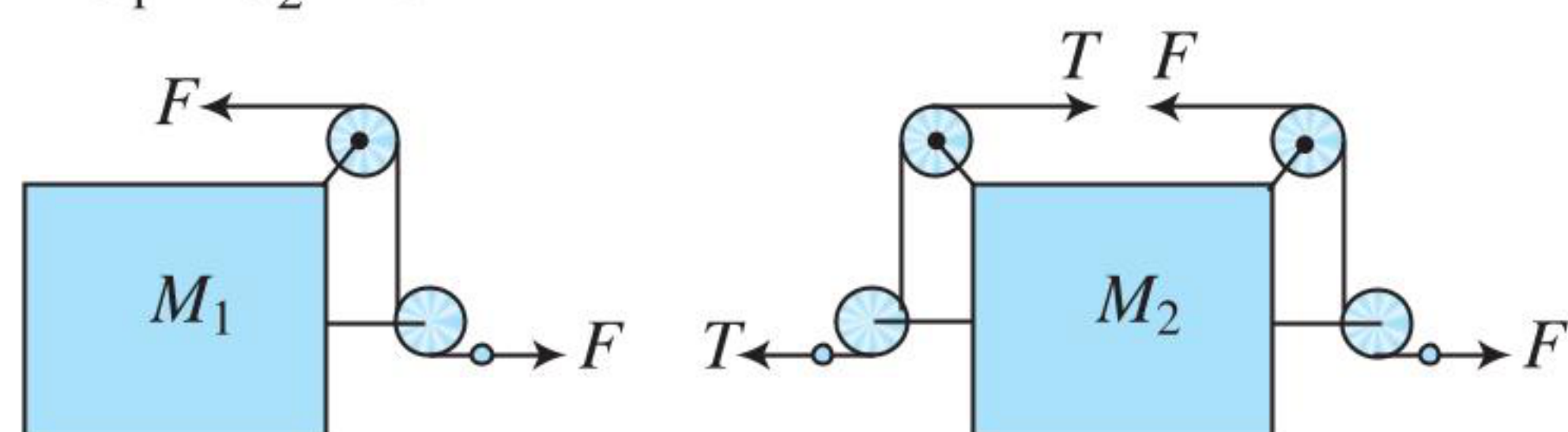
$$T + F_1 = m_1 g \quad \text{or} \quad F_1 = -25 \text{ N}$$

$$\therefore X_1 = \frac{-25}{k_1} = \frac{-25}{50} = -0.5 \text{ m}$$

Therefore, compression in first spring is -0.5 m . (negative sign indicates that it is extension)

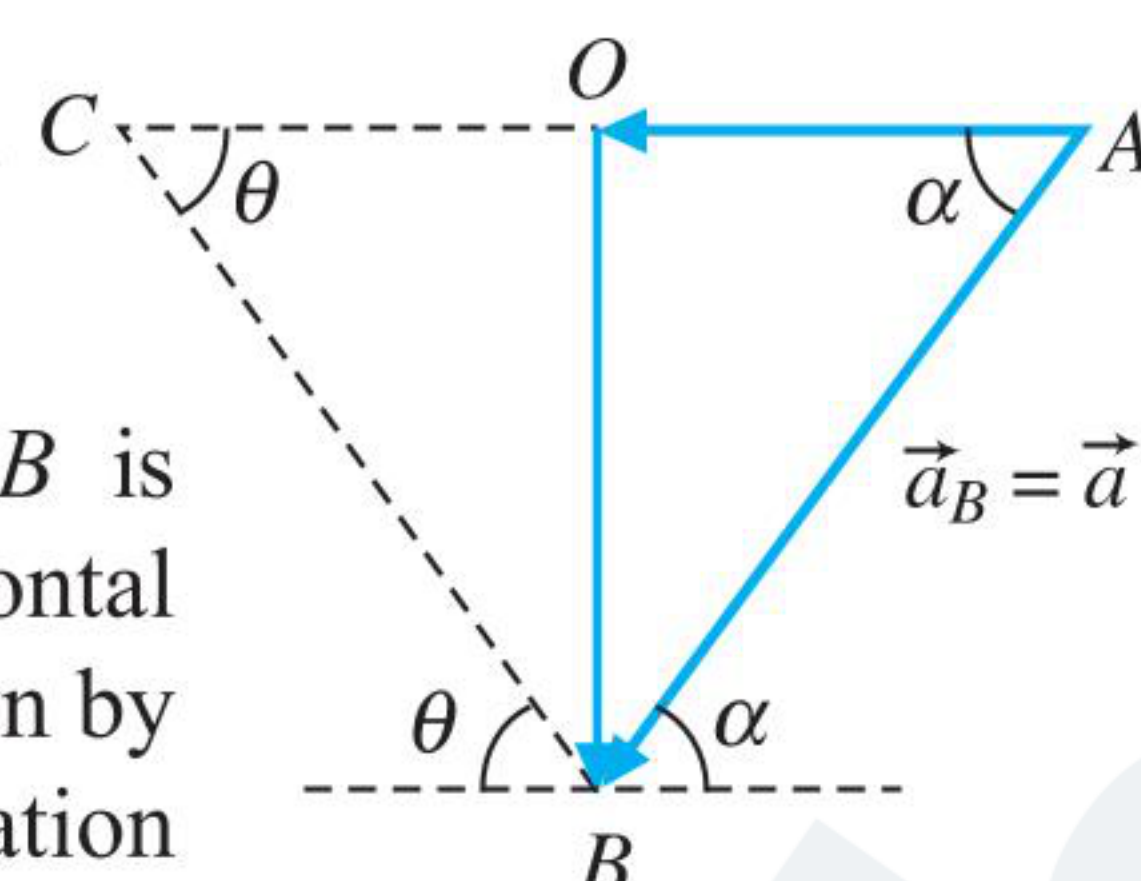
53. (2) From FBD, it is obvious that net force on each block is zero in horizontal direction. So

$$a_1 = a_2 = 0$$



54. (4) Direction of acceleration of B is along the fixed incline, and that of A is along horizontal towards left.

From diagram, acceleration of B is represented by $\vec{AB}$ while its horizontal and vertical components are shown by $\vec{AO}$ and $\vec{OB}$, respectively. Acceleration of A is represented by $\vec{AC}$



$$\vec{AC} = a(\sin \alpha \cot \theta + \cos \alpha)$$

55. (2) If the wedge moves leftward by x , then the block moves down the wedge by $4x$, i.e., w.r.t. wedge the block comes down by $4x$.

So, acceleration of block w.r.t wedge = $4a$ along the incline plane of wedge.

Acceleration of wedge with respect to ground = a , along left. So acceleration of block w.r.t. ground is the vector sum of the two vectors shown in figure. That is

$$|\vec{a}_{BG}| = \sqrt{a^2 + (4a)^2 + 2 \times a \times 4a \times \cos(\pi - \alpha)} \\ = (\sqrt{17 - 8\cos \alpha}) a \text{ m s}^{-2}$$

56. (4) Using constraint theory,

$$l_1 + l_2 + l_3 = \text{constant}$$

$$\Rightarrow v_1 + 2v_2 + v_3 = 0$$

Take downward as positive and upward as -ve.

$$\text{So, } 12 + 2(-4) + v_3 = 0$$

$$v_3 = \text{velocity of pulley } P = -4 \text{ m s}^{-1} \\ = 4 \text{ m s}^{-1} \text{ in upward direction}$$

$$\vec{v}_{AP} = -\vec{v}_{BP}$$

$$\Rightarrow v_{AP} = -(v_B - v_P)$$

$$v_B = v_P - v_{AP} = -4 - (-3) = -7 \text{ m s}^{-1}$$

i.e., block B will move up with speed 7 m s^{-1} .

57. (2) $\vec{u} = 4\hat{i} + 2\hat{j}$, $\vec{a} = \frac{\vec{F}}{m} = \hat{i} - 4\hat{j}$

Let at any time, the coordinates be (x, y) .

$$x - 2 = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x - 2 = 4t + \frac{1}{2} t^2 \quad \text{and} \quad y - 3 = 2t - \frac{1}{2} 4t^2$$

$$\Rightarrow y - 3 = 2t - 2t^2$$

When $y = 3 \text{ m}$, $t = 0, 1 \text{ s}$; when $t = 0$, $x = 2 \text{ m}$

When $t = 1 \text{ s}$, $x = 6.5 \text{ m}$

58. (3) Let the plank move up by x , then pulley 2 will move down by x . Let the end of string C moves down by a distance y .

Let the initial length of string passing over pulley 2,

$$l_1 + l_2$$

After displacements x and y mentioned above, the lengths become

$$(l_1 - 2x) + (l_2 + y - x)$$

Equating Eqs. (i) and (ii), we get $y = 3x$

Length of string that slips through A is $y + x = 4x$ and through B is $y = 3x$.

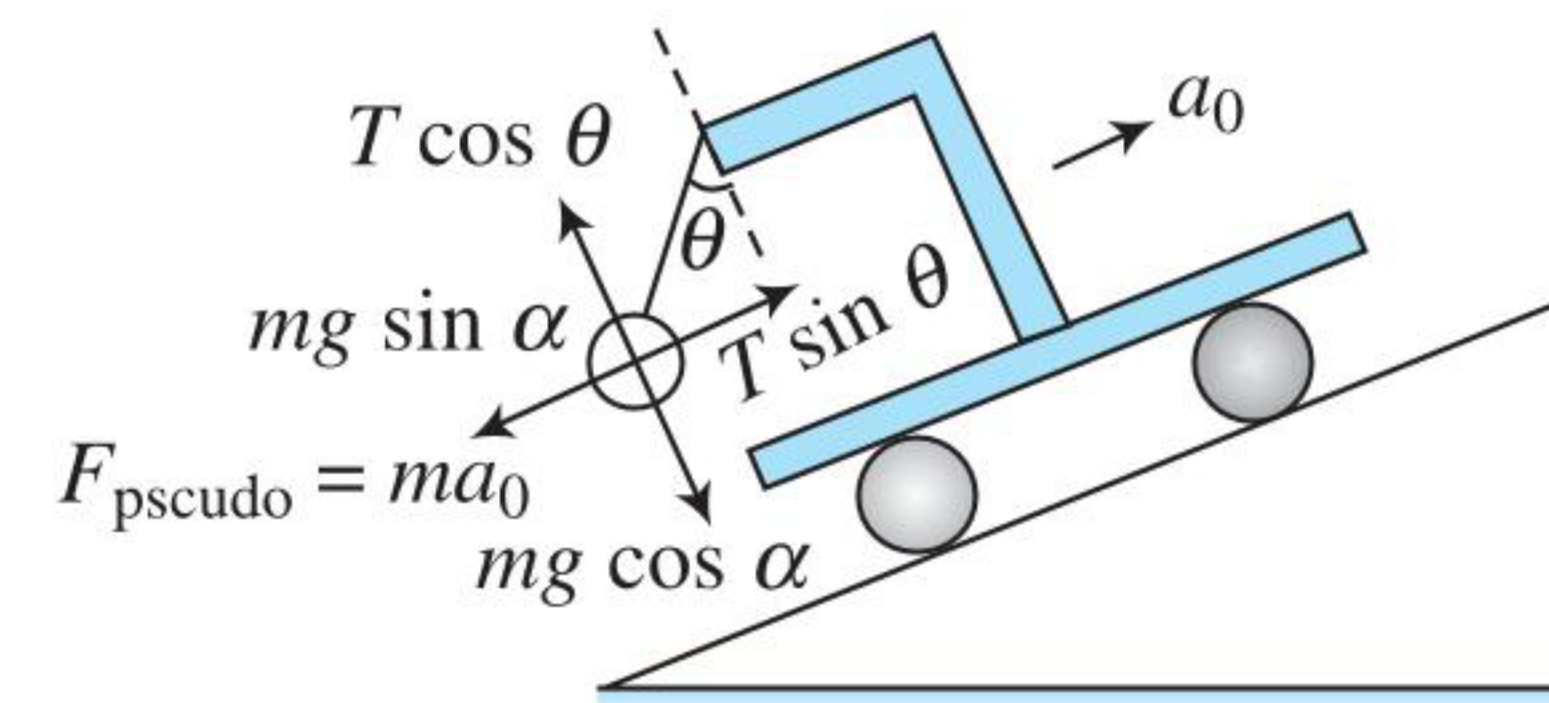
$$\text{Required ratio} = \frac{4x}{3x} = \frac{4}{3}$$

59. (3) $T = 2ma$

$$mg - T = ma$$

$$a = g/3$$

60. (4) As in Figure

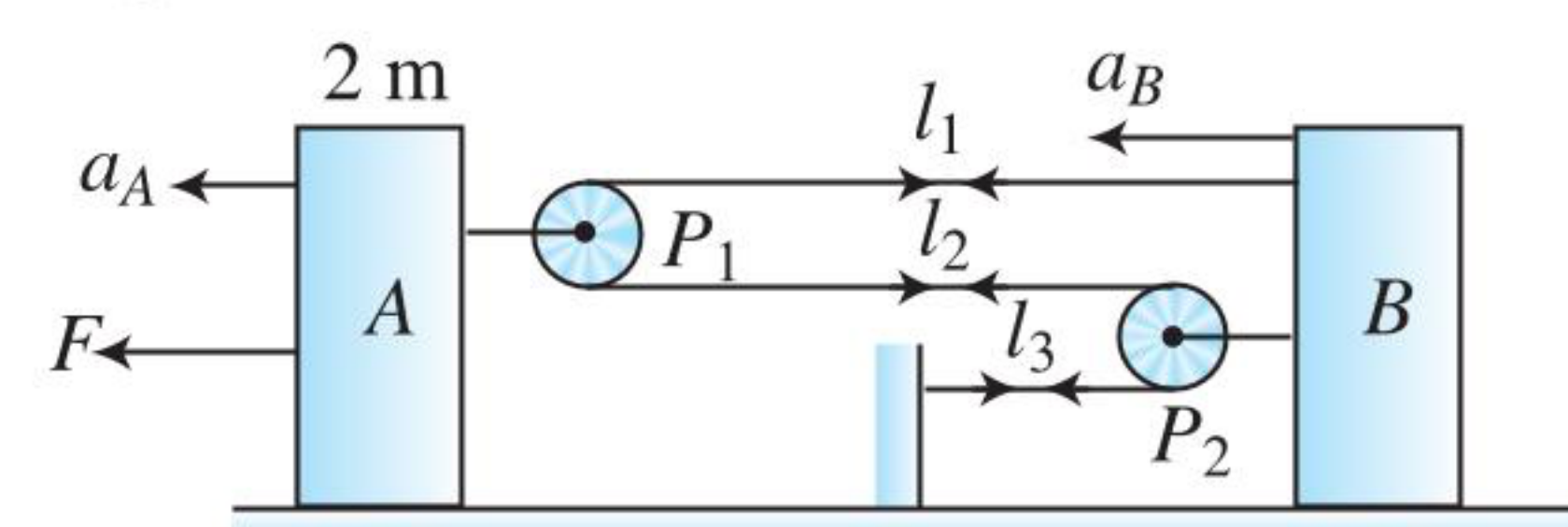


$$T \sin \theta = ma_0 + mg \sin \alpha$$

$$T \cos \theta = mg \cos \alpha$$

$$\tan \theta = \frac{a_0 + g \sin \alpha}{g \cos \alpha}$$

61. (2) As in Figure,



$$l_1 + l_2 + l_3 = C$$

$$l'_1 + l'_2 + l'_3 = 0 \Rightarrow -V_B + V_B - V_B + V_A - V_B = 0$$

$$3V_B = 2V_A \Rightarrow 3a_B = 2a_A$$

Applying Newton's law on A and B,

$$F - 2T = 2ma_A, 3T = 2ma_B$$

$$\text{Solve to get } a_B = \frac{3F}{13m}$$

62. (2) If initial acceleration of M towards right is A, then we can show that acceleration of m w.r.t. M down the incline is

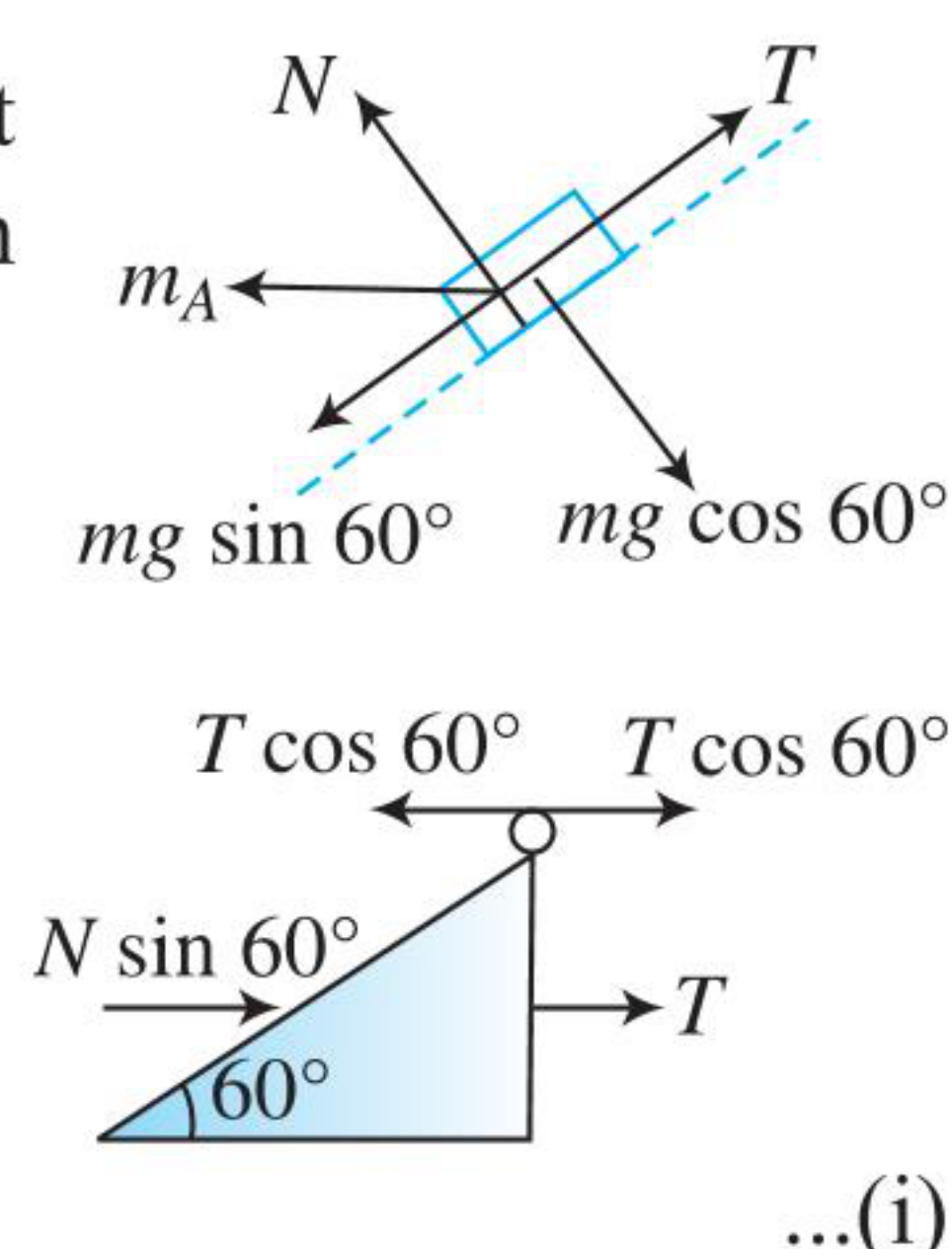
$$a = A(1 + \cos \theta) = \frac{3A}{2} \quad (\because \theta = 60^\circ)$$

FBD of block m (w.r.t. M) is shown below:

FBD of M (Figure)

Equation of motion:

$$\text{For } m: mg \frac{\sqrt{3}}{2} + mA \times \frac{1}{2} - T = m \frac{3}{2} A$$



... (i)

$$N + mA \frac{\sqrt{3}}{2} = mg \frac{1}{2} \quad \dots(ii)$$

$$\text{For } M: T + N \frac{\sqrt{3}}{2} = MA \quad \dots(iii)$$

$$\text{From Eqs. (i), (ii), and (iii)} \quad A = \frac{3\sqrt{3}g}{23} \text{ m s}^{-1}$$

63. (4) Acceleration of two mass systems is $a = \frac{F}{2m}$ leftwards. FBD of block A is shown in below.

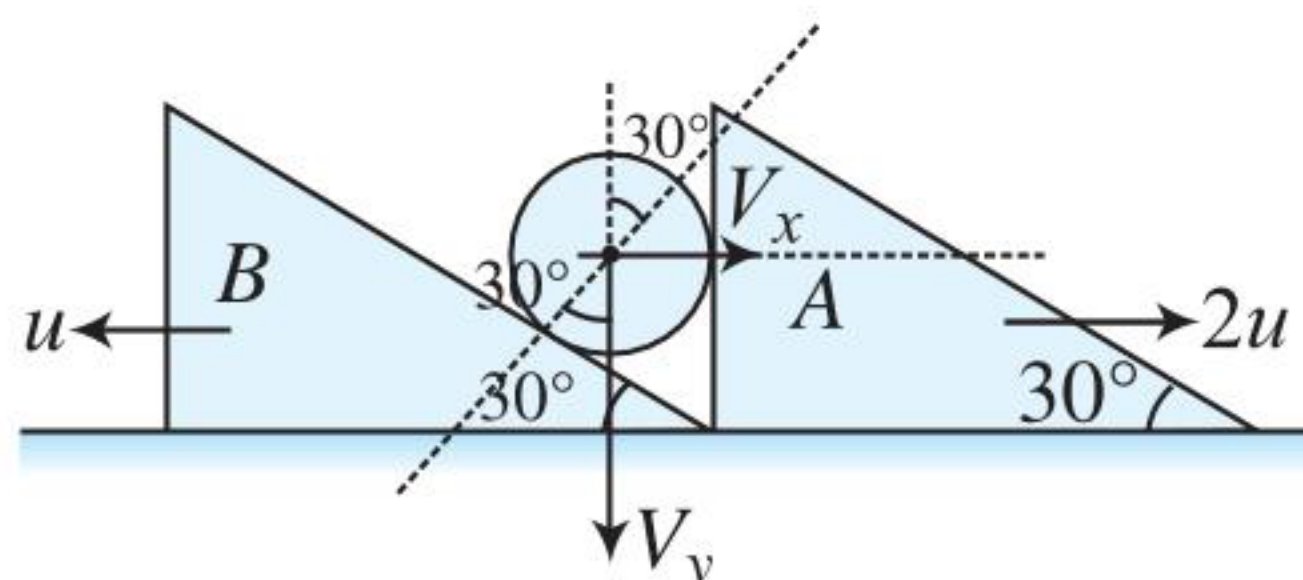
$$N \cos 60^\circ - F = ma = \frac{mF}{2m}$$

Solving, we get $N = 3F$

64. (4) **Method 1:** Velocity components perpendicular to the contact surface remain same.

As cylinder will remain in contact with wedge A

$$V_x = 2u$$



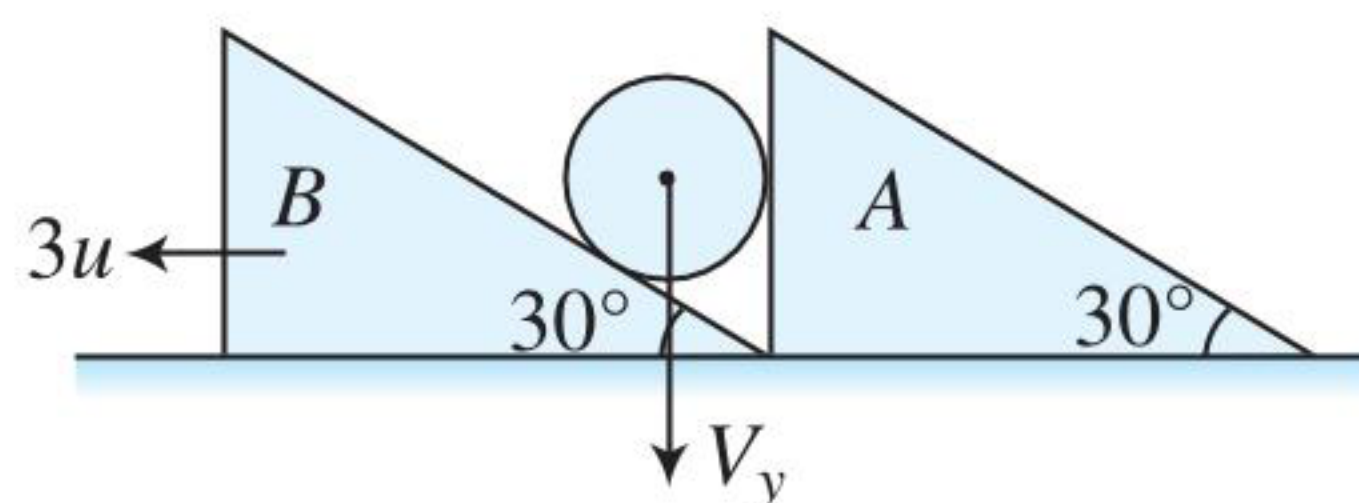
As it also remain in contact with wedge B,

$$u \sin 30^\circ = V_y \cos 30^\circ - V_x \sin 30^\circ$$

$$V_y = V_x \frac{\sin 30^\circ}{\cos 30^\circ} + \frac{u \sin 30^\circ}{\cos 30^\circ} = 3u \tan 30^\circ = \sqrt{3}u$$

$$\Rightarrow V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u$$

Method 2: In the frame of A



$$3u \sin 30^\circ = V_y \cos 30^\circ$$

$$V_y = 3u \tan 30^\circ = \sqrt{3}u \text{ and } V_x = 2u$$

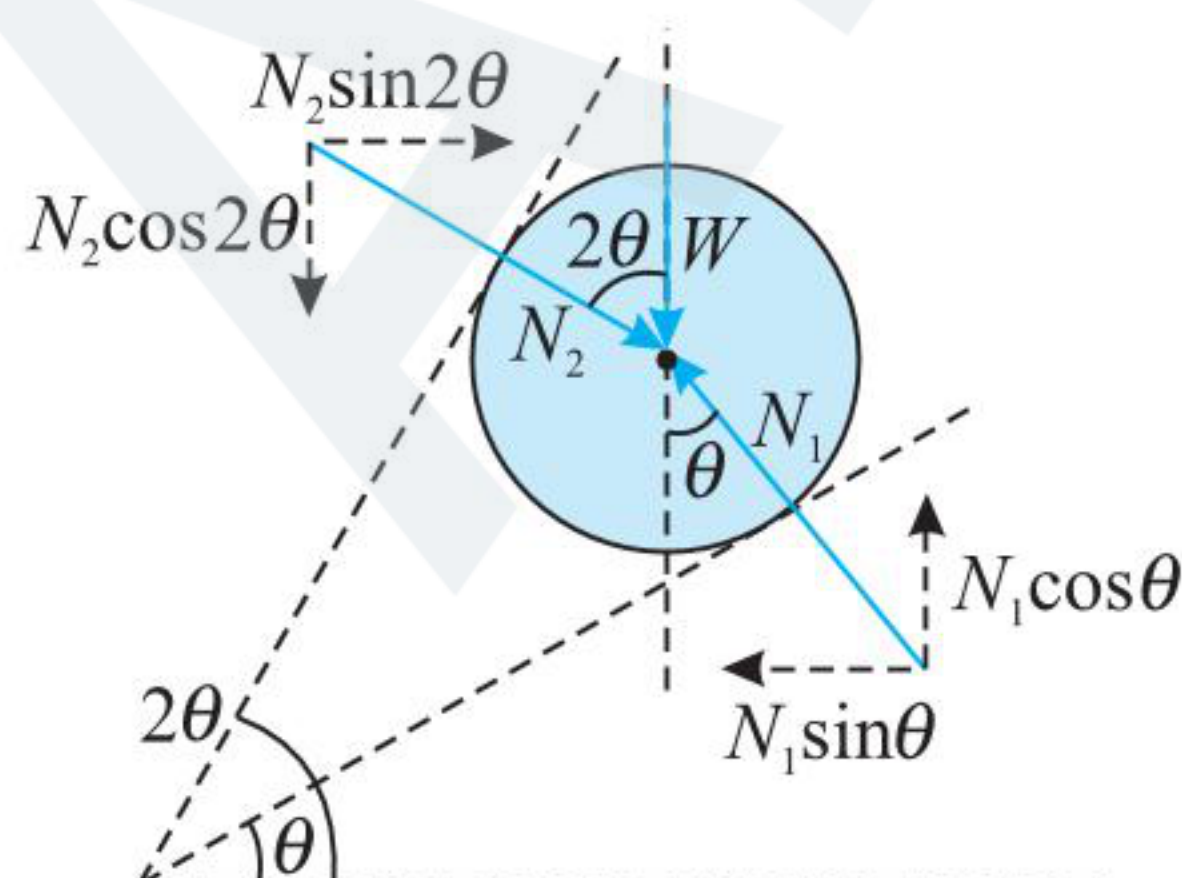
$$\Rightarrow V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u$$

$$65. (1) v_2 \cos \alpha + v_1 \cos \alpha = v_1 \Rightarrow v_2 = v_1 \left[\frac{2 \sin^2(\alpha/2)}{\cos \alpha} \right]$$

$$66. (3) \text{ In horizontal direction: } N_1 \sin \theta = N_2 \sin 2\theta$$

$$N_1 = N_2 \cos \theta$$

$$\frac{N_1}{N_2} = \cos \theta$$

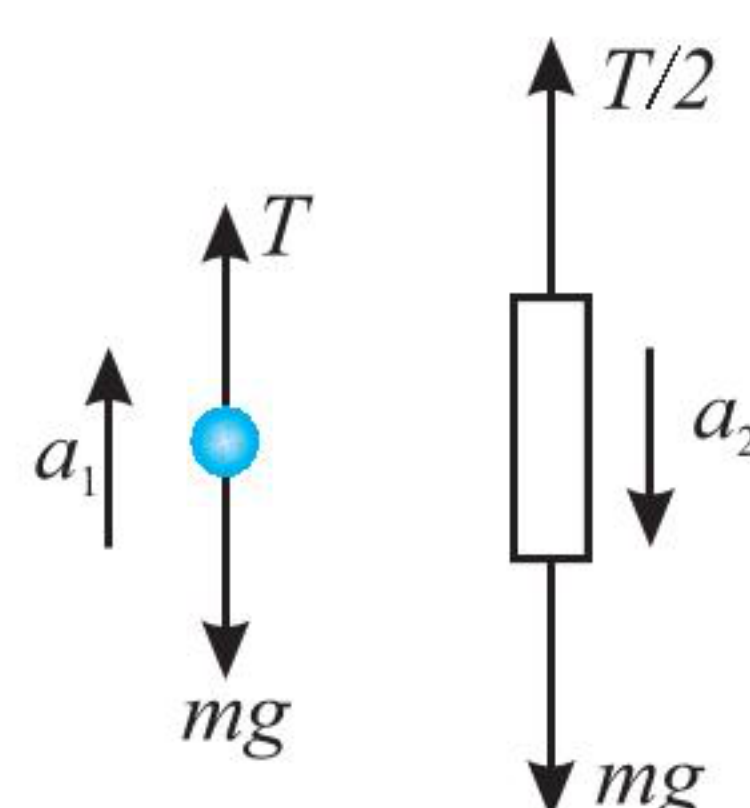


$$67. (1) a_2 = 2a_1 \quad \dots(i)$$

$$T - mg = ma_1 \quad \dots(ii)$$

$$mg - \frac{T}{2} = ma_2 \quad \dots(iii)$$

By solving (i), (ii) & (iii), we get



$$a_1 = \left(\frac{g}{5} \right) \text{ m/s}^2, a_2 = \left(\frac{2g}{5} \right) \text{ m/s}^2$$

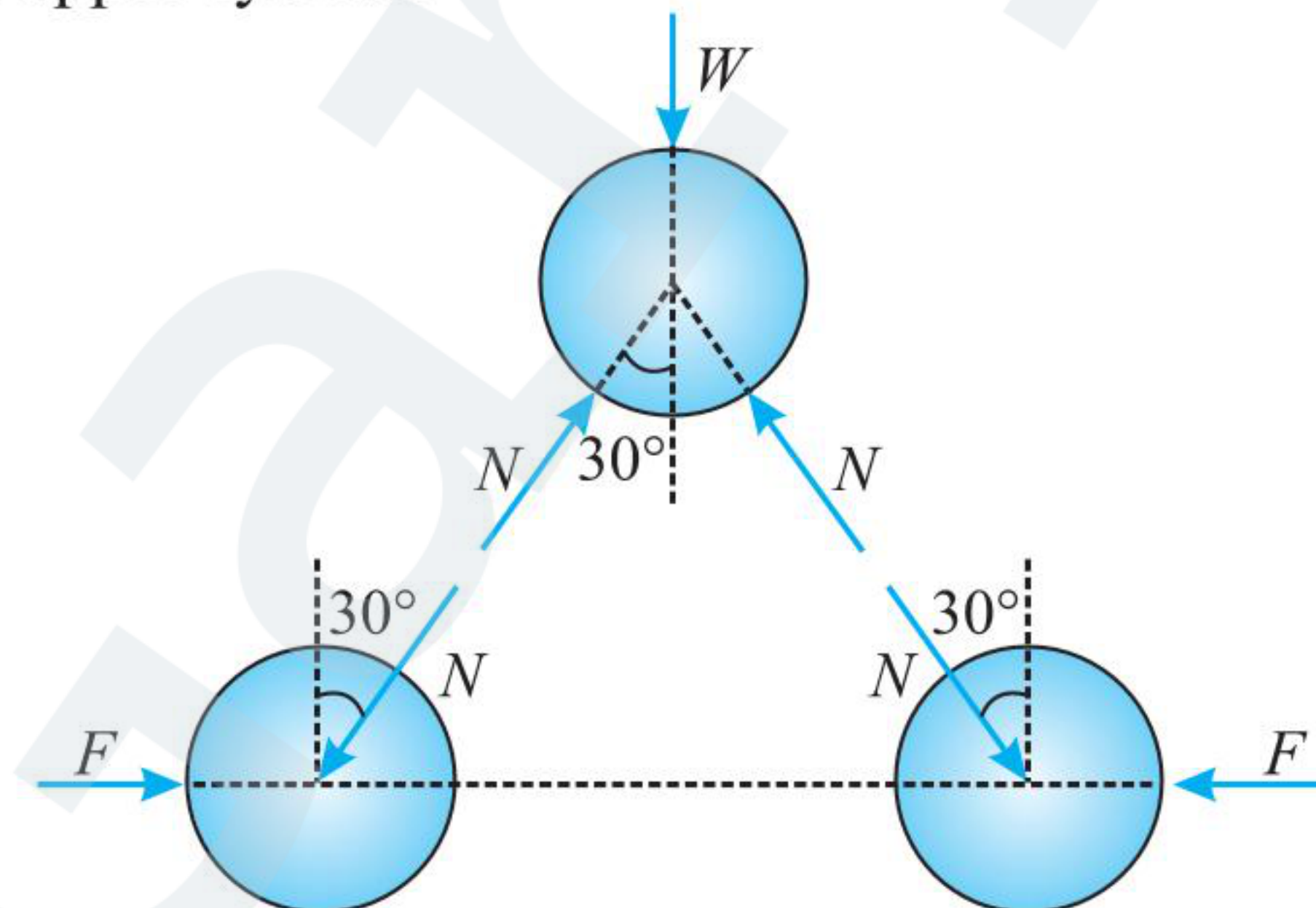
$$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = \frac{g}{5} - \left(-\frac{2g}{5} \right) = \frac{3g}{5}$$

$$\vec{s}_{12} = \vec{u}_{12} t + \frac{1}{2} \vec{a}_{12} t^2 \text{ and } \vec{s}_{12} = l$$

$$\therefore l = 0 + \frac{1}{2} \times \frac{3}{5} g t^2$$

$$\Rightarrow t = \sqrt{\frac{10l}{3g}}$$

68. (4) For upper cylinder



$$2N \cos 30^\circ = W \quad \dots(i)$$

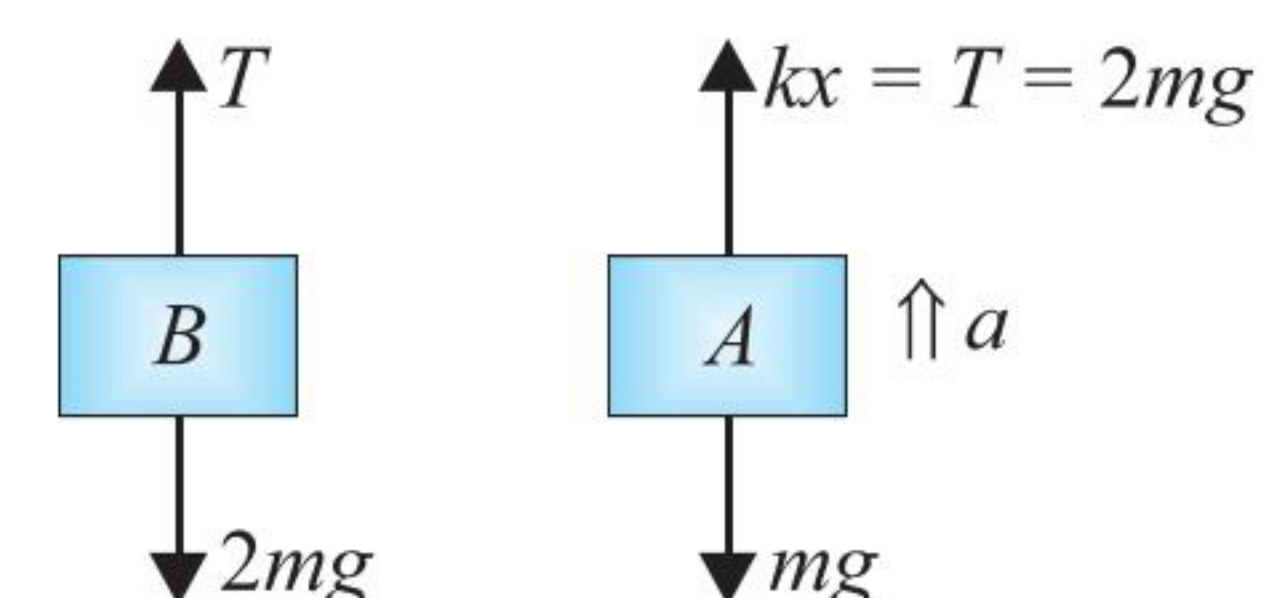
For lower block

$$N \cos 60^\circ = F \quad \dots(ii)$$

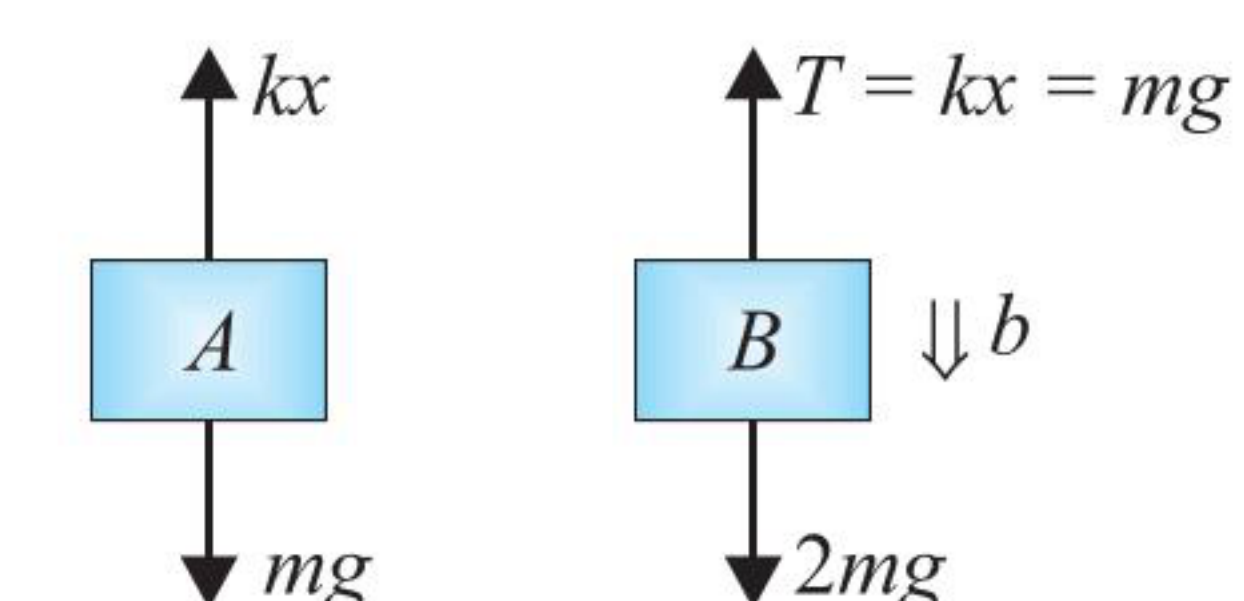
From (i) & (ii)

$$\frac{2 \cos 30^\circ}{\cos 60^\circ} = \frac{W}{F} \Rightarrow F = \frac{W}{2\sqrt{3}}$$

69. (1) In 1st case



In 2nd case

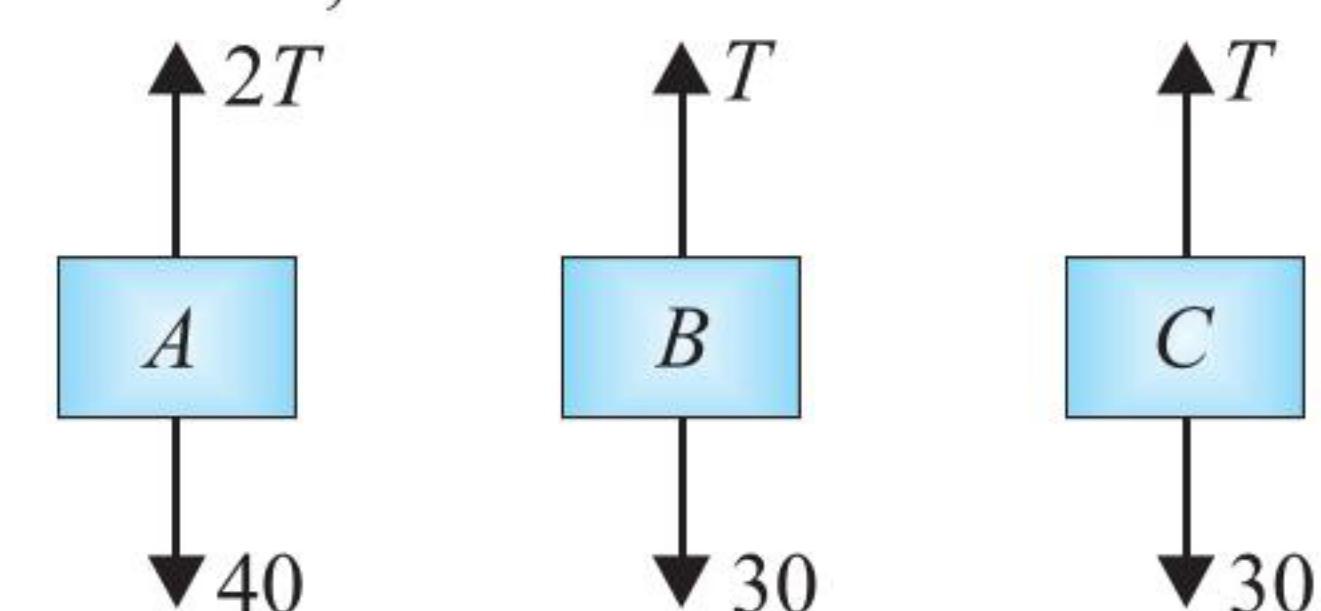


$$T = 2mg \text{ \& } a = \frac{2mg - mg}{m} \Rightarrow a = g \uparrow$$

$$kx = mg \text{ \& } b = \frac{2mg - mg}{2m} \Rightarrow b = \frac{g}{2} \uparrow$$

$$\therefore \frac{a}{b} = 2$$

70. (2) FBD of the blocks A, B and C



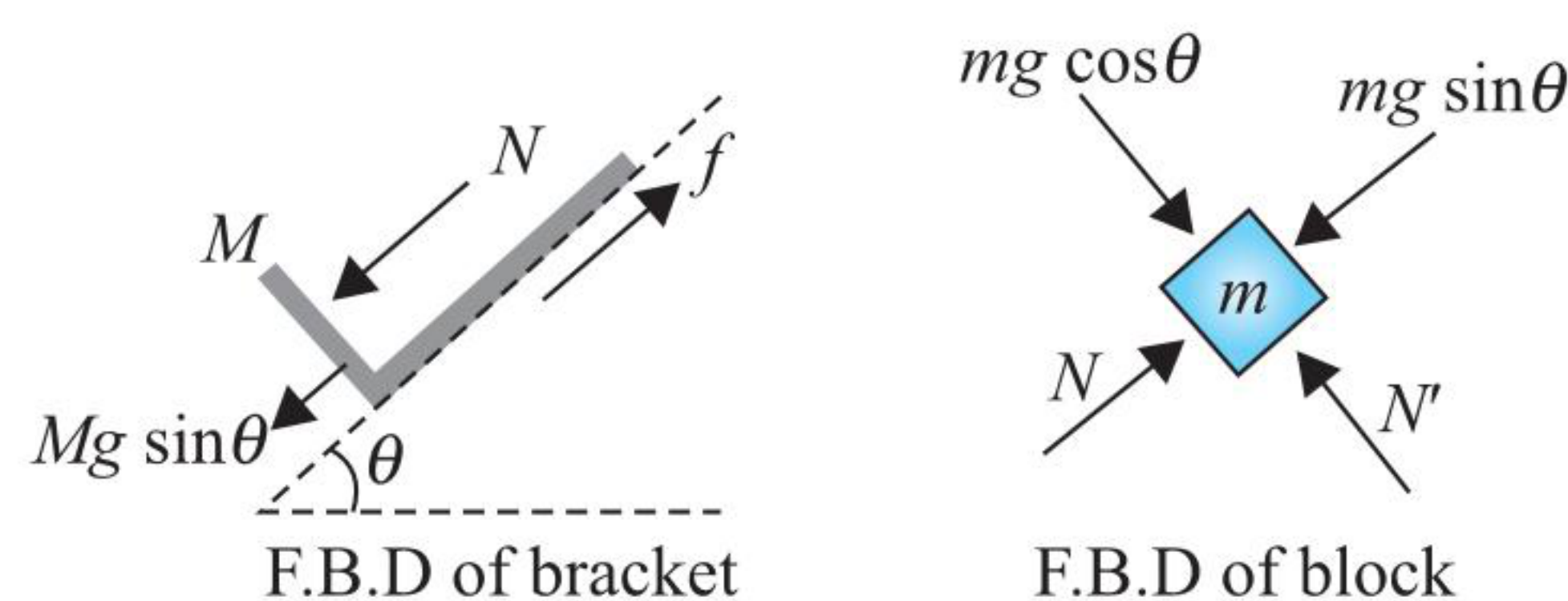
Conditions of forces on B & C are same so they both will move with same downward acceleration 'a'. The block A moves with the same acceleration a upwards.

$$\text{From block A: } 2T - 40 = 4a \quad \dots(i)$$

$$\text{From block B or C: } 30 - T = 3a \quad \dots(ii)$$

$$\text{From above two equations } a = 2 \text{ m/s}^2$$

71. (4)



$$N' = mg \cos \theta$$

$$\Rightarrow R = \sqrt{N^2 + N'^2} = \sqrt{(mg \sin \theta)^2 + (mg \cos \theta)^2} = mg$$

72. (4) $N = mg - m(\alpha t)$

$$N = (-m\alpha)t + mg$$

73. (2) Mass of block A: $m_A = M$

$$\text{Mass of block B; } m_B = M - m$$

Acceleration of B after the insect falls,

$$a_B = \left(\frac{m_A - m_B}{m_A + m_B} \right) g (\uparrow) = \frac{mg}{2M - m}$$

Acceleration of the insect = $g(\downarrow)$

The two objects separate with a relative acceleration of

$$a = g + \frac{mg}{2M - m} = \frac{2Mg}{2M - m}$$

$$\therefore \frac{1}{2}at^2 = L \quad \left(\frac{Mg}{2M - m} \right) t^2 = L$$

$$\Rightarrow t = \sqrt{\frac{(2M - m)L}{Mg}}$$

74. (1) Acceleration of the two blocks,

$$a = \frac{mg \sin 60^\circ - mg \sin 30^\circ}{3m} = \left(\frac{\sqrt{3} - 1}{4} \right) g$$

If the two blocks move a distance x along respective inclines in

$$\text{time } t, \quad x = \frac{1}{2}at^2$$

The centre of two blocks will be at equal height if

$$x \sin 60^\circ + x \sin 30^\circ = h$$

$$\therefore x \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) = h \Rightarrow \frac{1}{2} \left(\frac{\sqrt{3} - 1}{4} \right) gt^2 \left(\frac{\sqrt{3} + 1}{2} \right) = h$$

$$\therefore t^2 = \frac{16h}{(\sqrt{3} - 1)(\sqrt{3} + 1)g} = \frac{8h}{g}$$

$$\Rightarrow t = 2\sqrt{2} \sqrt{\frac{h}{g}}$$

75. (4) For pulley $2F = ma$

$$\text{For particle } F = ma'$$

$$\text{Hence, } a = 2a' = 2F/m$$

Acceleration of the point of application of the force is

$$= 2a + a' = \frac{5a}{2} = \frac{5F}{m}$$

Multiple Correct Answers Type

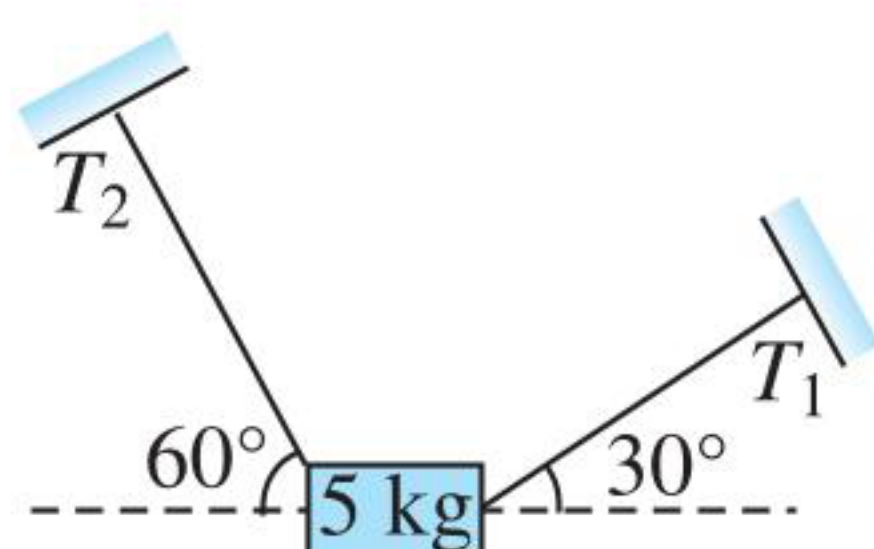
1. (1), (4)

$$T_2 \cos 60^\circ = T_1 \cos 30^\circ \quad \dots(i)$$

$$\text{and } T_2 \sin 60^\circ + T_1 \sin 30^\circ = 5g \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$T_1 = 25 \text{ N and } T_2 = 25\sqrt{3} \text{ N}$$



2. (2), (3), (4)

Acceleration of particle w.r.t. frame S_1 : $\vec{a}_p - \vec{a}_{s_1} = 2\hat{n}$ Acceleration of particle w.r.t. frame S_2 : $\vec{a}_p - \vec{a}_{s_2} = 2\hat{m}$ where $\hat{m}$ and $\hat{n}$ are unit vectors in any directions. Now relative acceleration of frames:

$$\vec{a}_{s_2} - \vec{a}_{s_1} = 2(\hat{n} - \hat{m})$$

Its magnitude can have any value between 0 to 4 ms^{-2} , depending upon the directions of $\hat{m}$ and $\hat{n}$.

3. (1), (3)

$$a_1 = \frac{2mg - mg}{m} = g$$

$$a_2 = \frac{mg + mg - mg}{2m} = g/2$$

$$a_3 = \frac{2mg - mg}{3m} = g/3$$

$$\text{Clearly } a_1 > a_2 > a_3$$

4. (1), (3), (4)

In first case, m will remain at rest. $a_M = F/M$

In second case, both will accelerate

$$a_m = a_M = F/(M + m)$$

In second case, force on $m = ma_m = mF/(M + m)$

5. (1), (3)

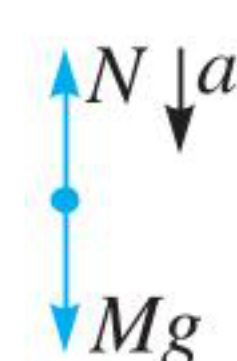
$$Mg - T = Ma \quad \dots(i)$$

$$T = ma \quad \dots(ii)$$

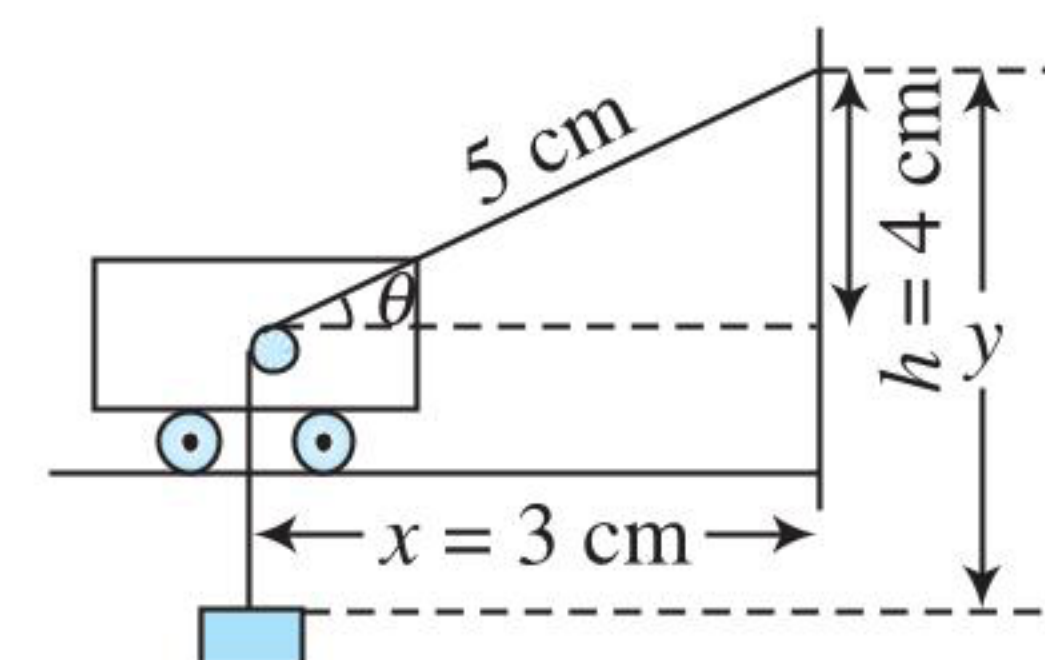
$$\text{Solving Eqs. (i) and (ii), } a = \frac{Mg}{M + m}$$

FBD of man,

$$Mg - N = Ma \Rightarrow N = \frac{Mmg}{(M + m)}$$



6. (3), (4)



$$(y - h) + \sqrt{x^2 + h^2} = l \quad \text{or} \quad \frac{dy}{dt} + \frac{x}{\sqrt{x^2 + h^2}} \frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{\sqrt{x^2 + h^2}} \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{3}{5}(-v_A)$$

$$v_B = \frac{3}{5}v_A \quad \dots(i)$$

$$\frac{d^2y}{dt^2} = \frac{v_A^2 h^2}{(x^2 + h^2)^{3/2}} \Rightarrow a_B = v_A^2 \frac{16}{(5)^3} = \frac{16}{125}v_A^2 \quad \dots(ii)$$

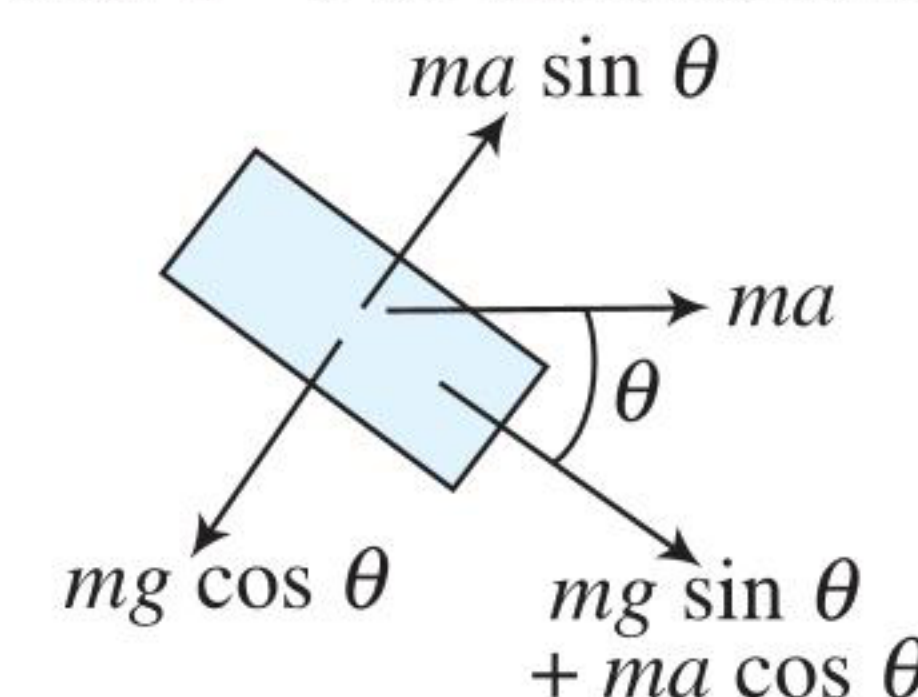
7. (1), (3)

$$M'g - T = M'a \quad \dots(i)$$

$$T = Ma \quad \dots(ii)$$

$$M'g = a(M + M')$$

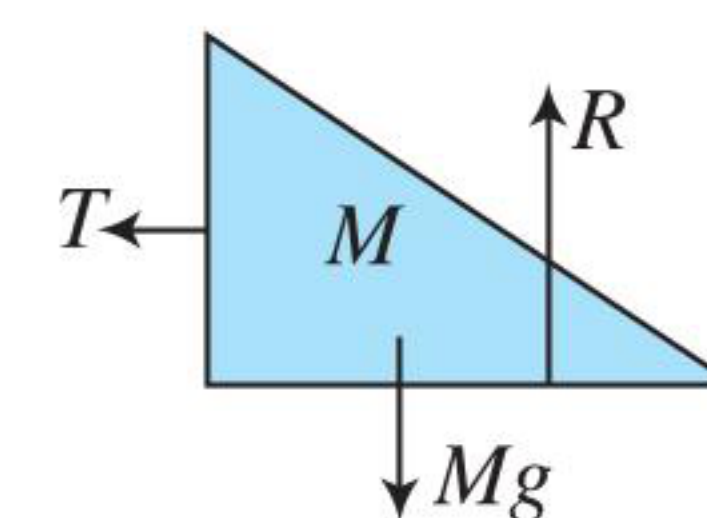
$$\Rightarrow a = \frac{M'g}{(M + M')}$$

 $ma \sin \theta = mg \cos \theta \rightarrow$ so that normal force is zero.

$$a = g \cot \theta$$

$$g \cot \theta = \frac{M'g}{(M + M')} \Rightarrow \cot \theta M + \cot \theta M' = M'$$

$$\text{or } M' = \frac{M \cot \theta}{(1 - \cot \theta)}, \quad T = Ma = Mg \cot \theta = Mg/\tan \theta$$



8. (1), (4)

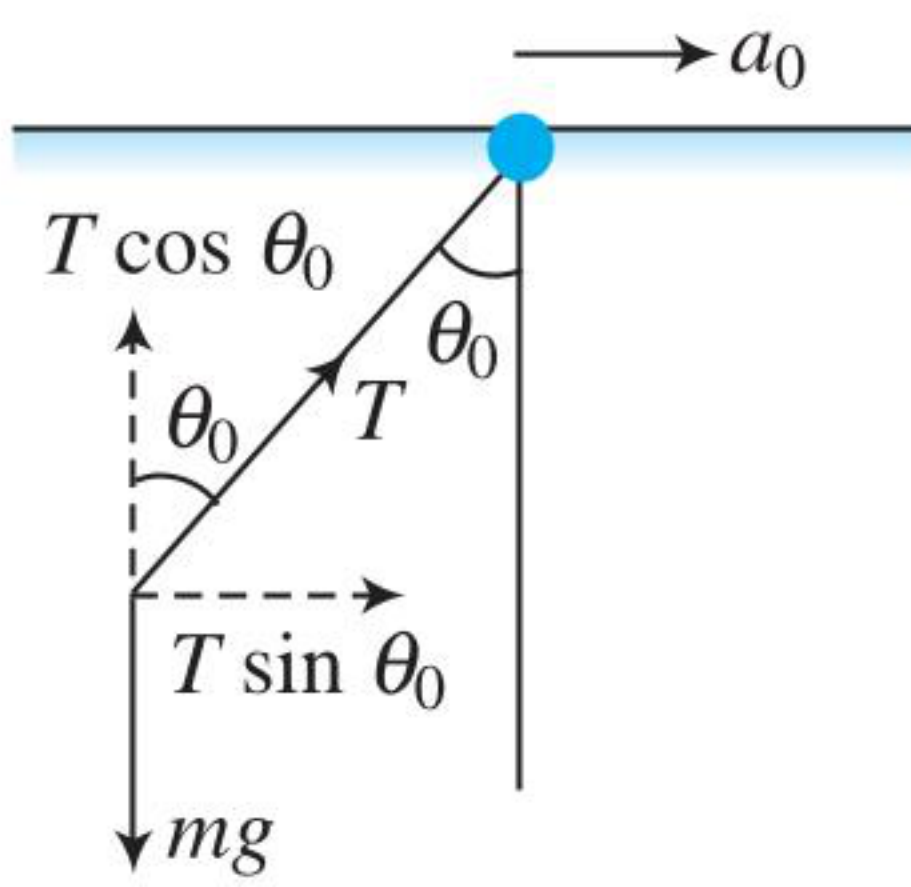
$$T \cos \theta_0 = mg \quad \dots(i)$$

$$T \sin \theta_0 = ma_0 \quad \dots(ii)$$

Dividing Eq. (ii) by (i), we get

$$\tan \theta_0 = \frac{a}{g} \Rightarrow \theta_0 = 30^\circ$$

$$T = \frac{mg}{\cos 30^\circ} = \frac{2mg}{\sqrt{3}}$$

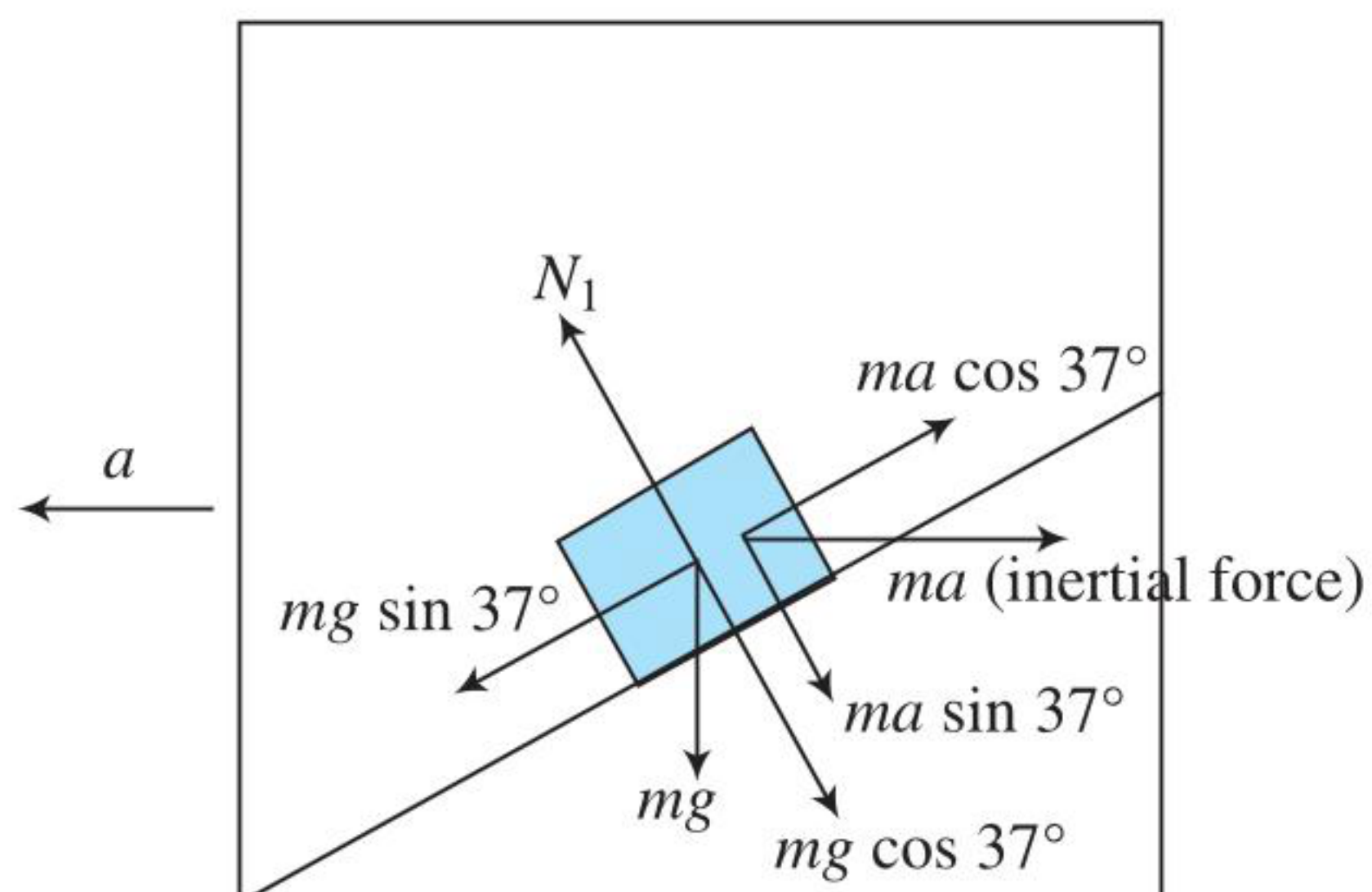


9. (1), (4)

$$\text{Acceleration of } M, a = \left(\frac{F}{M} \right)$$

$$l = \frac{1}{2} \frac{F}{M} t^2 \Rightarrow t = \sqrt{\frac{2Ml}{F}}$$

10. (1), (3)

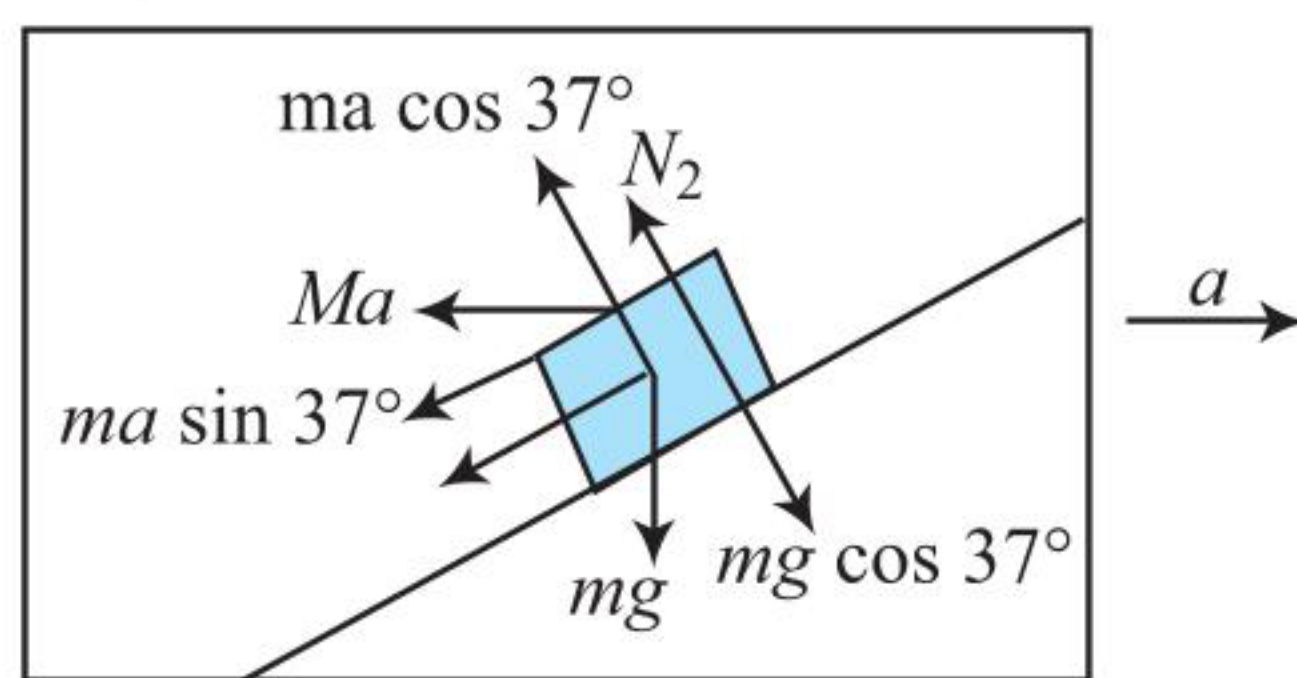


Balancing forces perpendicular to incline

$$N_1 = mg \cos 37^\circ + ma \sin 37^\circ$$

$$N_1 = \frac{4}{5} mg + \frac{3}{5} ma$$

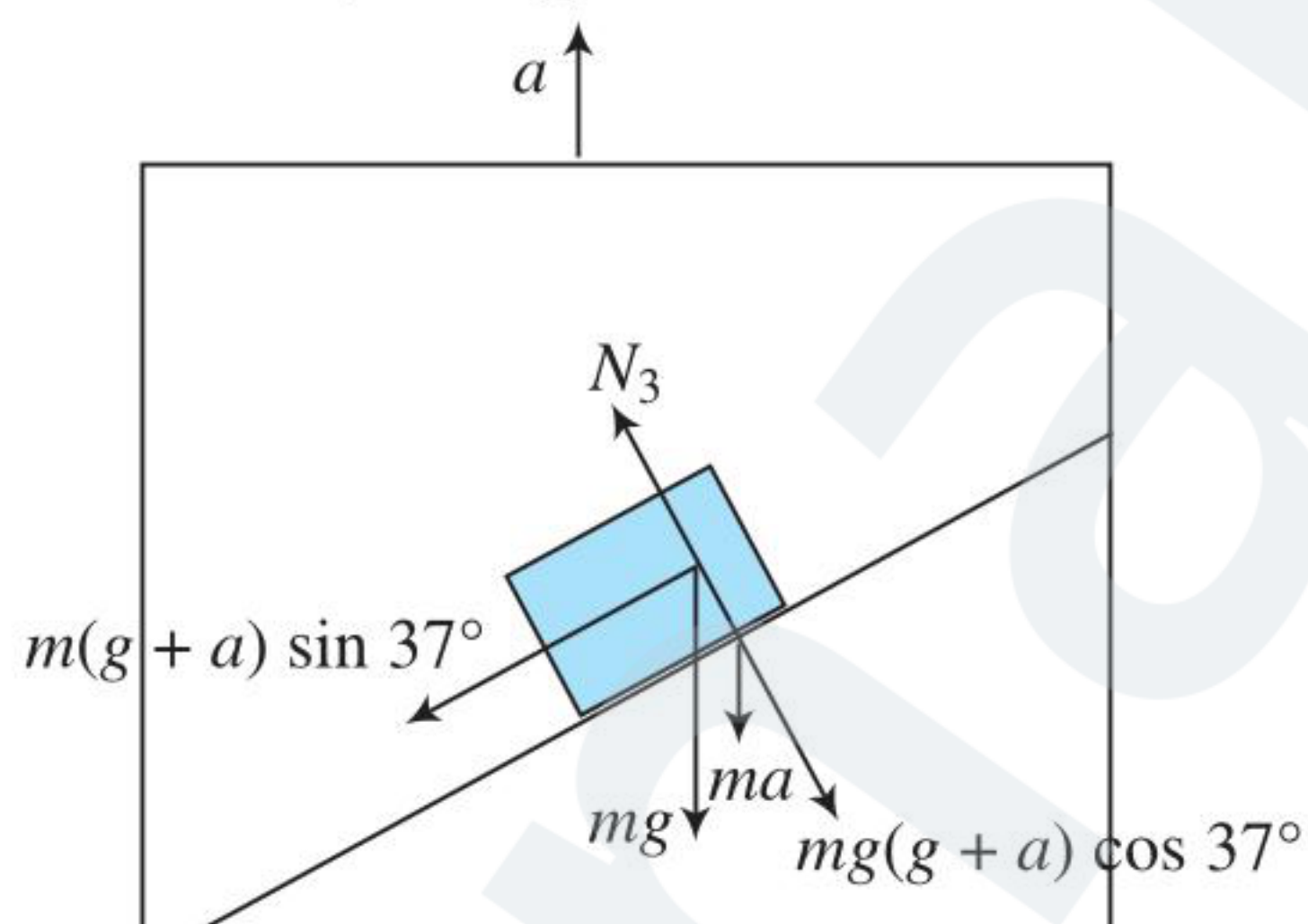
And along incline,



$$mg \sin 37^\circ - ma \cos 37^\circ = mb_1$$

$$b_1 = \frac{3}{5} g - \frac{4}{5} a$$

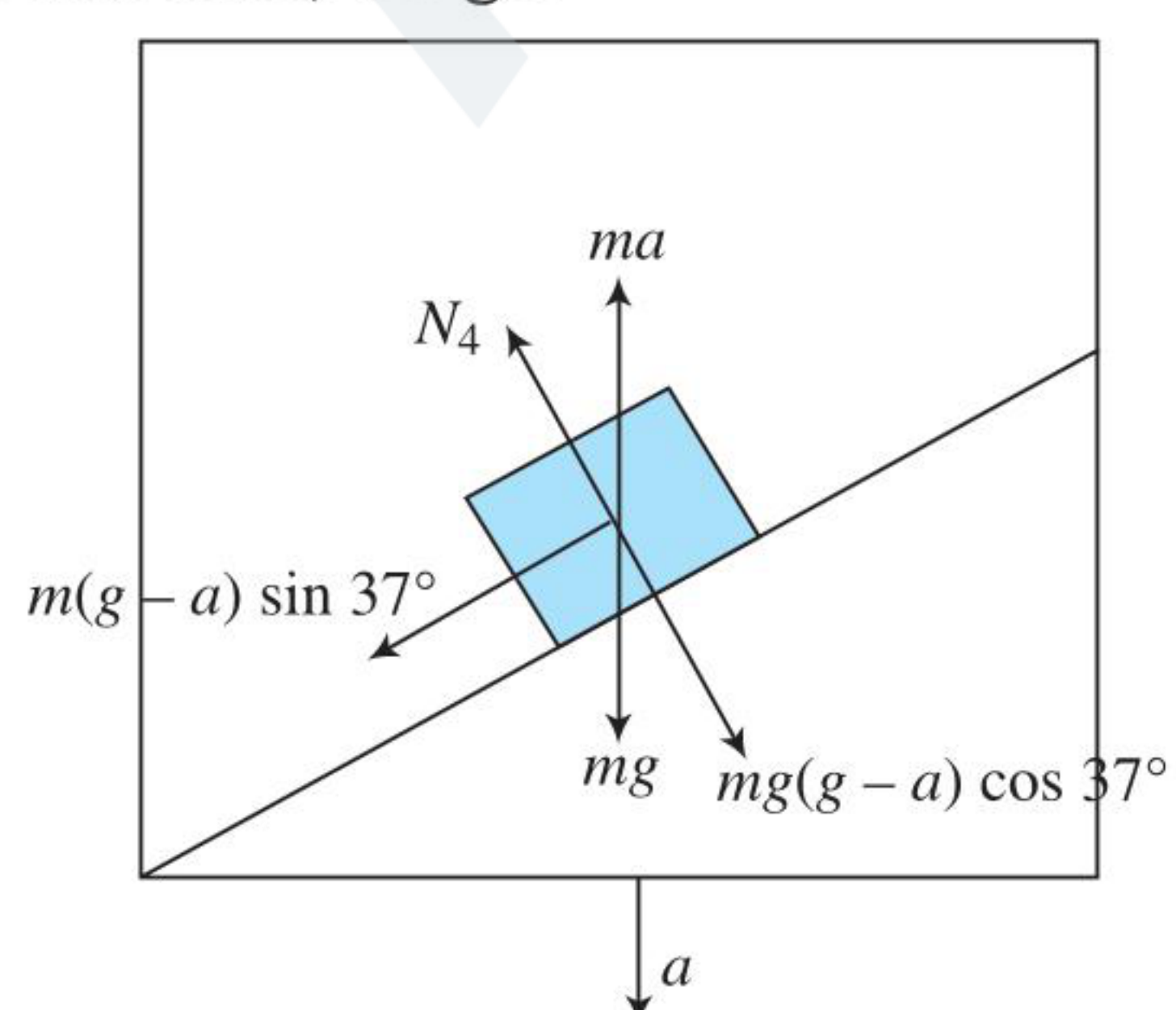
Similarly, for this case, we get



$$N_2 = \frac{4}{5} mg - \frac{3}{5} ma$$

$$\text{And } b_2 = \frac{3}{5} g + \frac{4}{5} a$$

Similarly, for this case, we get



$$N_3 = \frac{4}{5} mg + \frac{4}{5} ma$$

$$b_3 = \frac{3}{5} g + \frac{3}{5} a$$

Similarly, for this case, we get

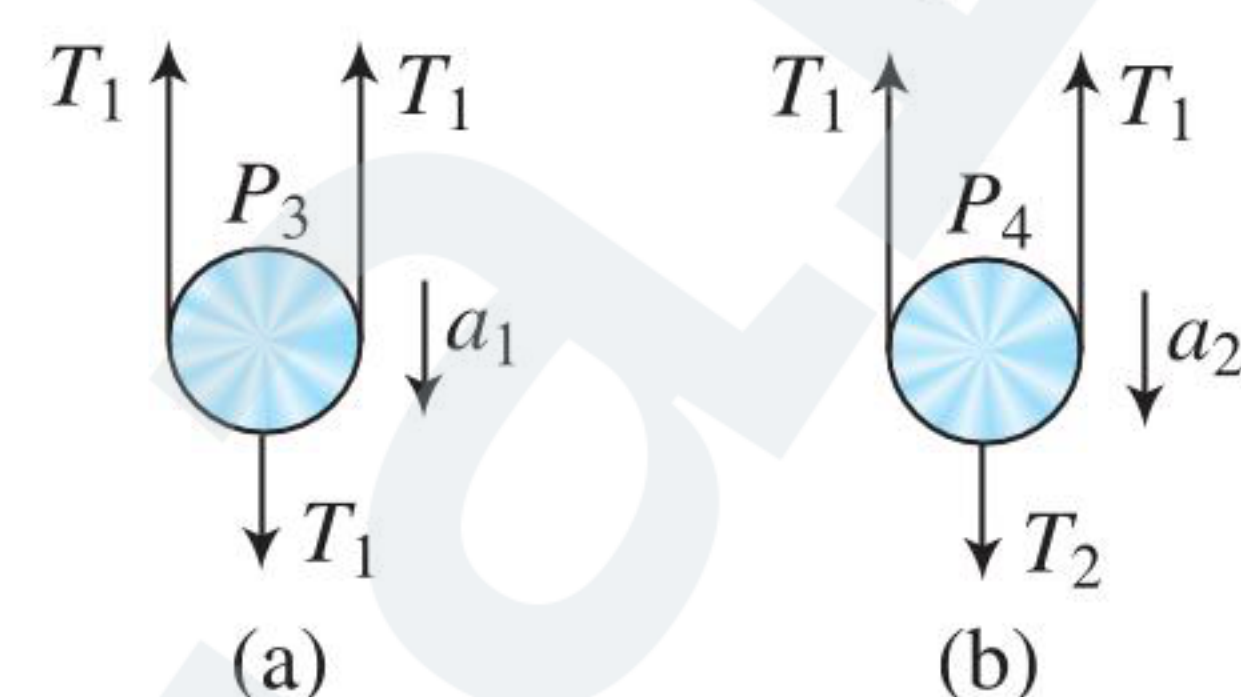
$$N_4 = \frac{4}{5} mg - \frac{4}{5} ma$$

$$\text{and } b_4 = \frac{3}{5} g - \frac{3}{5} a$$

11. (1), (3)

First of all, draw FBD of P_3 . Let tensions, in three strings be T_1 , T_2 and T_3 , respectively.

$$2T_1 - T_1 = 0 \times a \Rightarrow T_1 = 0$$

Now draw FBD of P_4 and P_5 .

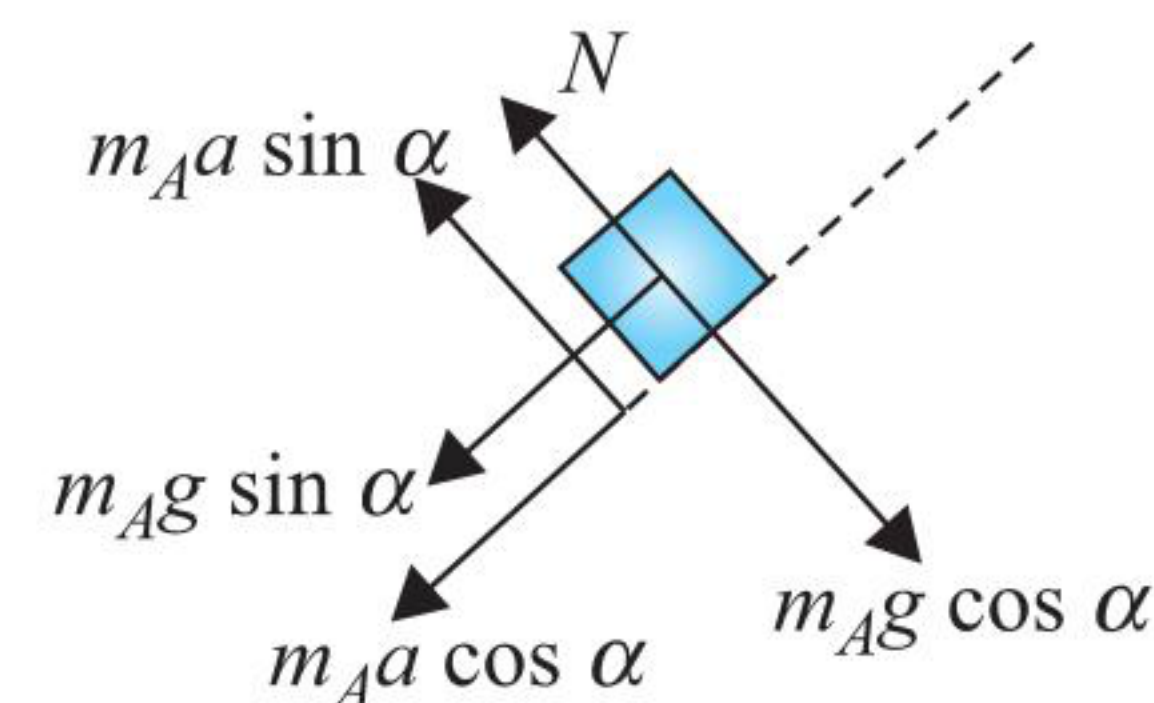
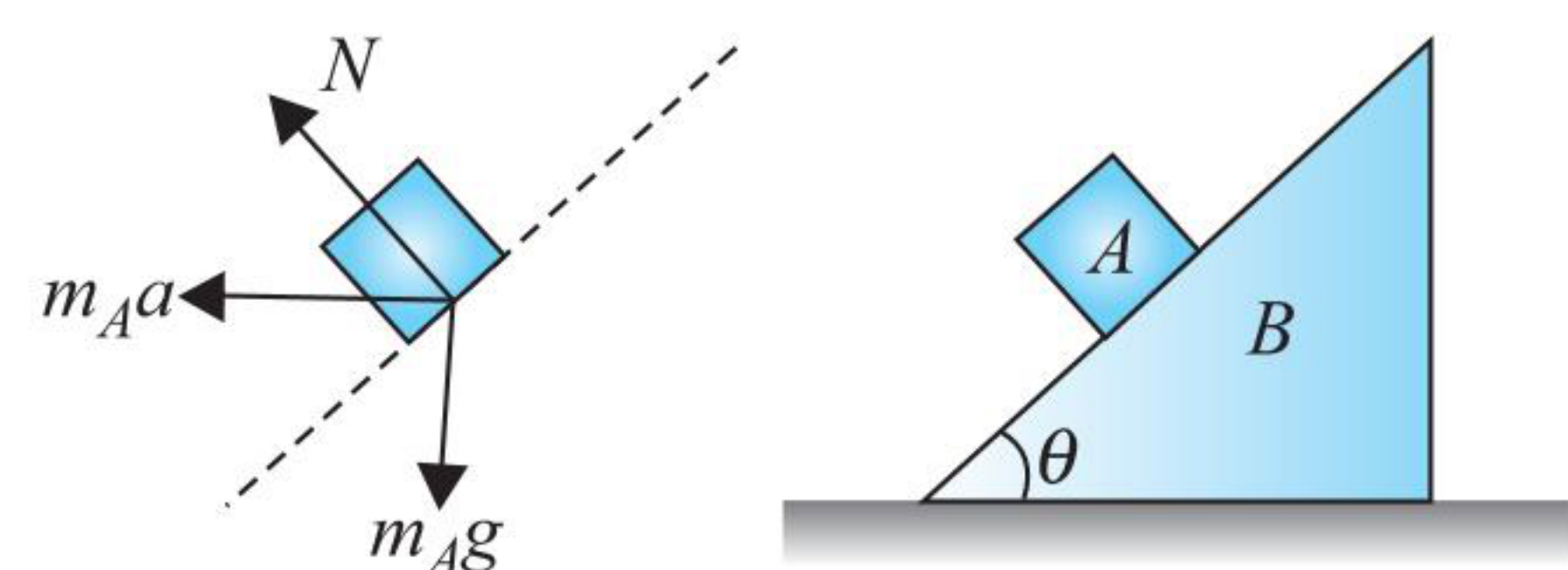
$$2T_1 - T_2 = 0 \Rightarrow T_2 = 0$$

$$2T_2 - T_3 = 0 \Rightarrow T_2 = T_3 = 0$$

So forces acting on P_6 and P_7 will be that of gravity and they will be in free fall. Hence, acceleration of each of them will be g downwards.

12. (1), (4)

$$\text{Force along inclined plane} = m_A g \sin \alpha + m_A a \cos \alpha$$



$$\text{Acceleration along inclined plane} = g \sin \alpha + a \cos \alpha$$

$$\text{So, } a_A > g \sin \alpha$$

$$N = m_A g \cos \alpha - m_A a \sin \alpha$$

$$N < m_A g \cos \alpha$$

13. (1), (2), (4)

Acceleration of 2 kg block

$$(a_2) = \frac{50 - 20}{2} = 15 \text{ m/s}^2 \uparrow$$

Acceleration of 4 kg block

$$(a_4) = \frac{50 - 40}{4} = 2.5 \text{ m/s}^2 \uparrow$$

14. (1), (3), (4)

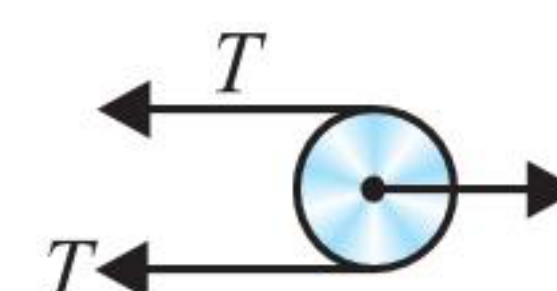
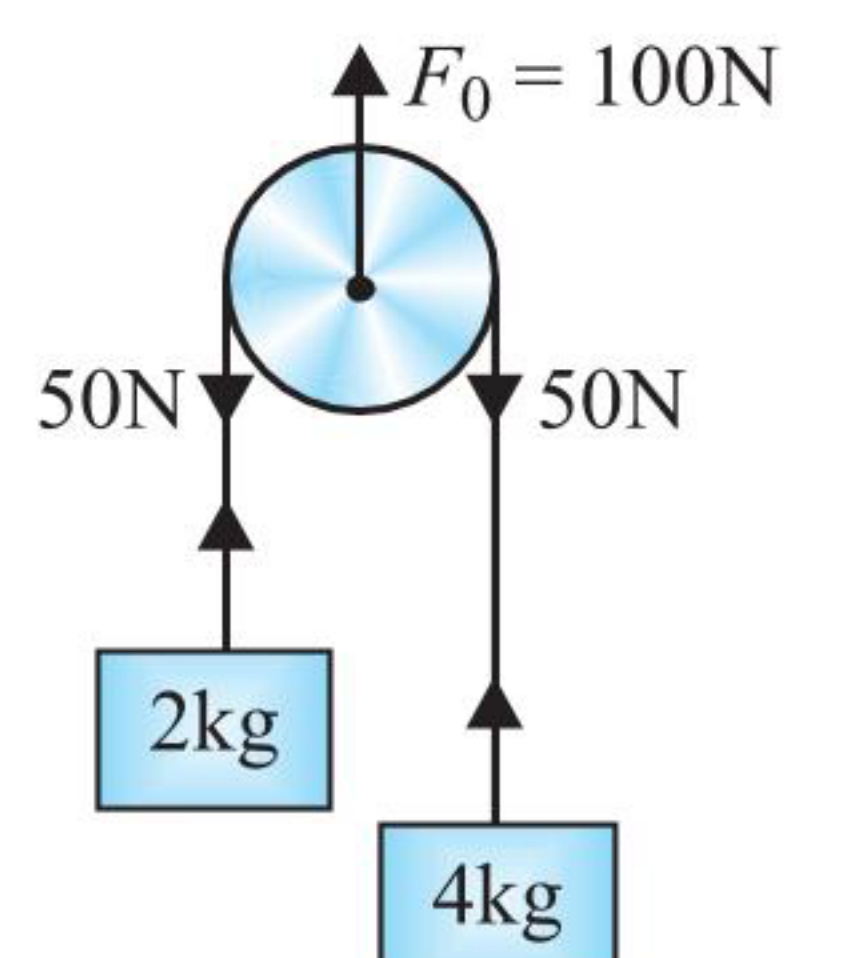
$$15 - T = 10a$$

$$T = 5a$$

$$a = 1 \text{ m/s}^2$$

$$T = 5 \text{ N}$$

$$2T = 10 \text{ N}$$



15. (1), (3), (4)

$$T = (m_B + m_C)a$$

$$T = 10a \quad \dots(i)$$

$$T = m_A a = m_A A$$

$$T + m_A (A - a) \quad \dots(ii)$$

$$T = 30(A - a)$$

$$\Rightarrow N = m_C a \quad \dots(iii)$$

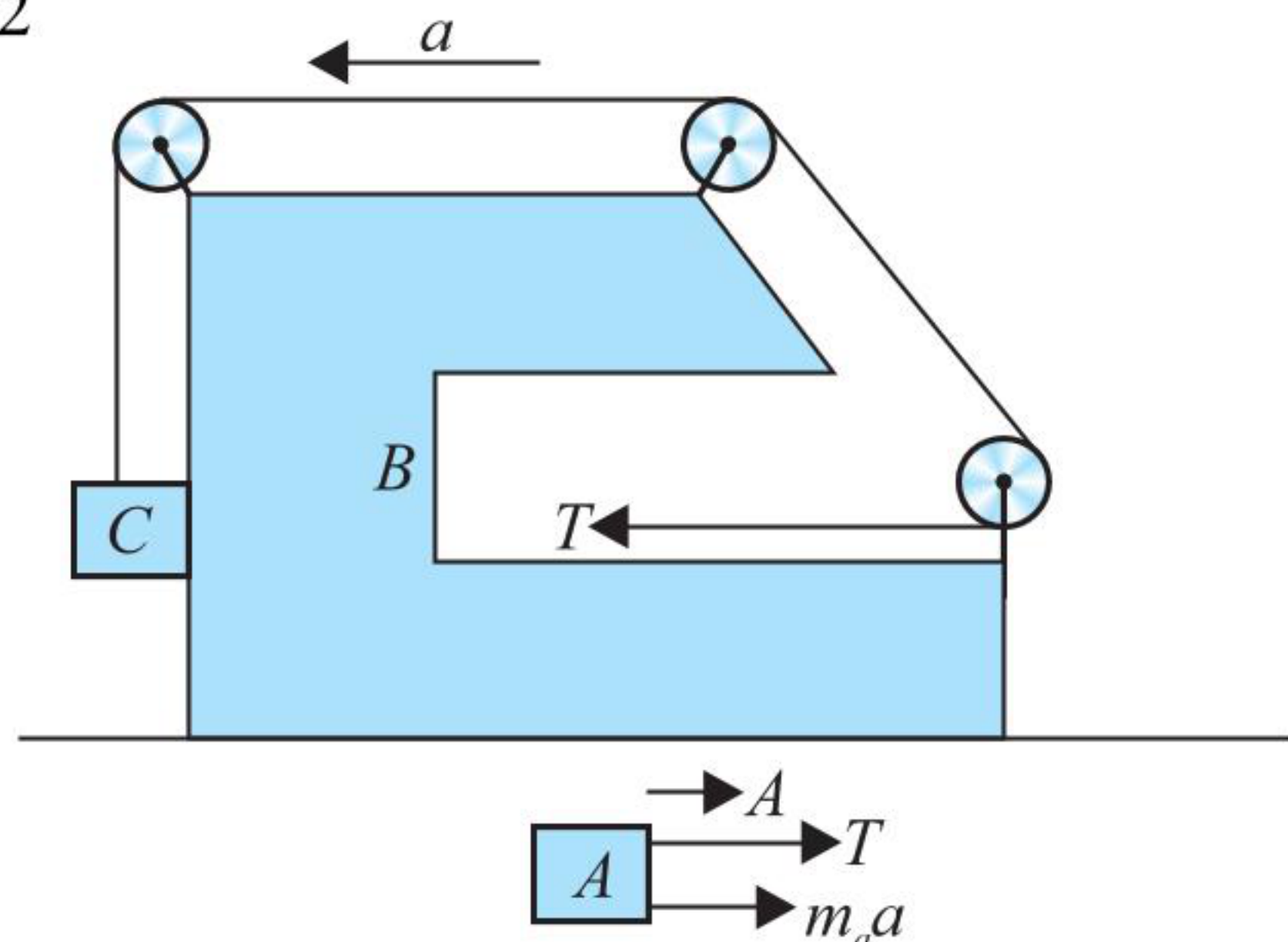
$$m_C g - T = m_C A \quad \dots(iv)$$

$$50 - T = 5A$$

$$A + 2a = 10 \text{ and } 10a = 30(A - a)$$

$$4a = 3A$$

$$A + \frac{3A}{2} = 10$$



$$A = 4, a = 3, T = 30 \text{ N}, N = 15 \text{ N}$$

Contact force between B and ground is 380 N

$$N_G = 5g + 30g + T = 380 \text{ N}$$

16. (1), (3), (4)

$$2T = 5g$$

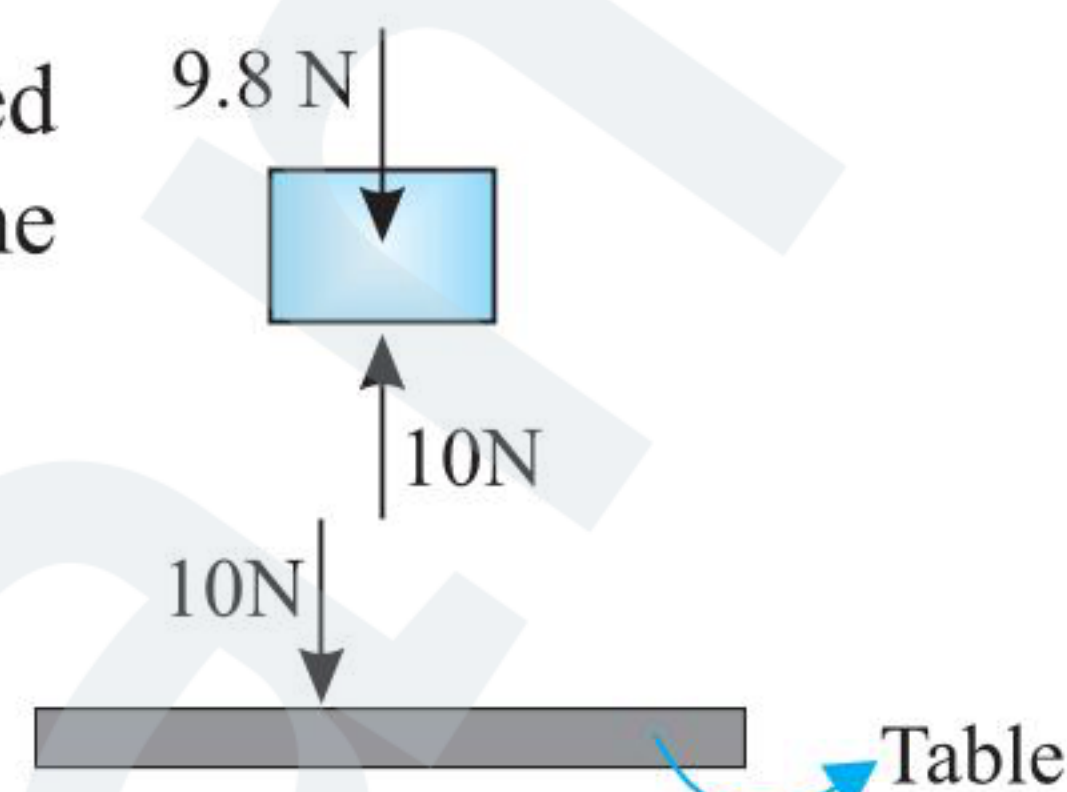
$$4T = 10g$$

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 2.5g$$

$$\Rightarrow \frac{m_1 m_2}{m_1 + m_2} = \frac{5}{4}g$$

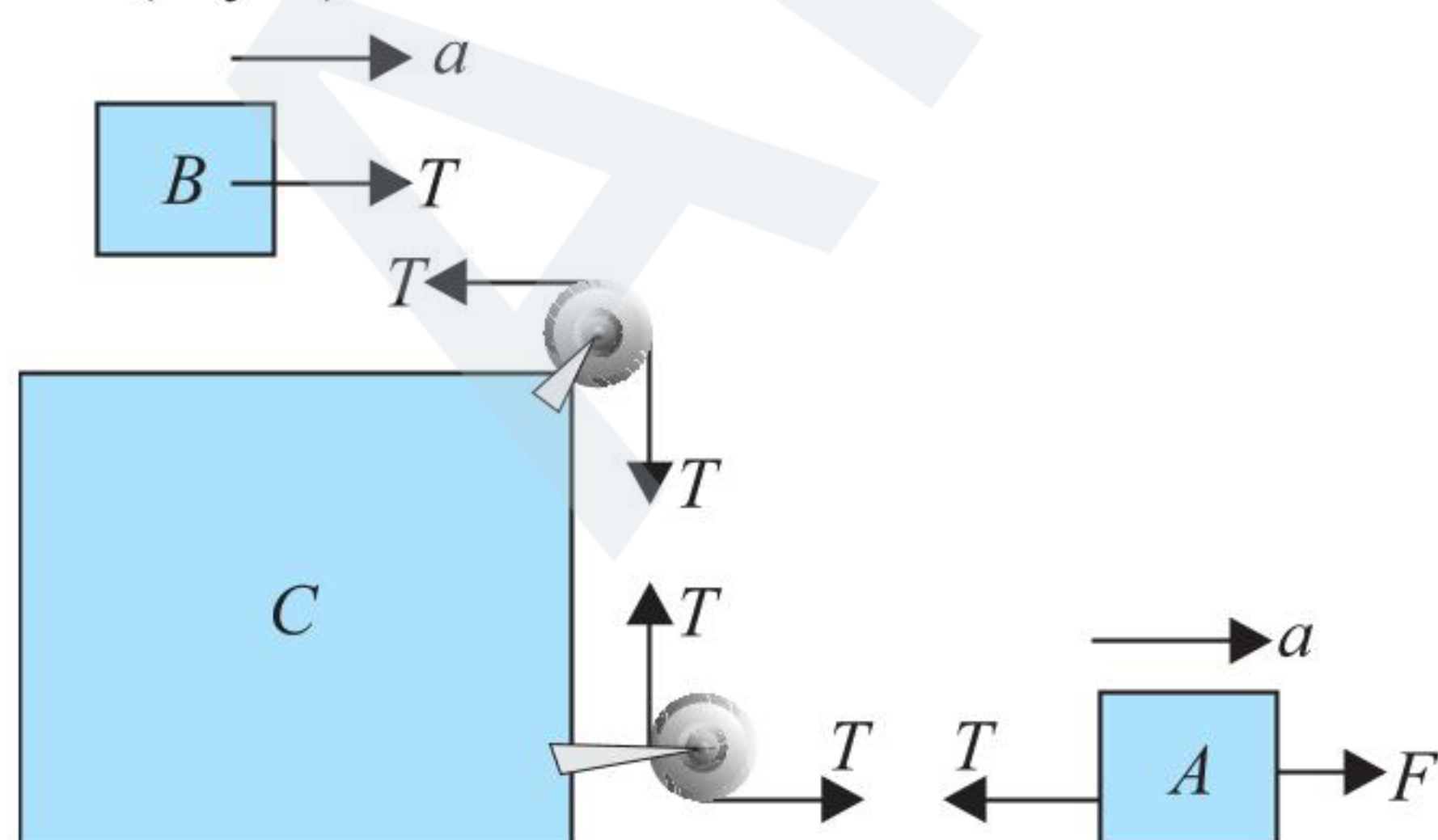
17. (1), (4)

Newton's third law states that force applied by one object on other is exactly of same magnitude that applied by other on first. As net force on the block is in upward direction hence the block accelerates in upward direction.



18. (2), (3)

From F.B.D, it is clear that no net force acts on the block C. Hence the acceleration of block C should be zero. The blocks A and B are connected directly by a string they will move with common acceleration (say a).



$$\text{For block B, } T = 10a \quad \dots(i)$$

$$\text{For block A, } F - T = 10a \quad \dots(ii)$$

$$\text{From (i) \& (ii), } a = \frac{F}{20} = \frac{20}{20} = 1 \text{ m/s}^2$$

$$\text{and } T = 10 \times 1 = 10 \text{ N}$$

19. (1), (2), (3), (4)

The acceleration of block A, w.r.t pulley P_1 ;

$$a_{AP_1} = (a_1 - a) \uparrow$$

$$a_{BP_1} = (a_B - a) \downarrow$$

The acceleration of block B, w.r.t pulley P_1 ;

$$\text{But, } |\vec{a}_{AP_1}| = -|\vec{a}_{BP_1}|$$

$$a_1 - a = a_B + a$$

$$\text{Hence, } a_B = 2a - a_1 \uparrow$$

The acceleration of block C w.r.t pulley P_2 ;

$$a_{CP_2} = (a_C + a) \uparrow$$

The acceleration of block D w.r.t pulley P_2 ;

$$a_{DP_2} = (a_D - a) \downarrow$$

$$a_C + a = a_D - a$$

$$a_D = (2a + a_1) \downarrow$$

$$\text{Hence, } \vec{a}_{BC} = \vec{a}_B - \vec{a}_C = 2(a_1 - a) \downarrow$$

$$\text{and } \vec{a}_{BD} = \vec{a}_B - \vec{a}_D = 4a \uparrow$$

20. (1), (3)

For equilibrium $T_1 = Mg$ and $T_2 = mg$

But $2T_2 = 2T_1$ (for equilibrium of pulley P_1)

$$\therefore m = M$$

When $m = 2M$, let acceleration of A be a ($\uparrow$).

Acceleration of pulley P_2 will be $\frac{a}{2}$ ($\downarrow$)

Acceleration of P_1 = acceleration of $P_2 = \frac{a}{2}$ ($\downarrow$)

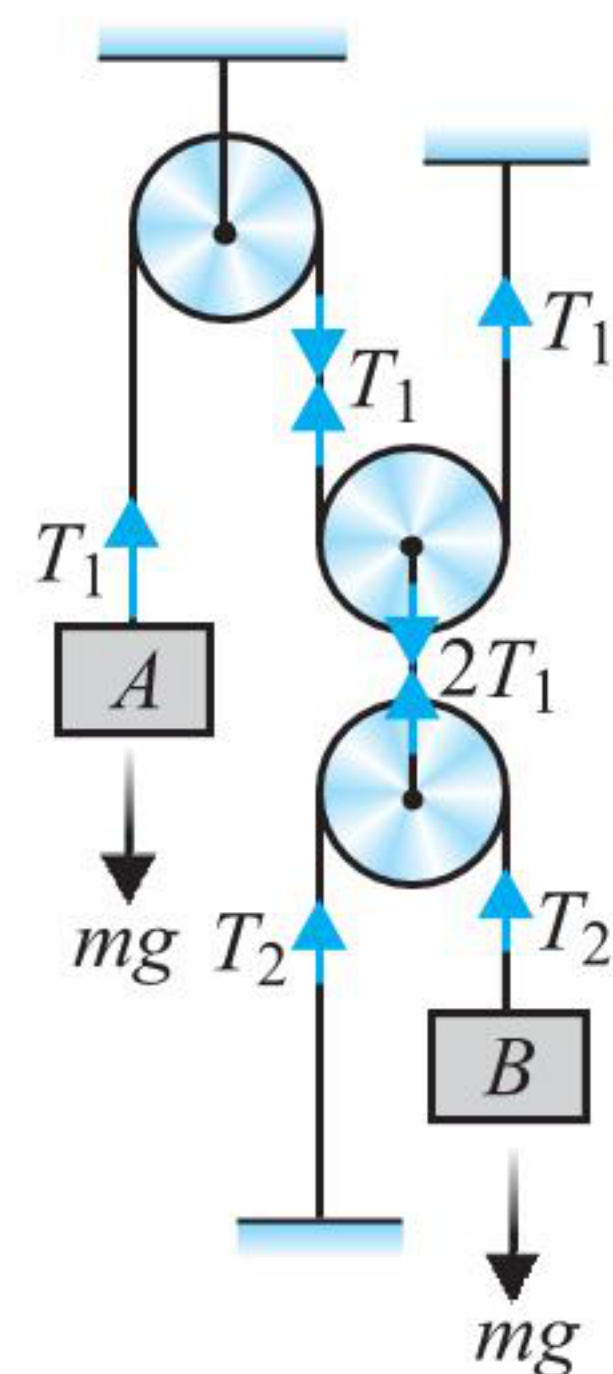
Acceleration of B = $2 \times$ acceleration of $P_2 = a$ ($\downarrow$)

For pulley P_1 ; $2T_1 = 2T_2 \Rightarrow T_1 = T_2$... (i)

For A; $T_1 - Mg = Ma$... (ii)

For B; $2Mg - T_2 = 2Ma$... (iii)

Solving (i), (ii) and (iii); $a = \frac{g}{3}$



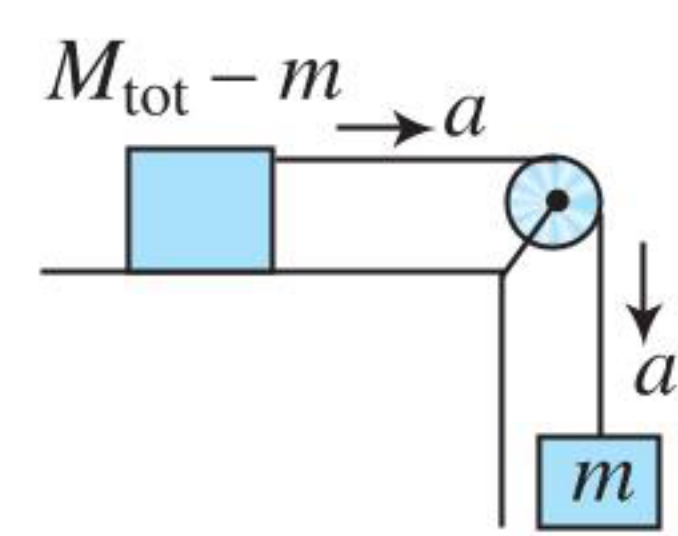
Linked Comprehension Type

For Problems 1 and 2

$$1. (1) \quad a = \left(\frac{m}{M_{\text{tot}}} \right) g$$

$$2. (4) \quad T = \frac{M_1 M_2 g}{M_1 + M_2} = \frac{m(M_{\text{tot}} - m)}{M_{\text{tot}}} g$$

$$= \left(\frac{mM_{\text{tot}} - m^2}{M_{\text{tot}}} \right) g$$



For Problems 3-5

3. (1) Let m_1 and m_2 do not accelerate up or down, then

$$F_1 = m_1 g, F_2 = m_2 g. \text{ But } m_1 \neq m_2, \text{ so } F_1 \neq F_2.$$

Hence net horizontal force on M is $F_1 - F_2$. So M cannot be in equilibrium. If M accelerates horizontally, then m_1 and m_2 also accelerate horizontally.

$$4. (3) \quad a = \frac{\text{Net force}}{\text{Net mass}} = \frac{F_1 - F_2}{M + m_1 + m_2} = \frac{m_1 g - m_2 g}{M + m_1 + m_2}$$

5. (4) In horizontal direction, both m_1 and m_2 will have same acceleration for any case. In vertical direction also, they should have same acceleration. Let it be a upwards for both, then

$$F_1 - m_1 g = m_1 a \quad \dots(i)$$

$$F_2 - m_2 g = m_2 a \quad \dots(ii)$$

From (i) and (ii), $\frac{F_1}{m_1} = \frac{F_2}{m_2}$

For Problems 6–8

6. (4) Initial velocity: $u = 0$

$$I = \int F dt \Rightarrow m(v - u) = \int_0^t (6t - 2t^2) dt$$

$$\Rightarrow 2v = 3t^2 - \frac{2}{3}t^3$$

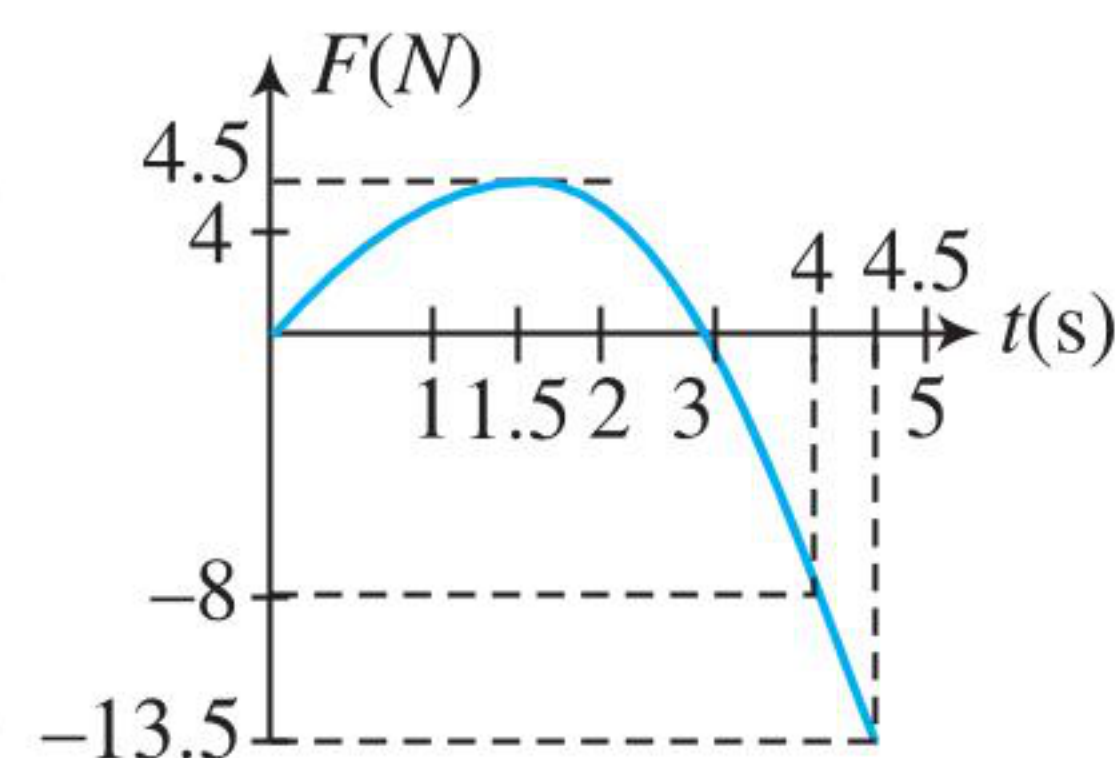
Put $v = 0$, we get $t = 4.5$ s

7. (2) If we draw $F-t$ graph, we can see that upto $t = 3$ s, force is positive, so velocity increase up to 3 s and after that velocity starts decreasing. So at 3 s, velocity is maximum.

8. (2) We can see from the figure above, when force becomes maximum (at $t = 1.5$ s) for the first time, velocity is not maximum at that instant. Hence, (1) is incorrect.

Option (2) is correct as explained in the above solution.

Option (3) is incorrect because when force becomes zero, velocity does not become zero.



For Problems 9–11

9. (3) For equilibrium of block A

$$F = N \sin \theta$$

$$N = F / \sin \theta$$

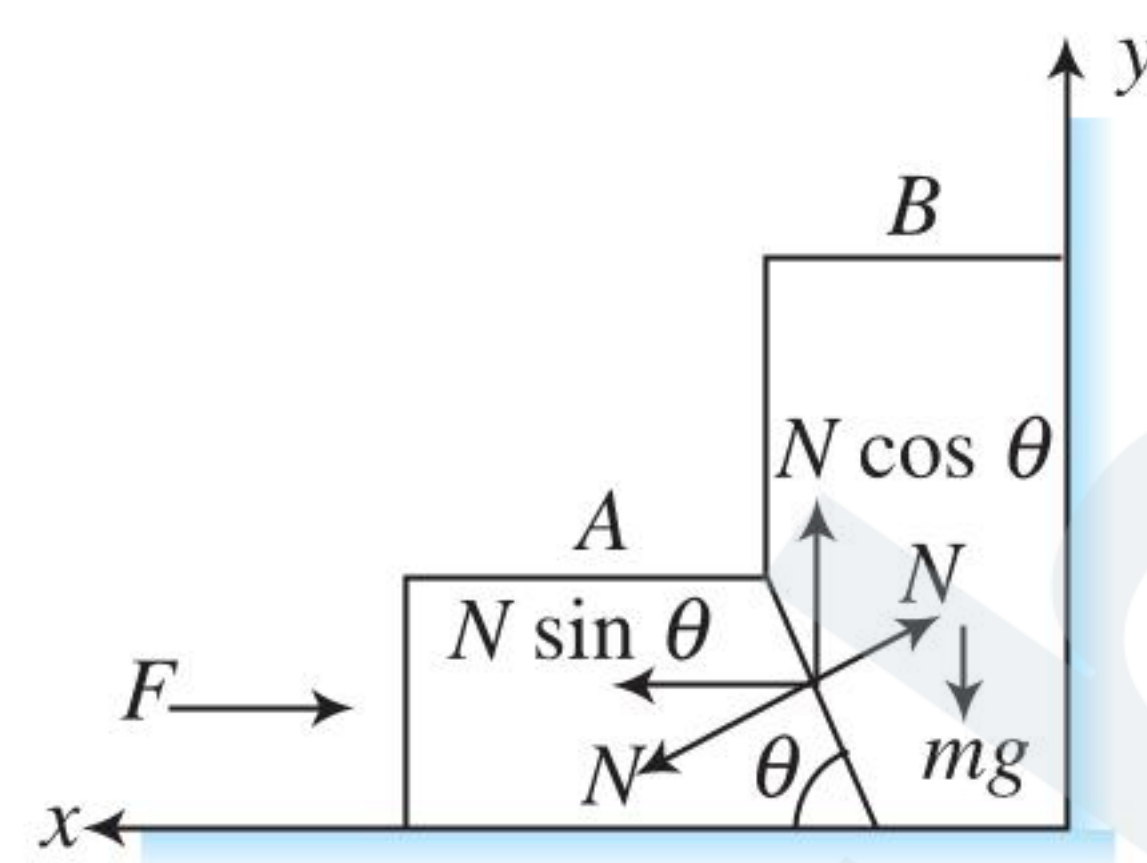
To lift block B from ground

$$N \cos \theta \geq mg$$

$$\Rightarrow \frac{F}{\sin \theta} \cos \theta \geq mg$$

$$F \geq mg \tan \theta = mg \left(\frac{3}{4} \right)$$

$$\text{So, } F_{\min} = \frac{3}{4} mg$$



10. (3) If both the blocks are stationary.

Balancing forces along x-direction

$$F = N \sin \theta$$

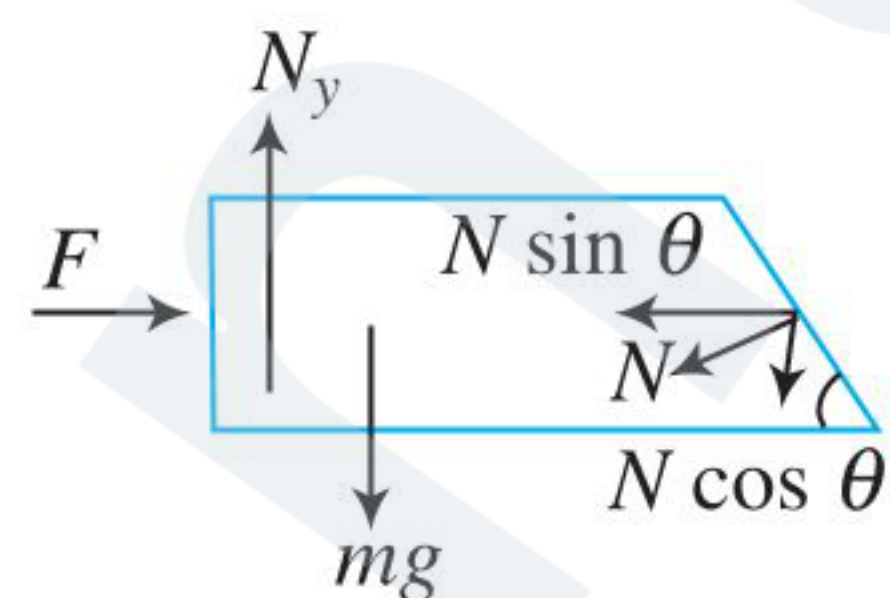
$$\Rightarrow N = F / \sin \theta$$

Balancing forces along y-direction

$$N_y = mg + N \cos \theta$$

$$= mg + \left(\frac{F}{\sin \theta} \right) \cos \theta = mg + F \cot \theta$$

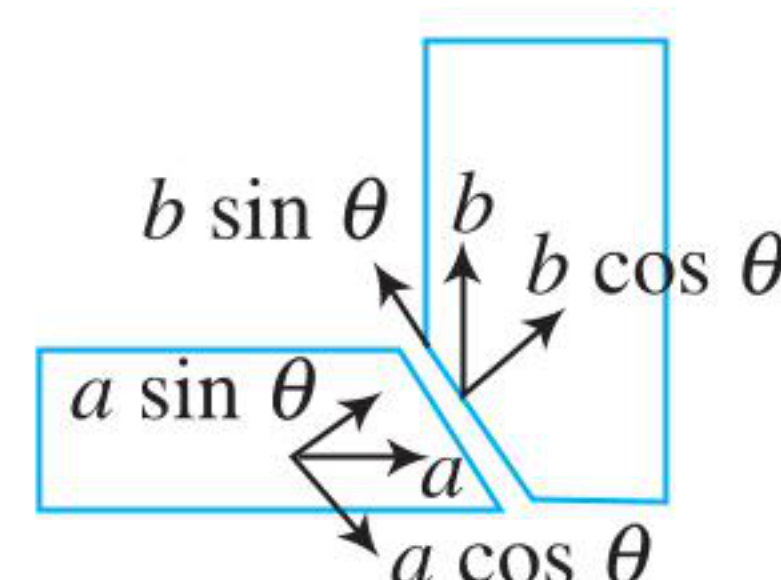
$$N_y = mg + \frac{4F}{3}$$



11. (1) To keep regular contact

$$a \sin \theta = b \cos \theta$$

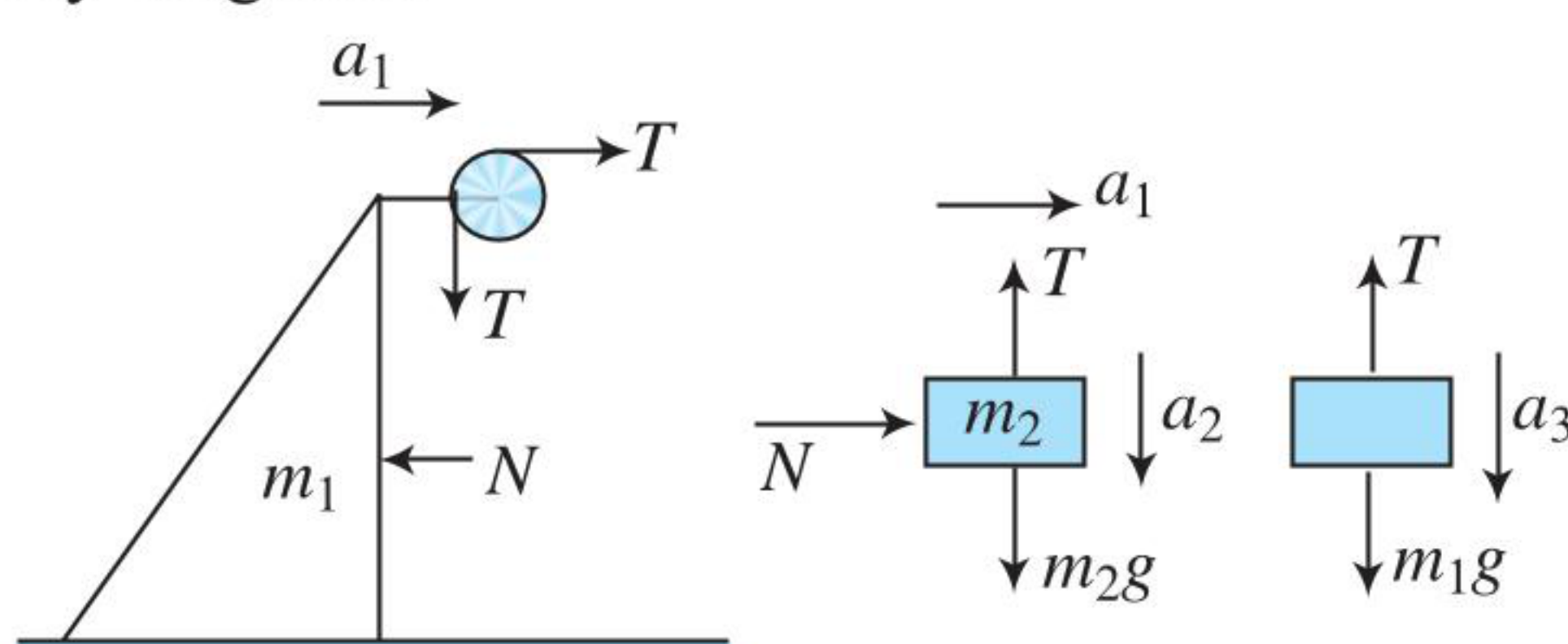
$$b = a \tan \theta = \frac{3}{4} a$$



For Problems 12–14

12. (1), 13. (4), 14. (2)

Free-body diagrams



Constraint relations:

$$x_1 = x_2 + x_3$$

$$a_1 = a_2 + a_3$$

Equation of motion:

$$T - N = m_1 a_1 \quad \dots(i)$$

$$N = m_2 a_1 \quad \dots(ii)$$

$$m_2 g - T = m_2 a_2 \quad \dots(iii)$$

$$m_3 g - T = m_3 a_3 \quad \dots(iv)$$

Using above equations, we can calculate the values.

For Problems 15–18

15. (1), 16. (2), 17. (3), 18. (3)

Acceleration of different objects is shown in figure. All the accelerations are w.r.t. ground

$$\text{Given: } a_3 = a_4 \quad \dots(i)$$

$$a_2 + a_1 = 1 \quad \dots(ii)$$

$$a_3 + a_1 = 5 \quad \dots(iii)$$

From figure, we can write

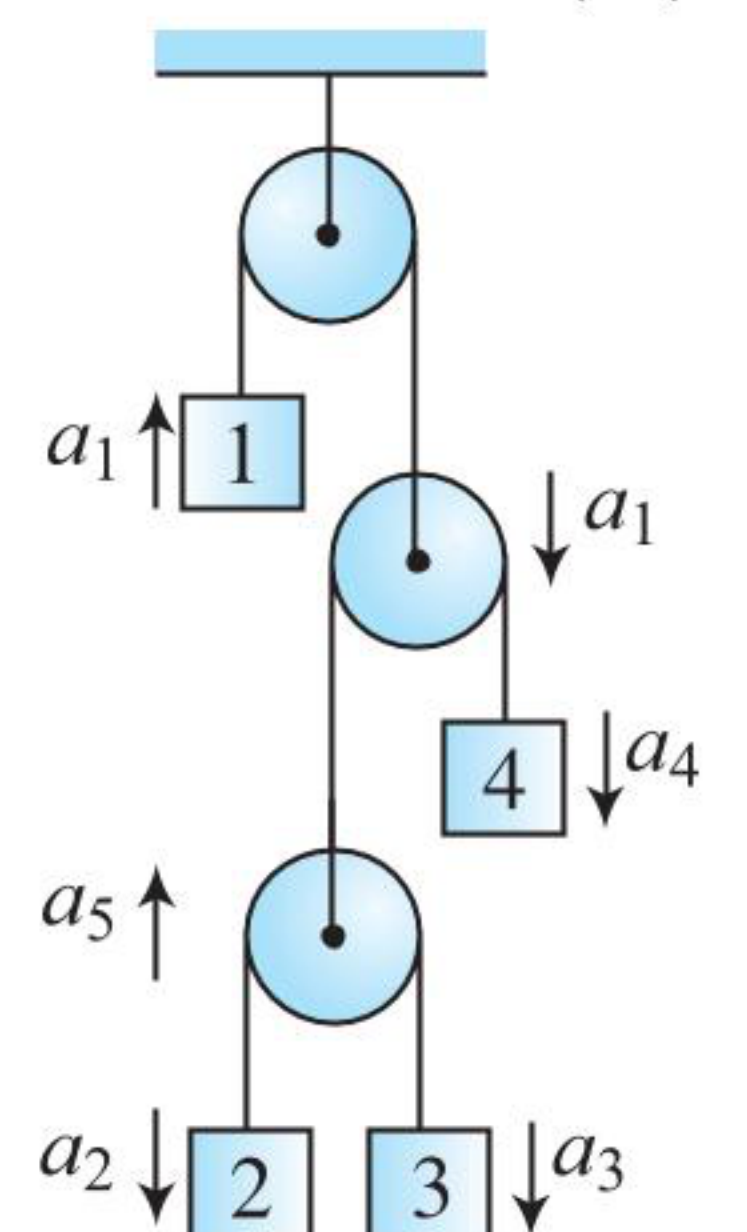
$$-a_5 = \frac{a_2 + a_3}{2} \quad \dots(iv)$$

$$a_1 = \frac{a_4 - a_5}{2} \quad \dots(v)$$

Solving the above equations, we get

$$a_1 = 2 \text{ m s}^{-2}, a_2 = -1 \text{ m s}^{-2},$$

$$a_3 = a_4 = 3 \text{ m s}^{-2}$$



For Problems 19–21

19. (2), 20. (1), 21. (4)

$$T_3 = 20t, T_1 = T_2 = 10t$$

For A to lose contact: $10t = 1g \Rightarrow t = 1$ s

For B to lose contact: $10t = 2g \Rightarrow t = 2$ s

For C to lose contact: $20t = 3g \Rightarrow t = 1.5$ s

$$a_A = \frac{T_1 - 1g}{1} \text{ Velocity of A when B loses contact}$$

$$V_1 = \int_1^2 a_A dt = \int_1^2 (10t - g) dt = 5 \text{ m s}^{-1}$$

$$\text{At } t = 2 \text{ s, } a_B = 0, a_A = \frac{10 \times 2 - 10}{1} = 10 \text{ m s}^{-2}$$

$$a_{A/B} = a_A - a_B = 10 - 0 = 10 \text{ m s}^{-2}$$

For Problems 22–24

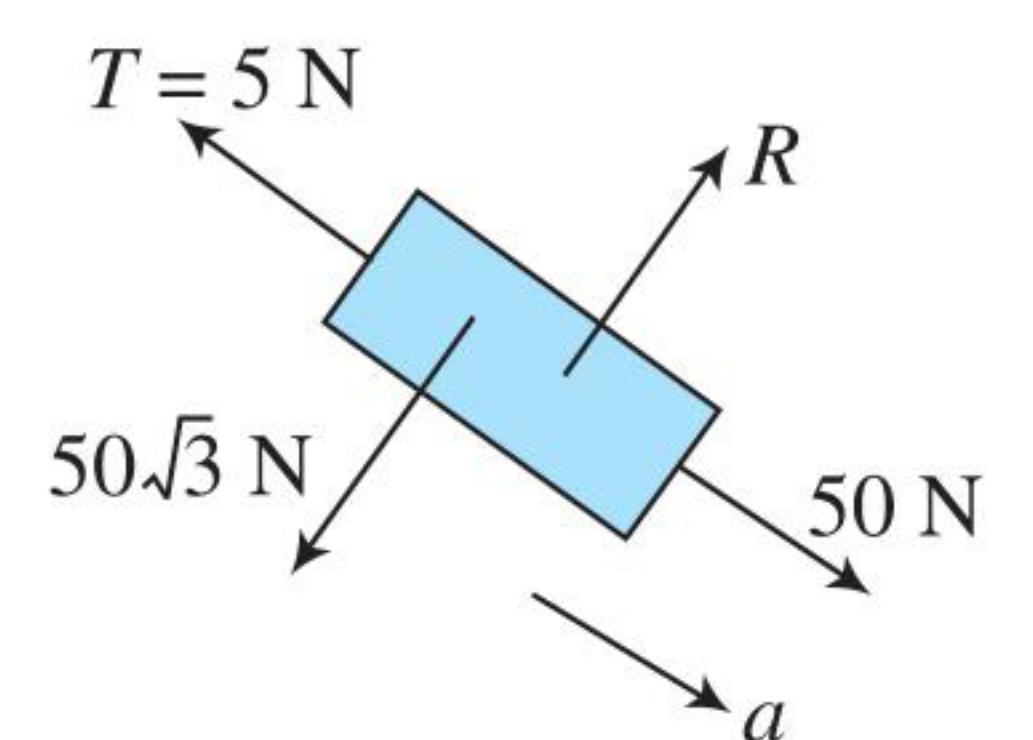
22. (4), 23. (1), 24. (1)

$$mg \sin 30^\circ - T = ma$$

$$50 - 5 = 10a$$

$$a = 4.5 \text{ m s}^{-2}$$

$$a_{P_4} = \frac{0 + 4.5}{2} = 2.25 \text{ m s}^{-1}$$



For Problems 25–27

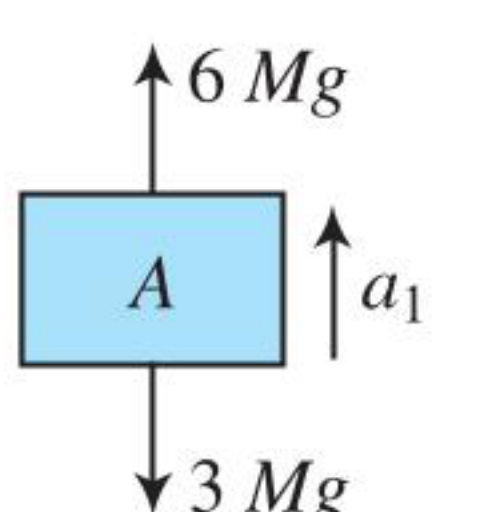
25. (4), 26. (2), 27. (1)

Force in spring can't change abruptly whereas tension in string can change.

25. When PQ is cut, no effect on the forces acting on C, hence its acceleration remains zero.

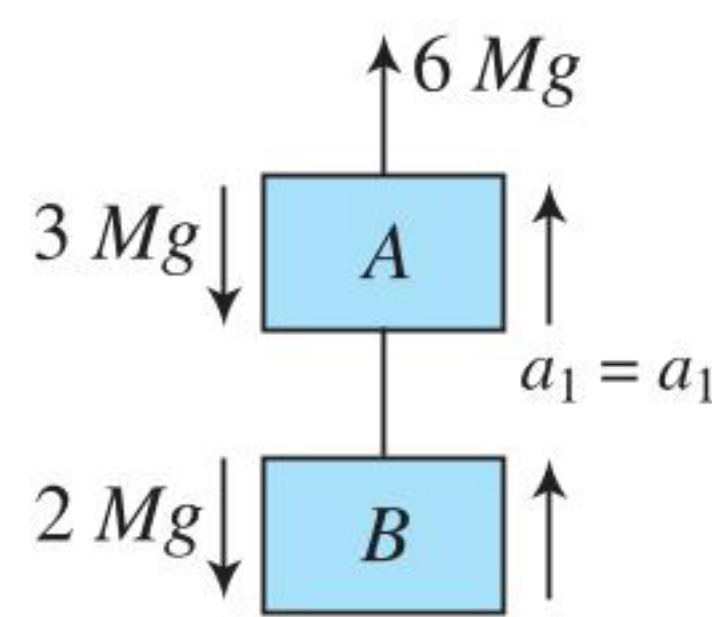
26. Tension in PQ is $6Mg$ before cutting RS.

$$\text{After cutting RS, } a_1 = \frac{6Mg - 3Mg}{3M} = g$$



27. After TU is cut

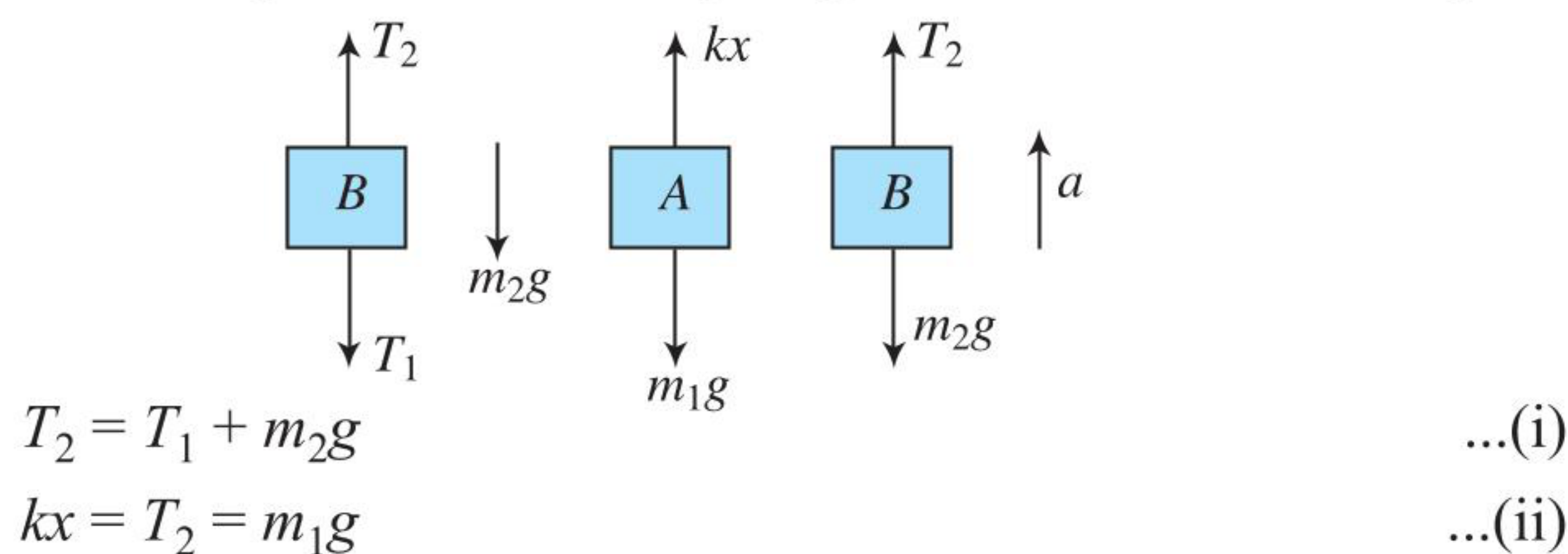
$$a_1 = a_2 = \frac{6Mg - 3Mg - 2Mg}{5M} = g/5$$



For Problems 28–30

28. (2), 29. (1), 30. (4)

Before burning BC , the free-body diagrams are shown in the figure.



where x is extension in the spring. Just after burning, T_1 will become zero, but T_2 will remain same.

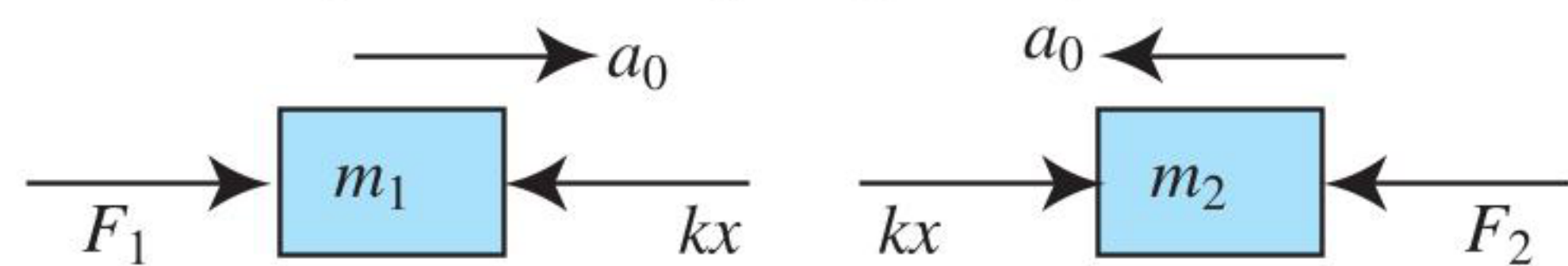
$$T_2 - m_2g = m_2a \Rightarrow a = \frac{(m_1 - m_2)g}{m_2}$$

As T_2 remains same, acceleration of block A will still remain zero.

For Problems 31–33

31. (2), 32. (4), 33. (3)

Let x be the compression in spring at any time



$$F_1 - kx = m_1a_0, F_2 - kx = m_2a_0$$

$$\text{Solve to get: } a_0 = \frac{F_1 - F_2}{m_1 - m_2} \text{ and } kx = \frac{m_1F_2 - F_1m_2}{m_1 - m_2}$$

Just after m_2 is removed:

$$a_2 = \frac{kx}{m_2} = \frac{F_2 - m_2a_0}{m_2} = \frac{F_2}{m_2} - a_0$$

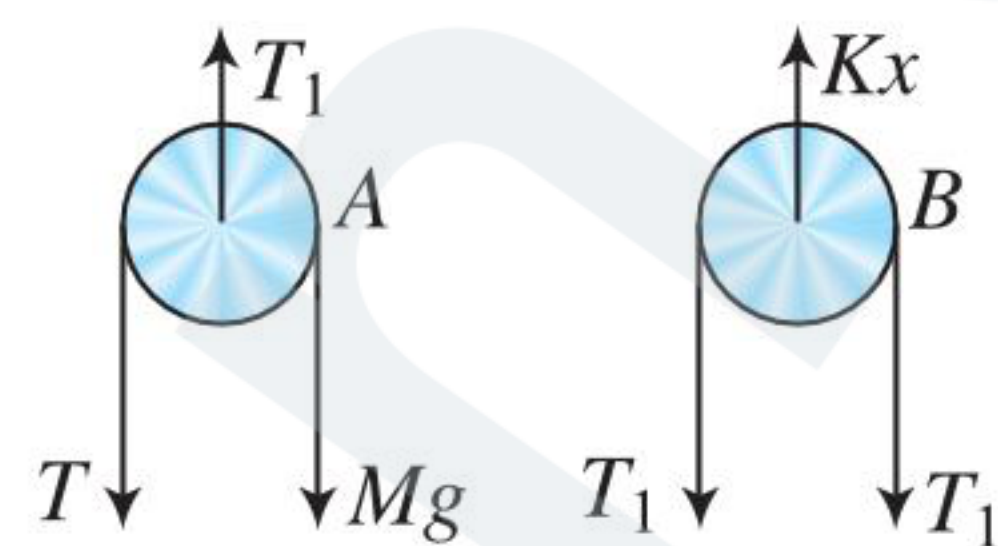
For Problems 34–36

34. (1) $T = Mg$

$$T_1 = T + Mg = 2Mg$$

$$Kx = 2T_1$$

$$\text{or } x = \frac{2T_1}{K} = \frac{4Mg}{K}$$



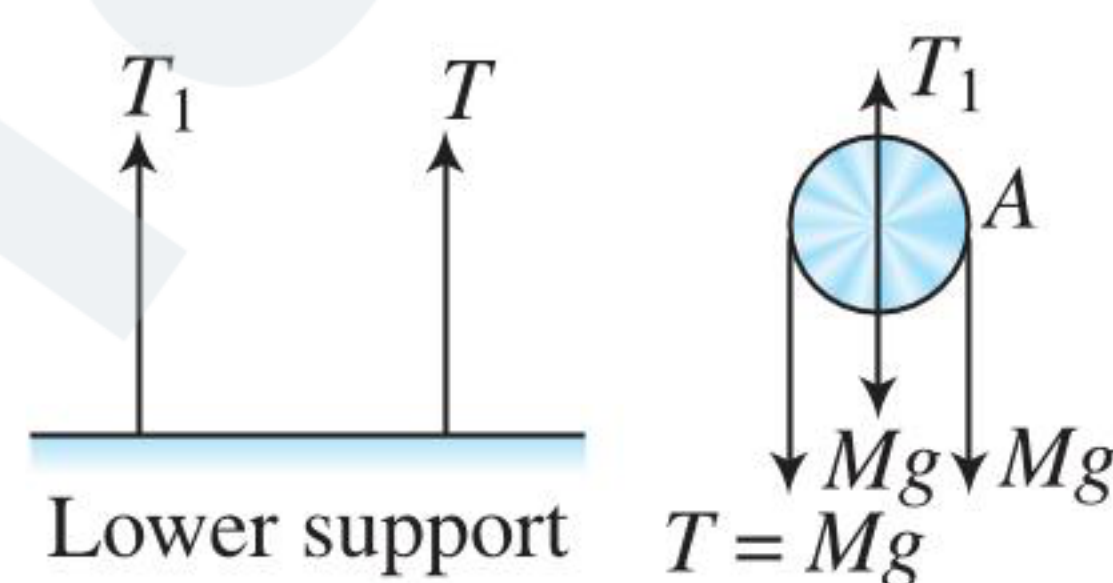
35. (3) Total tension on lower support:

$$T_1 + T = 2Mg + Mg = 3Mg$$

36. (4) $T_1 = 3Mg$

Net tension at lower support

$$T + T_1 = Mg + 3Mg = 4Mg$$



Matrix Match Type

1. i $\rightarrow$ c, d; ii $\rightarrow$ a, c; iii $\rightarrow$ a, c; iv $\rightarrow$ c, d.

For (i), it is not mentioned whether the object is accelerated or moving with constant velocity. So nothing can be predicted with surety.

If no net force is acting along east, then also it can move with constant velocity, and if no force is acting at all, then also it can move with constant velocity.

For (ii) and (iii): As the object is accelerated (whether uniform or non-uniform) a force must act on the object in such a manner that a component or whole of the force would be along east, and also the net force must be towards east.

For (iv): It is moving with constant velocity, so net force must be zero that implies no force may act on the object.

2. i $\rightarrow$ a, c; ii $\rightarrow$ b, c; iii $\rightarrow$ b, c; iv $\rightarrow$ b, d.

Acceleration of the whole system towards right:

$$a = \frac{F}{M + m}$$

$$F - mg \sin \theta = ma \cos \theta$$

$$= m \frac{F}{M + m} \cos \theta = \frac{(M + m)mg \sin \theta}{M + m - m \cos \theta}$$

Pseudo force on m as seen from the frame of M :

$$F_{s1} = ma = \frac{mF}{m + M}$$

$$= mg \sin \theta \left(\frac{m}{M + m(1 - \cos \theta)} \right) < mg \sin \theta$$

Pseudo force on M as seen from the frame of m :

$$F_{s2} = Ma = \frac{MF}{m + M} \left(> \frac{mF}{m + M} \right)$$

$$= mg \sin \theta \left(\frac{M}{M + m(1 - \cos \theta)} \right) < mg \sin \theta$$

$$\text{Now } mg \cos \theta - N = m \sin \theta \Rightarrow N = mg \cos \theta - m \sin \theta$$

Hence N is less than $mg \cos \theta$. Hence, it will also be less than $mg \sin \theta$, because $\theta = 45^\circ$.

Applying equation on m in horizontal direction:

$$F \cos \theta - N \sin \theta = ma$$

$$\Rightarrow F \cos \theta - N \sin \theta = m \frac{F}{M + m}$$

$$\Rightarrow N = \frac{mF}{M + m} \left(\frac{(M + m) \cos \theta - m}{m \sin \theta} \right)$$

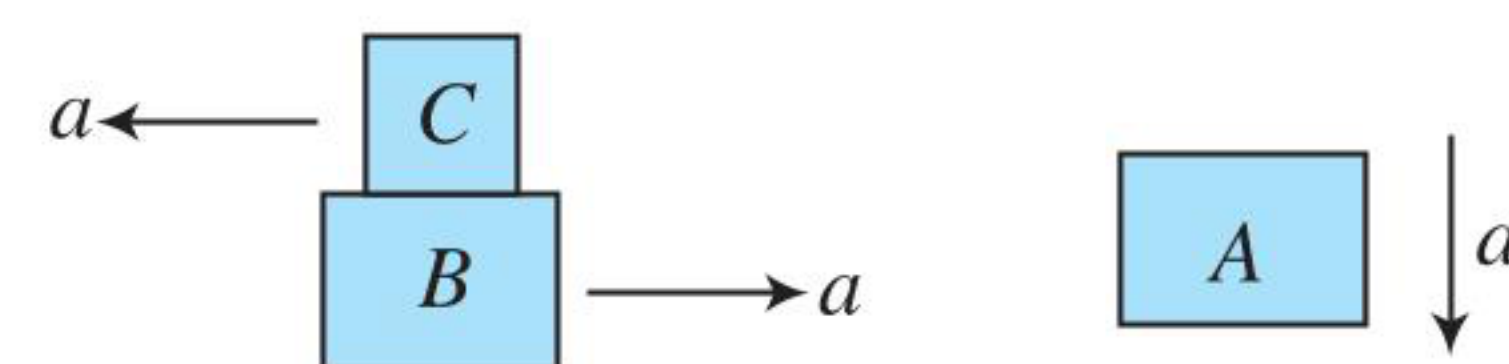
Put $\theta = 45^\circ$

$$\Rightarrow N = \frac{mF}{M + m} \left(\frac{M + m - \sqrt{2}m}{m} \right) > \frac{mF}{m + M}$$

Normal force between ground and M will be $(M + m)g$. It is greater than $mg \sin \theta$. It is also greater than $\frac{mF}{M + m}$, because $\frac{mF}{M + m}$ is less than $mg \sin \theta$.

3. i $\rightarrow$ b; ii $\rightarrow$ a; iii $\rightarrow$ c, d; iv $\rightarrow$ c, d.

The direction of accelerations of various blocks are as shown.



Acceleration of B is towards right. Hence, (i) $\rightarrow$ b. Acceleration of C w.r.t. B is towards left.

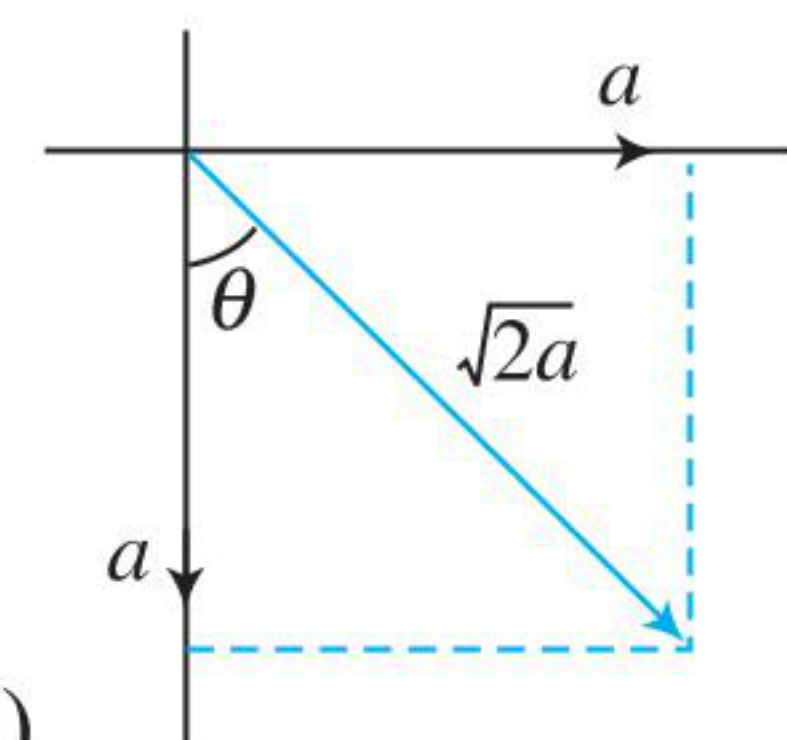
Hence, (ii) $\rightarrow$ a

Acceleration of A w.r.t. C :

$$= \vec{a}_{A/c} = \vec{a}_A - \vec{a}_C$$

$$= -a\hat{j} - (-a\hat{i}) = a\hat{i} - a\hat{j}$$

as shown below

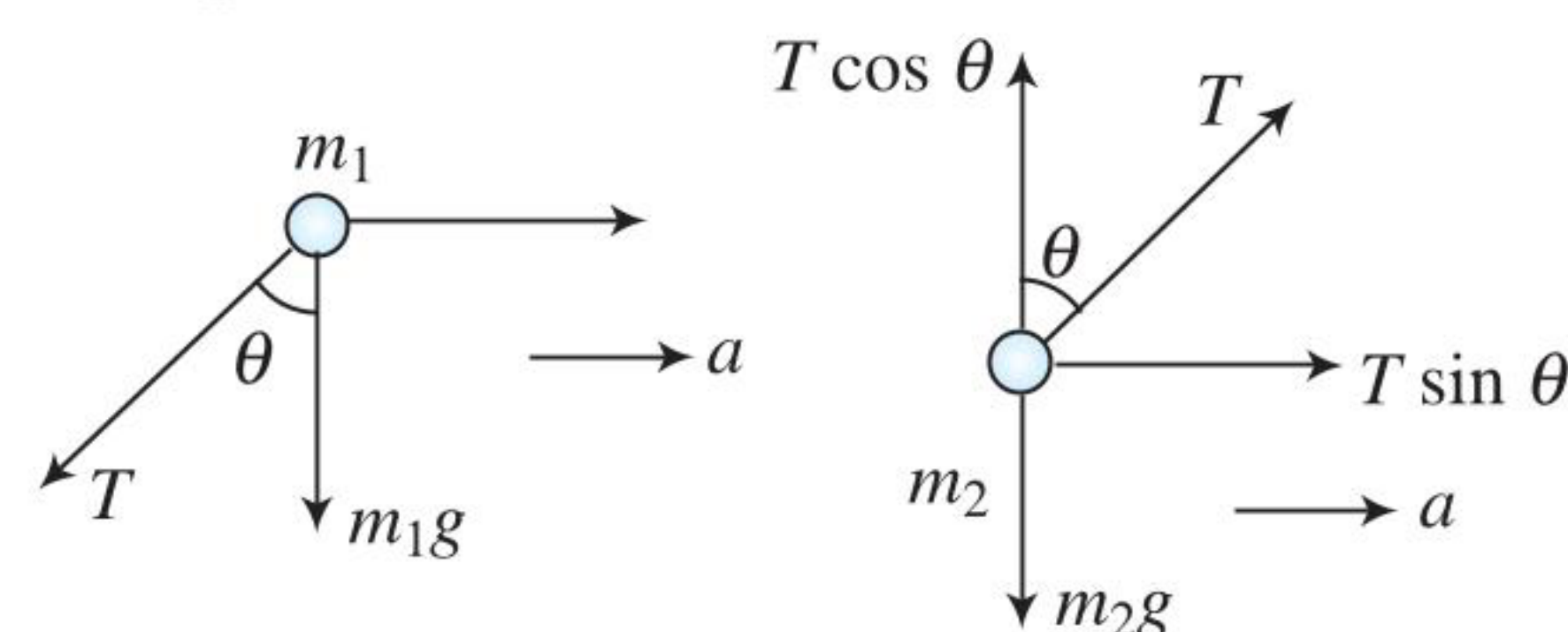


Hence, (iii) $\rightarrow$ (c, d), Similarly (iv) $\rightarrow$ (c, d).

4. i $\rightarrow$ d; ii $\rightarrow$ c; iii $\rightarrow$ b; iv $\rightarrow$ a.

$$F = (m_1 + m_2)a \quad \dots(i)$$

$$T \sin \theta = m_2 a \quad \dots(ii)$$



$$T \cos \theta = m_2g \Rightarrow T = m_2g \sec \theta \quad \dots(iii)$$

From, (ii) and (iii), $a = g \tan \theta$

Put in (i), $F = (m_1 + m_2)g \tan \theta$

Net force acting on $m_2 = m_2 a = \frac{m_2 F}{m_1 + m_2}$

Force acting on m_1 by wire:

$$m_1 g + T \cos \theta = m_1 g + m_2 g$$

5. i $\rightarrow$ b, c; ii $\rightarrow$ a, d; iii $\rightarrow$ d; iv $\rightarrow$ c.

Let the accelerations of various blocks are as shown in figure. Pulley P_2 will have downward acceleration a .

$$\text{Now } a = \frac{a_1 + a_2}{2} \Rightarrow a_2 = 2a - a_1 > 0$$

So acceleration of 2 is upwards

Hence, (i) $\rightarrow$ (b, c)

$$\text{and } a = \frac{-a_1 + a_4}{2} \Rightarrow a_4 = 2a + a_1 > 0$$

So, acceleration of 4 is downwards.

Hence (ii) $\rightarrow$ (a, d)

Acceleration of 2 w.r.t. 3:

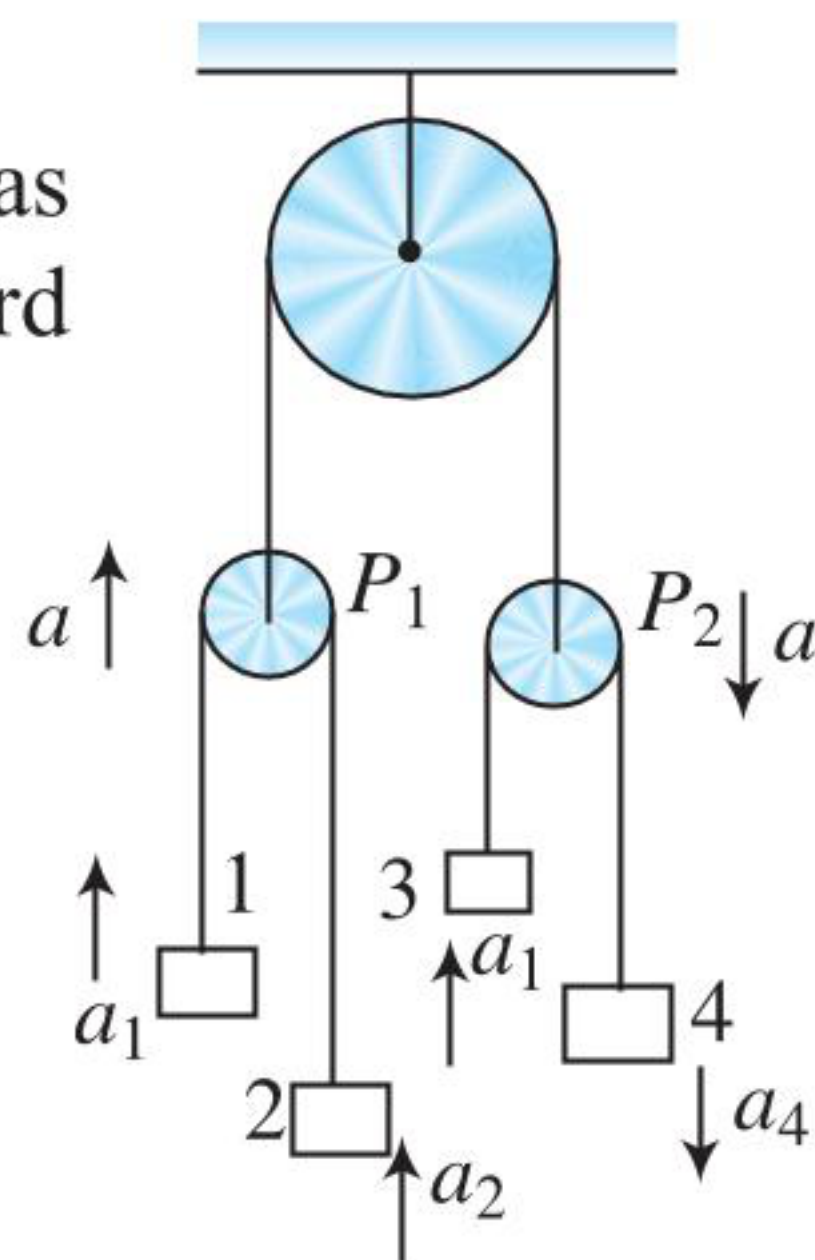
$$a_{2/3} = a_2 - a_3 = a_2 - a_1 = 2(a - a_1) < 0$$

This is downwards, Hence (iii) $\rightarrow$ (d)

Acceleration of 2 w.r.t. 4:

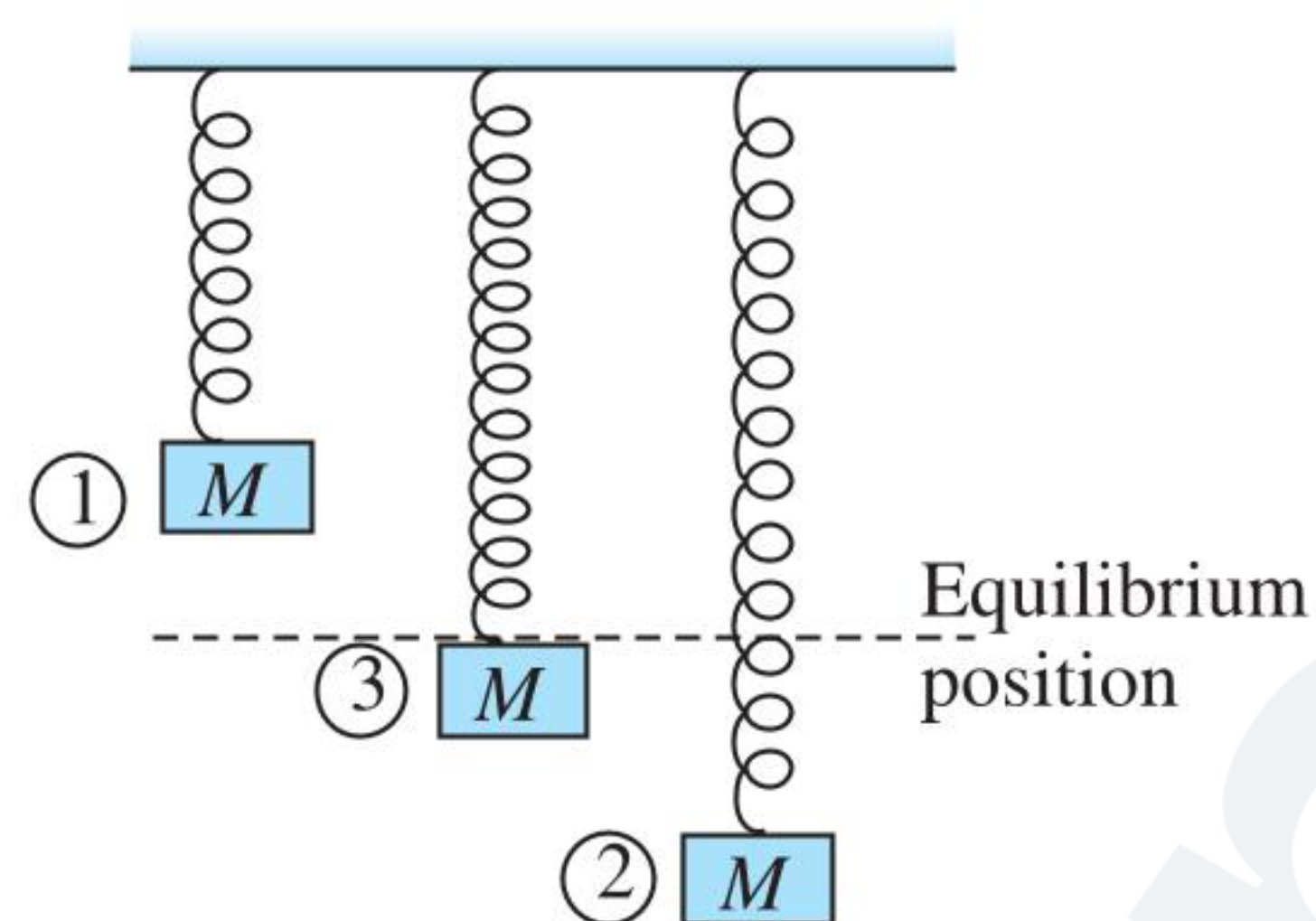
$$a_{2/4} = a_2 - (-a_4) = 4a > 0$$

This is upwards. Hence, (iv) $\rightarrow$ (c)



6. i $\rightarrow$ a; ii $\rightarrow$ c, d; iii $\rightarrow$ a, d; iv $\rightarrow$ b, d.

In figure is the equilibrium position where velocity is maximum and acceleration is zero. 1 and 2 are the extreme positions where velocity is zero and acceleration is maximum. 1 is the unstretched position.



When the block is at position 3, then $mg = kx$. So net force is zero. Hence, acceleration is zero. But velocity may be either in upward or downward direction.

Hence (ii) $\rightarrow$ (c, d)

When the block is between position 3 and 2, then $kx > mg$. So net force is in upward direction, hence acceleration is in upward direction. But velocity may be either in upward or downward direction.

Hence (iii) $\rightarrow$ (a, d)

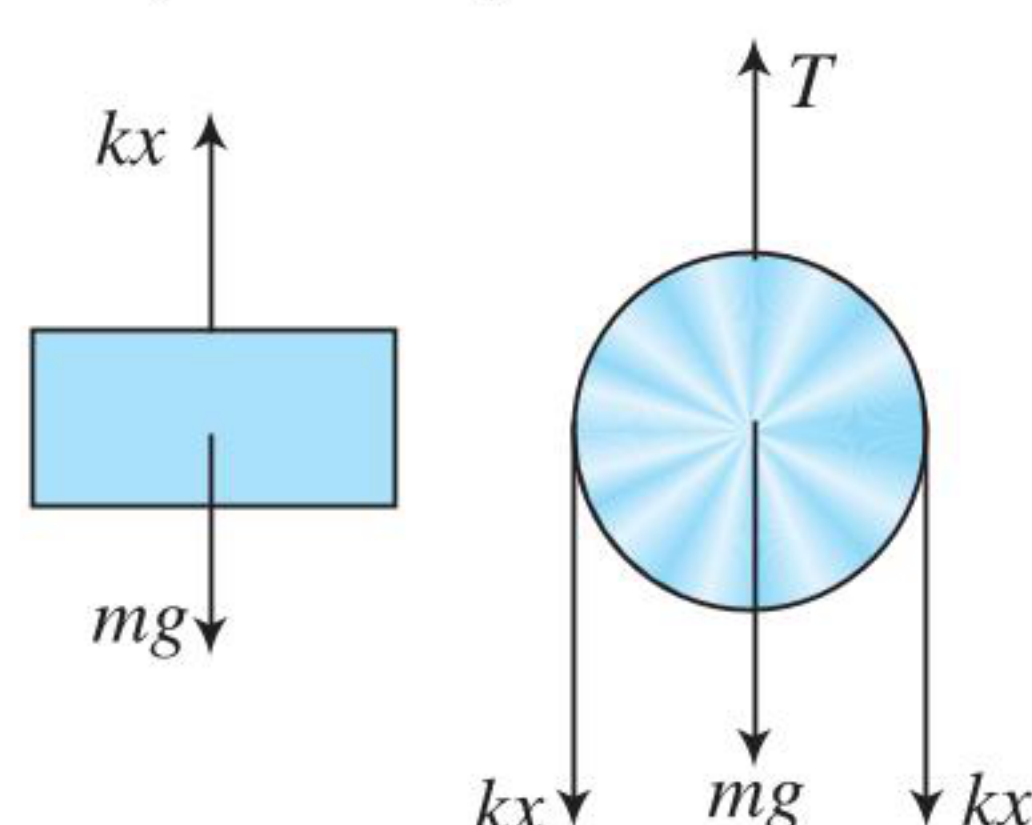
But if the block is at position 2, then velocity is zero and acceleration is in upward direction.

Hence (i) $\rightarrow$ (a)

When the block is between position 3 and 1, $mg > kx$. So net force is in downward direction, hence acceleration is in downward direction. But velocity may be either in upward or downward direction.

Hence (iv) $\rightarrow$ (b, d)

7. i $\rightarrow$ c; ii $\rightarrow$ c; iii $\rightarrow$ c; iv $\rightarrow$ b, d.



(i) Let the maximum downward displacement of m is x_0 , then

$$\frac{1}{2} kx_0^2 = mgx_0 \Rightarrow x_0 = 2mg/k$$

To lift the block (M): $kx_0 = Mg$

$$\Rightarrow 2mg = Mg \Rightarrow mg = Mg/2$$

Hence (i) $\rightarrow$ (c).

(ii) When m is in equilibrium,

$$kx = mg, T = 2kx + mg = 3mg = \frac{3M}{2}g$$

Hence (ii) $\rightarrow$ (a).

(iii) $N + kx = Mg$

$$N = Mg - kx = Mg - \frac{M}{2}g = \frac{M}{2}g$$

Hence (iii) $\rightarrow$ (c)

(iv) Tension $= kx_0 = Mg = 2mg$

Hence (iv) $\rightarrow$ (b, d)

8. i $\rightarrow$ c; ii $\rightarrow$ b; iii $\rightarrow$ b, d; iv $\rightarrow$ b.

(i), (ii)

After spring 2 is cut, tension in string AB will not change.

$$(T_{CD})_i = 4mg$$

$$(T_{CD})_f = m_D g + m_D \cdot \frac{m_A + m_B - m_C - m_D}{m_A + m_B + m_C + m_D} \cdot g$$

$$= 2mg \left(1 + \frac{1}{5} \right) = 2.4mg$$

Hence, T_{CD} decreases.

(iii), (iv)

After string between C and pulley is cut, tension in string AB will become zero.

$$(T_{CD})_i = (m_D + m_E)g = 4mg$$

Acceleration of C and D blocks is

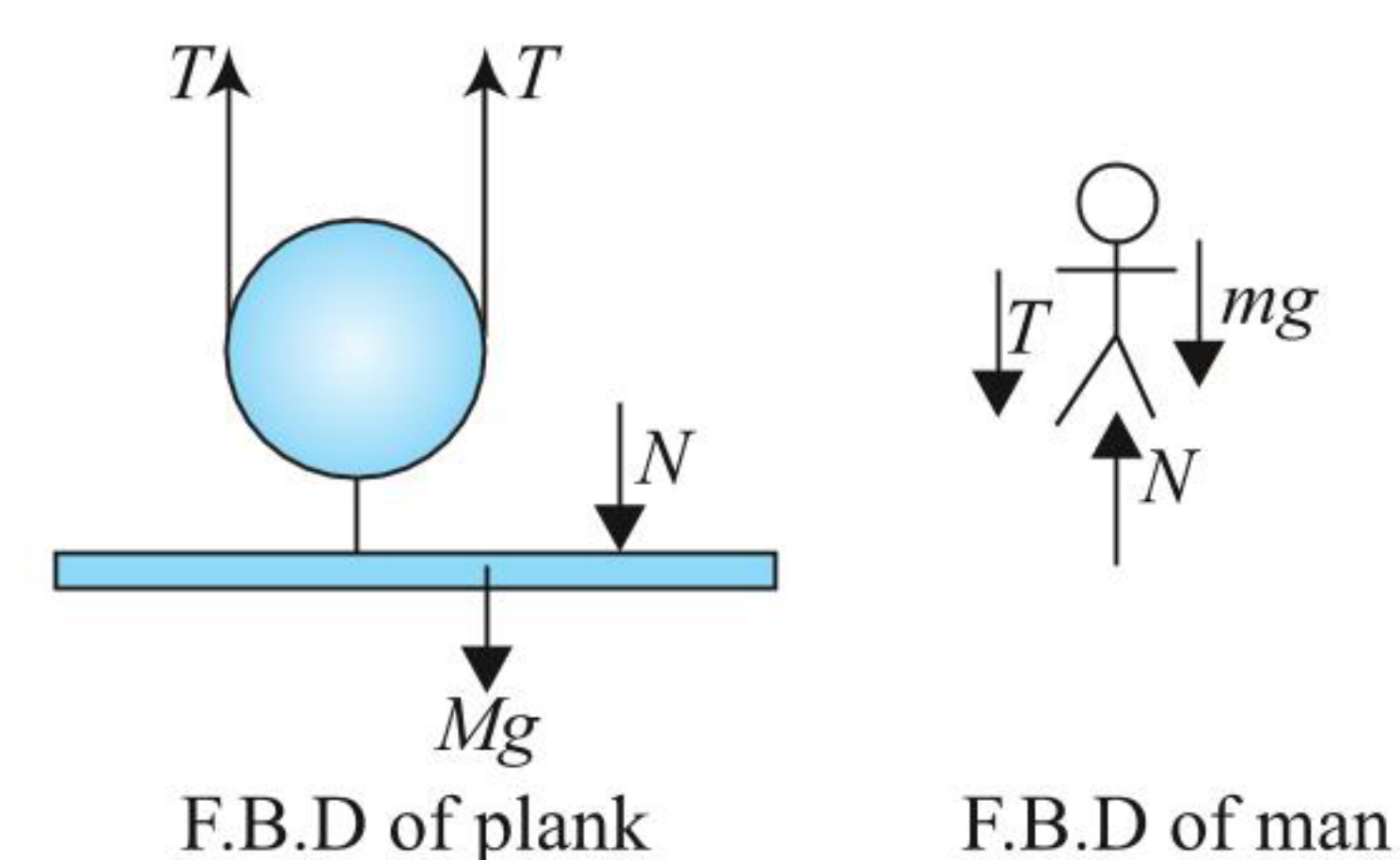
$$(m_C + m_D)g + m_E g = (m_C + m_D) \cdot a$$

$$a = \frac{6mg}{4mg} = \frac{3}{2}g, (T_{CD})_f + m_C g = m_C a$$

$$(T_{CD})_f = 2m \cdot \frac{3}{2}g - 2mg = mg$$

The tension decreases.

9. (2)



(i) At equilibrium: $2T = N + mg$... (i)

$$\text{and } N = T + mg \quad \dots (ii)$$

$$\text{From (i) and (ii) } T = (M + m)g \text{ and } N = (M + 2m)g$$

Hence (i) $\rightarrow$ a, b

(ii) When the system falls freely, $a = g$, then $N = 0$ and $T = 0$

Hence (ii) $\rightarrow$ c, e

(iii) When accelerating upward

$$\text{For platform: } 2T - N - Mg = Ma \quad \dots (i)$$

$$\text{For man: } N - T - mg = ma \quad \dots (ii)$$

$$\text{From (i) and (ii) } T = (M + m)(g + a) \text{ and } N = (M + 2m)(g + a)$$

$$\text{It means } N > (2m + M)g$$

Hence (iii) $\rightarrow$ d

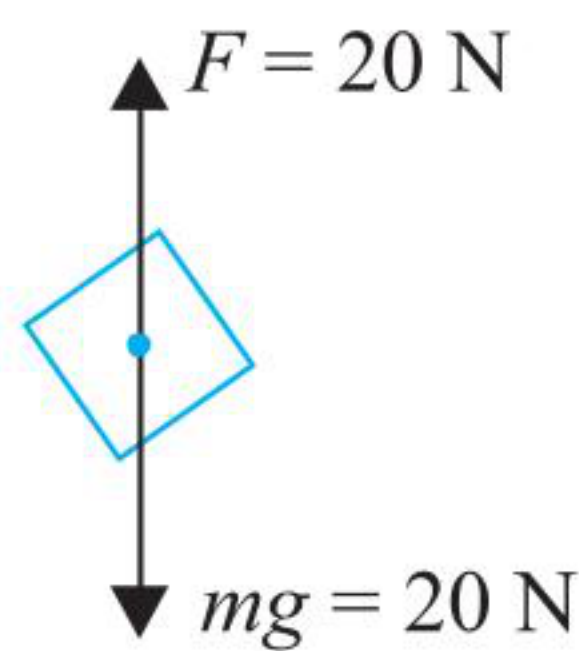
(iv) When the system accelerating downward

$$\text{Then } T = (M + m)(g - a) \text{ and } N = (M + 2m)(g - a)$$

It means (iv) $\rightarrow c, e$.

10. (4)

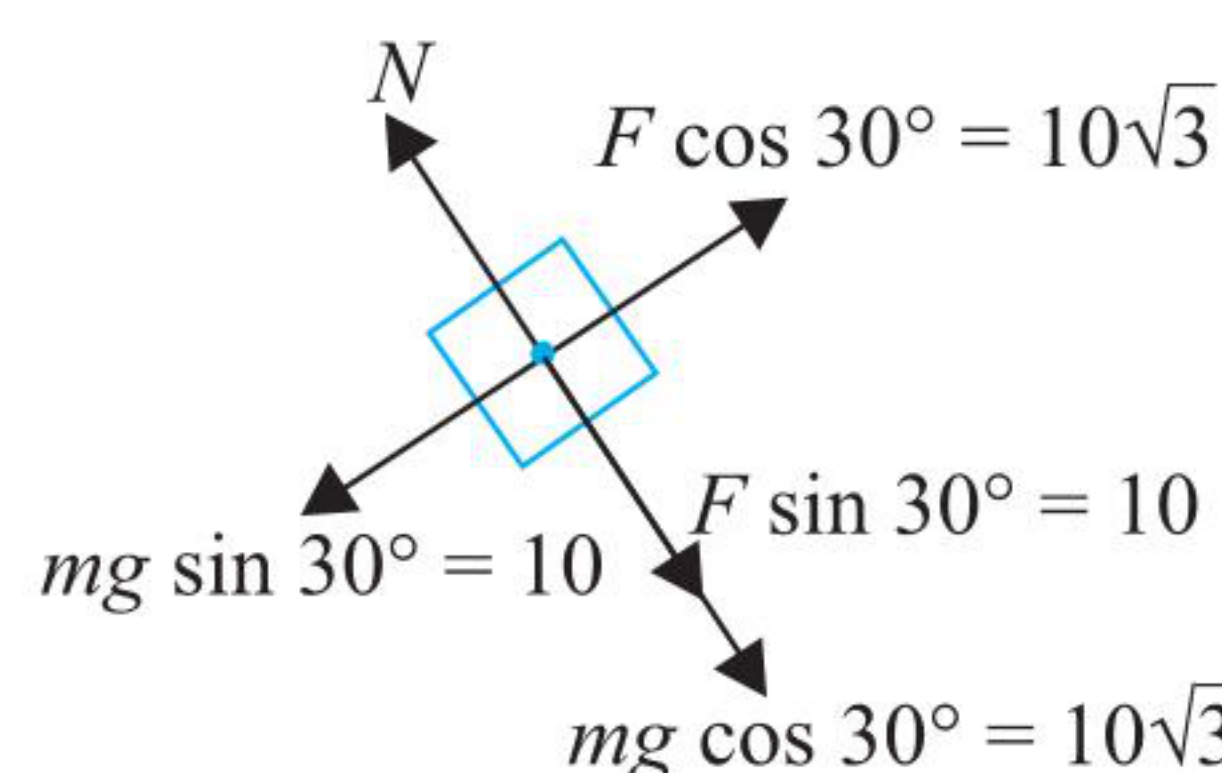
(a)



F exactly balances mg .

So body stays in its position without acceleration and with zero contact. So it does not have any match.

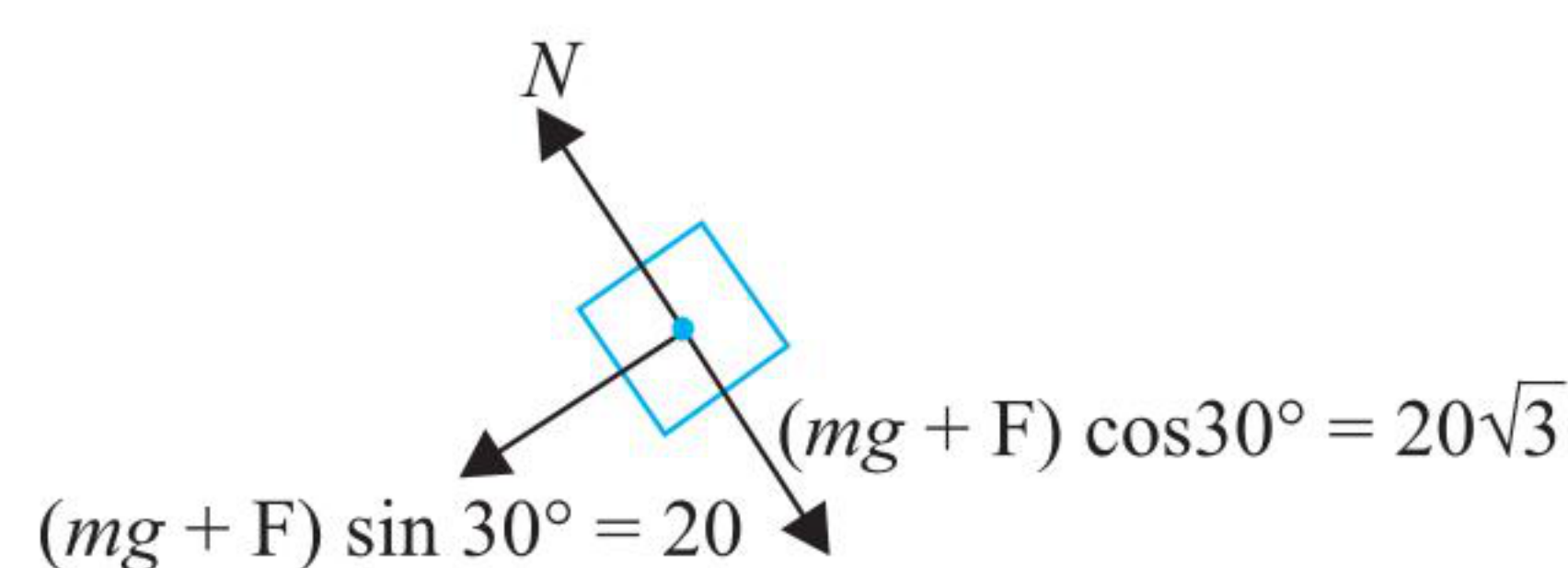
(b)



Slides up the inclined with acceleration $< 10 \text{ m/s}^2$;

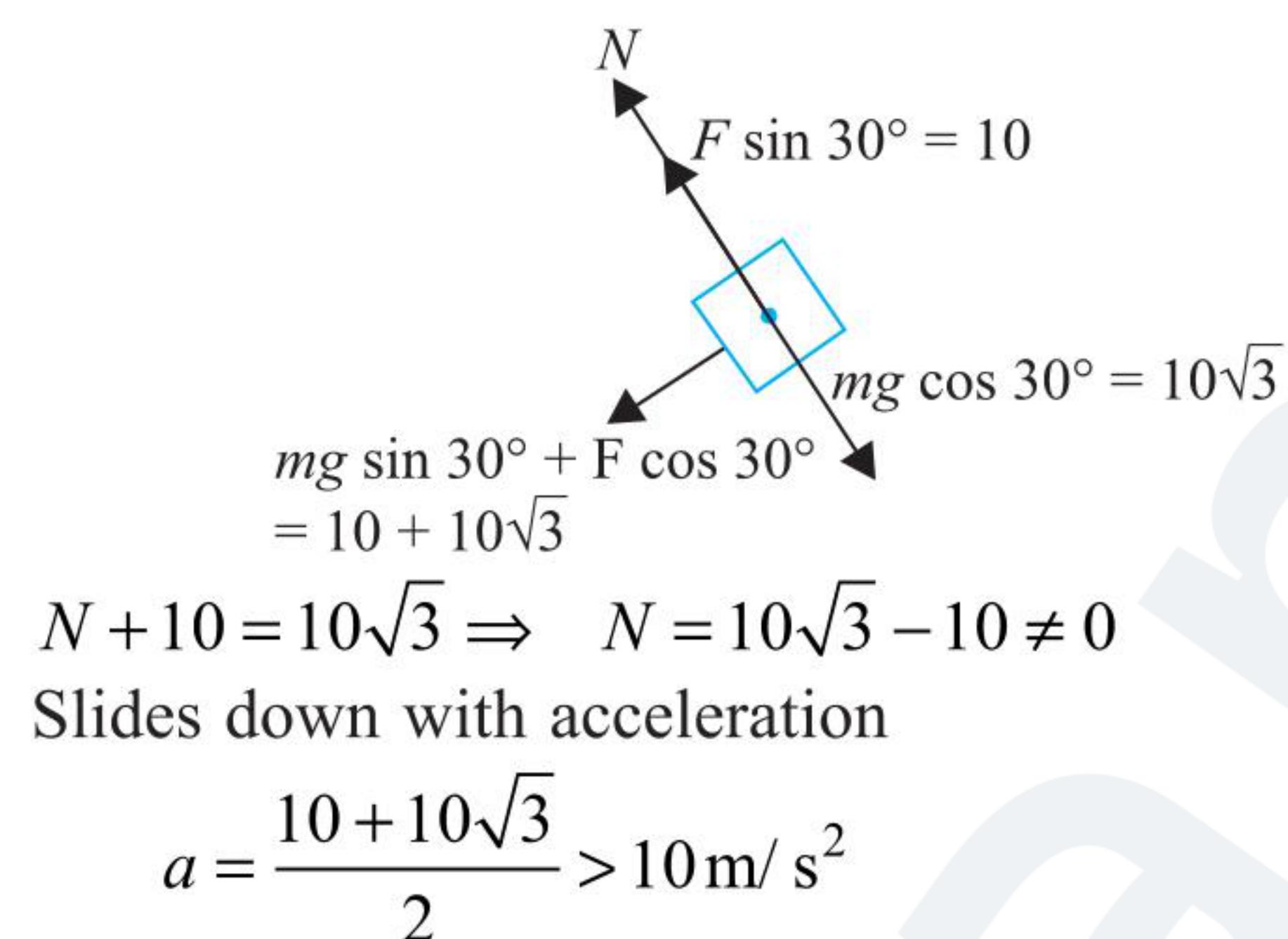
$N \neq 0 \Rightarrow$ matches with (i) only.

(c)



$N \neq 0 \Rightarrow$ Matches with (i); slides down the incline with acceleration $= 10 \text{ m/s}^2$ Matches (ii) also.

(d)



Numerical Value Type

1. (6) Consider the forces on the person:

$$\sum F_y = ma_y$$

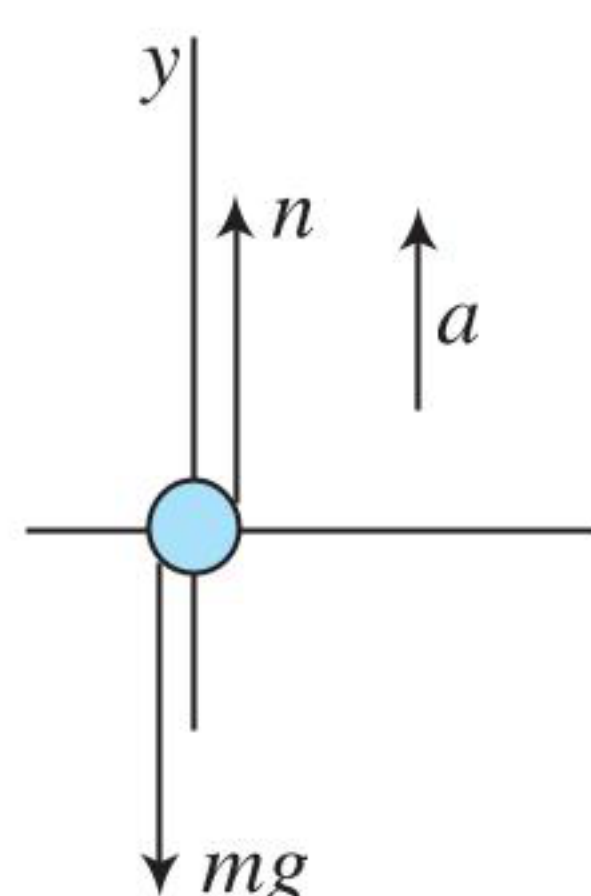
$$n - mg = ma$$

$$n = 1.6 mg$$

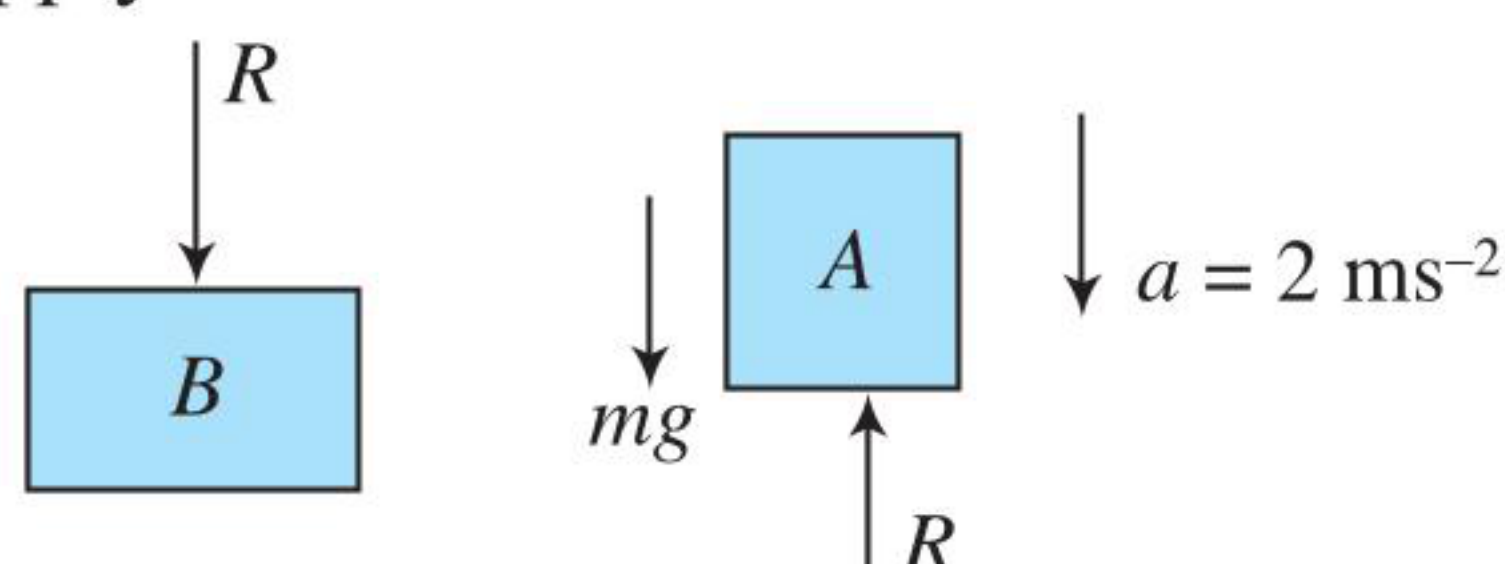
$$\text{So, } a = 0.60g = 6 \text{ m/s}^2$$

$$v^2 = u^2 + 2as = 0^2 + 2 \times 6 \times 3$$

$$\Rightarrow v = 6 \text{ m/s}^{-1}$$



2. (4) Let A apply a force R on B



Then B also applies an opposite force R on A as shown in figure

For A: $mg - R = ma$

$$\Rightarrow R = m(g - a) = 0.5[10 - 2] = 4 \text{ N}$$

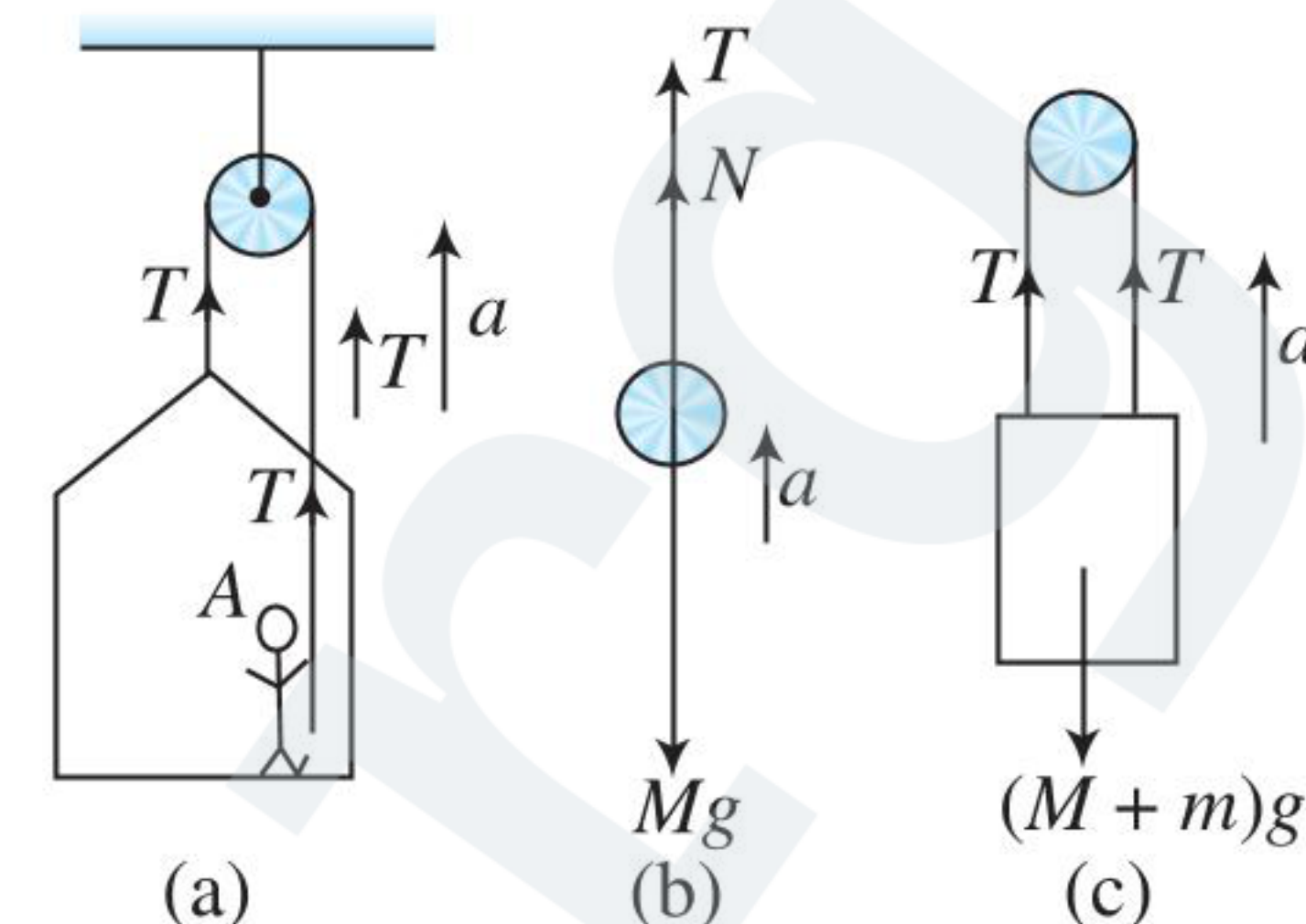
3. (2) Let $M =$ mass of painter $= 10 \text{ kg}$

$m =$ mass of crate $= 25 \text{ kg}$

Let F_A be the action force exerted by painter on crate, reaction force exerted by crate on man

$$N = F_A = 450 \text{ N}$$

The free-body diagram of painter is shown in figure (b)



Therefore, equation of motion of painter is

$$N + T - Mg = Ma \quad \dots(i)$$

The equation of motion of whole system is

$$2T - (M + m)g = (M + m)a \quad \dots(ii)$$

Multiplying (i) by 2, we get

$$2N + 2T - 2Mg = 2Ma \quad \dots(iii)$$

Subtracting (ii) from (iii), we get

$$2N - 2Mg + (M + m)g = (2M - M - m)a$$

$$\text{or } 2N - (M - m)g = (M - m)a$$

$$a = \frac{2N - (M - m)g}{M - m}$$

$$= \frac{2 \times 450 - (100 - 25) \times 10}{100 - 25} = \frac{900 - 750}{75} = 2 \text{ m/s}^2$$

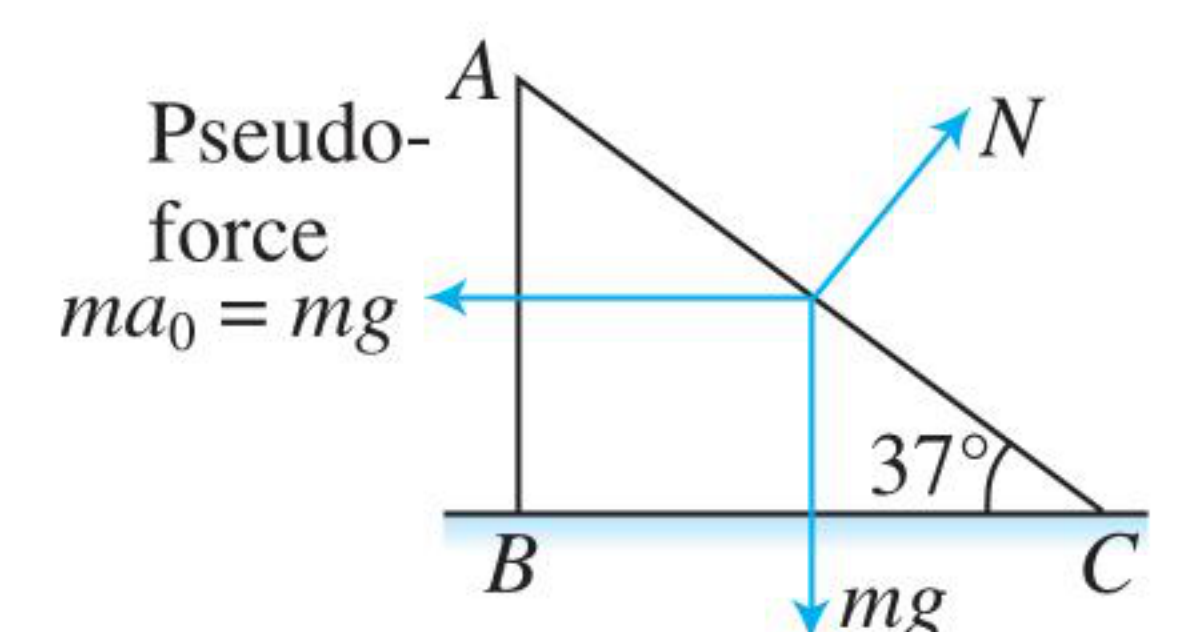
4. (1) Drawing the free-body diagram of block with respect to plane. Acceleration of the block up the plane is

$$a = \frac{mg \cos 37^\circ - mg \sin 37^\circ}{m}$$

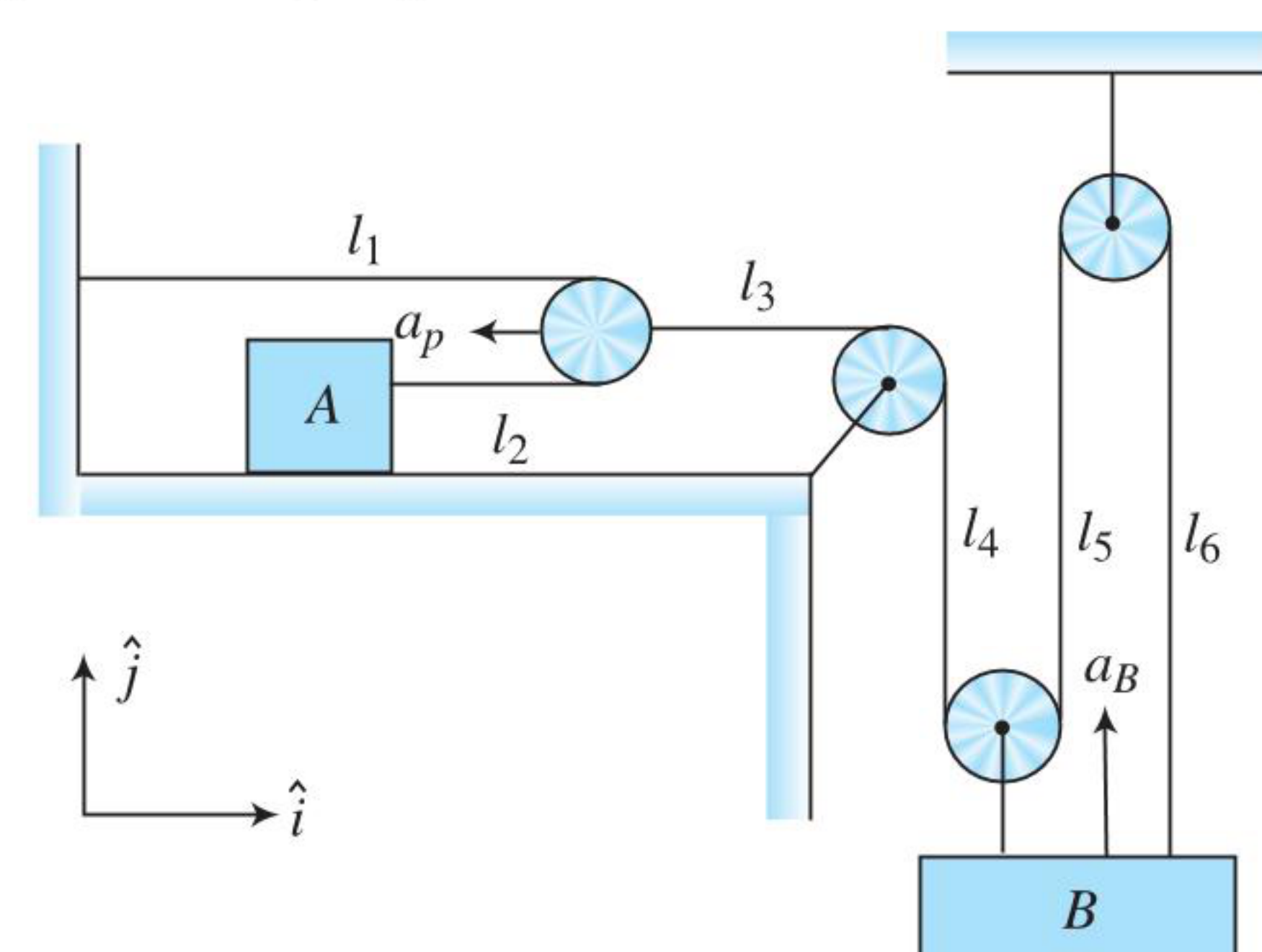
$$= g \left(\frac{4}{5} - \frac{3}{5} \right) = 2 \text{ m/s}^2$$

$$\text{Applying, } s = \frac{1}{2}at^2 \text{ we get}$$

$$\Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1}{2}} = 1 \text{ s}$$



5. (2) $l_1 + l_2 = C \Rightarrow l_1'' + l_2'' = 0$



$$\Rightarrow -a_p + (12 - a_p) = 0 \Rightarrow a_p = 6 \text{ m/s}^2$$

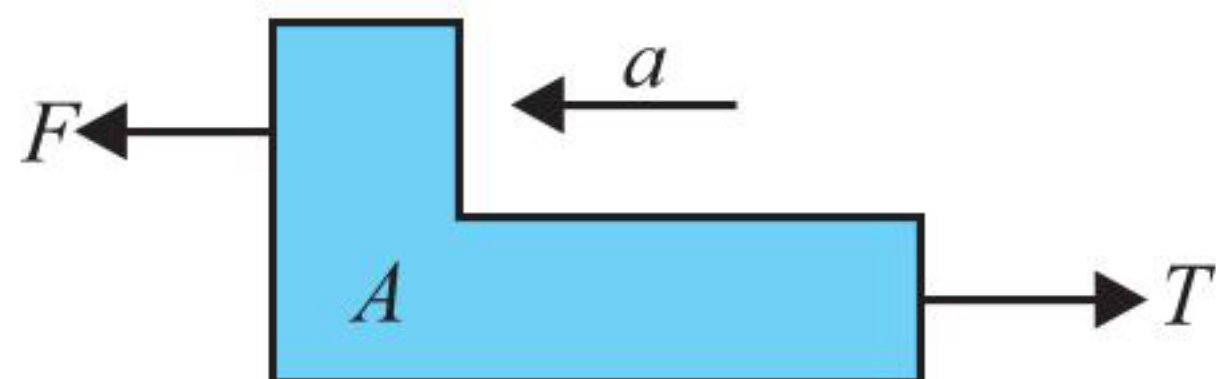
$$l_3 + l_4 + l_5 + l_6 = C$$

$$\Rightarrow l_3'' + l_4'' + l_5'' + l_6'' = 0$$

$$a_p - a_B - a_B - a_B = 0$$

$$\Rightarrow a_p = 3a_B \Rightarrow a_B = 2 \text{ m/s}^2$$

6. (3) $F - T = 1 \times a$



$$\Rightarrow a = 1.5 \text{ ms}^{-2}, a_{\text{rel}} = 1.5 + 4.5 = 3 \text{ ms}^{-2} \text{ here } u_{\text{rel}} = 0$$

$$\text{Hence, } s_{\text{rel}} = 0 + \frac{1}{2}(a_{\text{rel}})t^2$$

$$\Rightarrow t = \sqrt{\frac{2s_{\text{rel}}}{a_{\text{rel}}}} = \sqrt{\frac{2 \times 13.5}{3}} = 3 \text{ sec}$$

7. (5) At equilibrium $2T = m_1 g \Rightarrow T = 45 \text{ N}$... (i)

$$T - m_2 g = T_1 \quad \dots (ii)$$

As string is at cut then $T_1 = 0$

$$T - m_2 g = m_1 a$$

$$45 - 30 = 3 \times a$$

$$a = 5 \text{ m/s}^2$$

8. (2) Acceleration of A and B $= \left(\frac{\frac{11}{3} - 3}{\frac{11}{3} + 3} \right) g = 1 \text{ m/s}^2$

Relative acceleration of A and B $= 2 \text{ m/s}^2$

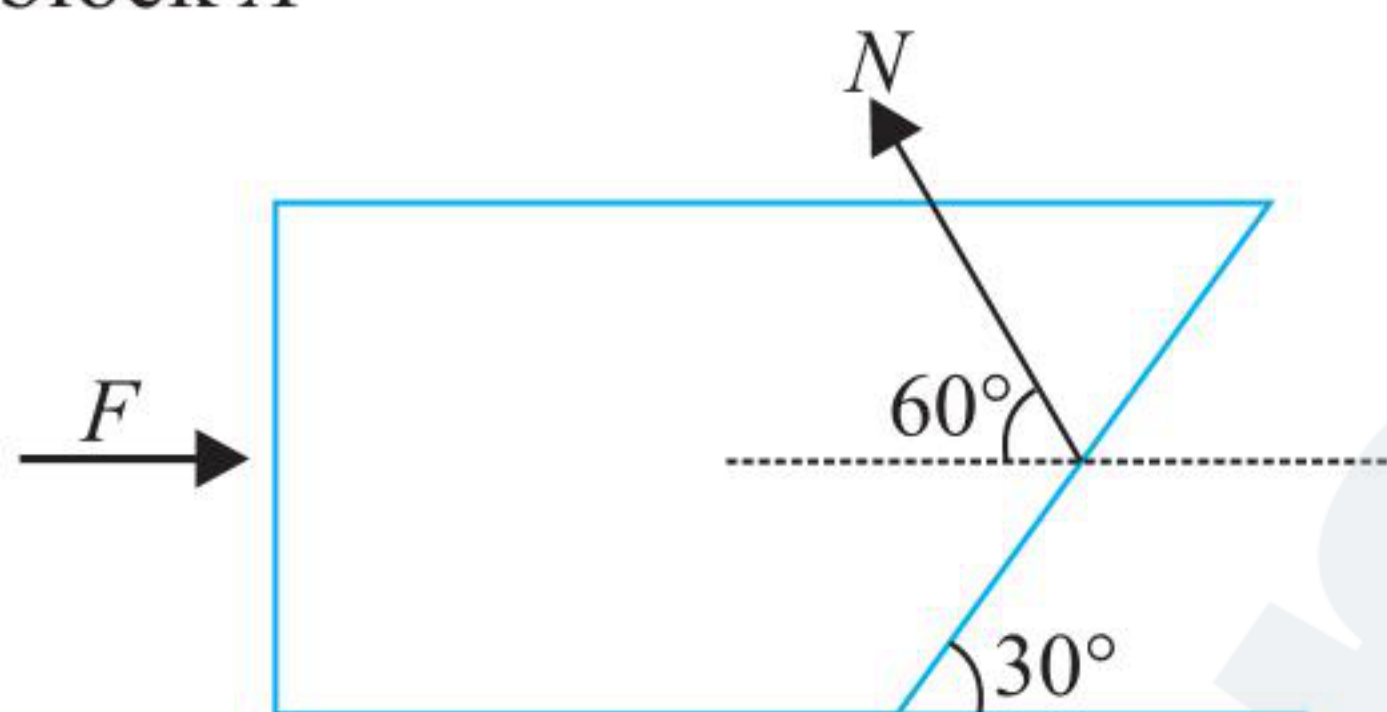
Now by using $s = ut + \frac{1}{2}at^2$;

$$4 = 0 + \frac{1}{2}(2)(t)^2$$

$$\Rightarrow t = 2 \text{ s}$$

9. (30) Acceleration of two mass system is $a = \frac{F}{2m}$ leftward.

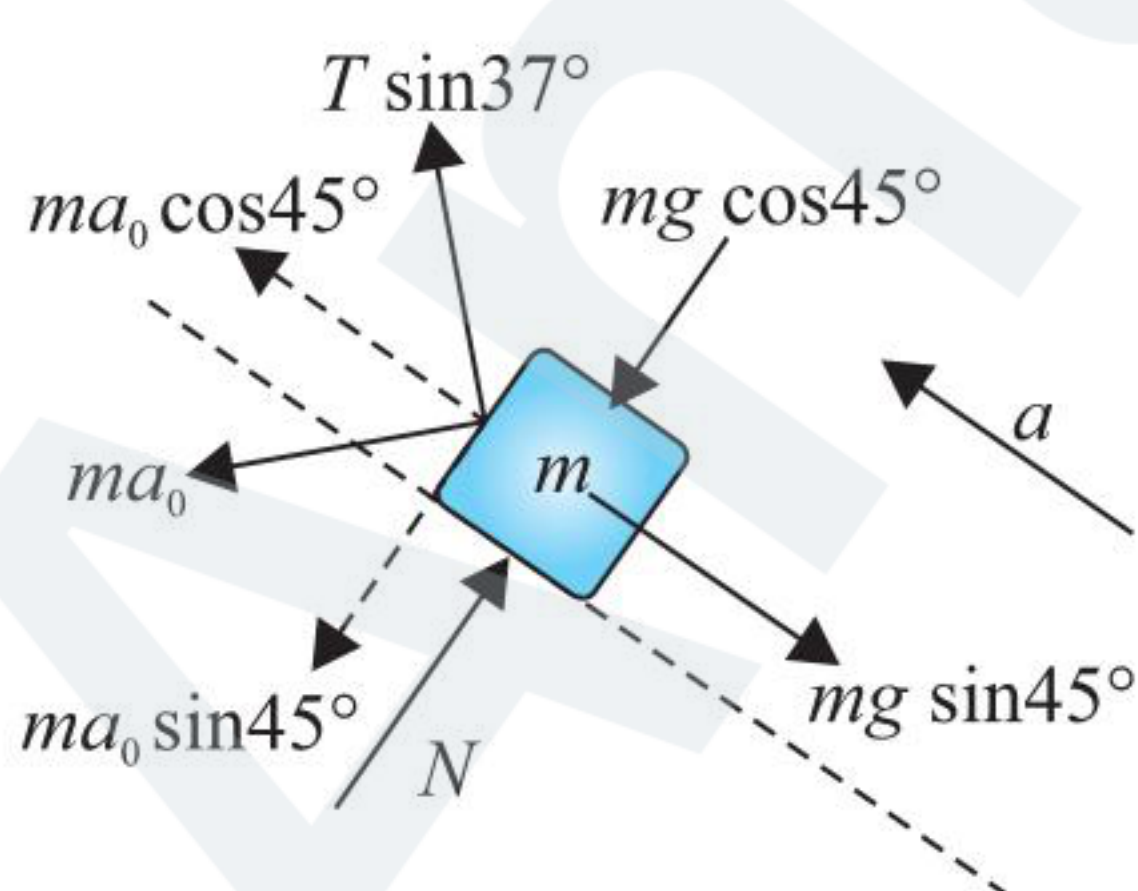
F.B.D of block A



$$N \cos 60^\circ - F = ma = \frac{mF}{2m}$$

Solving we get $N = 3F = 30 \text{ N}$

10. (2) Let a_0 be acceleration of wedge with respect to ground and 'a' be acceleration of block with respect to wedge.



From the frame of wedge

$$ma = ma_0 \cos 45^\circ - mg \sin 45^\circ$$

$$a = \frac{a_0 - g}{\sqrt{2}} \quad \dots (1)$$

$$N = ma_0 \sin 45^\circ + mg \cos 45^\circ$$

$$N = \frac{2g}{\sqrt{2}} + \frac{2a_0}{\sqrt{2}} \quad \dots (2)$$

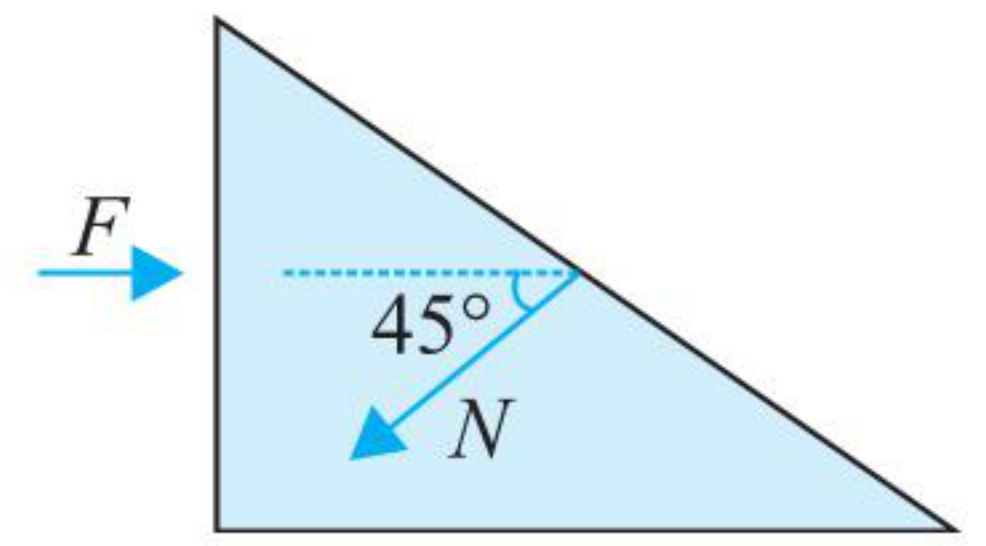
For the wedge $Ma_0 = 210 - N \sin 45^\circ$

$$a_0 = 20 \text{ m/s}^2$$

$$\text{From equation (1), } a = \frac{10}{\sqrt{2}} \text{ m/s}^2$$

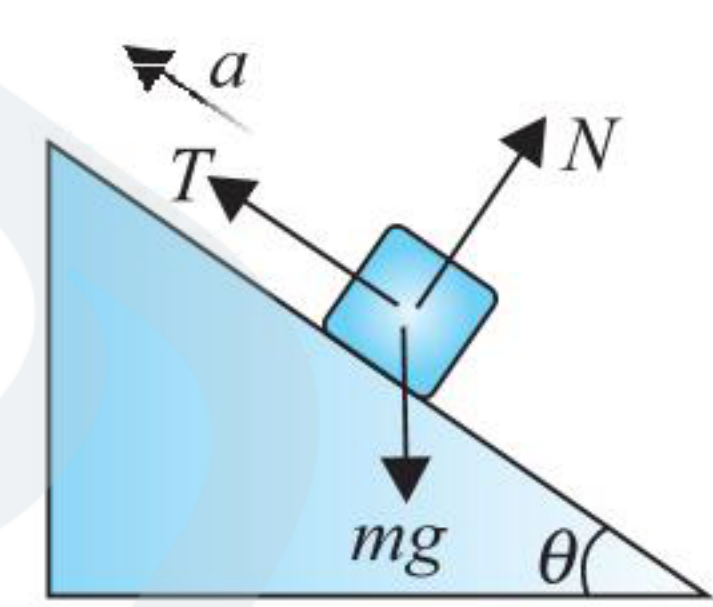
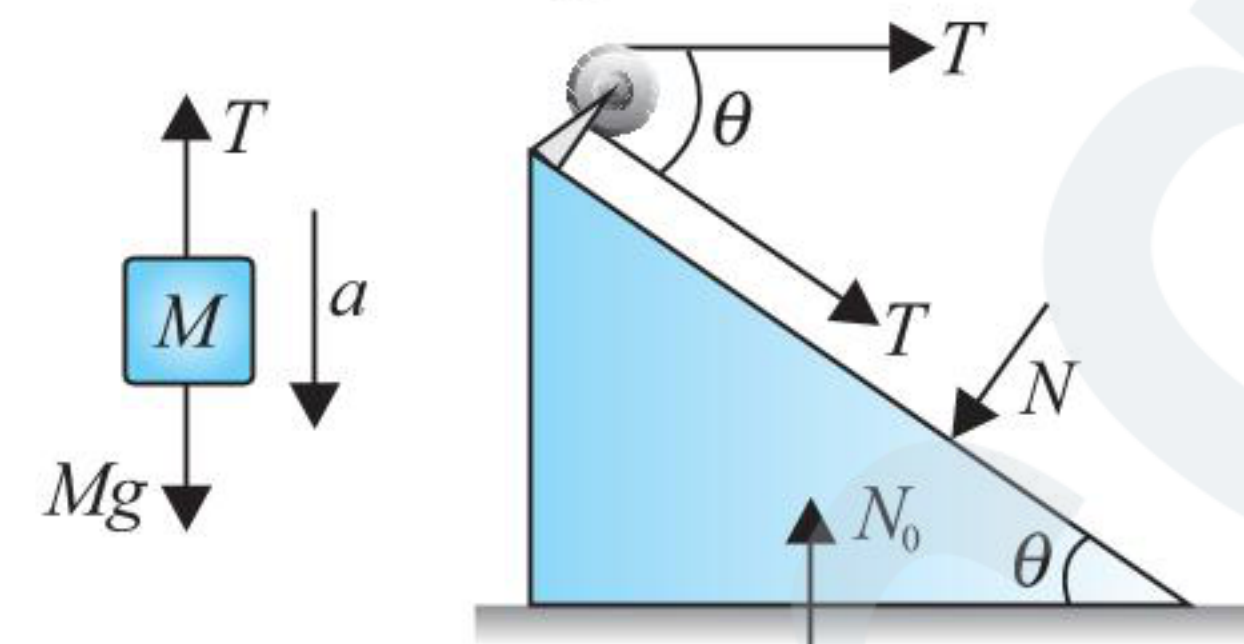
$$\text{Time taken is } 10\sqrt{2} = \frac{1}{2} \left(\frac{10}{\sqrt{2}} \right) t^2$$

$$t = 2 \text{ sec}$$



11. (5) For M: $Mg - T = Ma$... (i)

$$\text{For } m: T - mg \sin \theta = ma \quad \dots (ii)$$



$$\text{and } N = mg \cos \theta \quad \dots (iii)$$

$$\text{For } m_0: T + T \cos \theta = N \sin \theta \quad \dots (iv)$$

[In horizontal direction]

Eliminating a between (i) and (ii),

$$g - \frac{T}{M} = \frac{T}{m} - g \sin \theta$$

$$\Rightarrow T \left(\frac{1}{m} + \frac{1}{M} \right) = g(1 + \sin \theta)$$

$$\Rightarrow T = \frac{mMg}{m+M} (1 + \sin \theta)$$

Put this value of T and value of N from equation (iii) into equation (iv),

$$\frac{mMg(1 + \sin \theta)}{m+M} (1 + \cos \theta) = mg \cos \theta \cdot \sin \theta$$

$$\therefore \frac{M}{m+M} = \frac{\cos \theta \cdot \sin \theta}{(1 + \sin \theta)(1 + \cos \theta)}$$

$$= \frac{\frac{4}{5} \cdot \frac{3}{5}}{\left(1 + \frac{3}{5}\right) \left(1 + \frac{4}{5}\right)} = \frac{1}{6} \Rightarrow \frac{m}{M} = 5$$

12. (4) Let initial extension in S_1 be x_0

$$\text{Then, } kx_0 = Mg \quad \dots (i)$$

Let extension in S_2 be x_2 and further extension in S_1 be x_1 as point A moves down by L .

$$2x_1 + x_2 = L$$

For equilibrium of M

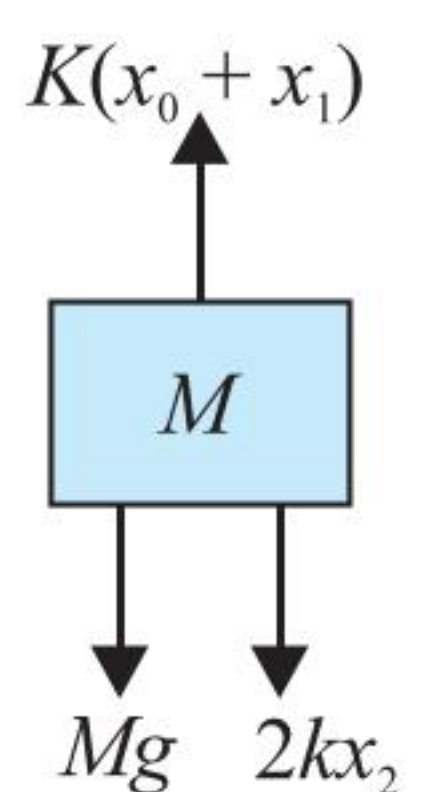
$$kx_0 + kx_1 = Mg + 2kx_2$$

$$\text{But } kx_0 = Mg$$

$$\therefore x_1 = 2x_2 \quad \dots (ii)$$

$$\text{Solving (i) and (ii) } x_2 = \frac{L}{5}, x_1 = \frac{2L}{5}$$

$$\text{Hence the displacement of the block } M, x_1 = \frac{2 \times 10}{5} = 4 \text{ cm}$$



13. (120)

Let T_1 be the tension in rope and T_2 the tension in the tail of monkey A. Let the force exerted by monkey A on the rope be F .

Let M_A and M_B be masses of monkeys A and B, respectively. The free-body diagrams of monkeys A and B are shown in figure (a) and (b). $F = T_1$

The equation of motion of monkeys A and B are

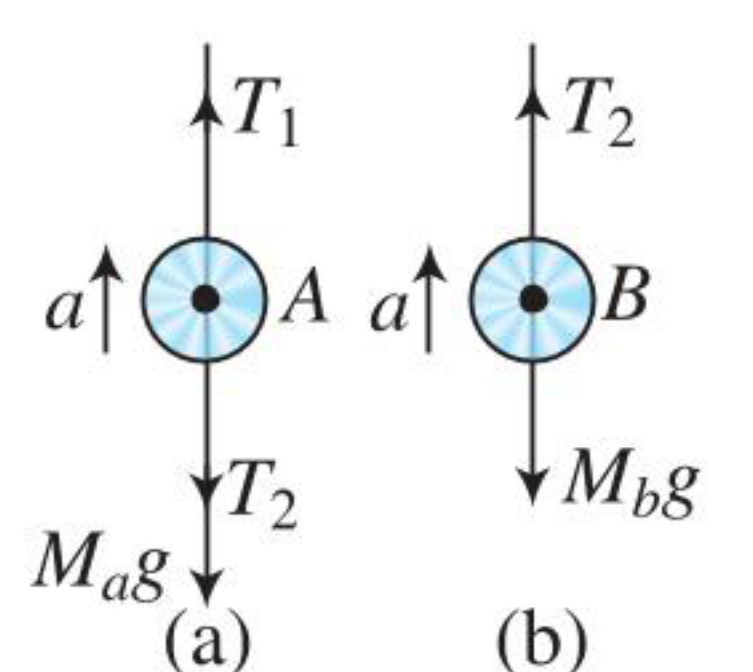
$$T_1 - M_A g - T_2 = M_A a \quad \dots (i)$$

$$T_2 - M_B g = M_B a \quad \dots (ii)$$

For maximum tension in tail

From Eq. (ii), we get

$$a = \frac{T_2 - M_B g}{M_B} = \frac{30 - 2 \times 10}{2} = 5 \text{ m/s}^2$$



Then from Eq. (i), we get

$$T_1 = T_2 + M_A(g + a) = 30 + 6(10 + 5) = 120 \text{ N}$$

14. (1.82)

Since blocks A and B remain stationary relative to block C , it means motion of blocks A and B is identical with block C or acceleration of all the three blocks is same as a horizontally (rightwards). Now considering FBDs w.r.t wedge C ,

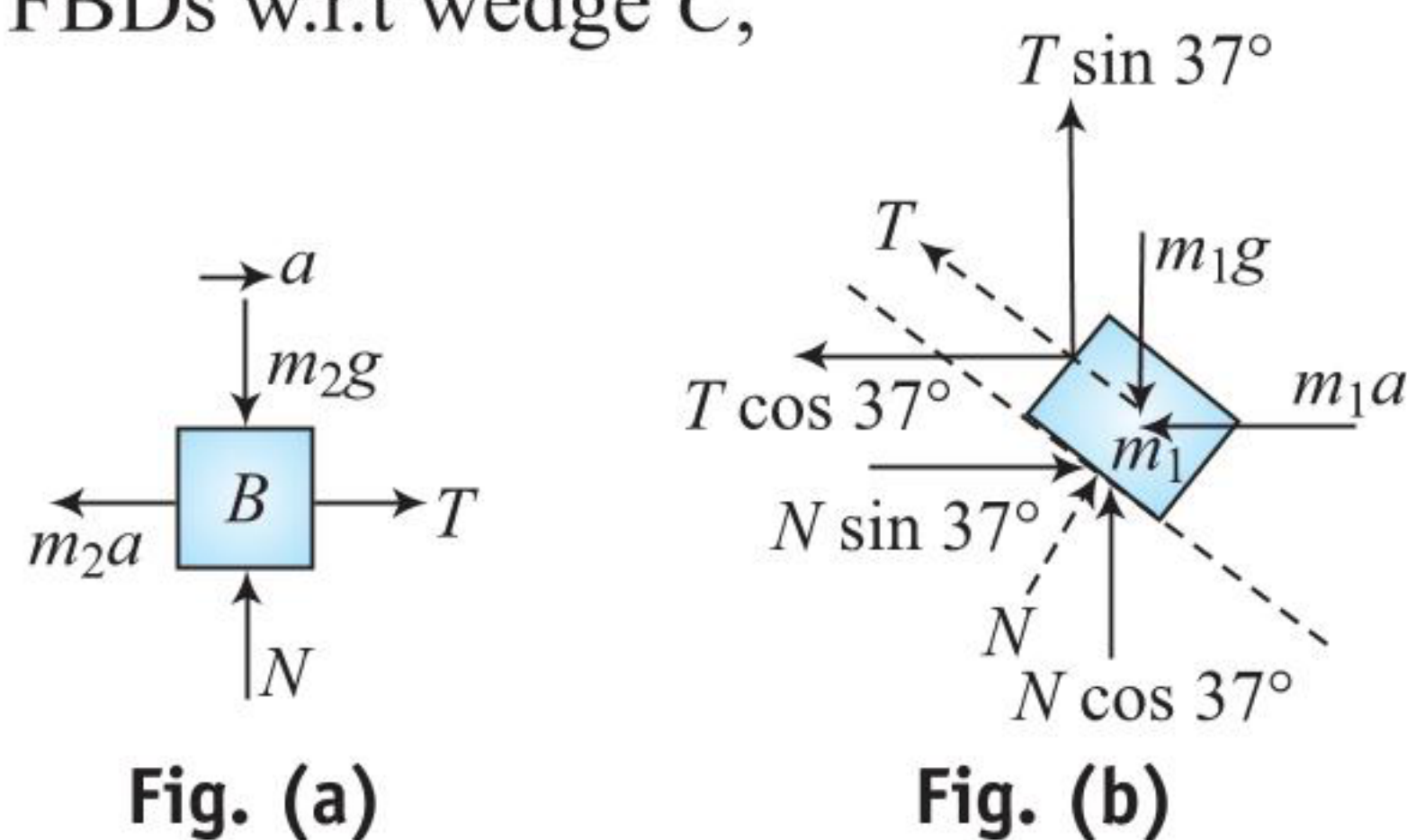


Fig. (a)

Fig. (b)

For blocks B ,

$$N_2 = m_2g \quad \text{or} \quad N_2 = 55 \text{ N}$$

$$T = m_2a$$

...(i)

For block A ,

$$N_1 \cos 37^\circ + T \sin 37^\circ = m_1g$$

...(ii)

$$N_1 \sin 37^\circ - T \cos 37^\circ = m_1a$$

...(iii)

Solving above equations,

$$T = 6.97 \approx 7 \text{ N}$$

$$N_1 = 12.25 \text{ N}$$

$$\text{and } a = 1.27 \text{ ms}^{-2}$$

15. (3) Let acceleration of block B relative to pulley E be b downward, the acceleration of block C relative to pulley E will be b upward and let acceleration of block A be a upward. Then acceleration of pulley E will also be a but downward. Hence, resultant accelerations of blocks B and C become $(a + b)$ and $(a - b)$, both downward, respectively.

Acceleration of B relative to A ,

$$a_{B,A} = (a + b) + a \Rightarrow a_{B,A} = (2a + b) = 5 \text{ ms}^{-2} \quad \text{...(i)}$$

And downward acceleration of C relative to A ,

$$a_{C,A} = (a - b) + a \Rightarrow a_{C,A} = (2a - b) = 3 \text{ ms}^{-2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$a = 2 \text{ ms}^{-2} \quad \text{and} \quad b = 1 \text{ ms}^{-2}$$

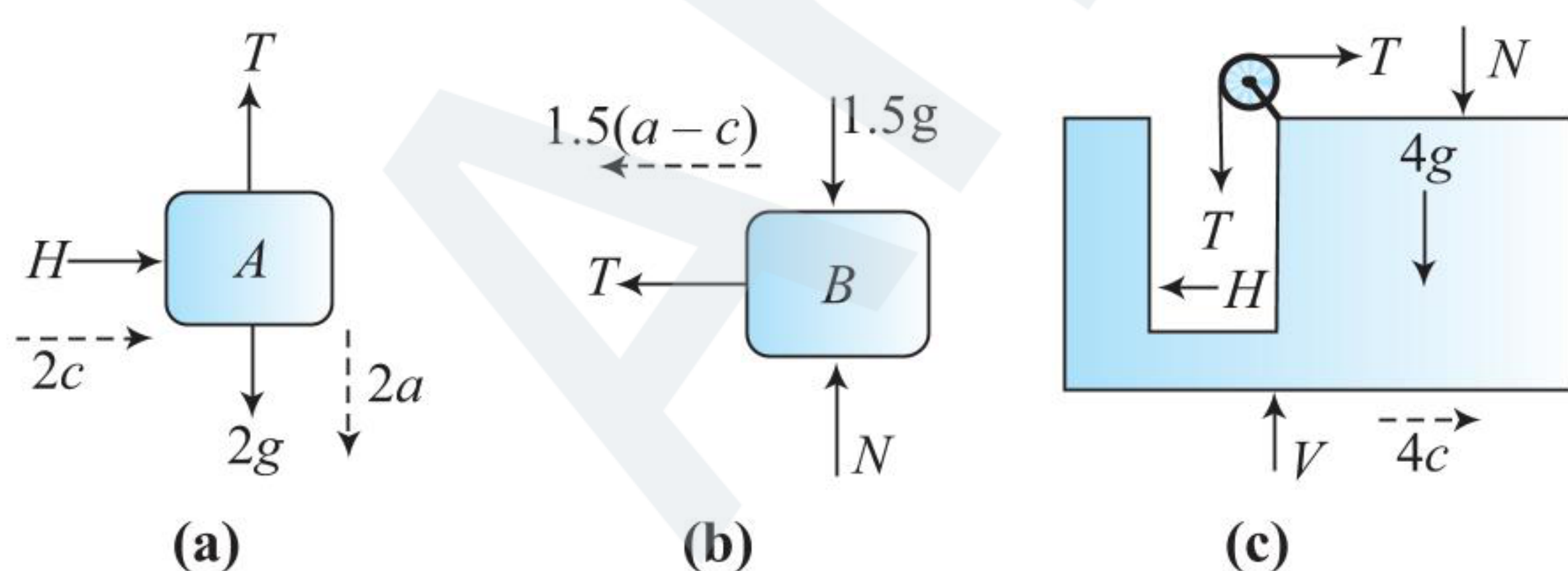
Resultant acceleration of $B = (a + b) = 3 \text{ ms}^{-2}$ (downward)

And that of $C = (a - b) = 1 \text{ ms}^{-2}$ (downward)

16. (5) Let vertically downward component of acceleration of A be a and let acceleration of C be c (rightward).

Then horizontal component of acceleration of A is c (rightward) and acceleration of B , relative to C is a (leftward). Hence, resultant acceleration of B is, $(a - c)$ leftward.

Now considering free body diagrams



(a)

(b)

(c)

For block A ,

$$H = 2c$$

...(i)

$$2g - T = 2a$$

...(ii)

For horizontal forces on block B ,

$$T = 1.5(a - c)$$

...(iii)

For horizontal forces on block C ,

$$T - H = 4c$$

...(iv)

From above equations,

$$a = 6.25 \text{ ms}^{-2} \quad \text{and} \quad c = 1.25 \text{ ms}^{-2}$$

Vertical acceleration of $A = a = 6.25 \text{ ms}^{-2}$ (downward)

Horizontal acceleration of $A = c = 1.25 \text{ ms}^{-2}$ (rightward)

Acceleration of $C = c = 1.25 \text{ ms}^{-2}$ (rightward)

Acceleration of $B = (a - c) = 5.00 \text{ ms}^{-2}$ (leftward)

17. (15)

$$N \cos 37^\circ = 1g$$

...(i)

$$N \sin 37^\circ = 1a$$

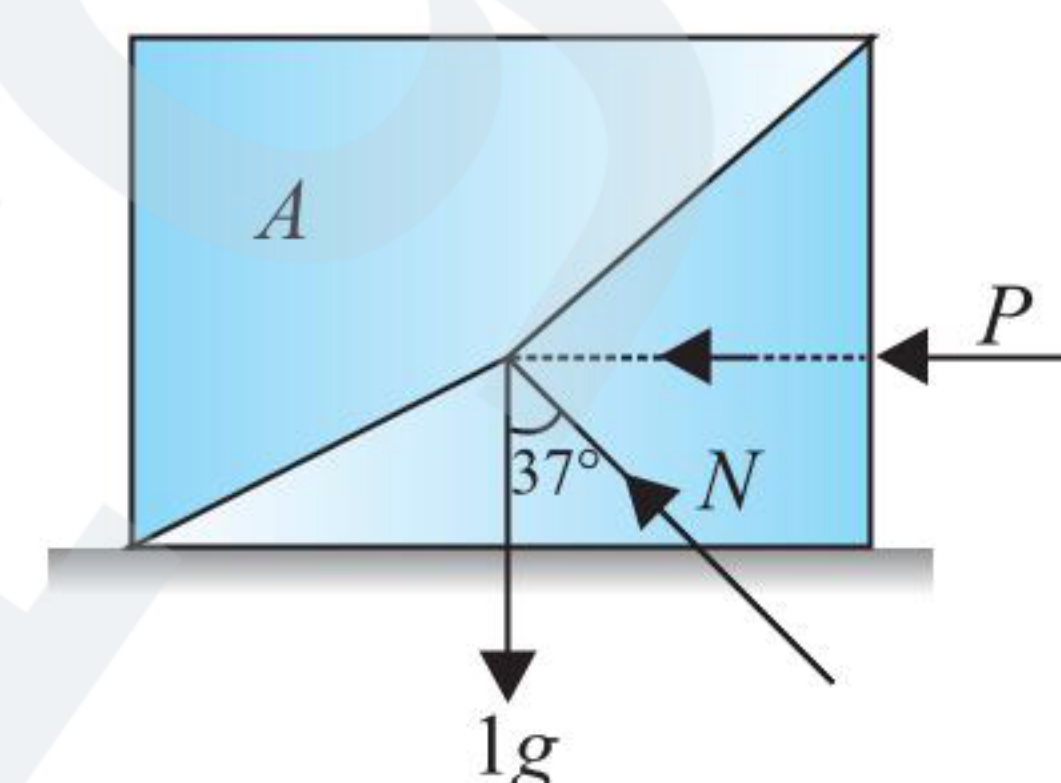
...(ii)

$$\frac{a}{g} = \tan 37^\circ$$

$$a = 10 \times \frac{3}{4} = 7.5 \text{ m/s}^2$$

$$P = 2a$$

$$P = 15 \text{ N}$$



Archives

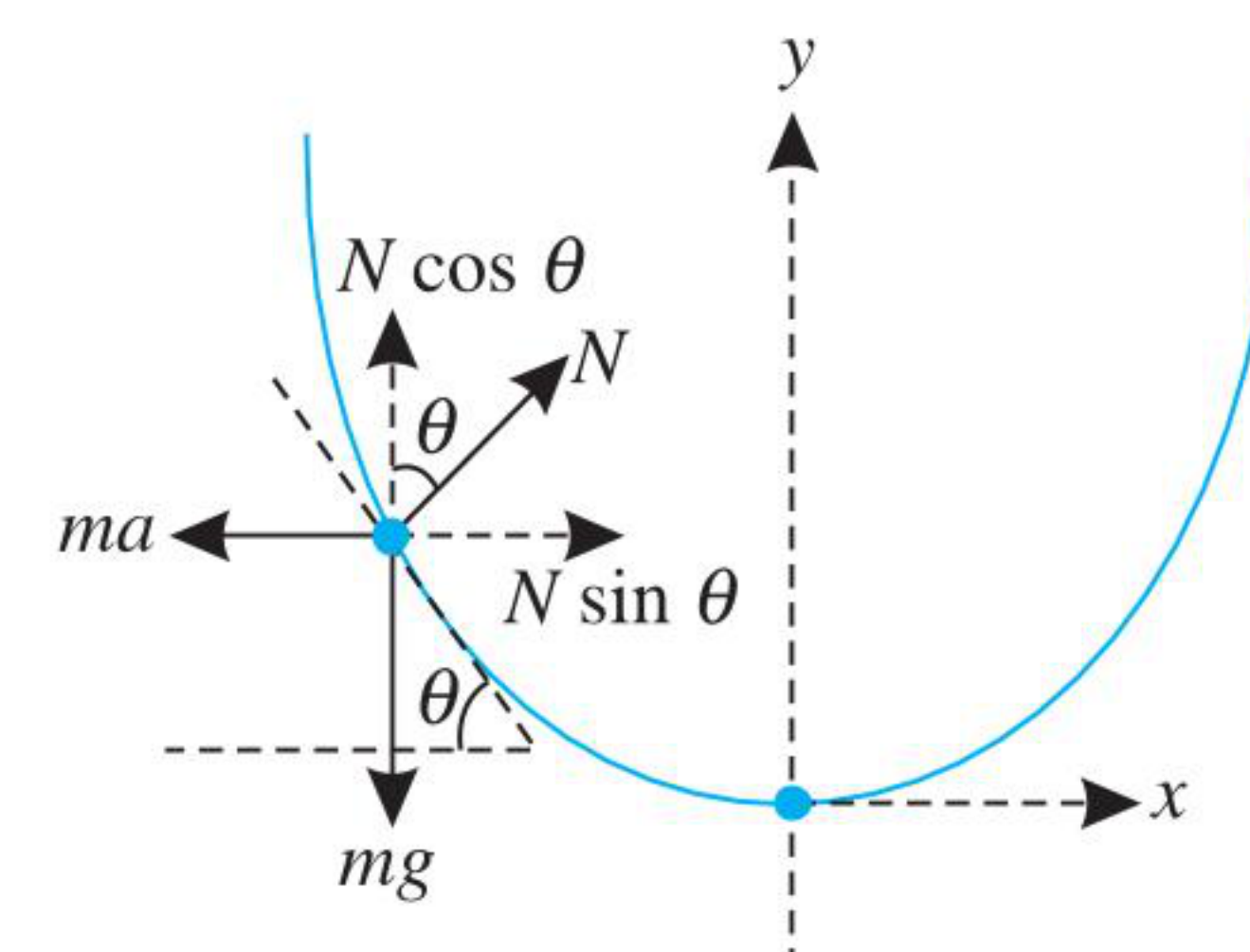
JEE Advanced

Single Correct Answer Type

$$1. (2) \quad \tan \theta = \frac{a}{g}$$

$$\tan \theta = \frac{dy}{dx} = 2kx$$

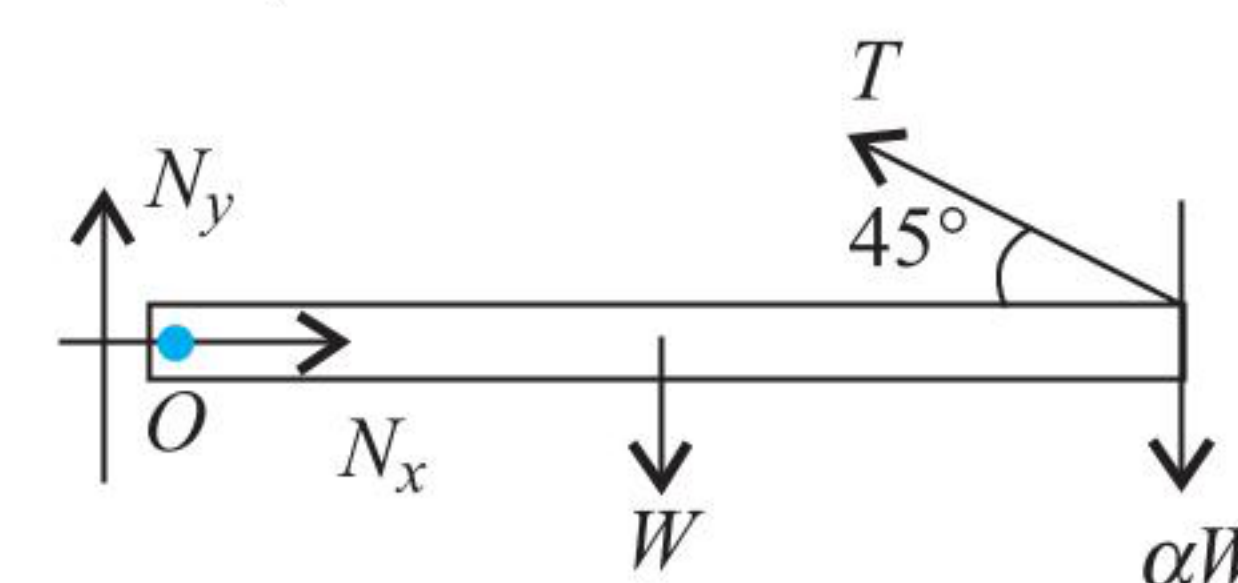
$$\Rightarrow x = \frac{a}{2gk}$$



Multiple Correct Answer Type

1. (1),(2),(4)

F.B.D. of the beam,



Considering rotational equilibrium about O.

$$T \sin 45^\circ \times l = \left(W \times \frac{l}{2} \right) + (\alpha W \times l)$$

$$T \times \frac{l}{\sqrt{2}} = \left(\frac{W}{2} + \alpha W \right) l$$

$$T = W \left(\frac{1}{\sqrt{2}} + \sqrt{2}\alpha \right)$$

- (4) To break the string $T > 2\sqrt{2} W$

$$\Rightarrow W \left(\frac{1}{\sqrt{2}} + \sqrt{2}\alpha \right) > 2\sqrt{2} W \Rightarrow \alpha > \frac{3}{2}$$

- (3) $\alpha = 0.5 = \frac{1}{2}$, $T = W \left(\frac{1}{\sqrt{2}} + \sqrt{2} \times \frac{1}{2} \right) = \sqrt{2} W$

- (2) $N_x = T \cos 45^\circ = \sqrt{2} W \times \frac{1}{\sqrt{2}}$

- (1) $N_y + T \sin 45^\circ = W + \alpha \cdot W$

$$N_y + \frac{W}{2} + \alpha W = W + \alpha W$$

$$N_y = \frac{W}{2} \quad \text{It does not depend on } \alpha$$